Top quark pair production at NNLO+NNLL′ in QCD including NLO EW contributions

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September 2019
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Top quark pairs at The LHC

Top quark physics is now a precision topic.
E.g. Total cross section for top pair production available at NNLO and with soft gluon resummation.

[Czakon, Fiedler, Mitov:1303.6254]

13 TeV

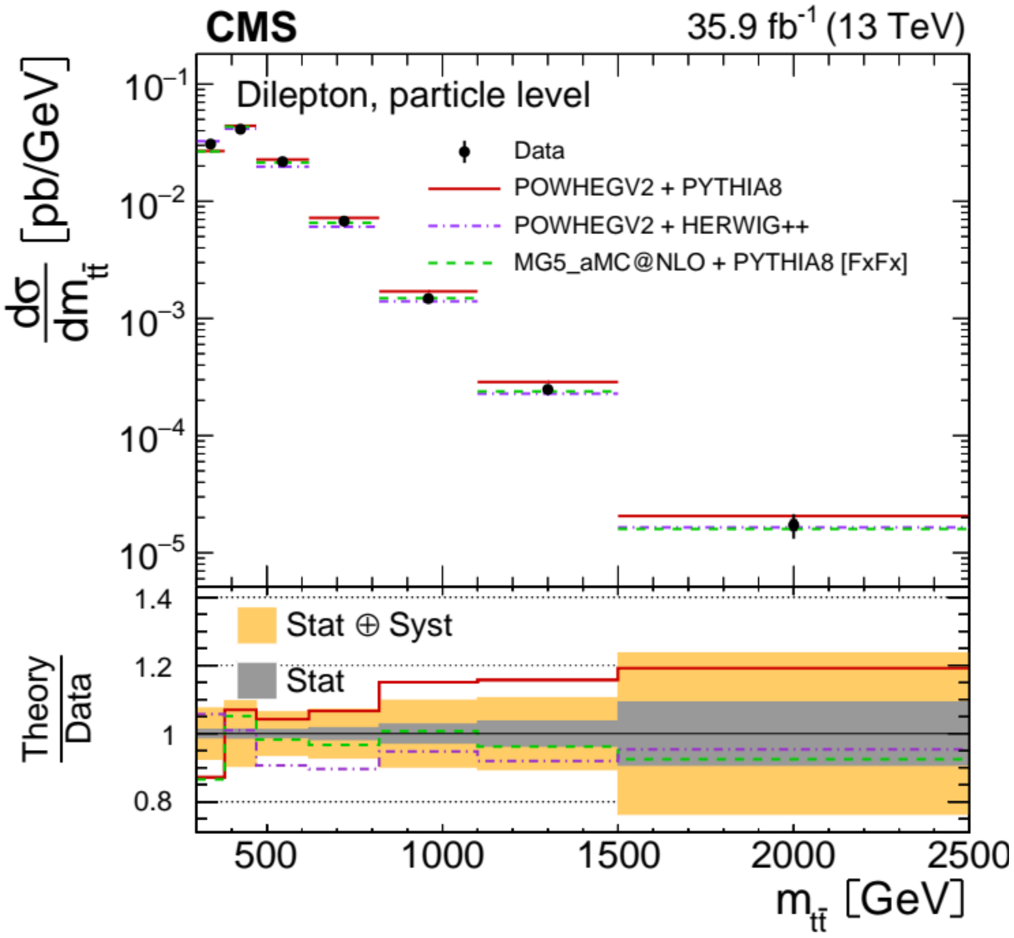
$$\sigma_{\rm NNLO}(pp \to t\bar{t} + X) \sim 800{\rm pb}$$
$$\sigma_{\rm NNLO+NNLL}(pp \to t\bar{t} + X) \sim 820{\rm pb}$$

[top++2.0]

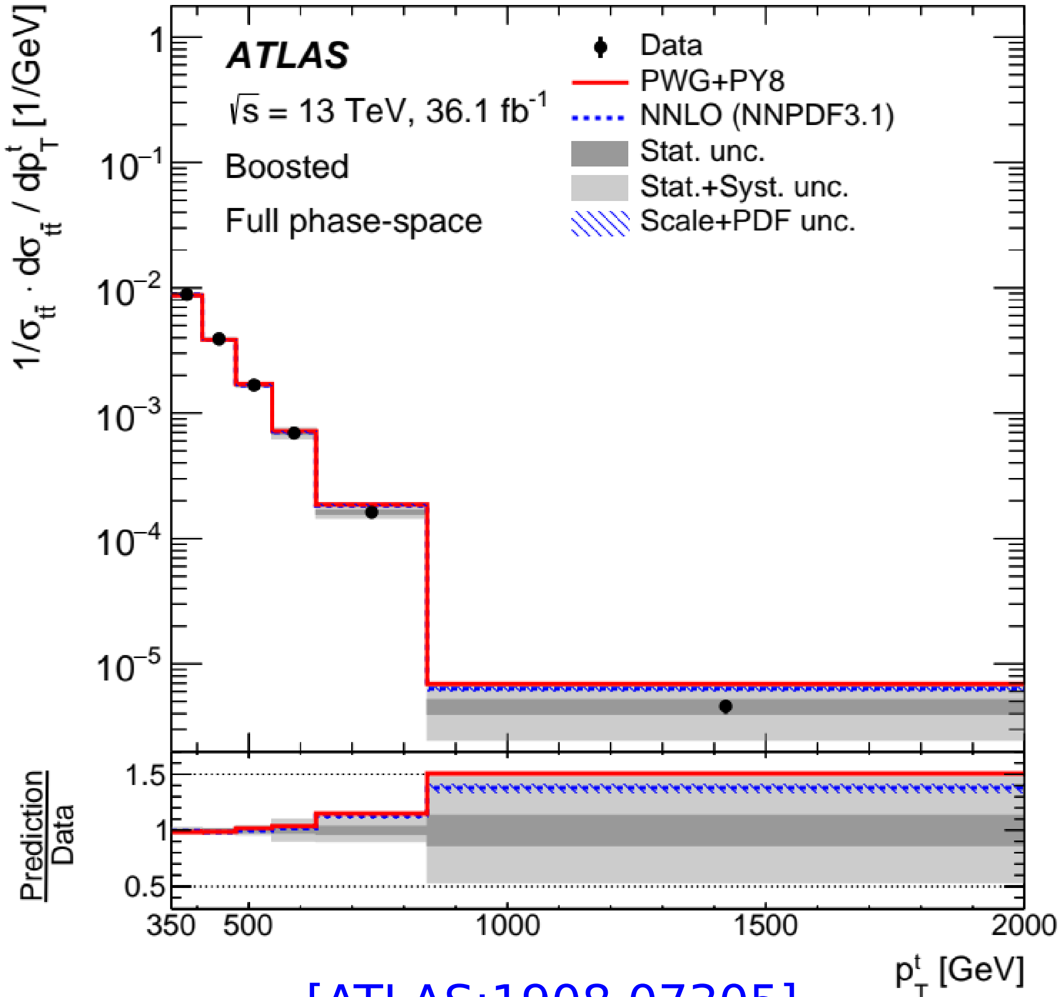
LHC will produce billions of top pairs over its operating lifetime.

Must have accurate predictions even in the tails of distributions.

Top quark pairs at The LHC



[CMS:1811.06625]



[ATLAS:1908.07305]

Top quark pairs and this talk

- The focus will mainly be on the impact of resummation on the fixed order results.

- Specifically, we present results at NNLO+NNLL’ accuracy in QCD + NLO EW corrections.

- Focus on distributions: top-pair invariant mass, transverse momentum, rapidity.

- Culmination of the work of many contributors:
  - NNLO QCD corrections
    [Bärnreuther, Czakon, Fiedler, Heymes, Mitov]
  - NLO EW corrections
    [Bernreuther, Si: 1003.3926, 1205.6580] [Hollik, Pagani: 1107.2606]
    [Pagani, Tsinikos, Zaro: 1606.01915] [Denner, Pellen :1607.05571]
    (+ many other studies and calculations!)
  - NNLL and NNLL’ resummation (soft gluons and mass logs)
    [Ferroglia, Pecjak, DS, Wang, Yang]

Top quark pairs and this talk

Outline

- Resummation: Soft gluons, and soft gluons in boosted limit
- Extension to include rapidity distributions
- Combining the resummed predictions
- Combination with NNLO QCD and lessons learned
- Inclusion of EW effects
- Results, effects on distributions

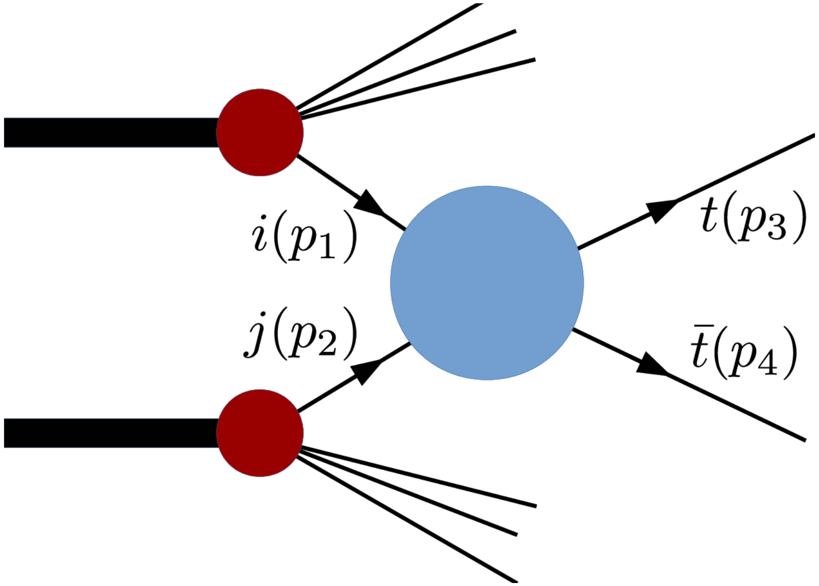
Fixed order calculations

Top pair production at hadron colliders

$$i(p_1) + j(p_2) \to t(p_3) + \bar{t}(p_4) + X(p_X)$$

$$\frac{d\sigma_{h_1 h_2 \to t\bar{t}X(\tau)}}{dM} = \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z, \mu_f) \frac{d\hat{\sigma}_{ij}}{dM}(z, \alpha_s(\mu_r), M, m_t, \mu_{f/r})$$

$$\mathcal{L}_{ij}(y) = \int_y^1 \frac{dx}{x} \phi_{h_1/i}(x) \phi_{h_2/j}(y/x)$$



Kinematic Quantities

$$\hat{s} = (p_1 + p_2)^2 \qquad t_1 = (p_1 - p_3)^2 - m_t^2$$

$$M^2 = M_{t\bar{t}}^2 = (p_3 + p_4)^2$$

$$\tau = \frac{M^2}{s} \qquad z = \frac{M^2}{\hat{s}}$$

Fixed order calculations

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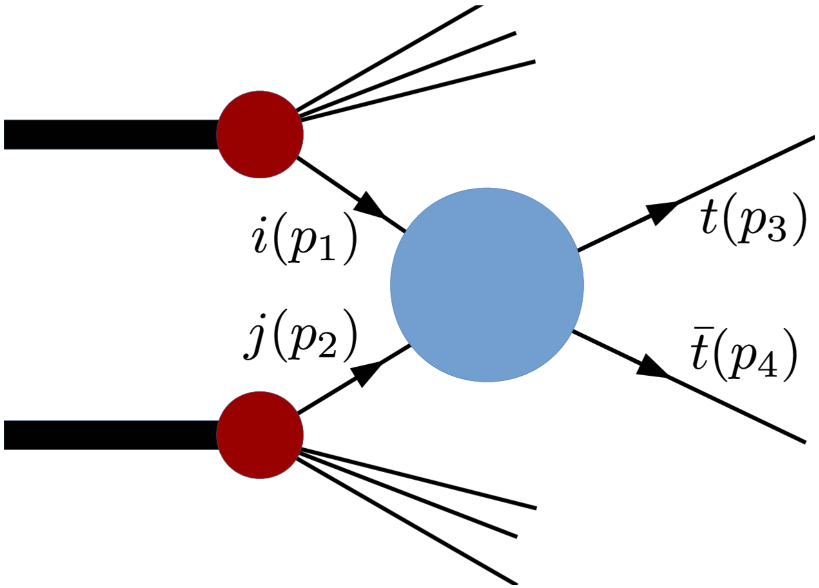

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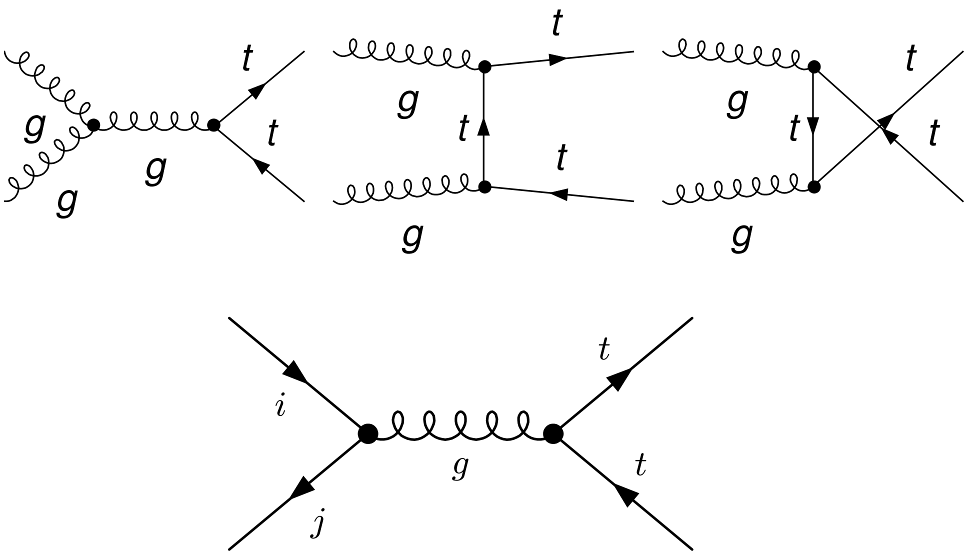
$$\tau = \frac{M^2}{s} \qquad z = \frac{M^2}{\hat{s}}$$

Fixed order calculations

Calculate perturbative corrections to the partonic cross section

$$d\hat{\sigma} = \alpha_s^2 \left(d\hat{\sigma}^{(0)} + \alpha_s d\hat{\sigma}^{(1)} + \alpha_s^2 d\hat{\sigma}^{(2)} \right)$$

LO



NLO:
[Dawson, Ellis, Nason ‘88]

NNLO:
[Czakon, Fiedler, Mitov ‘13]

Fully differential distributions available at NNLO
[Czakon, Heymes, Mitov ‘17]

Fixed order calculations

Calculate perturbative corrections to the partonic cross section

$$d\hat{\sigma} = \alpha_s^2 \left(d\hat{\sigma}^{(0)} + \alpha_s d\hat{\sigma}^{(1)} + \alpha_s^2 d\hat{\sigma}^{(2)} \right)$$

Corrections contain potentially large logarithms.
In particular...

Threshold logarithms:

$$\alpha_s^n \left[\frac{\ln^p(1-z)}{1-z} \right]_+ , \quad 0 \le p \le 2n-1$$

Large contribution as $z \to 1$

Small mass (collinear) logarithms:

$$\alpha_s \ln^2 \left(\frac{m_t}{M} \right)$$

We expect these to be important for “boosted tops”, $M^2 \gg m_t^2$

Factorization: soft (threshold)

Want to factorize different scales: $\hat{s}, M_{tt}^2, m_t^2 \gg \hat{s}(1-z)^2$

The partonic cross section factorizes in the threshold limit: $z \to 1$

- In Mellin moment space
  [Kidonakis, Sterman: 9705234]
- Using techniques from Soft Collinear Effective Theory (SCET)
  [Ahrens, Ferroglia, Neubert, Pecjak, Yang: 1003.5827]

$$\frac{d^2\sigma}{dM\ d\cos\theta} = \frac{8\pi\beta_t}{3sM} \sum_{ij=(\bar{q}q,gg)} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) C_{ij}(z, M, m_t, ..)$$

$$C_{ij} = \mathrm{Tr}[\mathbf{H}_{ij}^m(M_{t\bar{t}}, m_t, \mu_f, ..) \mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), m_t, \mu_f, ...)] + \mathcal{O}(1-z)$$

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Retained top mass

$$C_{ij} = \mathrm{Tr}[\mathbf{H}_{ij}^m(M_{t\bar{t}}, m_t, \mu_f, ..)\mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), m_t, \mu_f, ...)] + \mathcal{O}(1-z)$$

Factorization: soft (threshold)

$$C_{ij} = \text{Tr}[\mathbf{H}^m_{ij}(M_{t\bar{t}}, m_t, \mu_f, ..)\mathbf{S}^m_{ij}(\sqrt{\hat{s}}(1-z), m_t, \mu_f, ...)] + \mathcal{O}(1-z)$$

Factorization allows resummation!
We now have single scale* functions.

Derive and solve RGEs.
E.g. Hard function

$\mathbf{H}^m_{ij}$ – Hard Function. Related to virtual corrections

$\mathbf{S}^m_{ij}$ – Soft function. Related to real emission of soft gluons.

$$\mathbf{H}^m(\mu) = \mathbf{U}^m(\mu_h, \mu)\mathbf{H}^m(\mu_h)\mathbf{U}^{m\dagger}(\mu_h, \mu) \qquad \mu_h \sim M_{t\bar{t}}$$

Run the hard function between scales, resumming logarithms of the form:

$$\ln\left(\frac{\mu_h}{\mu}\right)$$

*Caveats later

Factorization: boosted-soft

Consider the boosted-soft limit: $z \to 1 \;\; M_{t\bar{t}}^2 \gg m_t^2$

$$\hat{s}, t_1 \gg m_t^2 \gg \hat{s}(1-z)^2 \gg m_t^2(1-z)^2$$

Further factorization occurs in this limit
[Ferroglia, Pecjak, Yang: 1205.3662]

$$C_{ij} = \mathrm{Tr}[\mathbf{H}_{ij}^m(M_{t\bar{t}}, m_t, \mu_f, ..)\mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), m_t, \mu_f, ...)] + \mathcal{O}(1-z)$$

$$\Downarrow \quad M_{t\bar{t}}^2 \gg m_t^2$$

$$\begin{aligned} C_{ij} =& C_D^2(m_t, \mu_f)\mathrm{Tr}\left[\mathbf{H}_{ij}(M, \mu_f, ..)\mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z), \mu_f, ...)\right] \\ & \otimes \mathbf{s}_D\left(m_t(1-z), \mu_f\right) \otimes \mathbf{s}_D\left(m_t(1-z), \mu_f\right) \otimes c_{ij}^t(z, m_t, \mu_f) \\ & + \mathcal{O}(1-z) + \mathcal{O}\left(m_t/M\right) \end{aligned}$$

Factorization: boosted-soft

$$C_{ij} = C_D^2(m_t, \mu_f)\mathrm{Tr}\left[\mathbf{H}_{ij}(M, \mu_f, ..)\mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z), \mu_f, ...)\right]$$
$$\otimes\, \mathbf{s}_D\left(m_t(1-z), \mu_f\right) \otimes \mathbf{s}_D\left(m_t(1-z), \mu_f\right) \otimes c_{ij}^t(z, m_t, \mu_f)$$
$$+\,\mathcal{O}(1-z) + \mathcal{O}\left(m_t/M\right)$$

- C_D and $\mathbf{s}_D$ related to soft/collinear emissions from tops
- **H** & **S** no longer depend on top mass

Aside: Heavy flavour matching coefficient, c_{ij}^t introduces additional $\ln(m_t)$ dependence which is not resummed. We add such contributions in fixed order.

Each of these matching functions is known to NNLO

$\mathbf{H}_{ij}$ – [Glover et. al: ‘00-’01]

$\mathbf{S}_{ij}$ – [Ferroglia, Pecjak, Yang: 1207.4798]

$C_D, \mathbf{s}_D$ – [Melnikov, Mitov: 0404143] [Becher, Neubert: 0512208]

Mellin space

Resummation performed in Mellin space.

$$\tilde{f}(N) = \int_0^1 dx\, x^{N-1} f(x)$$

- Convolutions become products: $d\tilde{\sigma}(N) = \tilde{\mathcal{L}}(N)\tilde{C}(N)$
- $z \to 1$ corresponds to $N \to \infty$

$$P_n(z) = \left[\frac{\ln^n(1-z)}{1-z}\right]_+$$

$$\bar{N} = N e^{\gamma_E}$$

$$\tilde{P}_0(N) = -\ln \bar{N} + \mathcal{O}(1/N)$$

$$\tilde{P}_1(N) = \frac{1}{2}\left(\ln^2 \bar{N} + \frac{\pi^2}{6}\right) + \mathcal{O}(1/N)$$

$$\tilde{P}_2(N) = -\frac{1}{3}\left(\ln^3 \bar{N} + \frac{\pi^2}{2}\ln \bar{N} + 2\zeta(3)\right) + \mathcal{O}(1/N)$$

Resummed cross sections

Two formulas for resummed cross sections:

<u>Threshold (soft gluon) resummation:</u>

$$\widetilde{C}_m(N) = \mathrm{Tr}\left[\widetilde{\mathbf{U}}^m_{ij}(\bar{N},\mu_f,\mu_h,\mu_s)\,\mathbf{H}^m_{ij}(\mu_h)\,\widetilde{\mathbf{U}}^{m\dagger}_{ij}(\bar{N},\mu_f,\mu_h,\mu_s)\widetilde{\mathbf{S}}^m_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2},\mu_s\right)\right] + \mathcal{O}\left(1/N\right)$$

<u>Boosted-soft resummation:</u>

$$\widetilde{C}_{b,ij}(N) = \mathrm{Tr}\left[\widetilde{\mathbf{U}}_{ij}(\bar{N},\mu_f,\mu_h,\mu_s)\,\mathbf{H}_{ij}(\mu_h)\,\widetilde{\mathbf{U}}^{\dagger}_{ij}(\bar{N},\mu_f,\mu_h,\mu_s)\widetilde{\mathbf{S}}_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2},\mu_s\right)\right] \times \widetilde{U}^2_D(\bar{N},\mu_f,\mu_{dh},\mu_{ds})\,C^2_D(m_t,\mu_{dh}) \times \widetilde{s}^2_D\left(\ln\frac{m_t}{\bar{N}\mu_{ds}},\mu_{ds}\right) + \mathcal{O}\left(1/N\right) + \mathcal{O}\left(\frac{m_t}{M}\right).$$

Transverse momentum

It is possible also to obtain results for the p_T distribution in addition to the M distribution.

Both factorization theorems derived in the soft limit – no hard emissions.

Top quarks always essentially in their Born configuration i.e. back-to-back (in the partonic c.o.m frame).

$$\hat{y} = \frac{1}{2} \ln \frac{1 + \beta_t \cos\theta}{1 - \beta_t \cos\theta}$$

Rapidity in partonic c.o.m frame

$\beta_t \sim \text{velocity}$

Thus relate: $p_T = \frac{M\beta_t}{2} \sin\theta$

And write: $\frac{d^2\widetilde{\sigma}(N)}{dp_T\, d\hat{y}} = 2\sin\theta \, \frac{d^2\widetilde{\sigma}(N)}{dM\, d\cos\theta}$

Rapidity distributions

Possible to recover the rapidity spectrum.

[Pecjak, DS, Wang, Yangi: 1811.10527]

Re-introduce a δ-function for the rapidity and eliminate x_1, x_2.

For Drell-Yan: [Bonvini, Forte, Ridolfi: 1009.5691]

$$\frac{d^3\sigma(\tau)}{dM\, d\cos\theta\, dY_{t\bar{t}}} = \frac{8\pi\beta_t}{3sM} \sum_{ij} \int dz\, d\xi\, \delta(\tau - z\xi)\, C_{ij}(z,\mu_f) \times f_{i/p}\left(\sqrt{\xi}e^{Y_{t\bar{t}}},\mu_f\right) f_{j/p}\left(\sqrt{\xi}e^{-Y_{t\bar{t}}},\mu_f\right)$$

Where in the soft limit:

$$Y_{t\bar{t}} = \frac{1}{2}\ln\frac{x_1}{x_2}$$

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$$\times f_{i/p}\left(\sqrt{\xi} e^{Y_{t\bar{t}}}, \mu_f\right) f_{j/p}\left(\sqrt{\xi} e^{-Y_{t\bar{t}}}, \mu_f\right)$$

$$L_{ij}(\xi, Y_{t\bar{t}}, \mu_f) \equiv f_{i/p}\left(\sqrt{\xi} e^{Y_{t\bar{t}}}, \mu_f\right) f_{j/p}\left(\sqrt{\xi} e^{-Y_{t\bar{t}}}, \mu_f\right)$$

Gives (in Mellin space):

$$\frac{d^3\tilde{\sigma}(N)}{dM\, d\cos\theta\, dY_{t\bar{t}}} = \frac{8\pi\beta_t}{3sM} \sum_{ij} \tilde{L}_{ij}(N, Y_{t\bar{t}}, \mu_f)\, \tilde{c}_{ij}(N, \mu_f)$$

Rapidity distributions

The rapidity of the (anti)top y_t is also accessible.

In the c.o.m frame (soft limit):

$$\hat{y} = \frac{1}{2} \ln \frac{1 + \beta_t \cos\theta}{1 - \beta_t \cos\theta}$$

Use that $y_{t/\bar{t}} = Y_{t\bar{t}} \pm \hat{y}$ to obtain

$$\frac{d^3\tilde{\sigma}(N)}{dM \, d\cos\theta \, dy_t} = \frac{8\pi\beta_t}{3sM} \sum_{ij} \tilde{L}_{ij}(N, y_t - \hat{y}, \mu_f) \, \tilde{c}_{ij}(N, \mu_f)$$

Show results for average: $\frac{d\sigma}{dy_{\mathrm{avt}}} \equiv \frac{1}{2} \left(\frac{d\sigma}{dy_t} + \frac{d\sigma}{dy_{\bar{t}}} \right)$

Resummation accuracy

The evolution functions have the generic form

$$\widetilde{U}(\{\mu\}) = \exp\left(L\, g_1(\{\mu\}) + g_2(\{\mu\}) + \alpha_s g_3(\{\mu\}) + \ldots\right)\mathbf{u}$$

← Matrix valued piece

g_i are $\mathcal{O}(1)$ functions $\sim \alpha_s L \sim 1$ $\qquad L = \ln\left(\frac{\mu_1}{\mu_2}\right)$

Resummation accuracy?

	g_i	$\mathbf{H}^{(m)}, \widetilde{\mathbf{s}}^{(m)}, c_D, \widetilde{s}_D$	$\alpha_s^n L^k$
NLL	$g_1,\, g_2$	LO	$2n-1 \le k \le 2n$
NNLL	$g_1,\, g_2,\, g_3$	NLO	$2n-3 \le k \le 2n$
NNLL$'$	$g_1,\, g_2,\, g_3$	NNLO	$2n-4 \le k \le 2n$

Resummation accuracy

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Resummation accuracy?

	g_i	$\mathbf{H}^{(m)}, \widetilde{\mathbf{s}}^{(m)}, c_D, \widetilde{s}_D$	$\alpha_s^n L^k$
NLL	g_1, g_2	LO	$2n-1 \le k \le 2n$
NNLL	g_1, g_2, g_3	NLO	$2n-3 \le k \le 2n$
NNLL′	g_1, g_2, g_3	NNLO	$2n-4 \le k \le 2n$

Pure threshold resummation at NNLL accuracy

“Boosted-Soft” resummation at NNLL’ accuracy

Combining resummed cross sections

Combine these calculations without double counting

$$d\sigma_{\text{res}} = \underbrace{d\sigma_b^{\text{NNLL'}}}_{\substack{\text{Missing parts subleading} \\ \text{in } m_t/M \text{ and } 1/\text{N}}} + \overbrace{\left(\underbrace{d\sigma^{\text{NNLL}}_{\text{Threshold}}}_{\substack{\text{Missing parts} \\ \text{subleading in } 1/N}} \underbrace{- d\sigma_b^{\text{NNLL}} \Big|_{\substack{\mu_{dh}=\mu_h \\ \mu_{ds}=\mu_s}}}_{\substack{\text{Removes double} \\ \text{counting}}} \right)}^{\substack{\text{Adds in parts subleading in } m_t/M \\ \text{but enhanced by } \ln N}}$$

Include NNLO QCD, subtracting logs already included.

$$d\sigma^{\text{NNLO+NNLL'}} = \underbrace{\left(d\sigma^{\text{NNLO}} - d\sigma_{\text{res}} \Big|_{\substack{\text{NNLO} \\ \text{expansion}}} \right)}_{\substack{\text{Adds exact NNLO results,} \\ \text{removes logs at same order}}}$$

Scale choices

In the boosted-soft resummed result, we have five separate scales to set...

$$\mu_f,\ \mu_h,\ \mu_s,\ \mu_{dh},\ \mu_{ds}$$

$$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$$

$$\mathbf{H},\ \mathbf{S},\ C_D,\ S_D$$

How should we pick these?

$C_D,\ S_D$ - Related to final state collinear emissions. Arise from soft limit of heavy quark fragmentation function.

$$\widetilde{D}_{t/t}(\bar{N}, m_t, \mu_f) = C_D(m_t, \mu_f)\widetilde{S}_D(m_t/\bar{N}, \mu_f) + \mathcal{O}(1/N)$$

Independent of hard scattering process.

$$\mu_{dh} = m_t,\ \mu_{ds} = m_t/\bar{N}$$

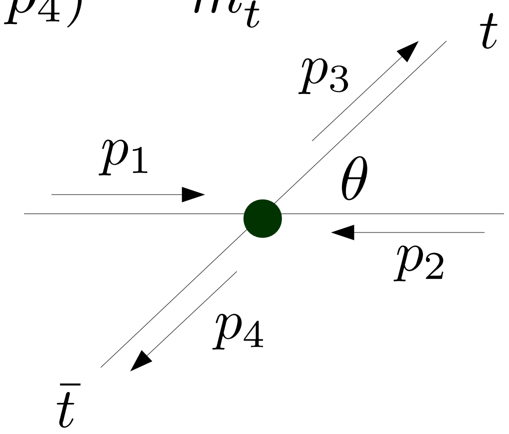
Scale choices

The hard and soft function are a little more subtle...

Depend on: $M_{t\bar{t}}^2, t_1, u_1, \cos\theta$

$$t_1 = (p_1 - p_3)^2 - m_t^2$$

$$u_1 = (p_1 - p_4)^2 - m_t^2$$



<u>Case 1:</u> $M_{t\bar{t}}^2 \sim |t_1| \sim |u_1|$

Setting $\mu_h \sim M_{t\bar{t}}$ frees the hard function of large logs.

<u>Case 2:</u> $M_{t\bar{t}}^2 \gg |t_1|$ or $M_{t\bar{t}}^2 \gg |u_1|$

Setting $\mu_h \sim M_{t\bar{t}}$

Setting $\mu_h \sim \sqrt{-t_1}$

Large logs $\ln\left(-\frac{t_1}{M_{t\bar{t}}^2}\right)$

Is the region $M_{t\bar{t}}^2 \gg |t_1|$ a concern?

Scale choices

For Born kinematics

$$t_1 = -\frac{M^2}{2}(1 - \beta_t \cos\theta) \qquad \beta_t = \sqrt{1 - \frac{4m_t^2}{M^2}}$$

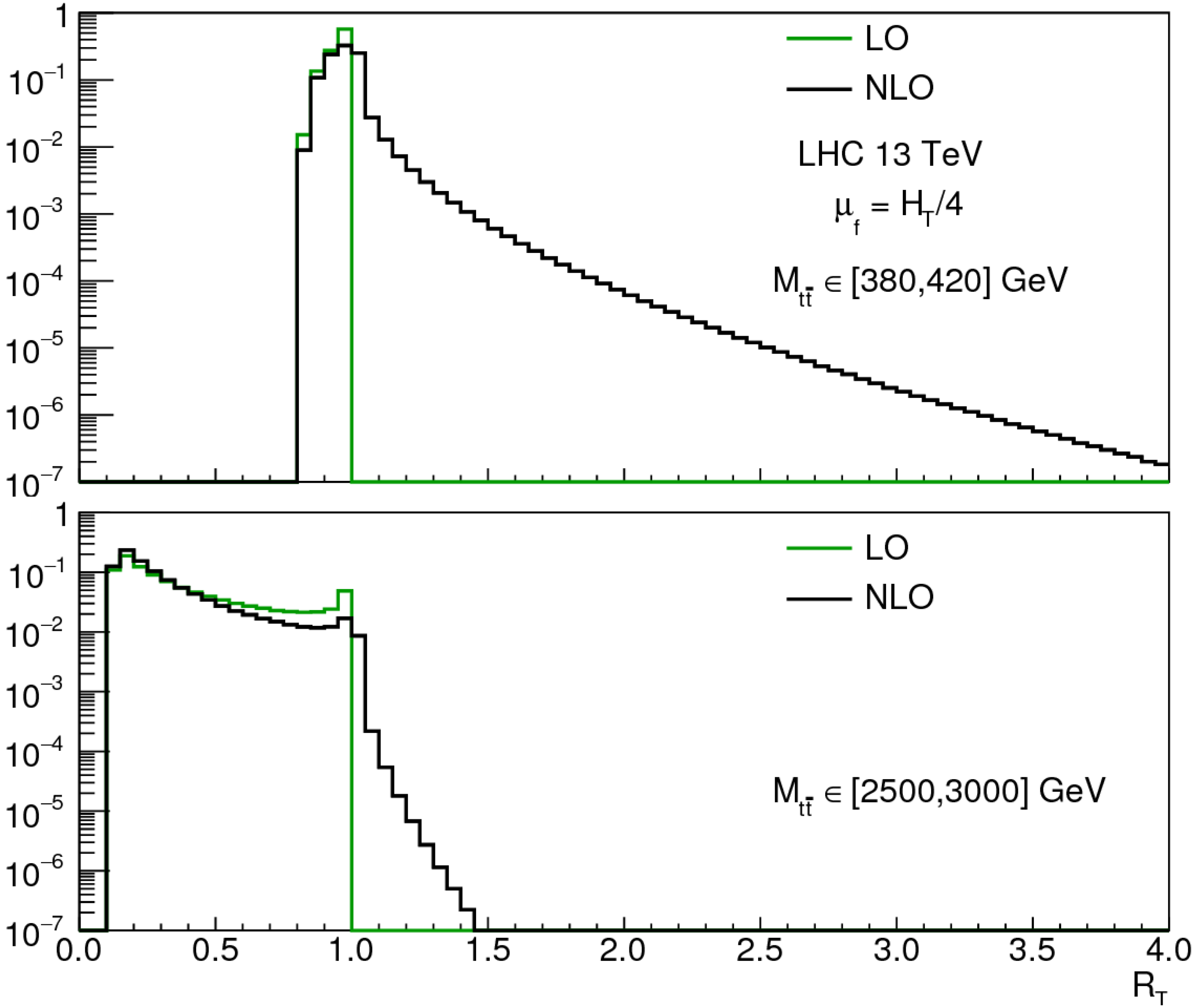
Expect a large scale separation $\beta_t \cos\theta \to 1$

$$-t_1 \xrightarrow{\beta_t \cos\theta \to 1} p_T^2 + m_t^2 \equiv m_T^2 = H_T^2/4 \qquad H_T = \sum_{i=t\bar{t}} m_{T,i}$$

And similarly for u_1.

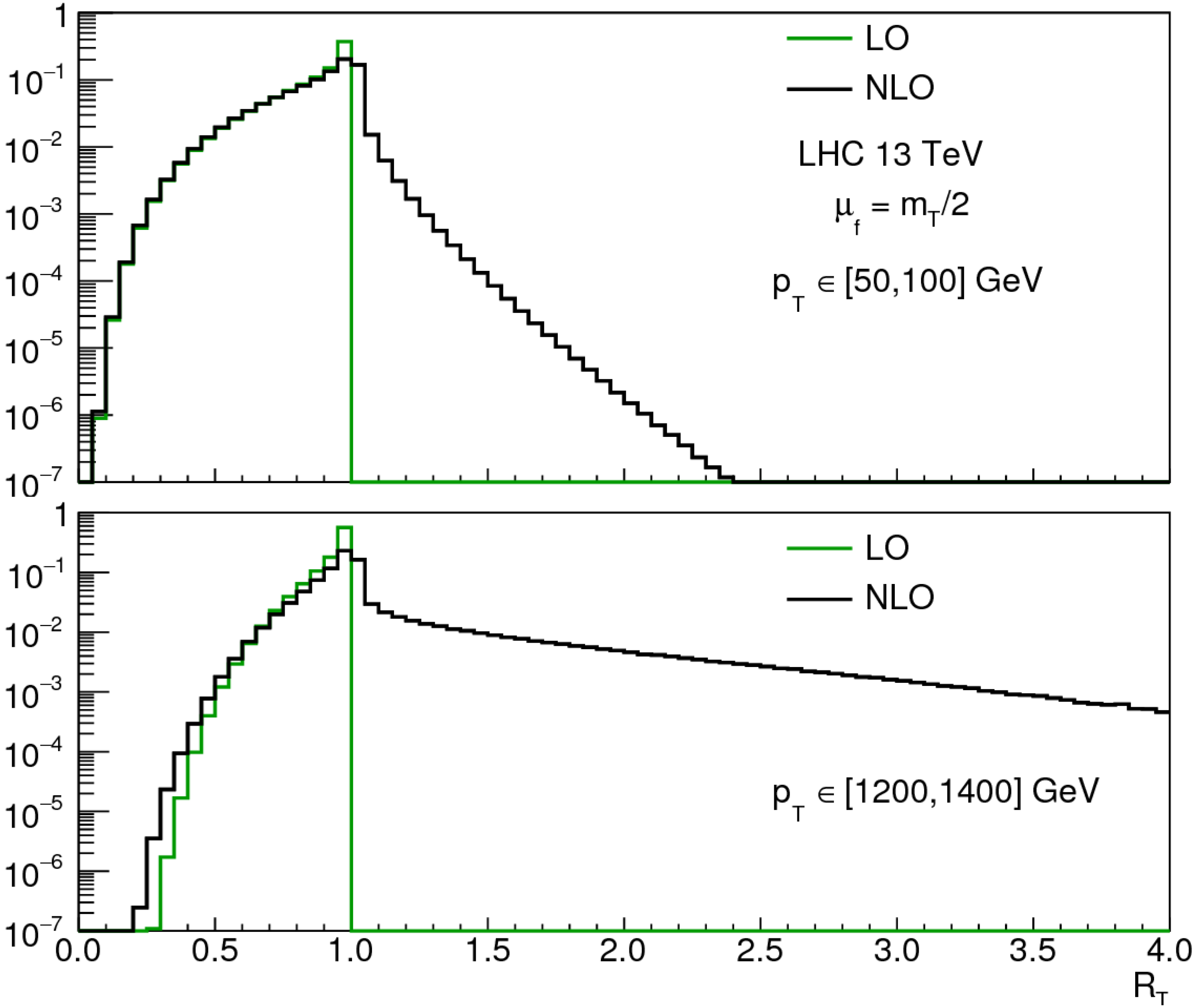
Useful to look at the quantity $R_T = \frac{H_T}{M_{t\bar{t}}}$

Scale choices



- Cross section in two sample $M_{t\bar{t}}$ bins as a function of R_T.
- Normalised to unity
- $R_T > 1$ Inaccessible at LO
- For large $M_{t\bar{t}}$, R_T distribution peaked at $R_T \ll 1$
- Clearly dominated by $|\cos\theta| \sim 1$

Scale choices

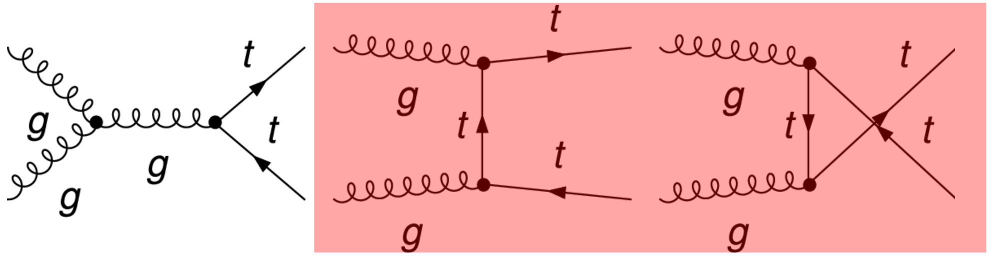


- Cross section in two sample p_T bins as a function of R_T.
- Normalised to unity
- $R_T > 1$ Inaccessible at LO
- Long tail for $R_T > 1$ at high p_T at NLO.

Scale choices

The peak at low R_T in the high energy $M_{t\bar{t}}$ bin can be explained by looking into the hard function itself. In the gluon-gluon channel, we have

$$H_{gg}^{(0)}\Big|_{t_1\to 0} = \frac{1}{2x_t}\begin{pmatrix} \frac{1}{N_c^2} & \frac{1}{N_c} & \frac{1}{N_c} \\ \frac{1}{N_c} & 1 & 1 \\ \frac{1}{N_c} & 1 & 1 \end{pmatrix}, \qquad x_t \equiv -t_1/M_{t\bar{t}}^2$$



t-channel diagrams

Since to obtain cross sections we integrate over $\cos\theta \in [-1, 1]$ the hard function receives large corrections from this region.

Scale choices

Thus, it would seem the most appropriate scale in the hard function is $\mu_h \sim H_T$ as opposed to $\mu_h \sim M_{t\bar{t}}$.

We can pick an appropriate constant from a careful K-factor analysis.
For the $M_{t\bar{t}}$ distributions we pick

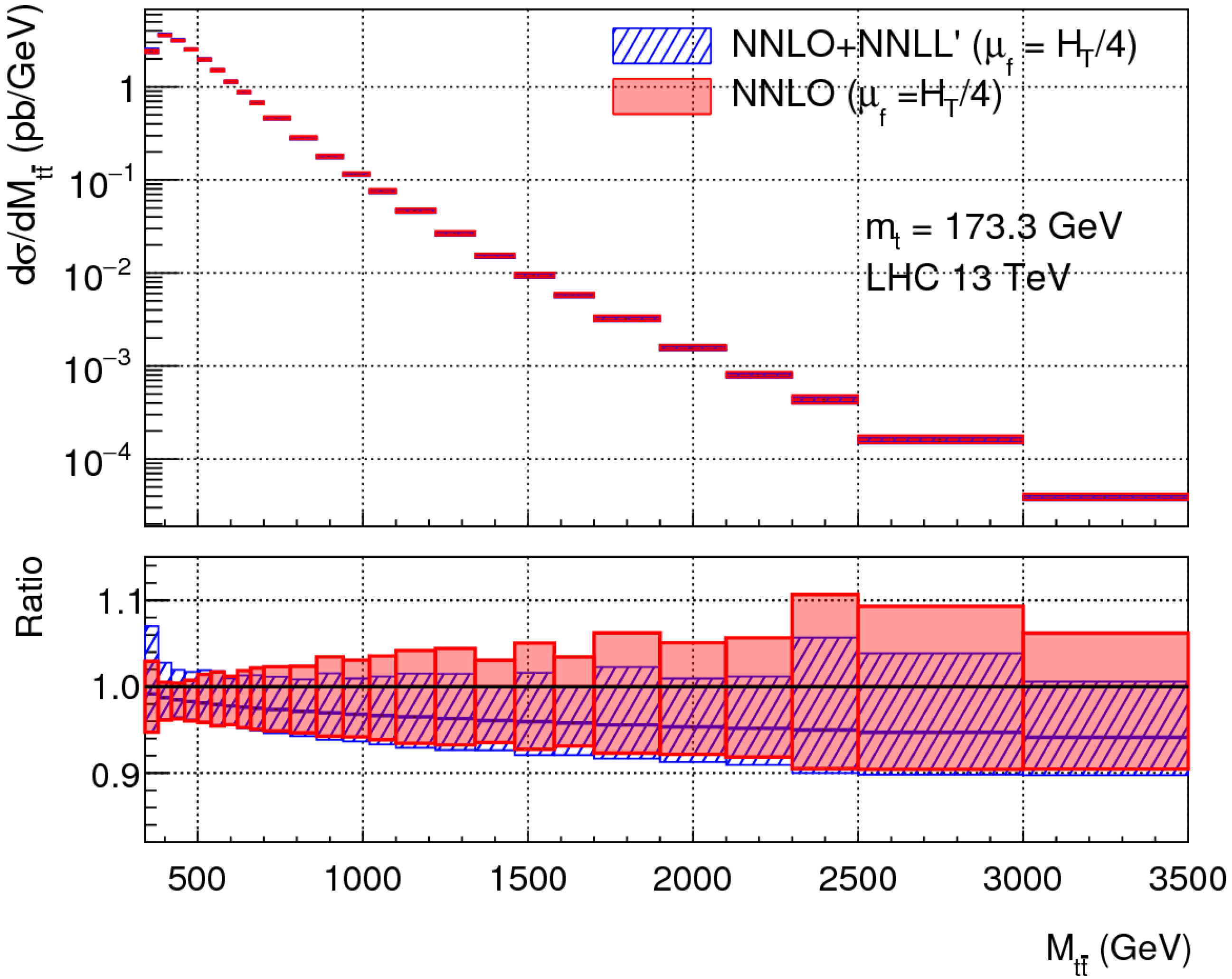
$$\mu_h = H_T/2 \qquad \mu_s = H_T/\bar{N}$$

For p_T distributions we use

$$\mu_h = m_T \qquad \mu_s = 2m_T/\bar{N}$$

The choice of μ_f is motivated by a similar K-factor analysis of the NNLO results.
We employ $\mu_f = H_T/4 \, \text{and} \, \mu_f = m_T/2$ for $M_{t\bar{t}}$ and p_T distributions respectively.

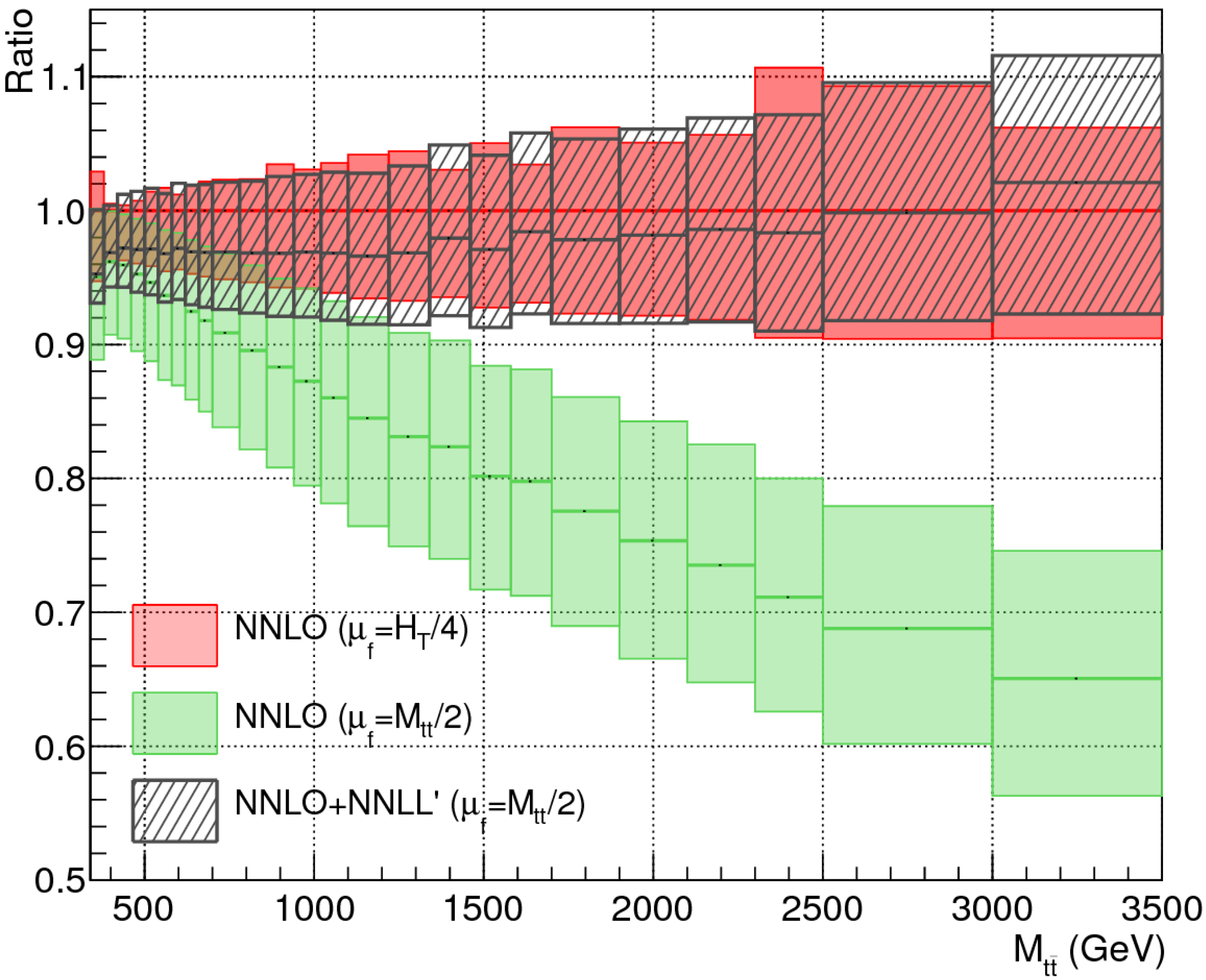
Pair invariant mass distribution



<u>NNLO+NNLL’ compared to NNLO:</u>

- Reduced uncertainties in the tails
- Slight suppression of cross section at large pair invariant mass

Pair invariant mass distribution



NNLO+NNLL' compared to NNLO:

- Evaluating at parametrically different choices for μ_f provides better stability.

$$\mu_f = \{M_{t\bar{t}}/2, H_T/4\}$$

- Perturbative corrections under good control.

Inclusion of electroweak corrections

So far we have only discussed QCD corrections.

We can also include NLO electroweak corrections.

Combining NNLO QCD and NLO EW

[Czakon, Heymes, Mitov, Pagani, Tsinikos, Zaro: 1705.04105]

- More than one way to combine QCD and EW corrections
- Additive and multiplicative approaches

	QCD	EW			
LO	α_s^2	$\alpha_s \alpha$	α^2		
NLO	α_s^3	$\alpha_s^2 \alpha$	$\alpha_s \alpha^2$	α^3	
NNLO	α_s^4	$\alpha_s^3 \alpha$	$\alpha_s^2 \alpha^2$	$\alpha_s \alpha^3$	α^4

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NNLO	α_s^4	$\alpha_s^3\alpha$	$\alpha_s^2\alpha^2$	$\alpha_s\alpha^3$	α^4

$= d\sigma^{\mathrm{QCD+EW}}$

Additive: Include all $\alpha_s^n\alpha^m$ for $m+n\leq 3$, & α_s^4

Leads to large corrections & uncertainties in high p_T tails.

Combining NNLO QCD and NLO EW

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$= d\sigma^{\text{QCD}\times\text{EW}}$

Multiplicative: Additive + approx $\alpha_s^3\alpha$

In high p_T limit, cross section dominated by Sudakov and soft logs.
These factorize, use K-factor to approximate $\sim (K_{\text{NLO}}^{\text{QCD}} - 1)\Sigma_{\text{EW}}^{\text{NLO}}$

Inclusion into resummed results

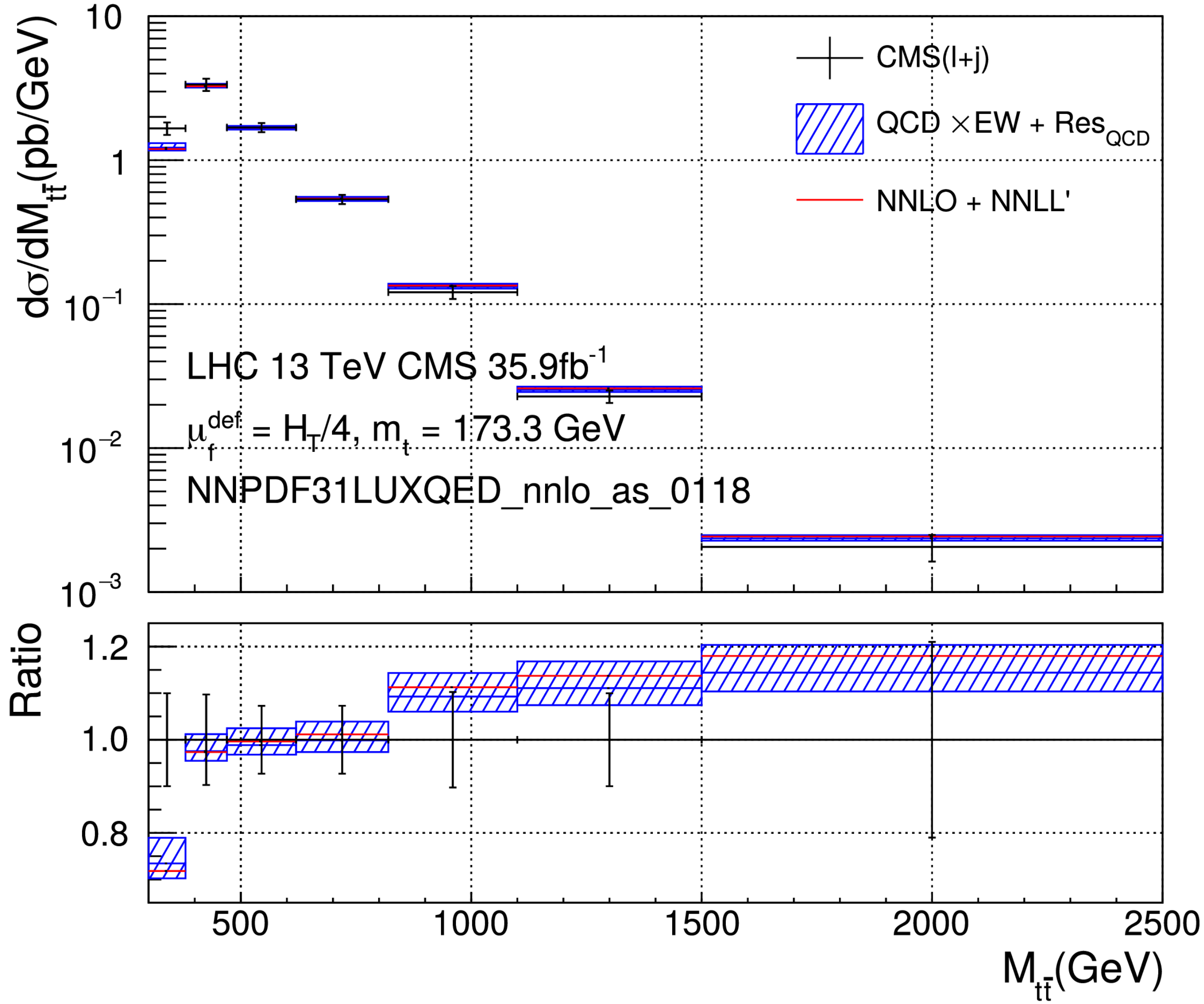
Combination proceeds as for the QCD case.

$$d\sigma^{\mathrm{QCD}\times\mathrm{EW+Res.}} = \left(d\sigma^{\mathrm{QCD}\times\mathrm{EW}} - d\sigma_{\mathrm{res}} \Big|_{\substack{\mathrm{NNLO} \\ \mathrm{expansion}}} \right)$$

The resummation knows nothing about the EW effects, there is no overlap.

Examine the effect on distributions.
We also compare against data in [CMS: 1811.06625]

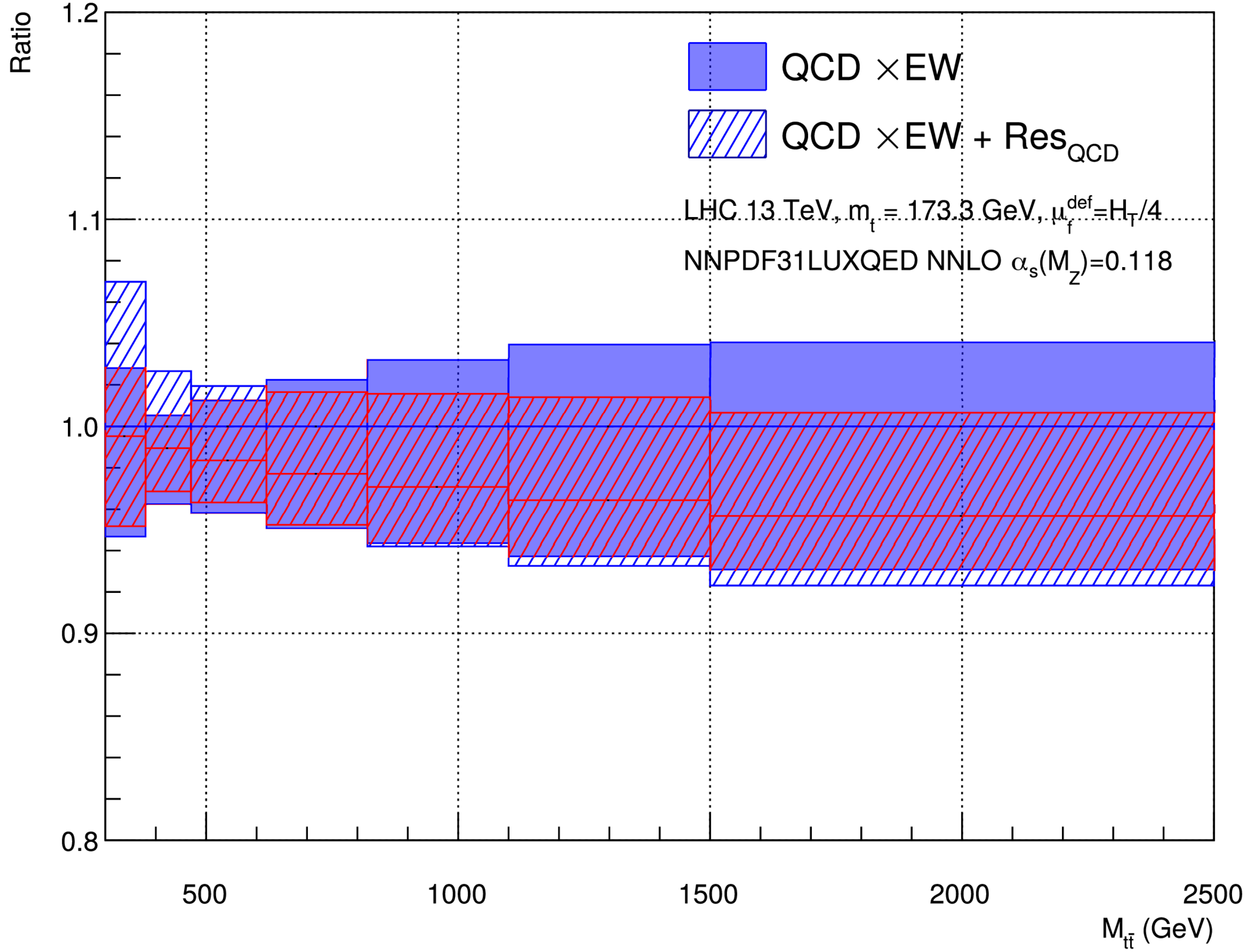
Pair invariant mass



Resummed QCD v Resummed QCDxEW:

- Electroweak corrections soften M spectrum at high M.

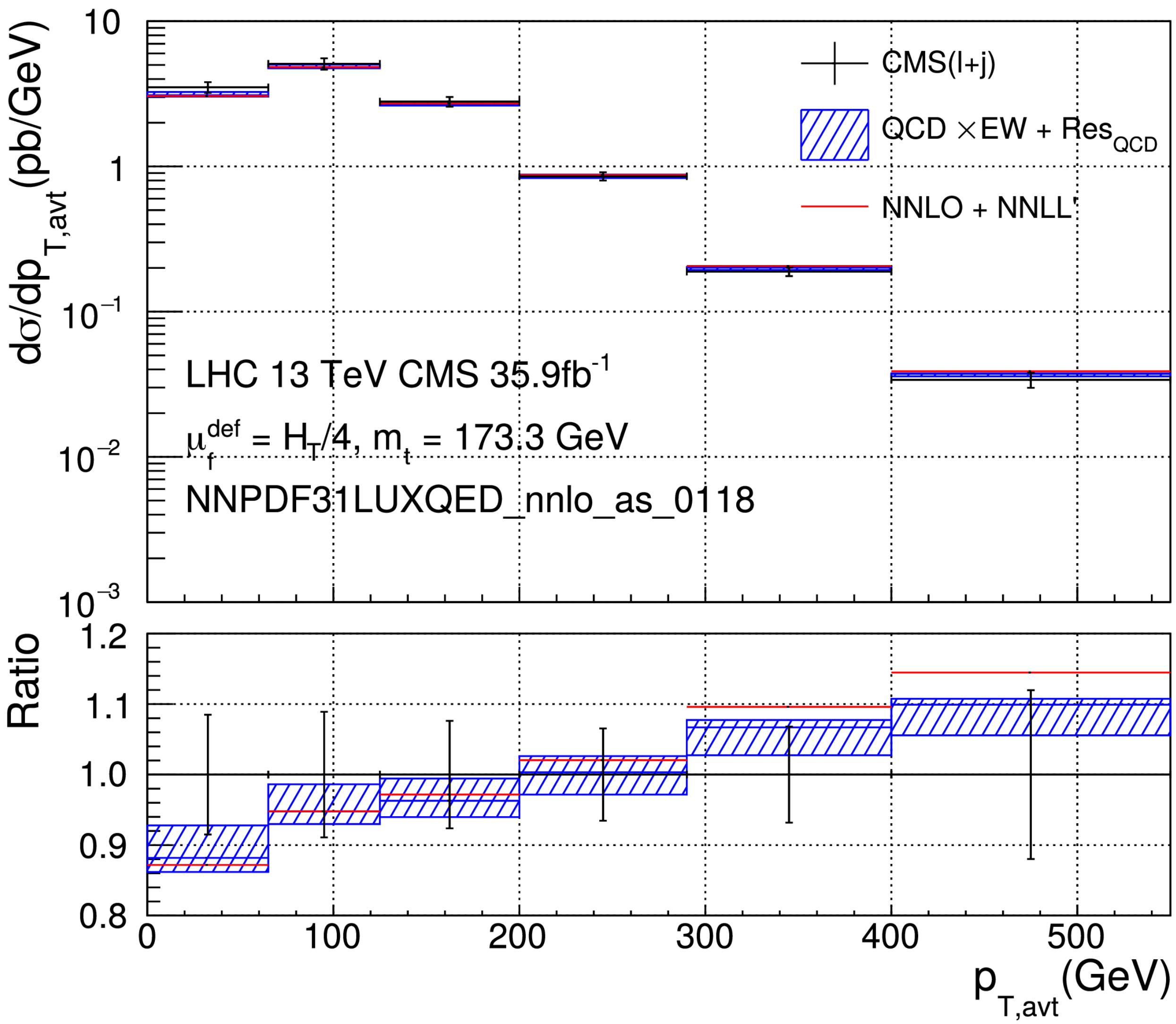
Pair invariant mass



<u>QCDxEW v Resummed QCDxEW:</u>

- Electroweak corrections soften M spectrum at high M.
- Resummation gives slight uncertainty reduction in tail. Slight softening of spectrum.

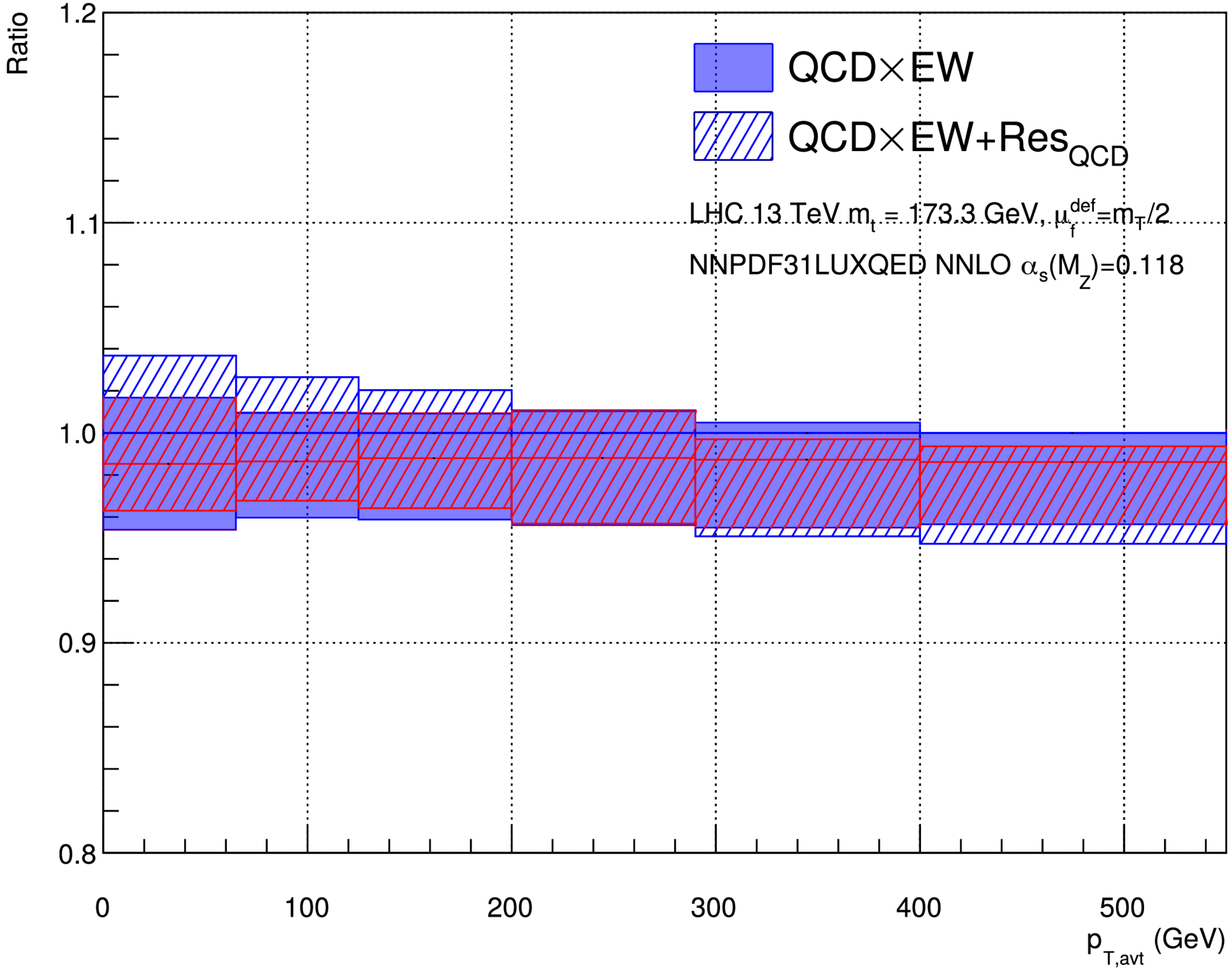
Transverse momentum



<u>Resummed QCD v Resummed QCDxEW:</u>

- Electroweak corrections soften p_T spectrum at high p_T.

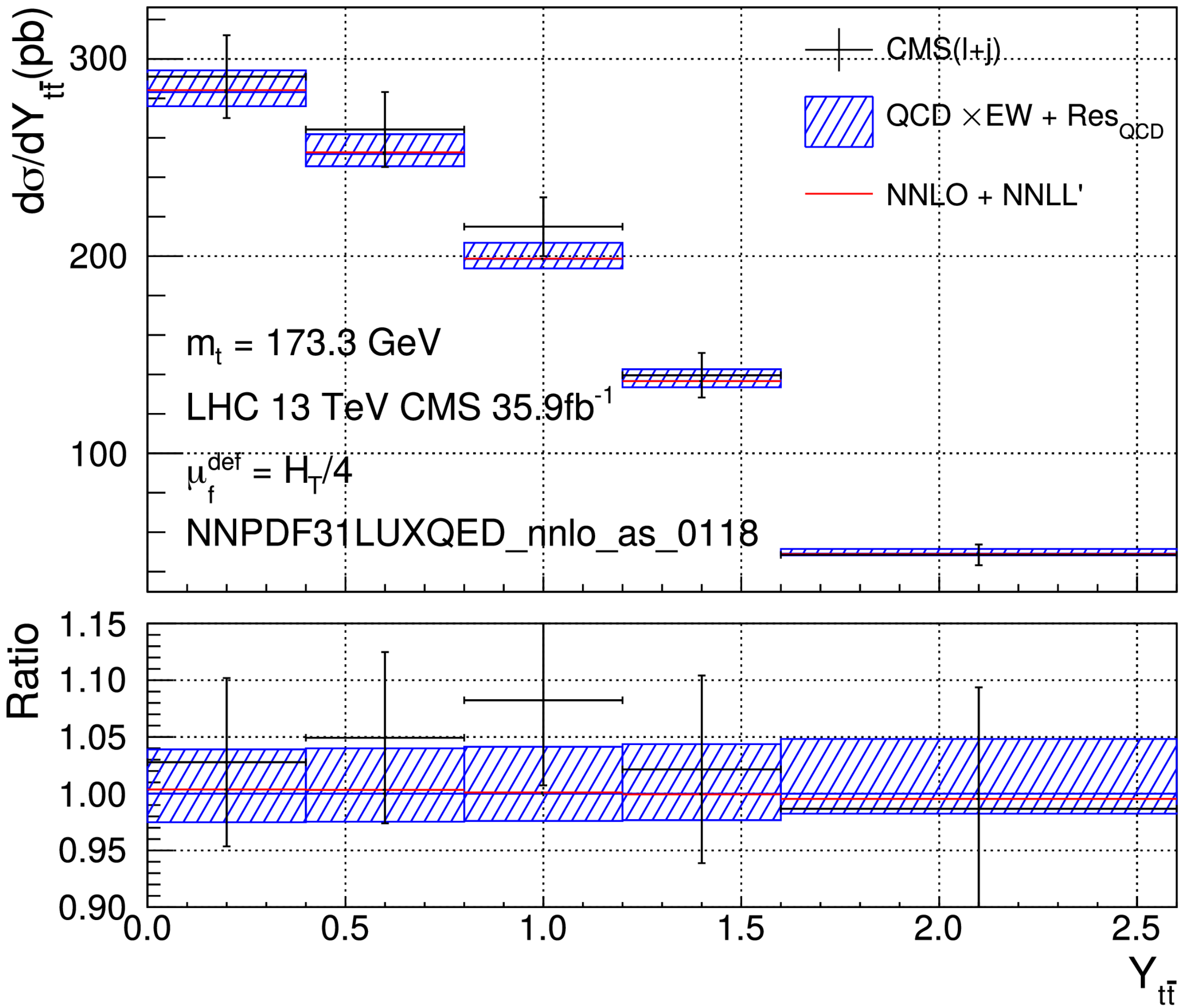
Transverse momentum



QCDxEW v Resummed QCDxEW:

- Electroweak corrections soften p_T spectrum at high p_T.
- Resummation: As in QCD only case. Very mild softening of spectrum.

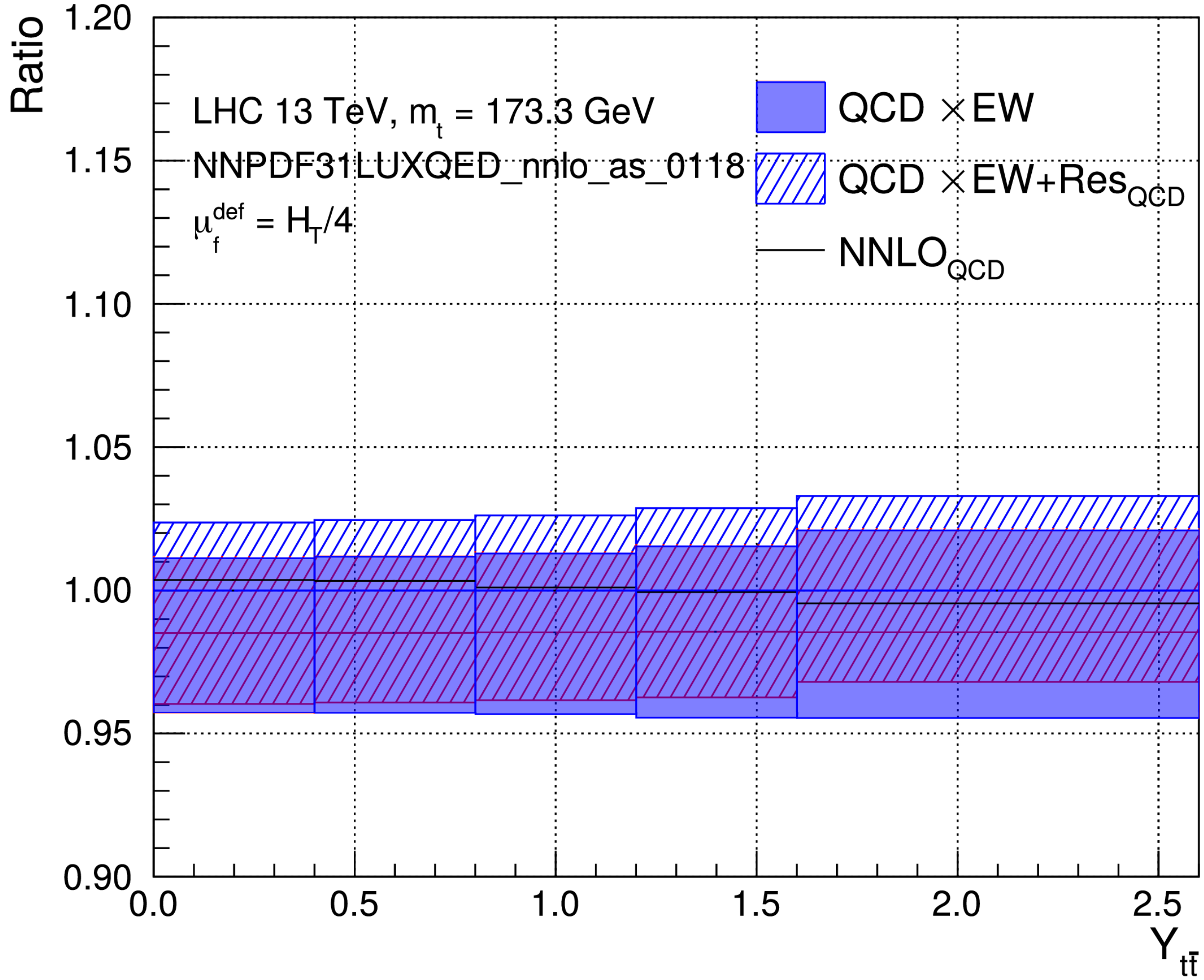
Rapidity



Resummed QCD v Resummed QCDxEW:

- Central value for resummed predictions very similar.
- Similar story for y_t distribution.

Rapidity



<u>QCD v QCDxEW v Resummed QCDxEW:</u>

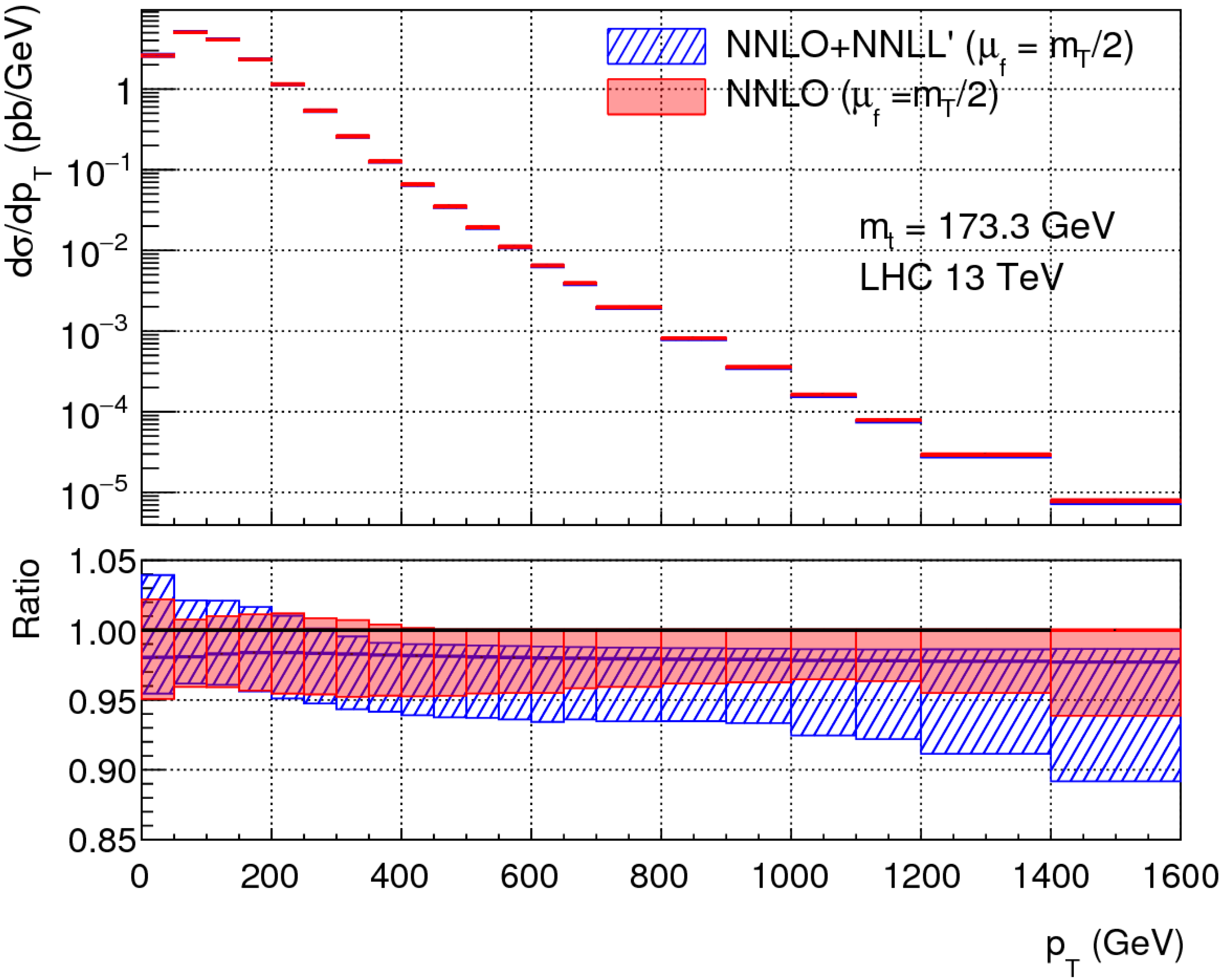
- Resummation leads to very slight softening of the spectrum.
- Similar story for y_t distribution.

Conclusions

- Presented results for top pair production at the LHC at NNLO+NNLL’ accuracy in QCD and as well as those including NLO EW effects.
- Resummation of soft gluon logs as well as small-mass logs in the soft limit.
- Insights into appropriate scale choices in top pair production.
- Predictions for invariant mass distributions under good perturbative control.
- The resummation has less impact on transverse momentum distributions – NNLO corrections most important here.

Backup

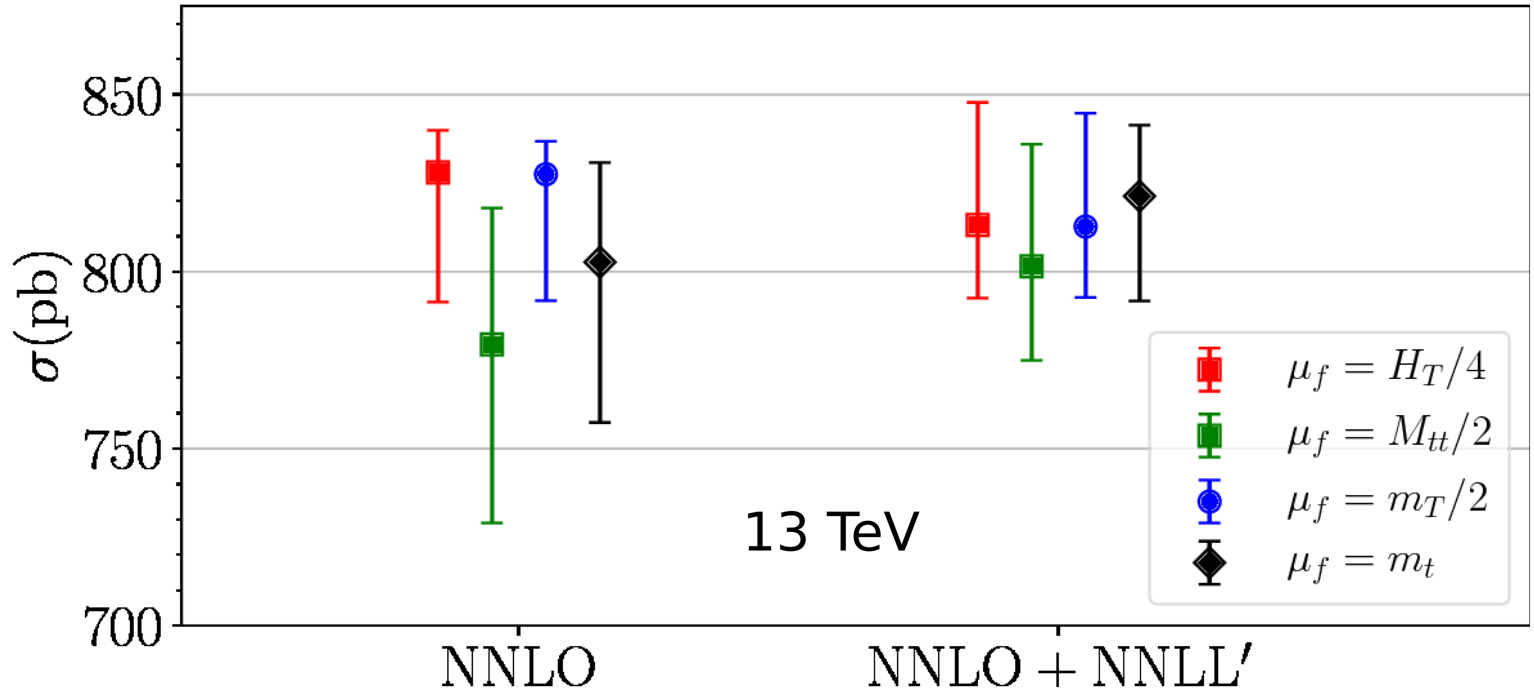
Transverse momentum distribution



NNLO+NNLL' compared to NNLO:

- Mild suppression of cross section at large transverse momentum

Total cross section



Resummed results agree with widely used fixed order ones.

- Can be used across phase space
- Useful for normalised distributions as well