Perturbative aspects of soft QCD dynamics

LFC 2019, 10 September 2019

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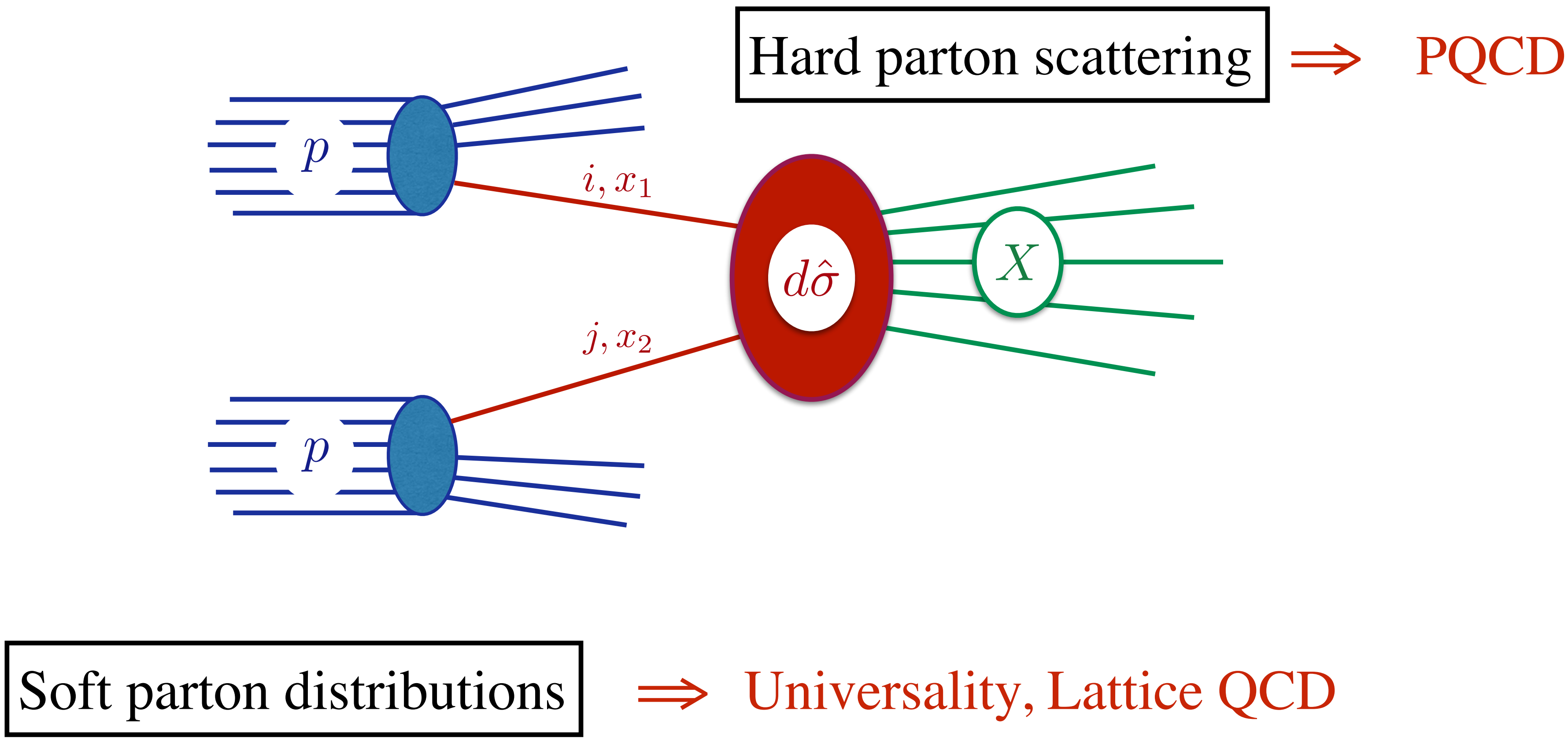


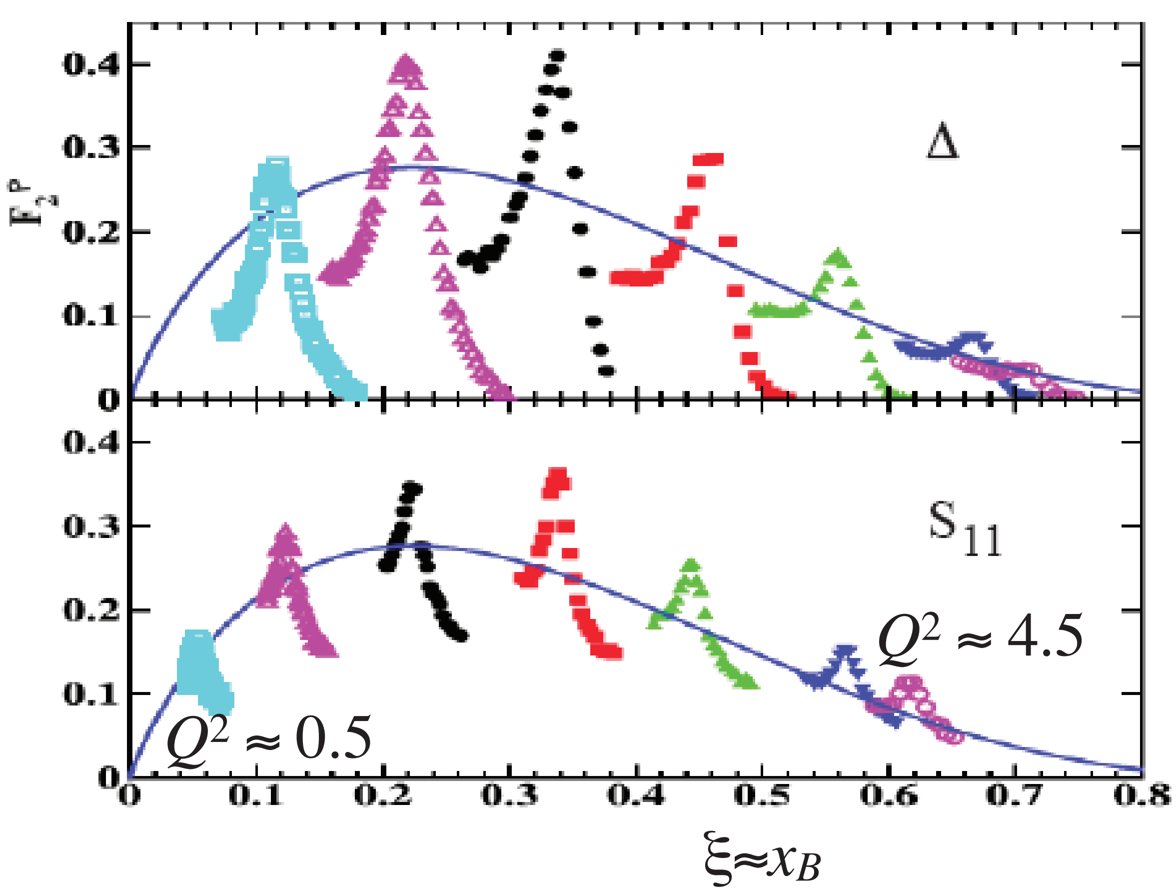
PH 1807.05598v2 and refs. therein

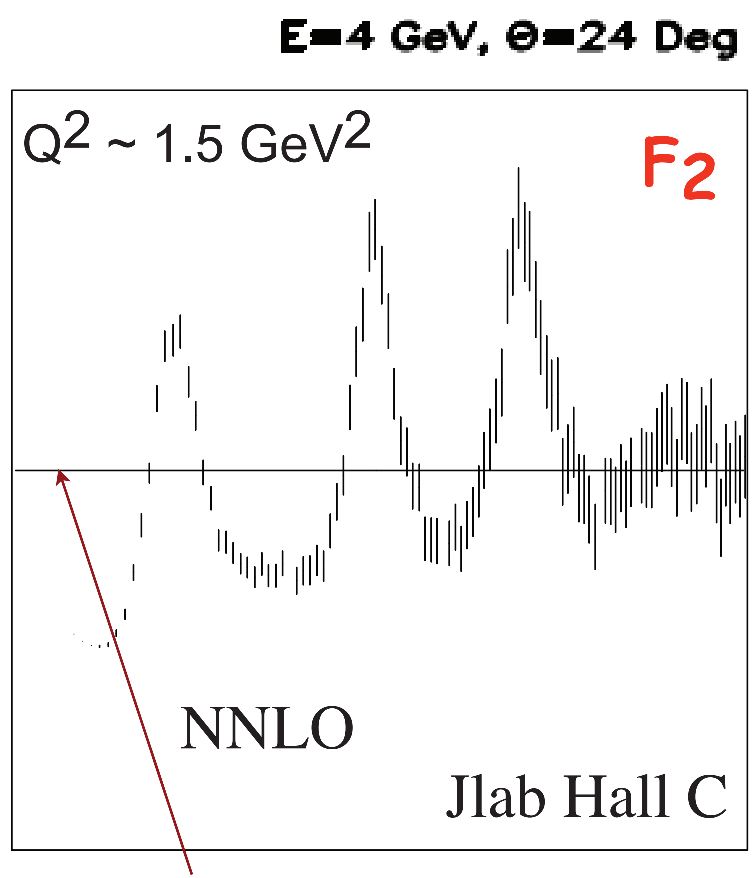
Resonances build the pdf's

Duality is a general and surprising feature of hadron dynamics.

Bloom-Gilman duality (1970): Resonances build the pdf's

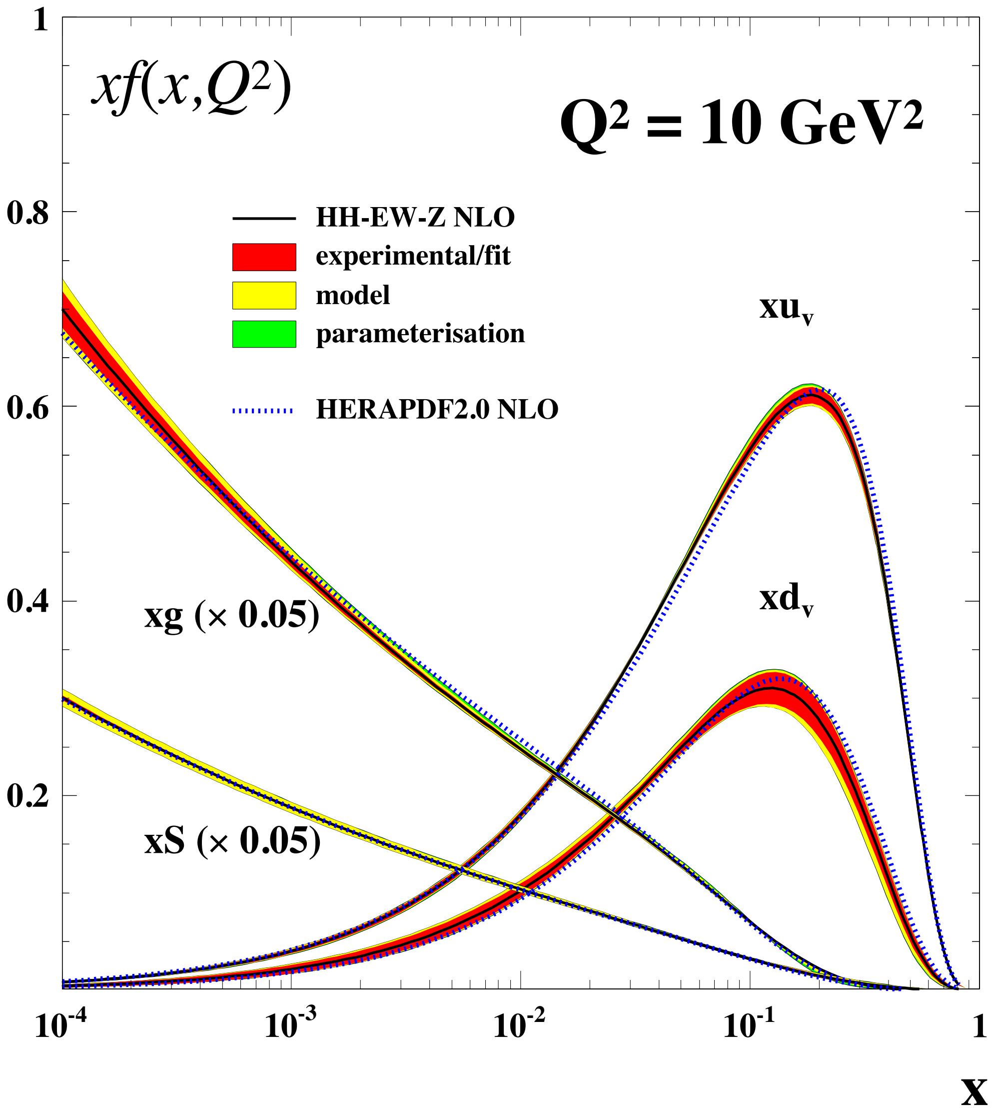
Jlab Hall C

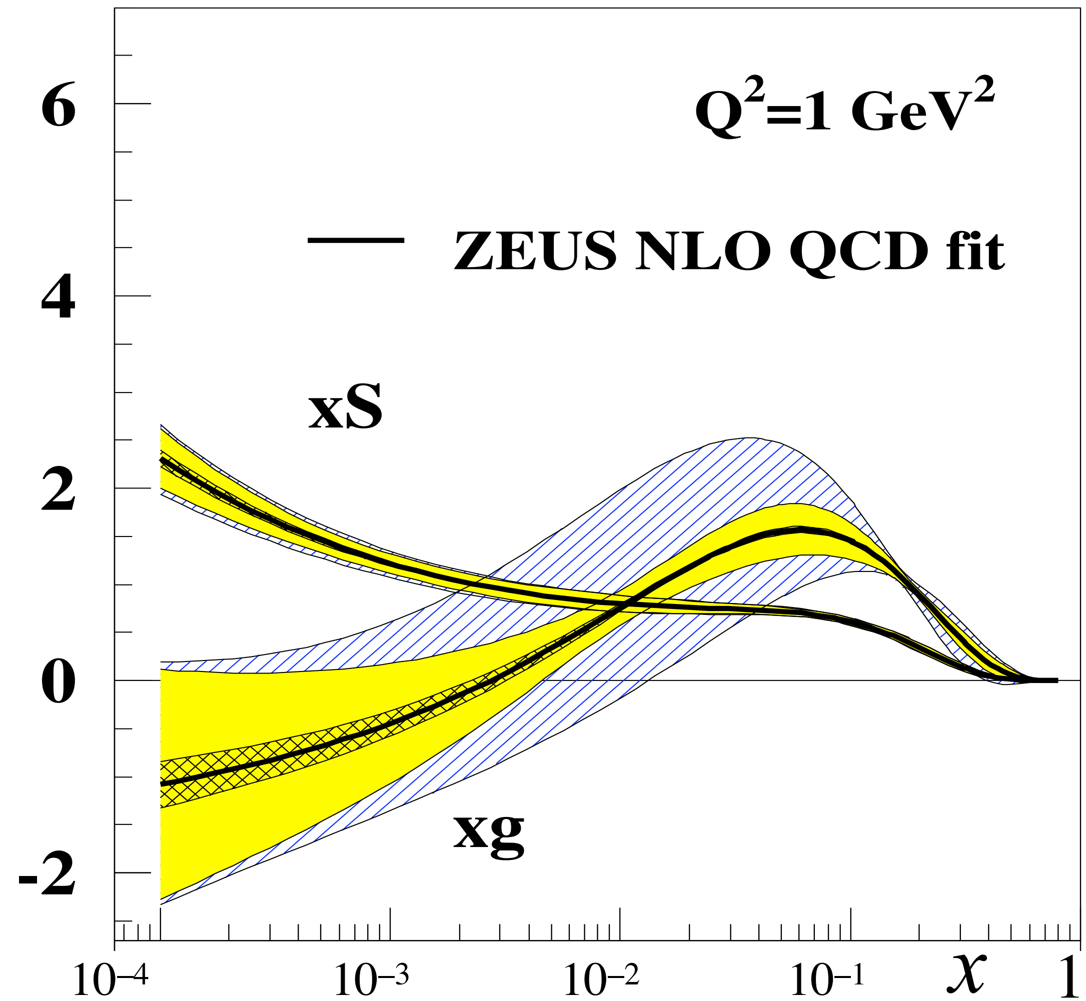




S. Alekhin, PRD 68 (2003) 014002

Gluons evolve away with decreasing Q^2





Resonances are not gluon dominated.

But the sea quarks remain at low *x*.

A. M. Cooper-Sarkar et al, 1206.0894, 1604.05083

The meaning of "non-perturbative"

Perturbative expansion diverges

Feynman diagrams lack essential features

Common view for soft QCD: $\alpha_s \gg 1$ $\Rightarrow$ Use lattice QCD (or models)

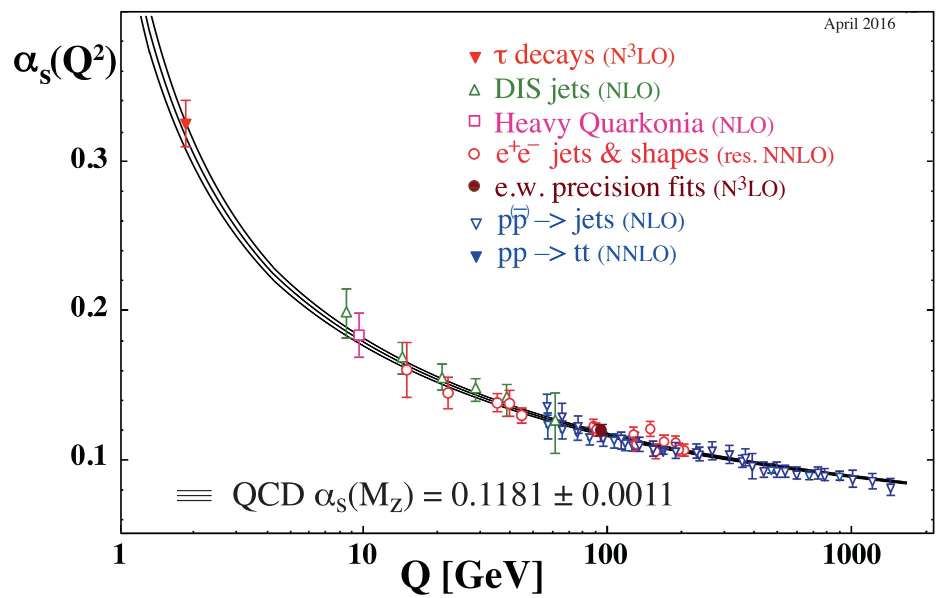
Alternative possibility: Coupling freezes, remains perturbative $\alpha_s(0)/\pi \approx 0.14$

★ ⟵ $\alpha_s{}^{crit} \approx 0.43$ Gribov

Divergence of perturbative expansion is due to low momentum transfers

This is the case for classical fields in QED

and for QED bound states $\alpha(0) \simeq 1/137$



Yuri Dokshitzer (2011):

infrared coupling

Theory + Phenomenology of 1/Q effects in event shape observables, both in **e^+e^-** annihilation and **DIS** systematically pointed at the ***average value*** *of the* ***infrared coupling***

$$\alpha_0 \equiv \frac{1}{2\ \mathrm{GeV}} \int_0^{2\ \mathrm{GeV}} dk\, \alpha_s(k^2) \sim 0.5$$

$\alpha_s = 0.1153 \pm 0.0017(\mathrm{exp}) \pm 0.0023(\mathrm{th})$

$\alpha_0 = 0.5132 \pm 0.0115(\mathrm{exp}) \pm 0.0381(\mathrm{th})$

T.Ghermann, M.Jaquier, G.Luisoni

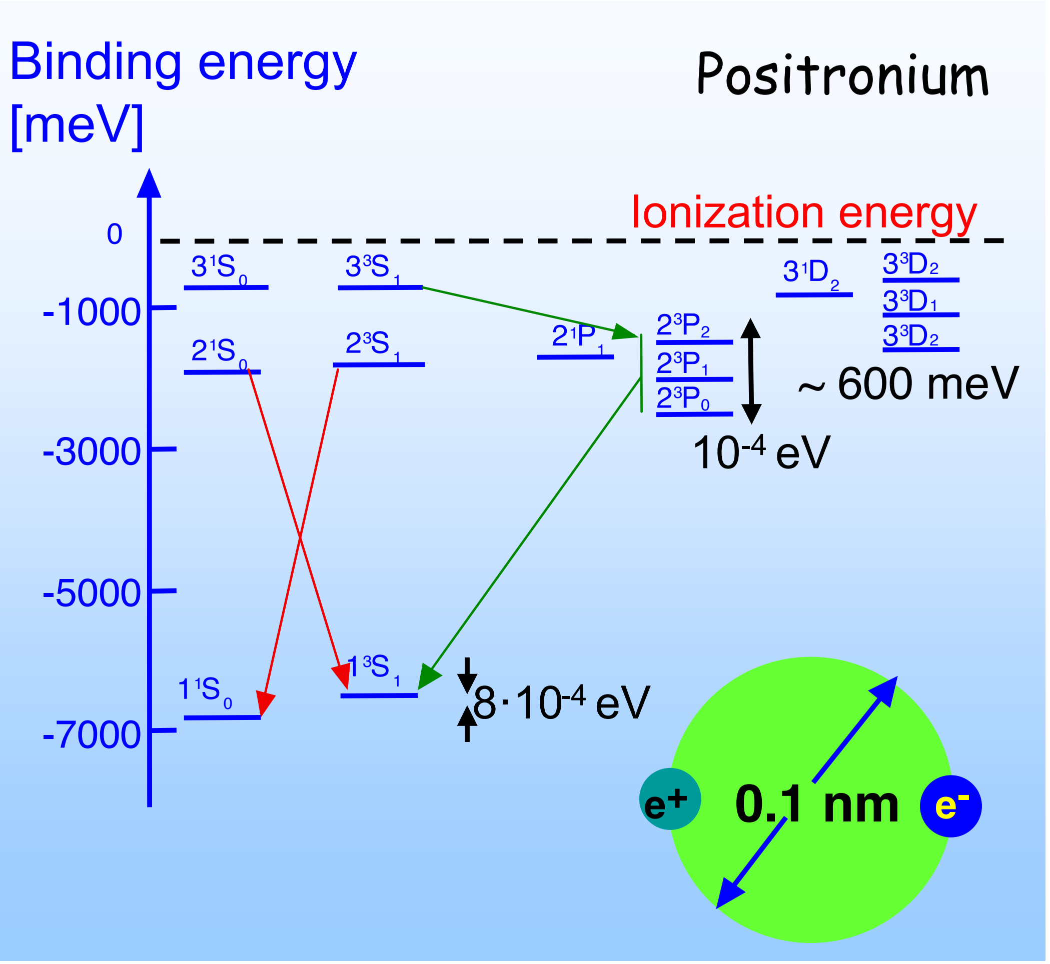
The main features of this result are as follows : the average IR coupling is

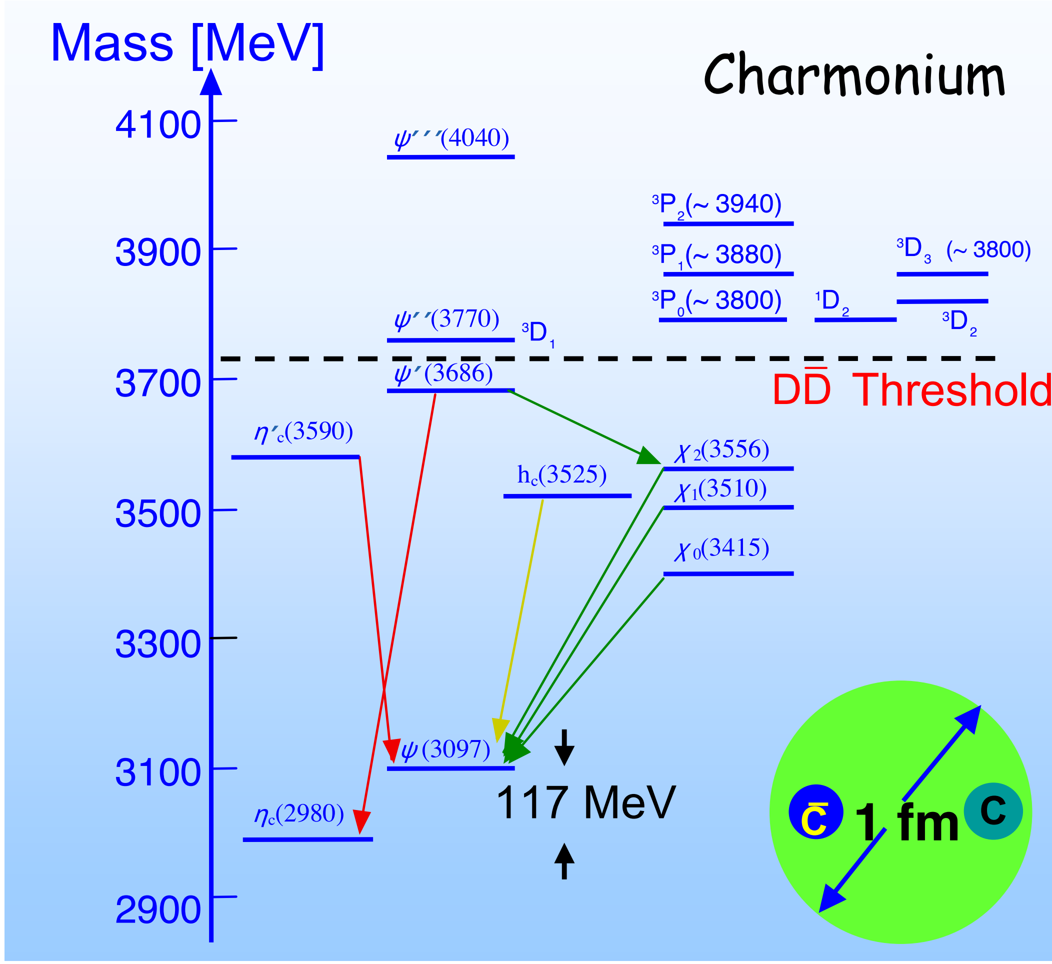
- Universal — holds to within $\pm 15\%$

  If not for the ***universality***,
  the whole game would made no sense : it would have meant just trading ***one unknown*** - non-perturbative "smearing" effects in a given observable (like in MC event generators) - for ***another unknown*** function - the shape of the coupling in the infrared...

- Reasonably small (which opens intriguing possibilities . . .)
- Comfortably above the Gribov's critical value $(\pi \cdot 0.137 \simeq 0.4)$

Similarity of quarkonia and atoms



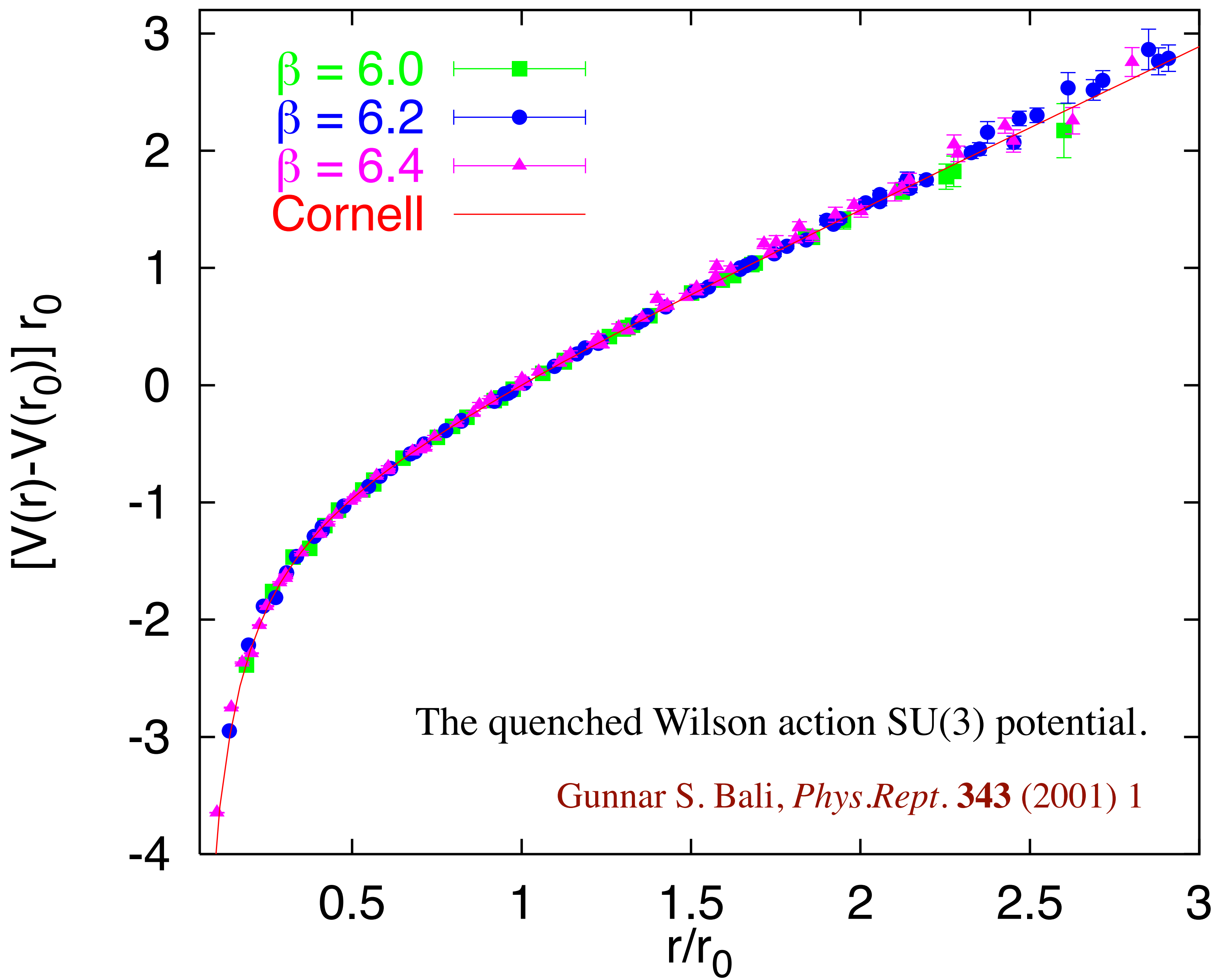


$$V(r) = -\frac{\alpha}{r}$$

$$V(r) = c\,r - \frac{4}{3}\frac{\alpha_s}{r}$$

"The J/ψ is the Hydrogen atom of QCD"

Cornell potential agrees with Lattice QCD



Perturbative expansions for atoms

PT for atoms start with an initial approximation, *e.g*., the Schrödinger eq.

Atomic wave functions are of $\mathcal{O}(\alpha^\infty)$: $\Psi(\boldsymbol{x}) \sim \exp(-\alpha m r/2)$

The wave function is not an observable (gauge dependent).

Binding energies are physical and they can be expanded in α and logα.

Example: Hyperfine splitting in Positronium

G. S. Adkins,
Hyperfine Interact. **233** (2015) 59

$$
\begin{aligned}
\Delta\nu_{QED} \;=\; & m_e\alpha^4\left\{\frac{7}{12}-\frac{\alpha}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)\right. \\
& +\frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha+\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{221}{144}\pi^2+\frac{1}{2}\right)\ln 2-\frac{53}{32}\zeta(3)\right] \\
& \left.-\frac{7\alpha^3}{8\pi}\ln^2\alpha+\frac{\alpha^3}{\pi}\ln\alpha\left(\frac{17}{3}\ln 2-\frac{217}{90}\right)+\mathcal{O}\left(\alpha^3\right)\right\} = 203.39169(41)\ \text{GHz}
\end{aligned}
$$

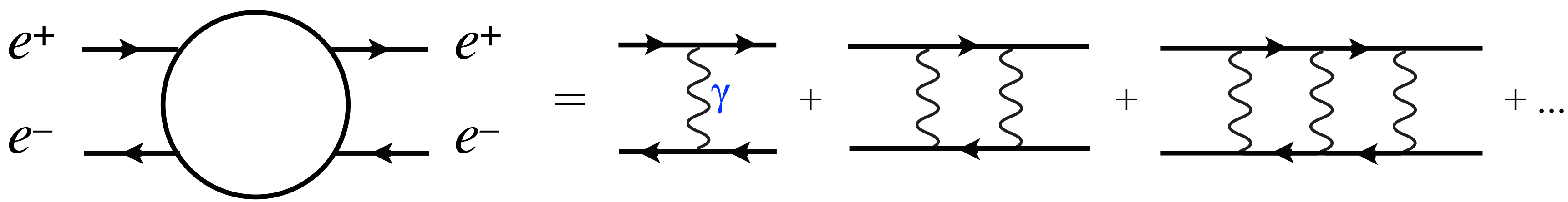
$\Delta\nu_{\text{EXP}}$ = 203.394± .002 GHz

Principles of bound state perturbation theory?

QED calculations start by postulating an initial wave function and potential.

An application to QCD requires a derivation of the Schrödinger eq. from L_{QED}.

Summing ladder diagrams is not the answer: *E.g.*, for $e^+ e^- \to e^+ e^-$



The divergence of the ladder sum gives rise to Positronium poles.

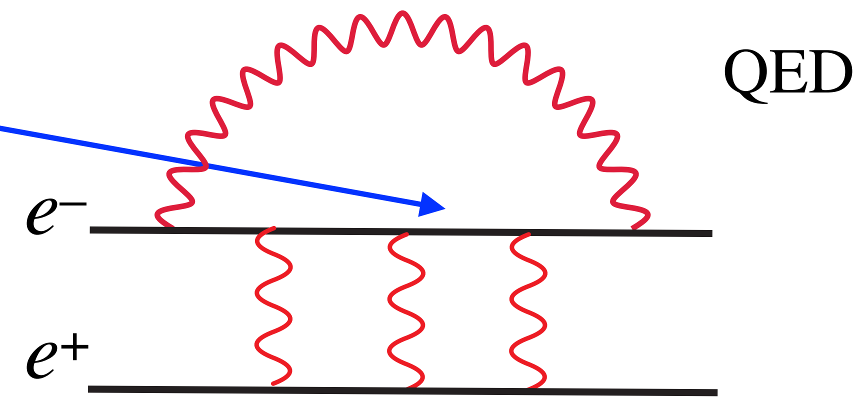
But: The free *in* and *out* states of PQED lack overlap with Positronia.

Free quark states at $t = \pm\infty$ are incompatible with confinement in QCD.

Beware of using Feynman diagrams, based on free propagation, for bound states!

Bound state constituents propagate in a field

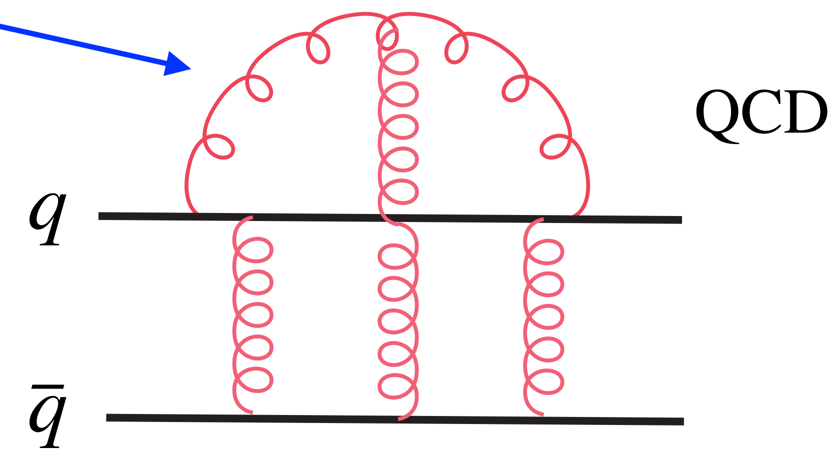
For QED lamb shift, need to calculate e^- propagator in the field of e^+



Lamb shift

In an NR approximation, this can be described by a fixed $-\alpha/r$ potential.

In QCD, relativistic gluons interact with colored quarks



Gluon and quark propagators depend on the state in which they propagate.

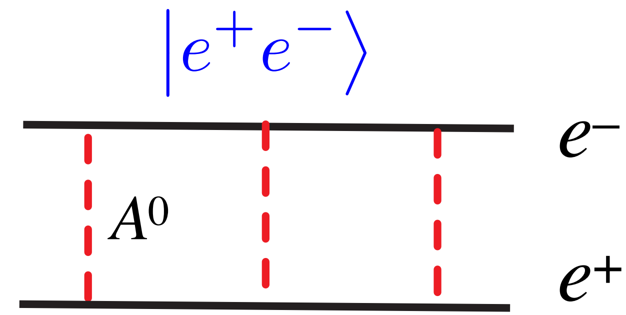
Cannot build bound states with constituents that have predetermined propagators.

$\Rightarrow$

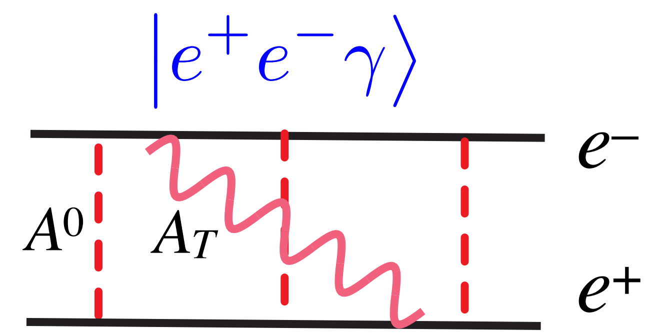
In gauge theories each Fock state defines an instantaneous field. Bound states are eigenstates of H.

Fock state expansion for Positronium (at rest)

The $|e^+e^-\rangle$ Fock state determines the binding energy at lowest order, $\mathcal{O}(\alpha^2)$.
Binding is due to instantaneous A^0 photons.
A^0 exchange is not suppressed by α.

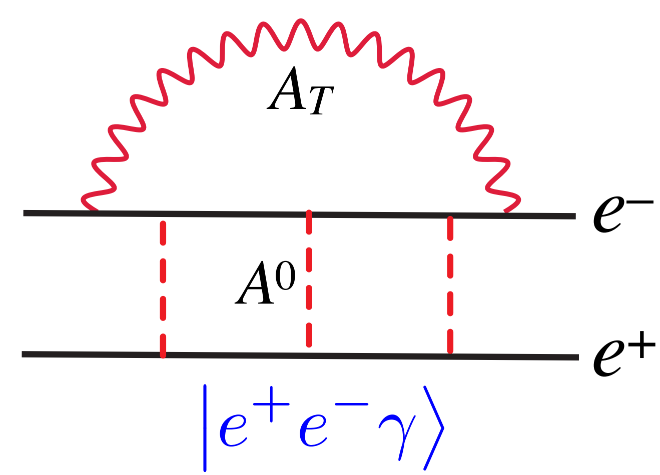


Spin dependence arises at $\mathcal{O}(\alpha^4)$ from states with a transverse photon, $|e^+e^-\gamma\rangle$.
A_T exchange is suppressed by powers of α.



The Lamb shift also arises from $|e^+e^-\gamma\rangle$.

The Fock expansion perturbatively generates the binding energy.



How can this be implemented in a Hamiltonian approach?

Canonical quantisation in temporal gauge: $A^0 = 0$

Avoids problem due to the missing conjugate field for A^0. No ghosts.

$$E^i = F^{i0} = -\partial_0 A^i \quad \text{conjugate to } -A^i \qquad (i = 1,2,3)$$

$$\left[E^i(t,\boldsymbol{x}), A^j(t,\boldsymbol{y})\right] = i\delta^{ij}\delta(\boldsymbol{x}-\boldsymbol{y}) \qquad \left\{\psi_\alpha^\dagger(t,\boldsymbol{x}), \psi_\beta(t,\boldsymbol{y})\right\} = \delta_{\alpha\beta}\,\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$H = \int d\boldsymbol{x}\left[\tfrac{1}{2}\boldsymbol{E}_L^2 + \tfrac{1}{2}\boldsymbol{E}_T^2 + \tfrac{1}{4}F^{ij}F^{ij} + \psi^\dagger(-i\alpha^i\partial_i - e\alpha^i A^i + m\gamma^0)\psi\right]$$

Gauss' operator does not vanish: $$G(x) \equiv \frac{\delta \mathcal{S}}{\delta A^0(x)} = \partial_i E_L^i(x) - e\psi^\dagger\psi(x)$$

$G(x)$ generates *time*-independent gauge transformations, consistent with $A^0 = 0$

Fix the gauge by constraining physical states: $G(x)\,|phys\rangle = 0$

This determines $E_L(x)$ for each state, imposing Gauss' law.

J. D. Bjorken, SLAC Summer Institute (1979)
G. Leibbrandt, Rev. Mod. Phys. 59, 1067 (1987)

Schrödinger equation for Positronium

$$G(x)\,|phys\rangle = 0 \quad \Rightarrow$$

$$\partial_i E_L^i(t,\boldsymbol{x})\,|phys\rangle = e\psi^\dagger\psi(t,\boldsymbol{x})\,|phys\rangle$$

$$E_L^i(t,\boldsymbol{x})\,|phys\rangle = -\partial_i^x \int d\boldsymbol{y}\,\frac{e}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\psi^\dagger\psi(t,\boldsymbol{y})\,|phys\rangle$$

For the component of Positronium with an electron at $\boldsymbol{x}_1$ and a positron at $\boldsymbol{x}_2$:

$$\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle = \bar{\psi}_\alpha(\boldsymbol{x}_1)\psi_\beta(\boldsymbol{x}_2)\,|0\rangle$$

$$E_L^i\,\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle = -\partial_i^x\,\frac{e}{4\pi}\Big(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|}-\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\Big)\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle$$

The instantaneous Hamiltonian $H_V \equiv \frac{1}{2}\int d\boldsymbol{x} E_L^i E_L^i(\boldsymbol{x})$ gives the classical potential:

$$H_V\,\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle = -\frac{\alpha}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle$$

The Schrödinger equation follows from $H\,\big|e^+e^-\big\rangle = (2m+E_b)\,\big|e^+e^-\big\rangle$

Removing the “art” from bound state calculations!

A Fock state expansion for QCD

The Fock expansion is compatible with the quark model of hadrons:

- Valence quantum numbers of mesons and baryons (lowest Fock state)
- Physical (transverse) gluon constituents contribute at $O(\alpha_s)$
- The E_L field is instantaneous also for relativistic constituents

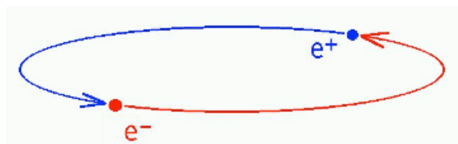
How can color confinement arise?

Gauss' law has no Λ_{QCD} scale

A crucial difference between QED and QCD

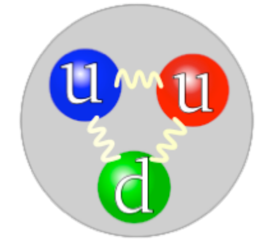
Global gauge invariance allows a classical gauge field for neutral atoms, but not a color octet gluon field for color singlet hadrons.

Positronium (QED)



$$E_L^i(\boldsymbol{x}) = -\partial_i^x\left(\frac{\alpha}{\boldsymbol{x}-\boldsymbol{x}_1} - \frac{\alpha}{\boldsymbol{x}-\boldsymbol{x}_2}\right)$$

Proton (QCD)



$$E_{L,a}^i(\boldsymbol{x}) = 0$$

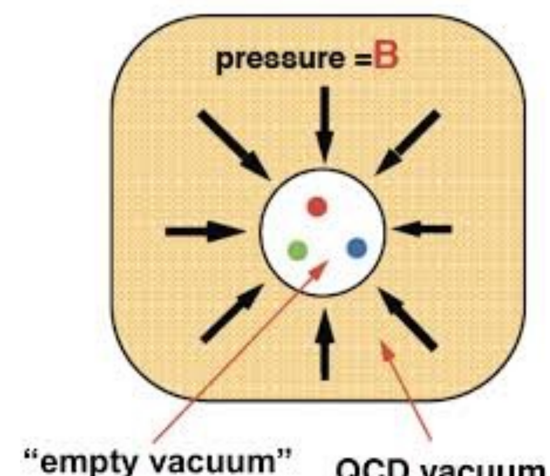
However:
The classical gluon field is non-vanishing for each color component C of the state

$$E_{L,a}^i(\boldsymbol{x}, C) \neq 0$$

$$\sum_C E_{L,a}^i(\boldsymbol{x}, C) = 0$$

$\Rightarrow$ Each color component C may generate a constant field energy density

Cf. bag model without a fixed boundary term:



Here: Constituents move in the QCD field, which gives a confining potential. The field is invisible to external observers.

Temporal gauge in QCD: $A_a^0 = 0$

Gauss' operator $G_a(x) \equiv \dfrac{\delta \mathcal{S}}{\delta A_a^0(x)} = \partial_i E_a^i(x) + g f_{abc} A_b^i E_c^i - g\psi^\dagger T^a \psi(x)$

generates time-independent gauge transformations, which keep $A_a^0 = 0$

The gauge is fully defined (in PT) by the constraint $G_a(x)\,|phys\rangle = 0$

$$\Rightarrow \qquad \partial_i E_{L,a}^i(\boldsymbol{x})\,|phys\rangle = g\big[-f_{abc}A_b^i E_c^i + \psi^\dagger T^a \psi(\boldsymbol{x})\big]\,|phys\rangle$$

In QED one solves for $\boldsymbol{E}_L$ requiring $\boldsymbol{E}_L(\boldsymbol{x}) \to 0$ for $|\boldsymbol{x}| \to \infty$

In QCD, for (globally) color singlet bound states: $\sum_C E_{L,a}^i(\boldsymbol{x}, C) = 0$

For each color component C we may consider homogeneous solutions of Gauss' law for E_L.

Translation invariance requires a constant field energy density (Λ_{QCD}) .

Poincaré invariance constrains the solution up to the single parameter Λ_{QCD}.

Including a homogeneous solution for $E^i_{L,a}$

$$E^i_{L,a}(\boldsymbol{x})\,|phys\rangle = -\partial_i^x \int d\boldsymbol{y}\Big[\kappa\,\boldsymbol{x}\cdot\boldsymbol{y} + \frac{g}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\Big]\mathcal{E}_a(\boldsymbol{y})\,|phys\rangle$$

where $\mathcal{E}_a(\boldsymbol{y}) = -f_{abc}A^i_b E^i_c(\boldsymbol{y}) + \psi^\dagger T^a \psi(\boldsymbol{y})$

$\kappa \neq \kappa(\boldsymbol{x},\boldsymbol{y})$ ensures $\partial_i \boldsymbol{E}^i(\boldsymbol{x}) = 0$ (a homogeneous solution)

The linear dependence on $\boldsymbol{x}$ makes $\boldsymbol{E}_L$ independent of $\boldsymbol{x}$, as required by translation invariance: The field energy density is spatially constant (cf. bag) .

The $\boldsymbol{E}_L$ contribution to the QCD Hamiltonian is

$$H_V = \int d\boldsymbol{y}d\boldsymbol{z}\Big\{\boldsymbol{y}\cdot\boldsymbol{z}\Big[\tfrac{1}{2}\kappa^2\int d\boldsymbol{x} + g\kappa\Big] + \tfrac{1}{2}\frac{\alpha_s}{|\boldsymbol{y}-\boldsymbol{z}|}\Big\}\mathcal{E}_a(\boldsymbol{y})\mathcal{E}_a(\boldsymbol{z})$$

The field energy ∝ volume of space is irrelevant only if it is universal. This relates the normalisation ϰ of all Fock components, leaving an overall scale $\Lambda_{\rm QCD}$ as the single parameter.

Examples: Fock state potentials (I)

$q\bar{q}$:

$$H_V\,|q(\boldsymbol{x}_1)\bar{q}(\boldsymbol{x}_2)\rangle = V_{q\bar{q}}\,|q(\boldsymbol{x}_1)\bar{q}(\boldsymbol{x}_2)\rangle$$

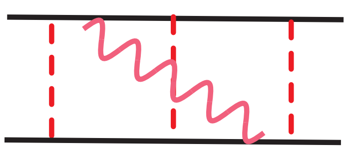
$$V_{q\bar{q}} = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - C_F\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$

"Cornell potential" also for relativistic quarks

$qg\bar{q}$:

$$V^{(0)}_{qgq}(\boldsymbol{x}_1,\boldsymbol{x}_g,\boldsymbol{x}_2) = \frac{\Lambda^2}{\sqrt{C_F}}\,d_{qgq}(\boldsymbol{x}_1,\boldsymbol{x}_g,\boldsymbol{x}_2)$$

(Λ as for $q\bar{q}$)



$$d_{qgq}(\boldsymbol{x}_1,\boldsymbol{x}_g,\boldsymbol{x}_2) \equiv \sqrt{\tfrac{1}{4}(N-2/N)(\boldsymbol{x}_1-\boldsymbol{x}_2)^2 + N(\boldsymbol{x}_g-\tfrac{1}{2}\boldsymbol{x}_1-\tfrac{1}{2}\boldsymbol{x}_2)^2}$$

$$V^{(1)}_{qgq}(\boldsymbol{x}_1,\boldsymbol{x}_g,\boldsymbol{x}_2) = \tfrac{1}{2}\,\alpha_s\Big[\frac{1}{N}\,\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|} - N\Big(\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_g|}+\frac{1}{|\boldsymbol{x}_2-\boldsymbol{x}_g|}\Big)\Big]$$

When q and g coincide:

$$V^{(0)}_{qgq}(\boldsymbol{x}_1=\boldsymbol{x}_g,\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| = V^{(0)}_{q\bar{q}}$$

$$V^{(1)}_{qgq}(\boldsymbol{x}_1=\boldsymbol{x}_g,\boldsymbol{x}_2) = V^{(1)}_{q\bar{q}}$$

Fock state potentials (II)

qqq :

$$V_{qqq} = \Lambda^2 d_{qqq}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) - \frac{2}{3}\alpha_s\Big(\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|} + \frac{1}{|\boldsymbol{x}_2 - \boldsymbol{x}_3|} + \frac{1}{|\boldsymbol{x}_3 - \boldsymbol{x}_1|}\Big)$$

$$d_{qqq}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) \equiv \frac{1}{\sqrt{2}}\sqrt{(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2 + (\boldsymbol{x}_2 - \boldsymbol{x}_3)^2 + (\boldsymbol{x}_3 - \boldsymbol{x}_1)^2}$$

gg :

$$V_{gg} = \sqrt{\frac{N}{C_F}}\,\Lambda^2\,|\boldsymbol{x}_1 - \boldsymbol{x}_2| - N\,\frac{\alpha_s}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}$$

The gg potential agrees with that of q$g\bar{\text{q}}$ when the quarks coincide:

$$V_{gg}(\boldsymbol{x}, \boldsymbol{x}_g) = V_{qg\bar{q}}(\boldsymbol{x}, \boldsymbol{x}_g, \boldsymbol{x})$$

It is straightforward to work out the instantaneous potential for any Fock state.

"Perturbative expansion of non-perturbative states"

A new approach to soft QCD:

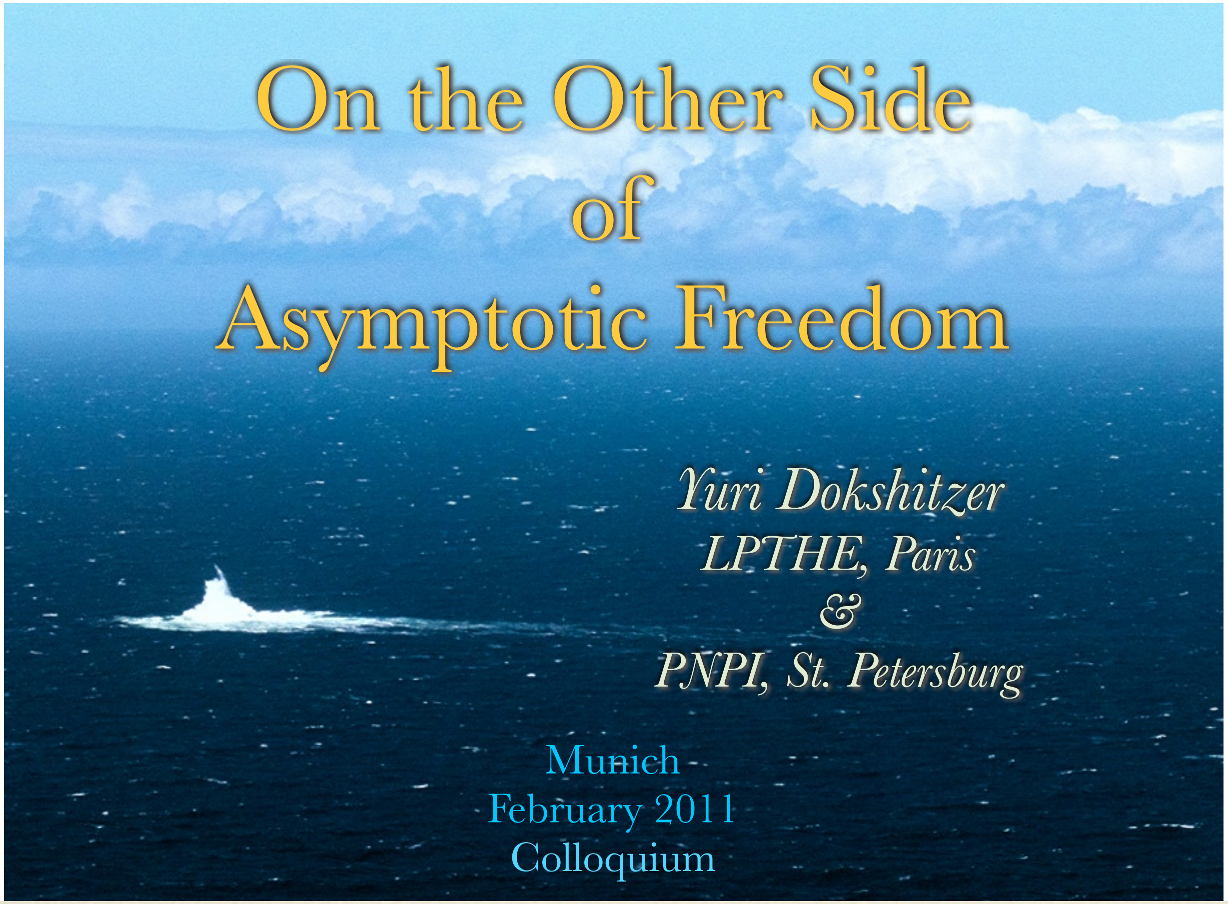
- The instantaneous $\mathcal{O}\left(\alpha_s^0\right)$ field binds the lowest Fock states
- The higher Fock states given by the Hamiltonian H_{QCD} are of $\mathcal{O}\left(\alpha_s\right)$
- Makes bound state calculations less of an art

For the approach to be viable the $\mathcal{O}\left(\alpha_s^0\right)$ dynamics must have:

Poincaré symmetry
Unitarity
Confinement
Chiral Symmetry Breaking (CSB)
Reasonable mass spectrum
. . .

Not all of these have been demonstrated, but the outlook is promising.

A new appearance of PQCD



PQCD can be relevant also for soft interactions.

$$\alpha_s/\pi \sim 0.14$$

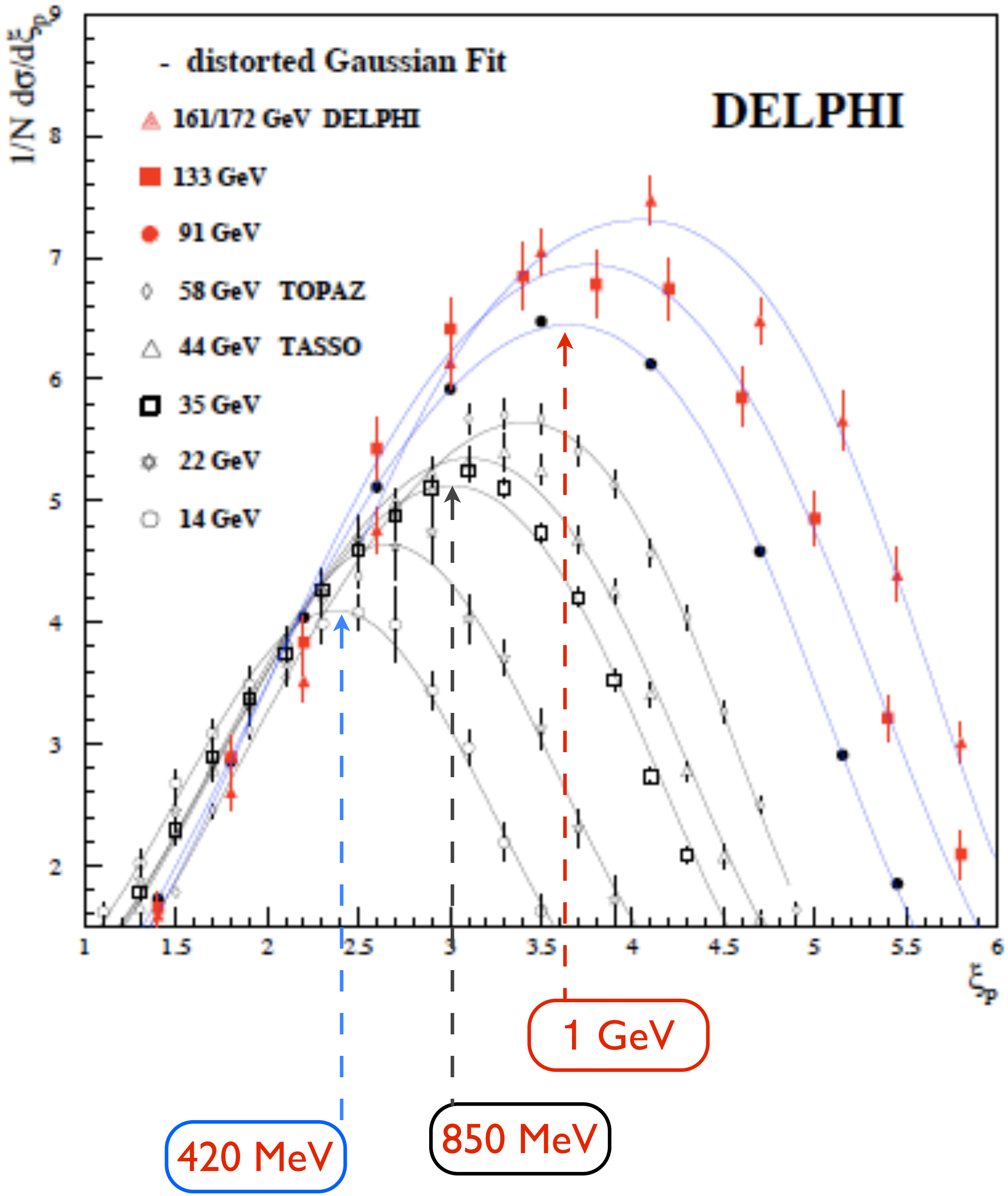
QCD *is about to undergo a* **faith transition**

QCD practitioners prepare themselves - slowly but steadily - to start using, in earnest, the language of ***quarks*** and ***gluons*** down into the region of small characteristic momenta - "**large distances**"

Extra slides

Soft Physics: **hadron production *inside* jets**

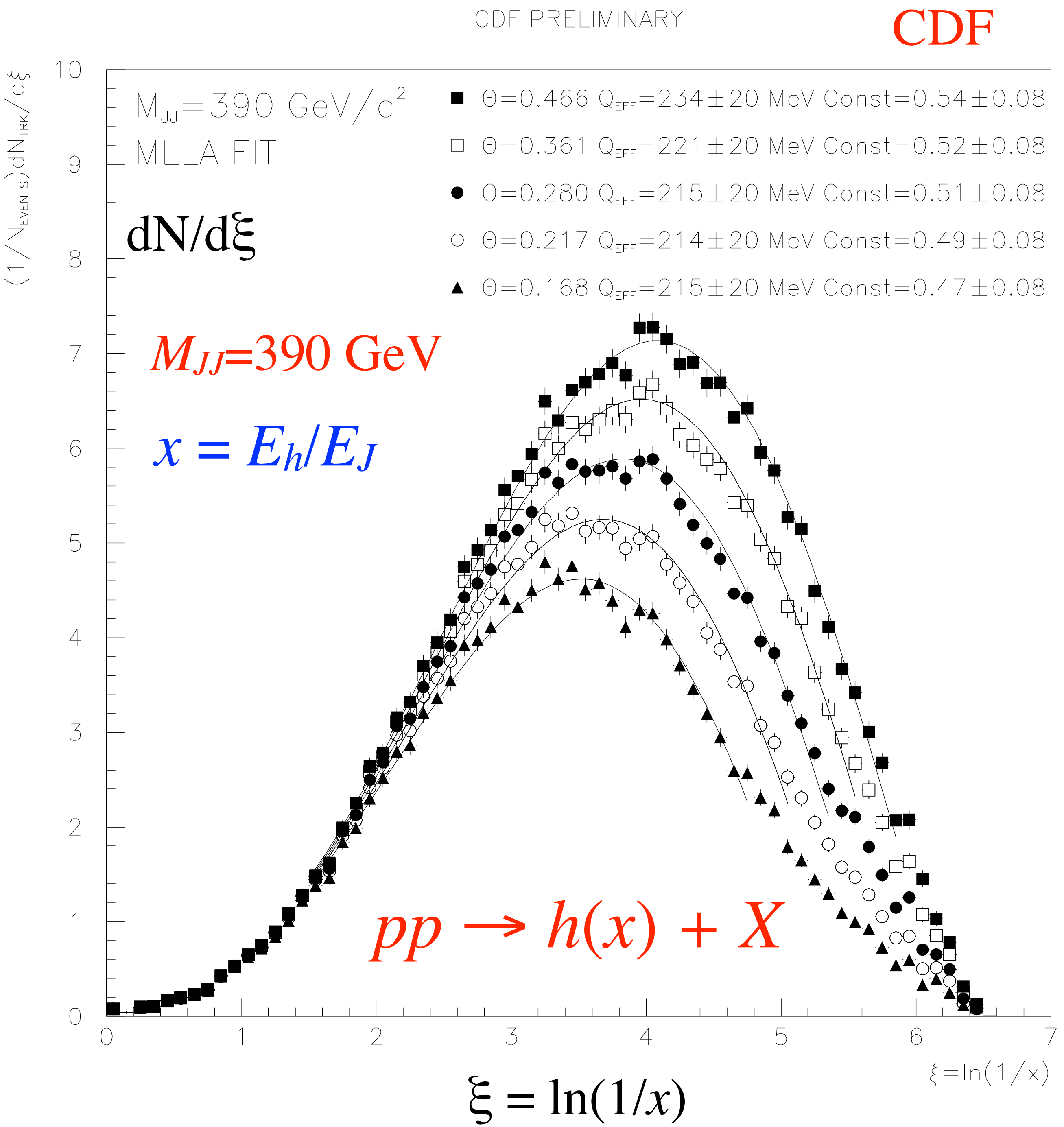
Dokshitzer 2011



$$\ln \frac{1}{x}$$

Dokshitzer (Les Houches 2008)

Hump-backed plateau



First confronted with theory in $e^+e^- \to h+X$.

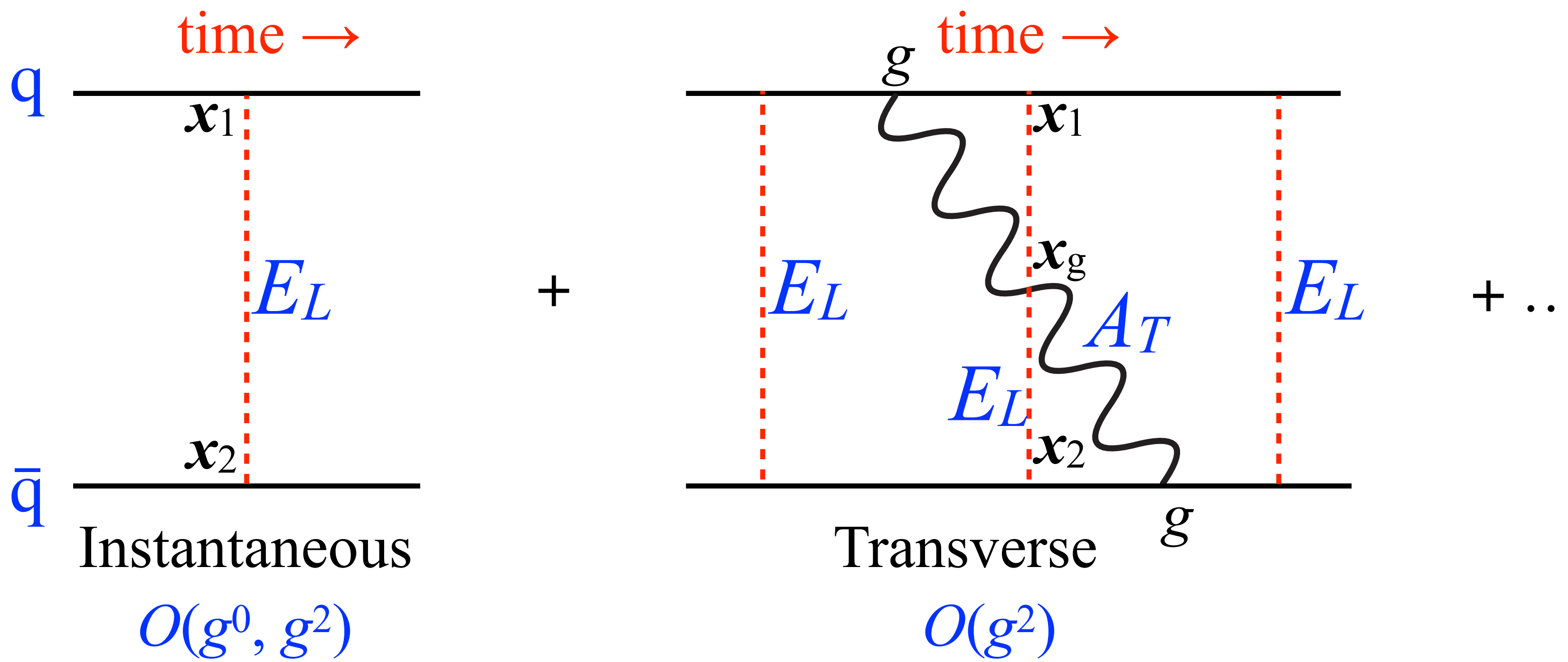
CDF (Tevatron)

$pp \to 2$ jets

Charged hadron yield as a function of $\ln(1/x)$ for different values of jet hardness, versus (MLLA) QCD prediction.

One free parameter – overall normalization (the number of final π's per extra gluon)

Perturbative expansion = Fock state expansion



The instantaneous Hamiltonian H_V determines the potential energy.

The $q\bar{q}A_T$ *etc*. terms in H determine couplings between Fock states.

The q and A_T kinetic terms determine the time evolution.

$\mathcal{O}\left(\alpha_s^0\right)$ $q\bar{q}$ bound states

The $\mathcal{O}\left(\alpha_s^0\right)$ meson is a superposition of $q\bar{q}$ Fock states with wave function Φ,

$$|M\rangle = \sum_{A,B;\alpha,\beta} \int d\boldsymbol{x}_1 d\boldsymbol{x}_2\, \bar{\psi}_\alpha^A(t=0,\boldsymbol{x}_1)\delta^{AB}\Phi_{\alpha\beta}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi_\beta^B(t=0,\boldsymbol{x}_2)\,|0\rangle$$

The bound state condition $H\,|M\rangle = M\,|M\rangle$ gives

$$\left[i\gamma^0\boldsymbol{\gamma}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0\right]\Phi(\boldsymbol{x}) + \Phi(\boldsymbol{x})\left[i\gamma^0\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}} - m\gamma^0\right] = \left[M - V(|\boldsymbol{x}|)\right]\Phi(\boldsymbol{x})$$

where $\boldsymbol{x} \equiv \boldsymbol{x}_1 - \boldsymbol{x}_2$ and $V(|\,\boldsymbol{x}\,|) = V'|\,\boldsymbol{x}\,| = \Lambda^2|\,\boldsymbol{x}\,|$.

In the non-relativistic limit ($m \gg \Lambda$) this reduces to the Schrödinger equation, and we may add the instantaneous gluon exchange potential.

$\Rightarrow$ The successful quarkonium phenomenology with the Cornell potential.

Relativistic $q\bar{q}$ bound states

$$i\boldsymbol{\nabla}\cdot\left\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\right\}+m\left[\gamma^0,\Phi(\boldsymbol{x})\right]=\left[M-V(\boldsymbol{x})\right]\Phi(\boldsymbol{x})$$

Expanding the 4×4 wave function in a basis of 16 Dirac structures $\Gamma_i(\boldsymbol{x})$

$$\Phi(\boldsymbol{x})=\sum_i \Gamma_i(\boldsymbol{x})F_i(r)Y_{j\lambda}(\hat{\boldsymbol{x}})$$

we may use rotational, parity and charge conjugation invariance to determine which $\Gamma_i(\boldsymbol{x})$ may occur for a state of given j^{PC}:

0^{-+} trajectory $[s=0,\ \ell=j]:$ $\quad -\eta_P=\eta_C=(-1)^j\quad \gamma_5,\ \gamma^0\gamma_5,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L}$

0^{--} trajectory $[s=1,\ \ell=j]:$ $\quad \eta_P=\eta_C=-(-1)^j\quad \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \boldsymbol{\alpha}\cdot\boldsymbol{L},\ \gamma^0\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

0^{++} trajectory $[s=1,\ \ell=j\pm 1]:$ $\quad \eta_P=\eta_C=+(-1)^j\quad 1,\ \boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

0^{+-} trajectory [exotic] : $\quad \eta_P=-\eta_C=(-1)^j\quad \gamma^0,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

$\Longrightarrow$ There are no solutions for quantum numbers that would be exotic in the quark model (despite the relativistic dynamics)

Example: 0⁻⁺ trajectory wf's

$$\Phi_{-+}(\boldsymbol{x}) = \Big[\frac{2}{M-V}(i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0) + 1\Big]\gamma_5\, F_1(r) Y_{j\lambda}(\hat{\boldsymbol{x}})$$

$$\eta_P = (-1)^{j+1}$$

$$\eta_C = (-1)^j$$

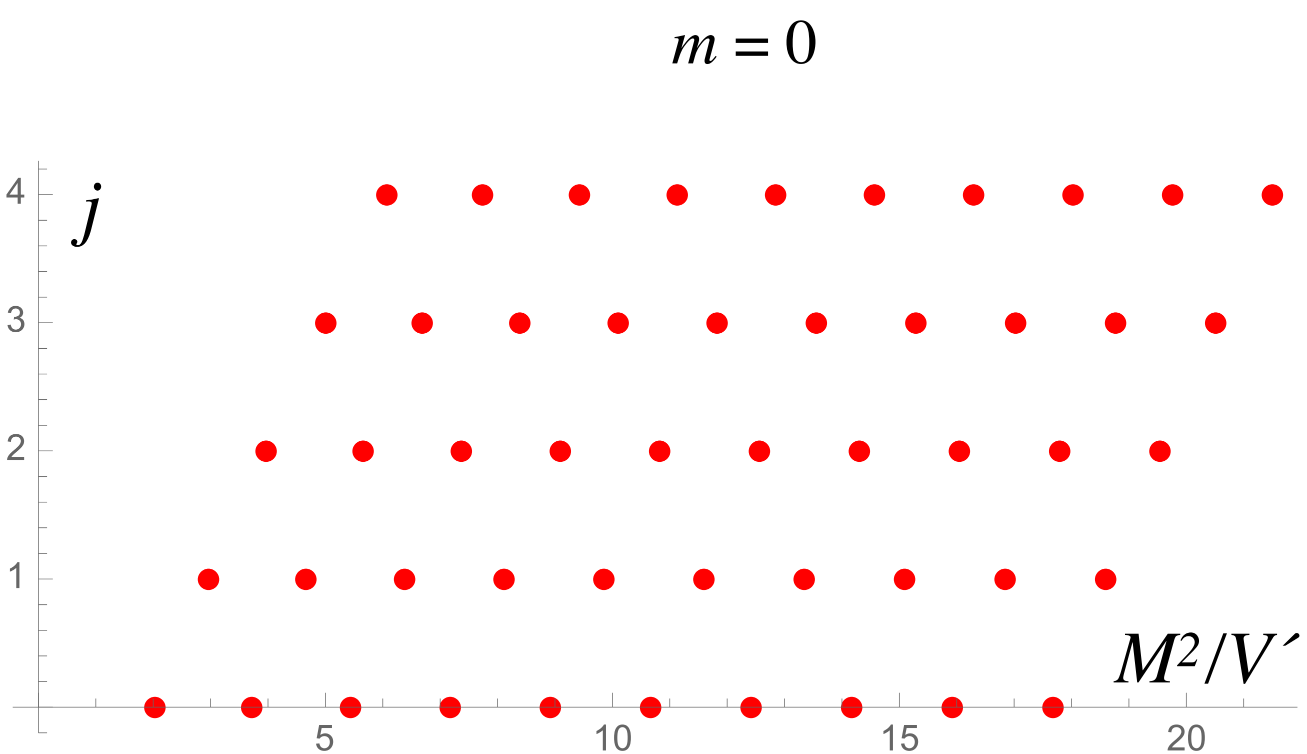
Radial equation: $F_1'' + \Big(\frac{2}{r} + \frac{V'}{M-V}\Big)F_1' + \Big[\frac{1}{4}(M-V)^2 - m^2 - \frac{j(j+1)}{r^2}\Big]F_1 = 0$

Local normalizability at $r = 0$ and at $V(r) = M$ determines the discrete M

Mass spectrum:

Linear Regge trajectories with daughters

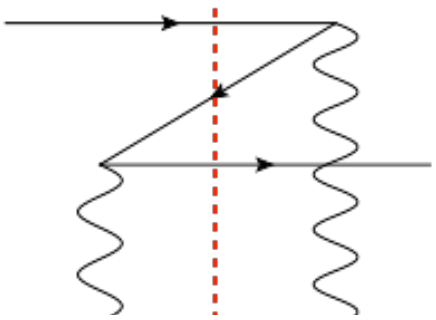
Spectrum similar to dual models



Sea quark contributions

Quark states in a strong field have E<0 components

Bogoliubov transformation, cf. Dirac states.



In time-ordered PT, these correspond to Z-diagrams, and interpreted as contributions from $q\bar{q}$ pairs.

This effect is manifest in the behavior of the wave function Φ for large V = V´|***x***| :

$$\lim_{\boldsymbol{x}\to\infty} |\Phi(\boldsymbol{x})|^2 = const.$$

The asymptotically constant norm reflects, via duality, pair production as the linear potential $V(|\,\boldsymbol{x}\,|)$ increases.

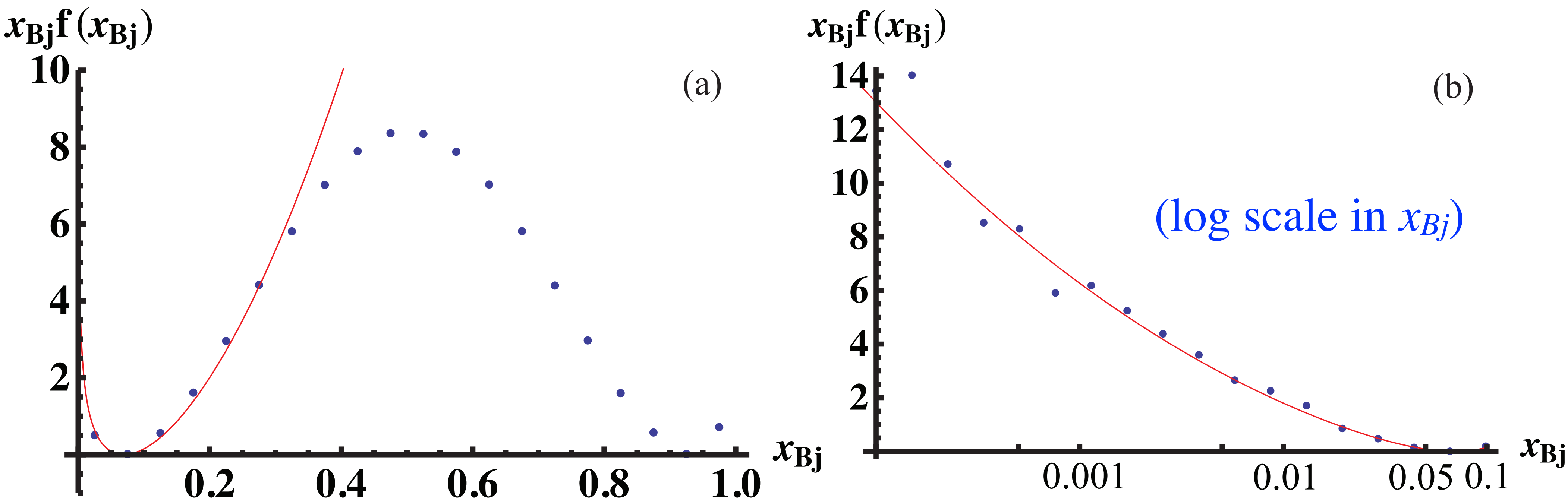
These sea quarks show up in the parton distribution measured in DIS.

Parton distributions have a sea component

In D=1+1 dimensions the sea component is prominent at low *m/e* :

$m/e = 0.1$

D. D. Dietrich, PH, M. Järvinen
arXiv 1212.4747



The red curve is an analytic approximation, valid in the $x_{Bj} \to 0$ limit.

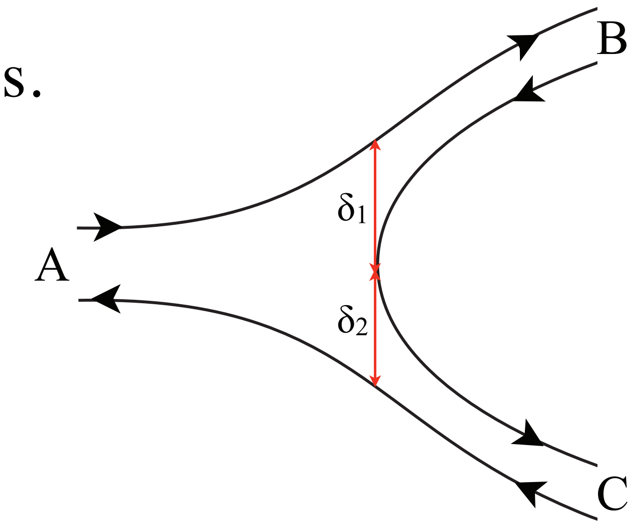
Note: Enhancement at low x is due to bd (sea), not to $b^\dagger d^\dagger$ (valence) component.

To be calculated in D=3+1 (and in various frames!)

Decays and hadron loops

The bound state equation determines zero-width states.

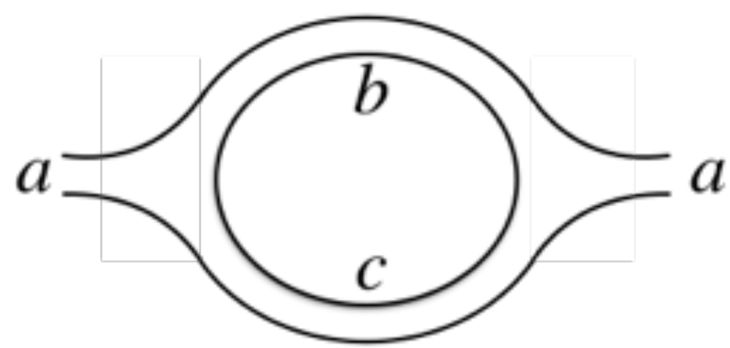
There is an $\mathcal{O}\left(1/\sqrt{N_C}\right)$ coupling between the states: string breaking



$$\langle B, C|A\rangle =$$

$$-\frac{(2\pi)^3}{\sqrt{N_C}}\delta^3(\boldsymbol{P}_A-\boldsymbol{P}_B-\boldsymbol{P}_C)\int d\boldsymbol{\delta}_1 d\boldsymbol{\delta}_2\, e^{i\boldsymbol{\delta}_1\cdot\boldsymbol{P}_C/2-i\boldsymbol{\delta}_2\cdot\boldsymbol{P}_B/2}\mathrm{Tr}\left[\gamma^0\Phi_B^\dagger(\boldsymbol{\delta}_1)\Phi_A(\boldsymbol{\delta}_1+\boldsymbol{\delta}_2)\Phi_C^\dagger(\boldsymbol{\delta}_2)\right]$$

When squared, this gives a $1/N_C$ hadron loop unitarity correction:



Unitarity should be satisfied at hadron level at each order of $1/N_C$.

Bound states in motion

An $\mathcal{O}\left(\alpha_s^0\right)$ $q\bar{q}$ bound state with CM momentum $\boldsymbol{P}$ may be expressed as

$$|M,\boldsymbol{P}\rangle = \int dx_1\, dx_2\, \bar{\psi}(t=0,x_1)\, e^{i\boldsymbol{P}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)/2}\, \Phi^{(\boldsymbol{P})}(x_1-x_2)\, \psi(t=0,x_2)\, |0\rangle$$

The instantaneous potential is $\boldsymbol{P}$-independent, $V(\boldsymbol{x}) = V'|\boldsymbol{x}|$, hence the BSE:

$$i\boldsymbol{\nabla}\cdot\left\{\boldsymbol{\alpha},\Phi^{(\boldsymbol{P})}(\boldsymbol{x})\right\} - \tfrac{1}{2}\boldsymbol{P}\cdot\left[\boldsymbol{\alpha},\Phi^{(\boldsymbol{P})}(\boldsymbol{x})\right] + m\left[\gamma^0,\Phi^{(\boldsymbol{P})}(\boldsymbol{x})\right] = \left[E - V(\boldsymbol{x})\right]\Phi^{(\boldsymbol{P})}(\boldsymbol{x})$$

The solution for $\Phi^{(P)}(\boldsymbol{x})$ is not simply Lorentz contracting in $\boldsymbol{x}$.

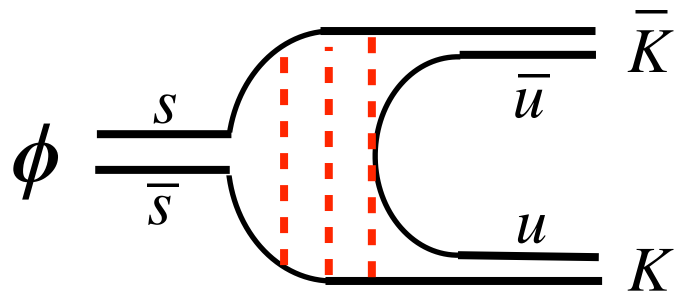
States with general $\boldsymbol{P}$ are needed for:

- $\boldsymbol{P}$-dependence of angular momentum ($P\to\infty$ frame).
- EM form factors (gauge invariance has been verified)
- Parton distributions
- Hadron scattering
- . . .

The OZI rule

Connected diagrams: Unsuppressed, string breaking from confining potential

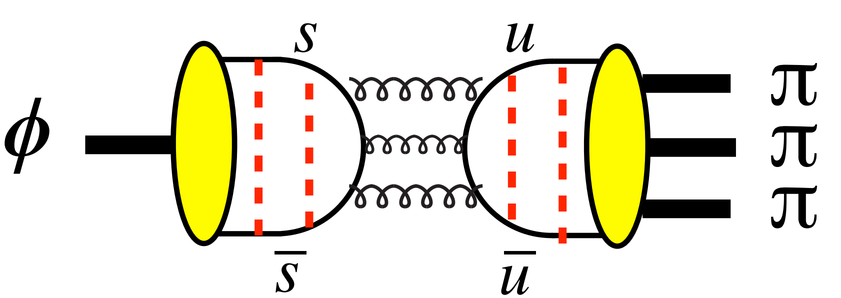
$\phi(1020) \to K\bar{K}$



	ΔE	Br
$\phi(1020) \to K\bar{K}$	26 MeV	83.1 %
$\phi(1020) \not\to \pi\pi\pi$	610 MeV	15.3 %

Disconnected, perturbative diagrams are suppressed

$\phi(1020) \not\to \pi\pi\pi$



This suggests that perturbative corrections are small even in the soft regime.