AdS/QCD bottom-up models and application to thermalisation

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Outline

- Soft-wall model for QCD

- AdS/CFT correspondence for out-of-equilibrium systems and QGP

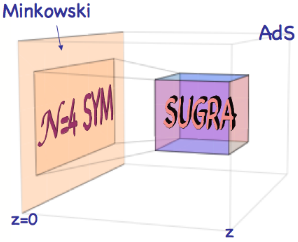
AdS/QCD correspondence

AdS/CFT correspondence [Maldacena, '97]

Type IIB string theory on $AdS_5 \times S^5$	$\Leftrightarrow$ $g_s = g^2_{\text{YM}}$ $R^4 = 4\pi g_s N \alpha'$	$\mathcal{N}$=4 SYM theory on 4d Minkowski
SUGRA limit $g_s \to 0$ $R \to \infty$	$\Leftrightarrow$	large N + NP limit $N \to \infty$ $\lambda = g^2_{\text{YM}} N \to \infty$

How can the theories be linked?

$\rightarrow$ dictionary [Adv. Theor. Math. Phys. 2, 253 (1998), Phys. Lett. B 428, 105 (1998)]



$$ds^2 = \frac{R^2}{z^2}(dt^2 - d\bar{x}^2 - dz^2)$$

$$z > 0$$

1. $5d$ fields $\leftrightarrow$ $4d$ operators

d	$d+1$
operator $\mathcal{O}(x)$	field $\phi(x,z)$
Δ	m^2_{d+1}

p-form: $m^2_{d+1}R^2 = (\Delta - p)(\Delta + p - d)$

2. Boundary value of field is the source of operator $\phi_0(x)$

3. $\langle \mathrm{e}^{\int d^dx\, \phi_0(x)\mathcal{O}(x)} \rangle = Z_S[\phi_0(x)] \approx \mathrm{e}^{i\mathcal{S}_{OS}}$

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2) \rangle = \left. \frac{\delta^2 \mathcal{S}_{OS}}{\delta\phi_0(x_1)\delta\phi_0(x_2)} \right|_{\phi_0=0}$$

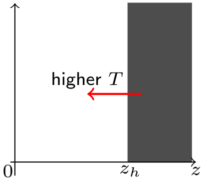
Finite temperature: metric with black hole

$$ds^2 = \frac{R^2}{z^2}\left(f(z)dt^2 - d\bar{x}^2 - \frac{dz^2}{f(z)}\right)$$

$0 < z < z_h$ z_h = BH horizon

$$f(z) = 1 - \frac{z^4}{z_h^4}$$ with $f(z_h) = 0$

$$T = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z_h} = \frac{1}{\pi z_h}$$



Finite temperature and density: Reissner Nordström metric

$$f(z) = 1 - \left(\frac{1}{z_h^4} + Q^2 z_h^2\right) z^4 + Q^2 z^6$$

charged black hole

add $A_0(z)$ U(1) gauge field dual to $\mu\bar{q}\gamma^0 q$

- Temperature: $T = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z_h}$
- Chemical potential: $\mu = A_0(0)$

Application to QCD: break conformal invariance

Bottom-up approach: specify an extra-dimensional spacetime geometry and the fields that propagate based on the properties of QCD to be incorporated

1. Hard-wall model [PRL 95, 261602 (2005)]: $z \leqslant z_m$ with $z_m \sim \mathcal{O}(\Lambda_{QCD}^{-1})$

Observable	Measured	Hard wall
m_π	139.6 $\pm$ 0.0004 MeV	139.6 MeV *
m_ρ	775.8 $\pm$ 0.5 MeV	775.8 MeV *
m_{a_1}	1230 $\pm$ 40 MeV	1363 MeV
f_π	92.4 $\pm$ 0.35 MeV	92.4 MeV *
$F_\rho^{1/2}$	345 $\pm$ 8 MeV	329 MeV
$F_{a_1}^{1/2}$	433 $\pm$ 13 MeV	486 MeV
$g_{\rho\pi\pi}$	6.03 $\pm$ 0.07 MeV	4.48 MeV

parameters z_m, m_q, σ fixed by *

2. Soft-wall model [PRD 74, 015005 (2006)]: "dilaton" in the metric or action

$$\mathrm{e}^{-\varphi(z)} \qquad \varphi(z) = c^2 z^2$$

Also modified versions of soft wall, comprising a dynamical dilaton

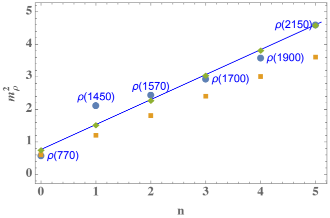
Main features of soft wall

- Regge trajectories

Example: action for scalar mesons

$$\mathcal{S} = -\frac{1}{2k}\int d^5x\sqrt{g}\,e^{-\varphi(z)}\ \mathrm{Tr}\left[g^{MN}\partial_M S^A \partial_N S^A + m_5^2 S^A S^A\right]$$

Vector mesons	$m_n^2 = c^2(4n+4)$
Scalar mesons	$m_n^2 = c^2(4n+6)$
Scalar glueballs	$m_n^2 = c^2(4n+8)$
Hybrid mesons	$m_n^2 = c^2(4n+8)$
Oddballs	$m_n^2 = c^2(4n+16)$



$m_\rho = 0.776$ GeV $\rightarrow$ $c = 0.388$ GeV (orange points)

Fit from trajectory $\rightarrow$ $c = 0.438$ GeV (green points)

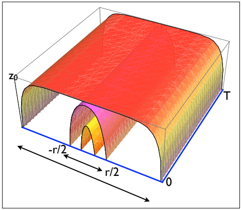
- $Q\bar{Q}$ potential [PRD 74, 025023 (2006)]

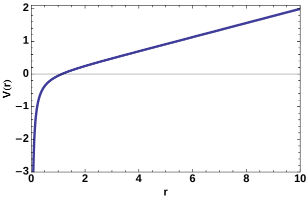
  dilaton in metric (string frame)

  $$\langle W_{\mathcal{C}} \rangle = \mathrm{e}^{-\mathcal{S}_{NG}} \qquad\qquad V(r) = \lim_{T\to\infty} \frac{1}{T} S_{NG}$$

  $\mathcal{S}_{NG}$ = Nambu Goto action = area of ws spanned by the string attached to $\mathcal{C}$

  $$S_{NG} = \frac{1}{2\pi\alpha'} \int d^2\xi \sqrt{\det\left[g_{MN}\,\partial_\alpha X^M\,\partial_\beta X^N\right]}$$

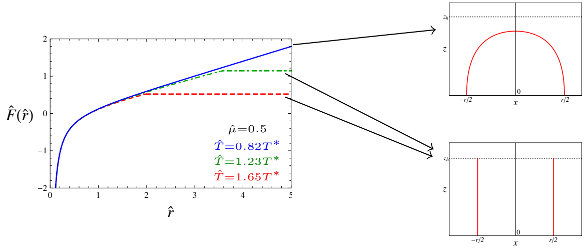




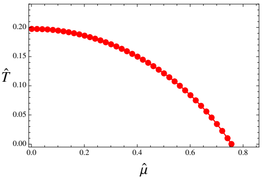
- $Q\bar{Q}$ free energy $F(r,T)$ in plasma [PRD 83, 035015 (2011)]

  work in Euclidean space: $\langle \mathcal{P}(\vec{x}_1)\mathcal{P}^\dagger(\vec{x}_2)\rangle = e^{-\frac{1}{T}F(r)}$

  From AdS/QCD correspondence: $F(r) = T\mathcal{S}_{NG}$



possible string configurations



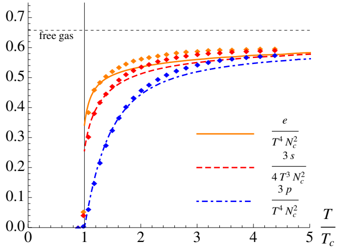
(μ, T) at which $F(r)$ becomes flat at large r

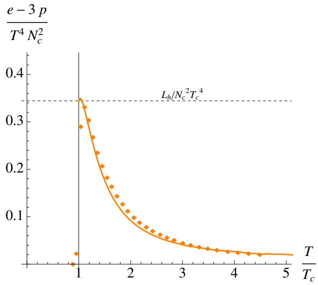
hadron phase near the origin, deconfined phase beyond the curve

With mass scale from ρ mass:
$T_c \sim 134$ MeV, $\mu_c \sim 248$ MeV

- Thermodynamic functions [Nucl.Phys. B 820, 148 (2009)]

  Model with dynamical dilaton, 3 parameters (in dilaton potential) fixed from β function and lattice data on latent heat and pressure at $2T_c$

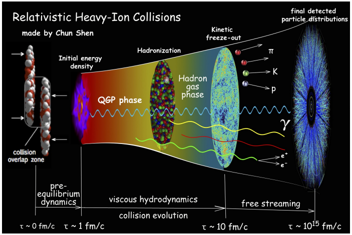




Plasma out of equilibrium

Study relaxation towards the hydrodynamic regime of a boost-invariant non Abelian plasma taken out of equilibrium.

Application: possibility to study the system produced in ultrarelativistic HIC, as those taking place at RHIC and at LHC.



data from RHIC and LHC:
fast thermalisation $\mathcal{O}(1\,fm/c)$

HIC produce a plasma-like system whose properties are similar to the ones expected for the QGP

QGP features:

- it contains deconfined quarks and gluons
- near perfect fluid
- matter flows collectively like a fluid in local equilibrium
- it can be described by hydrodynamics with low viscosity
- strongly interacting

Result from the duality: $\eta/s = \dfrac{1}{4\pi}$ [PRL 87, 081601 (2001)]

Use AdS/CFT correspondence: introduce perturbation for short time intervals in the boundary 4d metric and find the corresponding 5d metric at varying time.

Properties of 4d metric: boost-invariance along collision axis (x_3), translation invariance and $O(2)$ rotation invariance in the orthogonal plane $x_\perp = \{x^1, x^2\}$

$$ds^2 = -d\tau^2 + e^{\gamma(\tau)} dx_\perp^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$$

$x^0 = \tau \cosh y$, $x_3 = \tau \sinh y$

Solve Einstein equations to find 5d metric (Eddington Finkelstein coordinates)

$$ds^2 = 2drd\tau - Ad\tau^2 + \Sigma^2 e^B dx_\perp^2 + \Sigma^2 e^{-2B} dy^2$$

$$R_{MN} - \frac{1}{2} g_{MN}(R - 2\Lambda) = 0$$

Boundary condition: for $r \to \infty$ the $4d$ metric is reproduced, initial condition is AdS metric.

For any distortion $\gamma(\tau)$, as soon as the perturbation starts, a horizon is formed in the $5d$ space, meaning that a black hole has appeared.

To study thermalisation, use local and nonlocal probes:

- Local probes: boundary energy-momentum tensor.
- Nonlocal probes: equal time two-point correlation function, expectation value of Wilson loop.

Local observables:

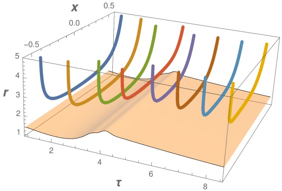
- Energy-momentum tensor is the operator dual to the metric tensor
  $T^{\mu}_{\nu} = \frac{N_c^2}{2\pi^2}\,\mathrm{diag}(-\epsilon, p_\perp, p_\perp, p_\parallel)$
  It can be computed from holographic renormalisation recipe [de Haro et al. 2000]
  1. Start from metric in this form: $ds^2 = \frac{g_{\mu\nu}(x,z)dx^\mu dx^\nu + dz^2}{z^2}$
  2. expand metric near boundary $z \to 0$: $g_{\mu\nu}(x,z) = \eta_{\mu\nu} + z^4 g^{(4)}_{\mu\nu}(x) + \ldots$
  3. compute EMT: $T_{\mu\nu}(x) = \frac{N_c^2}{2\pi^2}\, g^{(4)}_{\mu\nu}(x)$
- Effective temperature can be defined from the event horizon r_h
  $T_{eff} = \frac{r_h}{\pi}$

Nonlocal observables:

- Equal-time two-point correlation function of operators with large Δ

$$\langle \mathcal{O}(t,\boldsymbol{x})\mathcal{O}(t,\boldsymbol{x}')\rangle \simeq e^{-\Delta\,\mathcal{L}}$$

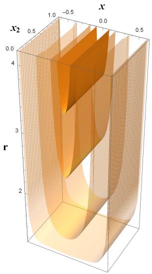
$\mathcal{L}$ is the length of the extremal string connecting the points on the boundary

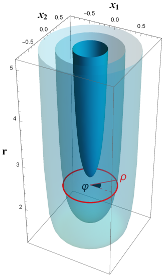


- Expectation value of spatial Wilson loop (circular or rectangular)

$$\langle W_{\mathcal{C}}\rangle \sim e^{-\mathcal{S}_{NG}}$$

$\mathcal{S}_{NG}$ is the Nambu Goto action, i.e. the area of the surface spanned by the extremal string attached to the contour $\mathcal{C}$





Compare observables with viscous hydrodynamics

Local and nonlocal observables computed in $5d$ model dual to viscous hydrodynamics, in which deviations from the ideal behaviour described by late time expansion [PRD 76, 025027 (2007); JHEP 0804, 100 (2008)]

- Energy-momentum tensor

$$\epsilon(\tau) = \frac{3\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}}\left[1 - \frac{2c_1}{(\Lambda\tau)^{2/3}} + \frac{c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left((\Lambda\tau)^{-2}\right)\right]$$

$$p_\parallel(\tau) = \frac{\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}}\left[1 - \frac{6c_1}{(\Lambda\tau)^{2/3}} + \frac{5c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left((\Lambda\tau)^{-2}\right)\right]$$

$$p_\perp(\tau) = \frac{\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}}\left[1 - \frac{c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left((\Lambda\tau)^{-2}\right)\right]$$

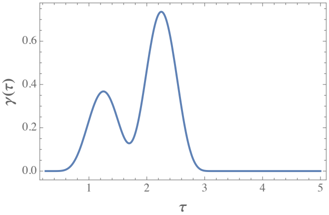
$$c_1 = \frac{1}{3\pi},\ c_2 = \frac{1+2\log 2}{18\pi^2}$$

- Effective temperature:

$$T_{eff}(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}}\left[1 - \frac{1}{6\pi(\Lambda\tau)^{2/3}} + \frac{-1+\log 2}{36\pi^2(\Lambda\tau)^{4/3}} + \mathcal{O}\left((\Lambda\tau)^{-2}\right)\right]$$

- Nonlocal probes computed numerically

Model with two pulses of different intensity, describing phenomena where a small number of collisions takes place before the system starts evolving to thermal equilibrium [JHEP 1507, 053 (2015)]

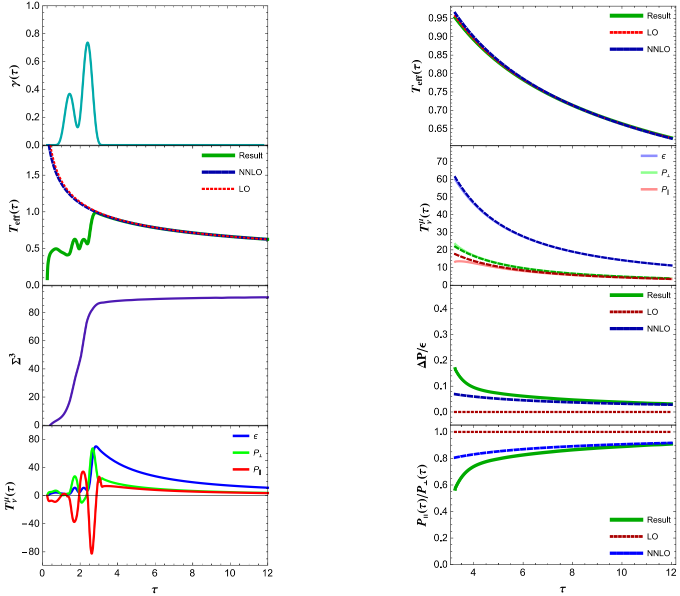


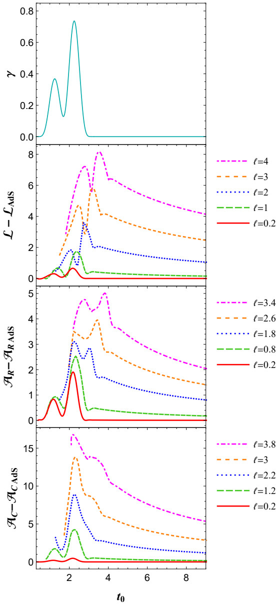
boundary metric

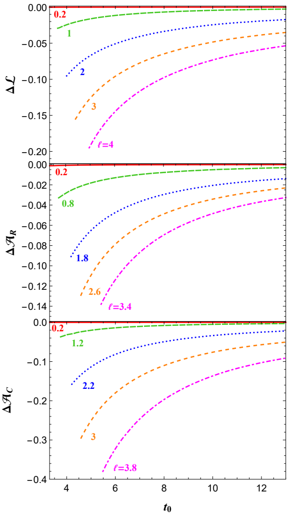
$$ds^2 = -d\tau^2 + e^{\gamma(\tau)} dx_\perp^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$$

Perturbation ends at $\tau_f = 3.25$

Λ fitted from T_{eff} $\quad \rightarrow \quad \Lambda = 1.73$


$\gamma(\tau)$
$T_{\text{eff}}(\tau)$
Result
NNLO
LO
Σ^3
$T^{\mu}_{\nu}(\tau)$
ϵ
$P_{\perp}$
$P_{\parallel}$
τ
$T_{\text{eff}}(\tau)$
Result
LO
NNLO
$T^{\mu}_{\nu}(\tau)$
ϵ
$P_{\perp}$
$P_{\parallel}$
$\Delta P/\epsilon$
Result
LO
NNLO
$P_{\parallel}(\tau)/P_{\perp}(\tau)$
Result
LO
NNLO
τ





slight delay in the onset of thermalization, depending on size [PRD 94, 025005 (2016)]

Soon at the end of perturbation $\epsilon \sim \epsilon_{hydro}$
Define equilibration time τ^* from

$$\left|\frac{\epsilon(\tau^*) - \epsilon_H(\tau^*)}{\epsilon(\tau^*)}\right| = 0.05 \quad \rightarrow \quad \tau^* = \tau_f$$

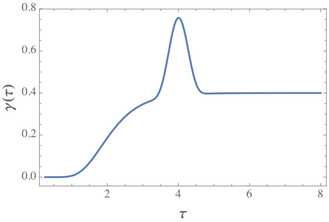
After a short time interval pressure anisotropy similar to expected by viscous hydrodynamics
Define isotropisation time τ_p from

$$\left|\frac{p_{||}(\tau_p)/p_\perp(\tau_p) - (p_{||}(\tau_p)/p_\perp(\tau_p))_H}{p_{||}(\tau_p)/p_\perp(\tau_p)}\right| = 0.05 \quad \rightarrow \quad \tau_p = 6$$

Fix energy scale such that $T_{eff} = 500$ MeV at the end of perturbation
$\rightarrow \quad \tau_p - \tau^* \sim 1.03$ fm/c

Nonlocal probes: thermalisation depends on the size of the probe in the boundary theory (distance between the points at which the correlation function is evaluated, the size of Wilson loops)

Model with one pulse plus slow deformation, effects with different time scales

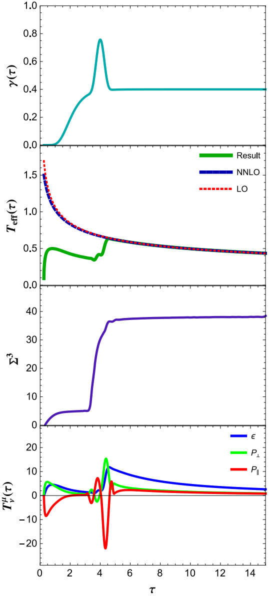


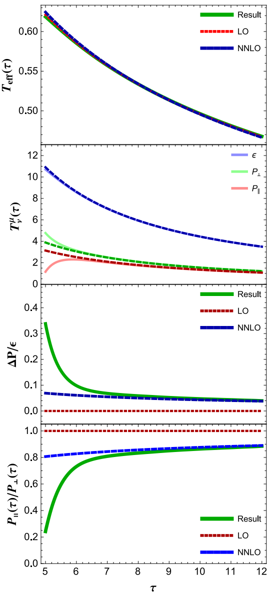
boundary metric

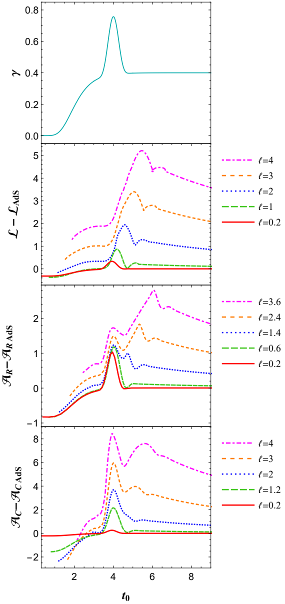
$$ds^2 = -d\tau^2 + e^{\gamma(\tau)} dx_\perp^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$$

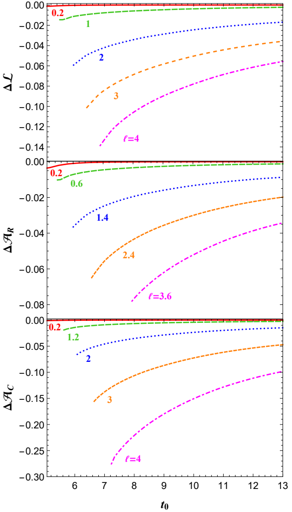
Perturbation almost constant at $\tau_f = 5$

In this case, $\Lambda = 1.12$


γ(τ)
Result
NNLO
LO
T_eff(τ)
Σ^3
ϵ
P_⊥
P_∥
T^μ_ν(τ)
τ


Result
LO
NNLO
T_eff(τ)
ϵ
P_⊥
P_∥
T^μ_ν(τ)
ΔP/ϵ
P_∥(τ)/P_⊥(τ)
τ


γ
$\mathcal{L}-\mathcal{L}_{\rm AdS}$
$\mathcal{A}_R-\mathcal{A}_{R\,\rm AdS}$
$\mathcal{A}_C-\mathcal{A}_{C\,\rm AdS}$
t_0
ℓ=4
ℓ=3
ℓ=2
ℓ=1
ℓ=0.2
ℓ=3.6
ℓ=2.4
ℓ=1.4
ℓ=0.6
ℓ=0.2
ℓ=4
ℓ=3
ℓ=2
ℓ=1.2
ℓ=0.2


$\Delta\mathcal{L}$
$\Delta\mathcal{A}_R$
$\Delta\mathcal{A}_C$
t_0
0.2
1
2
3
ℓ=4
0.2
0.6
1.4
2.4
ℓ=3.6
0.2
1.2
2
3
ℓ=4

Soon at the end of perturbation $\epsilon \sim \epsilon_{hydro}$ and $T_{rh} \sim T_\epsilon$
Define equilibration time τ^* from

$$\left|\frac{\epsilon(\tau^*) - \epsilon_H(\tau^*)}{\epsilon(\tau^*)}\right| = 0.05 \quad \rightarrow \quad \tau^* = \tau_f$$

After a short time interval pressure anisotropy similar to expected by viscous hydrodynamics
Define isotropisation time τ_p from

$$\left|\frac{p_{||}(\tau_p)/p_\perp(\tau_p) - (p_{||}(\tau_p)/p_\perp(\tau_p))_H}{p_{||}(\tau_p)/p_\perp(\tau_p)}\right| = 0.05 \quad \rightarrow \quad \tau_p = 6.74$$

Fix energy scale such that $T_{eff} = 500$ MeV at the end of perturbation
$\rightarrow \quad \tau_p - \tau^* \sim 0.42$ fm/c

Nonlocal probes: thermalisation depends on the size of the probe in the boundary theory (distance between the points at which the correlation function is evaluated, the size of Wilson loops)

Conclusions

- Reproduce key features of QCD
- Able to reach finite density
- Able to study a process out of equilibrium, with main results:
  - isotropisation and thermal equilibration have different time scales
  - hydrodynamic behaviour reached after a time of a few fm/c in all the considered models
  - energetic (local) modes equilibrate first, as in strongly coupled theories
- Future: study hadronisation and confinement/deconfinement phase transition