# Heavy Flavour in high-energy nuclear collisions

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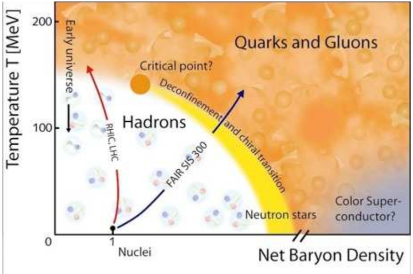
INFN
Istituto Nazionale di Fisica Nucleare

# Heavy-ion collisions: exploring the QCD phase-diagram




QCD phases identified through the *order parameters*

- Polyakov loop $\langle L \rangle \sim e^{-\beta \Delta F_Q}$ energy cost to add an isolated color charge
- Chiral condensate $\langle \overline{q}q \rangle \sim$ effective mass of a "dressed" quark in a hadron

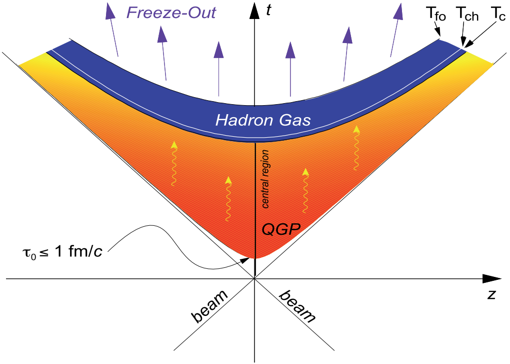
Region explored at LHC: *high-T/low-density* (early universe, $n_B/n_\gamma \approx 0.6 \cdot 10^{-9}$)

- From QGP (color deconfinement, chiral symmetry restored)
- to hadronic phase (confined, chiral symmetry breaking[1])

NB $\langle \overline{q}q \rangle \neq 0$ responsible for most of the baryonic mass of the universe: *only $\sim$35 MeV of the proton mass from $m_{u/d} \neq 0$*

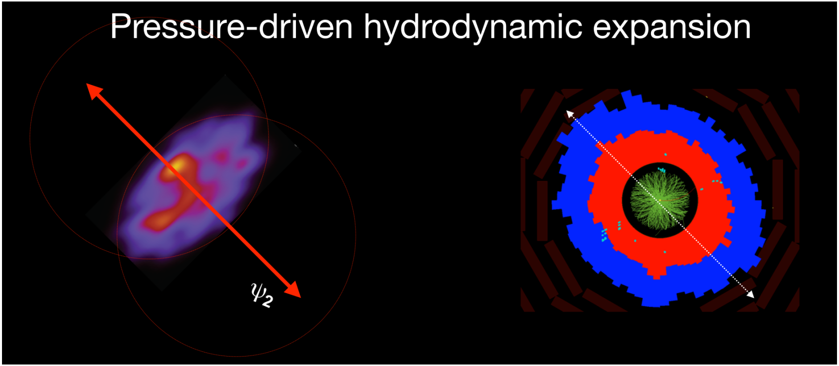
[1] V. Koch, *Aspects of chiral symmetry*, Int.J.Mod.Phys. E6 (1997)

# Heavy-ion collisions: a cartoon of space-time evolution




- Soft probes (low-$p_T$ hadrons): collective behavior of the *medium*;
- Hard probes (high-$p_T$ particles, heavy quarks, quarkonia): produced in *hard pQCD processes* in the initial stage, allow to perform a tomography of the medium

# A medium displaying a collective behavior



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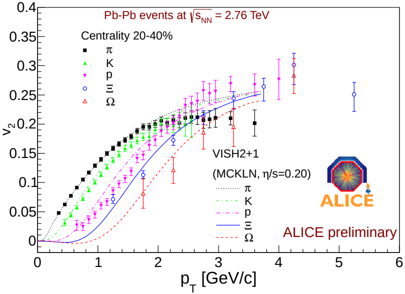
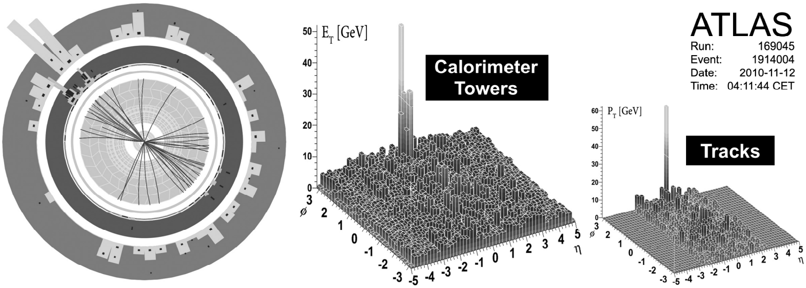



Anisotropic azimuthal distribution of hadrons as a response to pressure gradients quantified by the *Fourier coefficients* $v_n$

$$\frac{dN}{d\phi} = \frac{N_0}{2\pi}\left(1 + 2\sum_n v_n \cos[2(\phi - \psi_n)] + \dots\right)$$

$$v_2 \equiv \langle \cos[2(\phi - \psi_2)] \rangle$$

# A medium inducing energy-loss to colored probes



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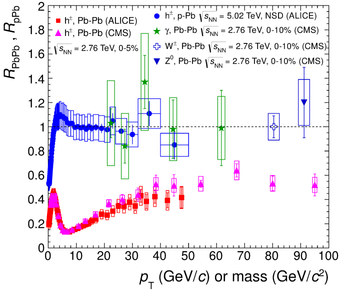
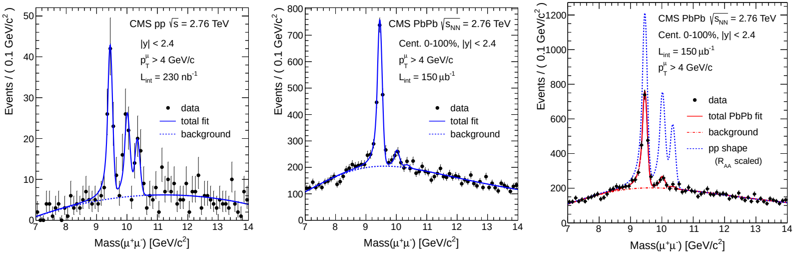



Medium-induced suppression of high-momentum hadrons
quantified through the *nuclear modification factor*

$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

# A medium screening the $Q\overline{Q}$ interaction




Suppression of $\Upsilon$ production in Pb-Pb collisions at the LHC, in particular its excited (weaker binding, larger radius!) states

# Heavy Flavour in the QGP: the conceptual setup

- Description of soft observables based on hydrodynamics, assuming to deal with a system close to local thermal equilibrium (no matter why): *collective behaviour* of the medium;

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NB At high-$p_T$ the interest in heavy flavor is no longer related to thermalization, but to the study of the mass and color charge dependence of jet-quenching (not addressed in this talk)

# Heavy quarks as probes of the QGP

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  - However, *a source of systematic uncertainty for studies of parton-medium interaction*;
- Hadronic rescattering (e.g. $D\pi \to D\pi$), from effective Lagrangians, but no experimental data the on relevant cross-sections

# HF transport: the relativistic Langevin equation

Most transport calculations are based on the Langevin equation allowing one to follow the in-medium dynamics of each individual quark arising from the pQCD Monte Carlo simulation of the initial $Q\overline{Q}$ production.
*In the LRF of the fluid* one performs the update of the HQ momentum

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} \quad ,$$

with the properties of the noise encoded in

$$\langle\xi^i(\mathbf{p}_t)\rangle = 0 \quad \langle\xi^i(\mathbf{p}_t)\xi^j(\mathbf{p}_{t'})\rangle = b^{ij}(\mathbf{p})\frac{\delta_{tt'}}{\Delta t} \quad b^{ij}(\mathbf{p}) \equiv \kappa_L(p)\hat{p}^i\hat{p}^j + \kappa_T(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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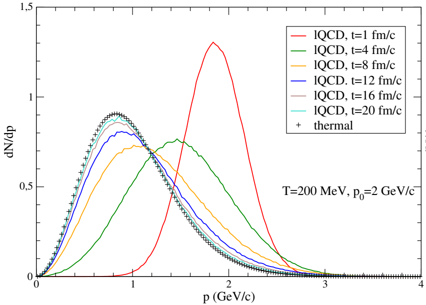
One needs to know the *transport coefficients*:

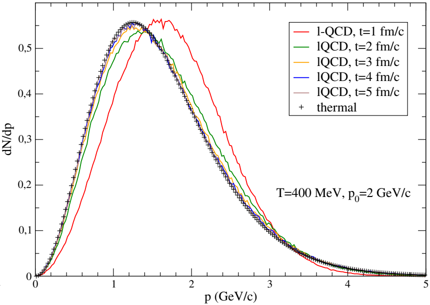
- *Momentum diffusion*: $\kappa_T \equiv \frac{1}{2}\frac{\langle\Delta p_T^2\rangle}{\Delta t}$ and $\kappa_L \equiv \frac{\langle\Delta p_L^2\rangle}{\Delta t}$
- *Friction* term, in the *Ito pre-point discretization scheme*,

$$\eta_D^{\text{Ito}}(p) = \frac{\kappa_L(p)}{2TE_p} - \frac{1}{E_p^2}\left[(1-v^2)\frac{\partial\kappa_L(p)}{\partial v^2} + \frac{d-1}{2}\frac{\kappa_L(p)-\kappa_T(p)}{v^2}\right]$$

fixed in order to ensure approach to equilibrium (Einstein relation)

# Consistency check I: thermalization in a static medium




(Test with a sample of $c$ quarks with $p_0 = 2$ GeV/c).

For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution

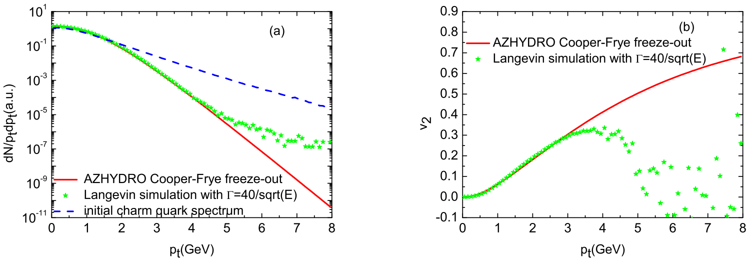
$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \int d^3p\, f_{\rm MJ}(p) = 1$$

The larger $\kappa$ ($\kappa \sim T^3$), the faster the approach to thermalization.

# Consistency check II: thermalization in a static medium

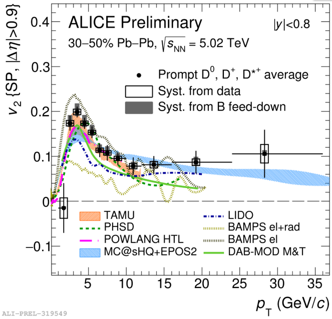
In the limit of large transport coefficients heavy quarks should reach local thermal equilibrium and decouple from the medium as the other light particles, according to the Cooper-Frye formula:

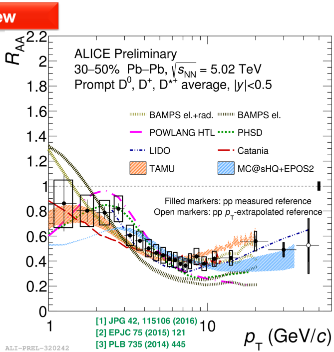
$$E(dN/d^3p) = \int_{\Sigma_{\rm fo}} \frac{p^\mu \cdot d\Sigma_\mu}{(2\pi)^3} \exp[-p \cdot u / T_{\rm fo}]$$




This was verified to be actually the case (M. He, R.J. Fries and R. Rapp, PRC 86, 014903).

# Theory-to-data comparison: a snapshot of recent results




$$v_n \equiv \langle \cos[n(\phi - \Psi_n)] \rangle \qquad R_{AA} \equiv \frac{dN/dp_T|_{AA}}{\langle N_{coll} \rangle \, dN/dp_T|_{pp}}$$

In spite of their large mass, also the D-mesons turn out to be quenched and to have a sizable $v_2$. Does also charm reach local thermal equilibrium? Transport calculations are challenged to consistently reproduce this rich phenomenology.

# HQ transport coefficients: non-perturbative definition

One consider the non-relativistic limit of the Langevin equation for a HQ

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t)\xi^j(t')\rangle = \delta^{ij}\delta(t-t')\kappa$$

in which the strength of the noise is given by a single number, the momentum-diffusion coefficient $\kappa$. Hence, in the $p \to 0$ limit:

$$\kappa = \frac{1}{3}\int_{-\infty}^{+\infty} dt \langle \xi^i(t)\xi^i(0)\rangle_{\mathrm{HQ}} \approx \frac{1}{3}\int_{-\infty}^{+\infty} dt \underbrace{\langle F^i(t)F^i(0)\rangle_{\mathrm{HQ}}}_{\equiv D^>(t)},$$

For a static ($M=\infty$) HQ the force is due to the color-electric field:

$$\mathbf{F}(t) = g\int d\mathbf{x} Q^\dagger(t,\mathbf{x}) t^a Q(t,\mathbf{x}) \mathbf{E}^a(t,\mathbf{x})$$

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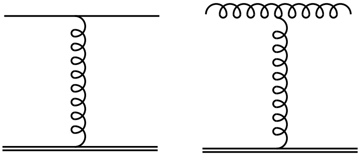
The above non-perturbative definition, referring to the $M\to\infty$ limit, is the starting point for a thermal-field-theory evaluation based on

- weak-coupling calculations (up to NLO);
- gauge-gravity duality ($\mathcal{N}=4$ SYM)
- lattice-QCD simulations

# HQ momentum diffusion: weak-coupling calculation



HQ momentum diffusion is due to scattering with light quarks and gluons.

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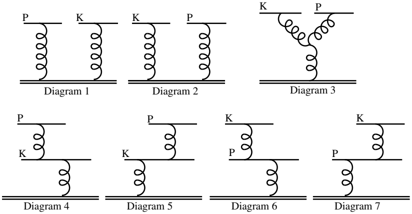


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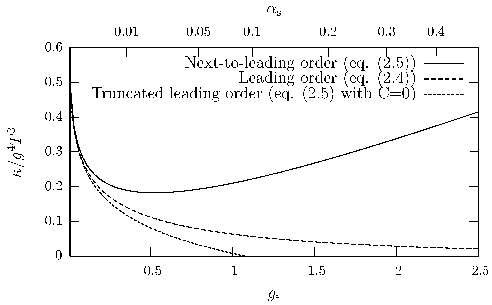


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$$\kappa = \frac{16\pi}{3}\alpha_s^2 T^3 \left(\ln\frac{1}{g} + 0.07428 + 1.9026g + \mathcal{O}(g^2)\right)$$

For realistic values of the coupling $\alpha_s \sim 0.3$ NLO corrections to $\kappa$ are large!

# HQ momentum diffusion: lattice-QCD

The $(p \to 0)$ HQ momentum-diffusion coefficient

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is given by the $\omega \to 0$ limit of the FT of the electric-field correlator $D^>$. In a thermal ensemble, from the periodicity of the bosonic fields, one has $\sigma(\omega) \equiv D^>(\omega) - D^<(\omega) = (1 - e^{-\beta\omega})D^>(\omega)$, so that

$$\kappa \equiv \lim_{\omega\to 0} \frac{D^>(\omega)}{3} = \lim_{\omega\to 0} \frac{1}{3}\frac{\sigma(\omega)}{1 - e^{-\beta\omega}} \underset{\omega\to 0}{\sim} \frac{1}{3}\frac{T}{\omega}\sigma(\omega)$$

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On the lattice one evaluates then the *euclidean electric-field correlator* $(t = -i\tau)$

$$D_E(\tau) = -\frac{\langle {\rm Re\,Tr}[U(\beta,\tau)gE^i(\tau,\mathbf{0})U(\tau,0)gE^i(0,\mathbf{0})]\rangle}{\langle {\rm Re\,Tr}[U(\beta,0)]\rangle}$$

and from the latter one extract the spectral density according to

$$D_E(\tau) = \int_0^{+\infty}\frac{d\omega}{2\pi}\frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)}\sigma(\omega)$$

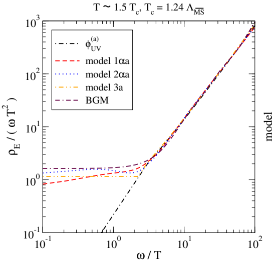
# HQ momentum diffusion: lattice-QCD

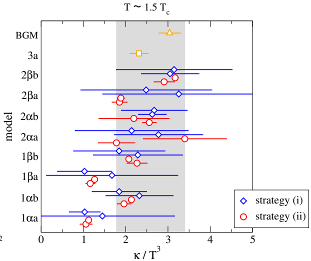
The direct extraction of the spectral density from the euclidean correlator

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)} \sigma(\omega)$$

is a ill-posed problem, since the latter is known for a limited set ($\sim 20$) of points $D_E(\tau_i)$, and one wishes to obtain a fine scan of the the spectral function $\sigma(\omega_j)$. A direct $\chi^2$-fit is not applicable. Possible strategies:

- Bayesian techniques (Maximum Entropy Method)
- Theory-guided ansatz for the behaviour of $\sigma(\omega)$ to constrain its functional form (A. Francis *et al.*, PRD 92 (2015), 116003)




From the different ansatz on the functional form of $\sigma(\omega)$ one gets a systematic uncertainty band:

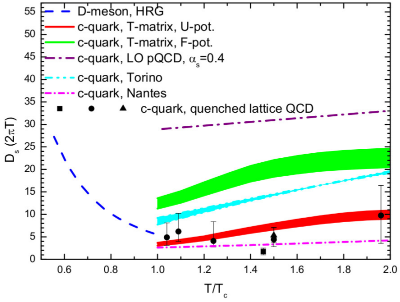
$$\kappa/T^3 \approx 1.8 - 3.4$$

# From momentum broadening to spatial diffusion

In the *non-relativistic* limit an excess of HQ's initially placed at the origin will diffuse according to

$$\langle \vec{x}^2(t)\rangle \underset{t\to\infty}{\sim} 6D_s t \quad \text{with} \quad D_s = \frac{2T^2}{\kappa}.$$

For a strongly interacting system spatial diffusion is very small! Theory calculations for $D_s$ have been collected (F. Prino and R. Rapp, JPG 43 (2016) 093002) and are often used by the experimentalists to summarize the difference among the various models (BUT momentum dependence, not captured by $D_s$, is important!)




- lattice-QCD

$$(2\pi T)D_s^{lQCD} \approx 3.7 - 7$$

- $\mathcal{N}=4$ SYM:

$$(2\pi T)D_s^{SYM} = \frac{4}{\sqrt{g^2_{SYM}N_c}} \approx 1.2$$

for $N_c = 3$ and $\alpha_{SYM} = \alpha_s = 0.3$.

# Collisional broadening in the non-static case

In the case of experimental interest HQ's have a large but finite mass and most of the $p_T$-bins for which data are available refer to quite fast, or even relativistic, HF hadrons: extending the estimates for the HQ transport coefficients to finite momentum is mandatory to provide theoretical predictions relevant for the experiment.

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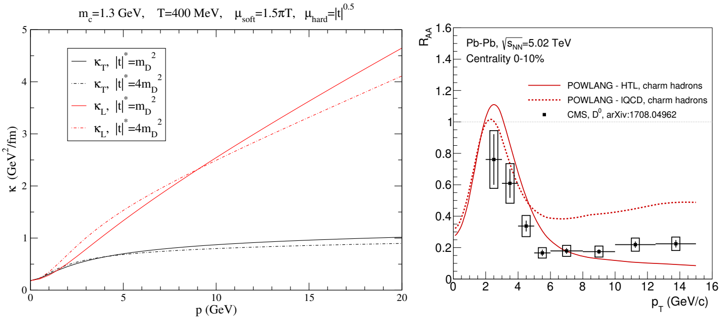


For the same hydro background, simulations with momentum dependent transport coefficients $\kappa_{T/L}$ (left panel: weak-coupling HTL calculation) leads to quite different D-meson $p_T$-distributions wrt to the static lattice-QCD results (A.B. *et al.*, JHEP 1802 (2018) 043).

# Outline of recent developments

- HQ's and quarkonia as Open Quantum Systems;
- Event-by-event fluctuations: odd harmonics ($v_3$) and event-shape engineering;
- Directed flow $v_1$: access to initial conditions, thermalization and magnetic field?
- In-medium hadronization and HF hadro-chemistry

# HQ's and quarkonia as Open Quantum Systems

Attempts to provide a consistent description of HQ's and quarkonia in the medium through the language of Open Quantum Systems, coupled to an Environment $E$[2]:

$$H = H_S \otimes \mathbb{1}_E + \mathbb{1}_S \otimes H_E + H_I$$

The problem is formulated in terms of the density matrix, given by

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but this is no longer the case for the *reduced density matrix* of the system, obtained taking a trace over the environment E:

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To solve the equation, for the initial condition one takes the factorized ansatz (HQ's have not yet interacted with the environment)

$$\rho(0) = \rho_S(0) \otimes \rho_E \qquad \text{with} \qquad \rho_E \equiv e^{-\beta H_E}/Z_E$$

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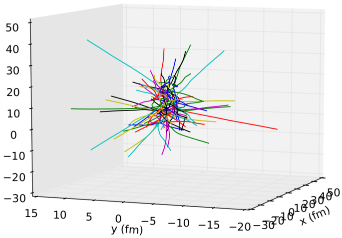
# Application I: evolution of a sample of $Q\overline{Q}$ pairs




Taking a *semi-classical approximation* (J.P. Blaizot and M.A. Escobedo, JHEP 1806 (2018) 034) one can formulate the problem in terms of a set of Langevin equations, accounting for the interactions of the HQ's with the environment and among each others:

$$M\ddot{\mathbf{r}}_a = -C_F\gamma\dot{\mathbf{r}}_a + \mathbf{\Xi}_a(t) + \sum_{b\neq a}^{N_Q} \mathbf{\Theta}_{ab}(\mathbf{r}_{ab}) + \sum_{\hat{b}}^{N_Q} \mathbf{\Theta}_{a\hat{b}}(\mathbf{r}_{a\hat{b}}, t)\,,$$

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Some of the pairs will evolve into bound states, some others will move far apart

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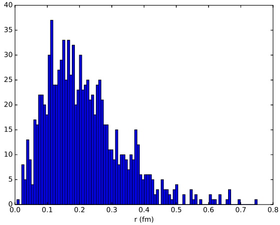


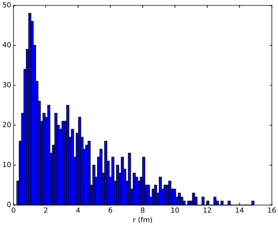


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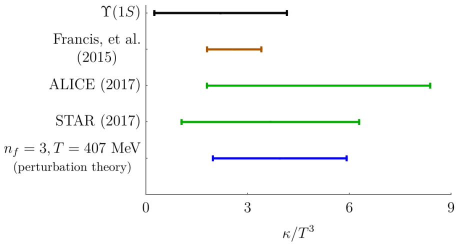
Some of the pairs will evolve into bound states, some others will move far apart

# Application II: momentum-diffusion coefficient $\kappa$

Still in the Open Quantum System approach it is possible to derive a relation between the thermal width of the quarkonium ground state $\Gamma(1S)$, the imaginary part of the $Q\overline{Q}$ self-energy in the colour-singlet channel and the momentum diffusion coefficient $\kappa$ (Brambilla *et al.*, arXiv:1903.08063):

$$\Gamma(1s) = -2\langle \mathrm{Im}(-\mathrm{i}\Sigma_\mathrm{s})\rangle = 3a_0^2\kappa,$$

where $a_0$ is the quarkonium Bohr radius $a_0 \equiv \frac{2}{MC_F\alpha_s(1/a_0)}$.

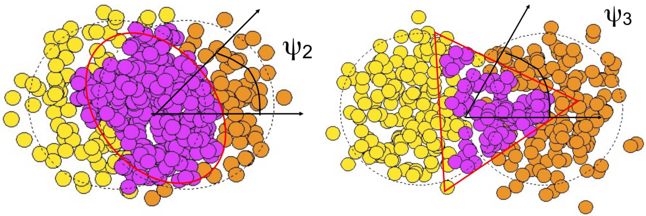



Taking for $\Gamma(1S)$ the estimates from l-QCD simulations (Aarts *et al.* 1109.4496 and Kim *et al.* 1808.08781) one gets

$$0.24 \lesssim \frac{\kappa}{T^3} \lesssim 4.2$$

Large systematic uncertainties, but conceptually interesting!

# Event-by-event fluctuations




- The random distribution of nucleons can lead to different geometric deformations (elliptic, triangular...) for the same impact parameter. Odd anisotropies (triangular, pentagonal...) can only arise from EBE fluctuations;

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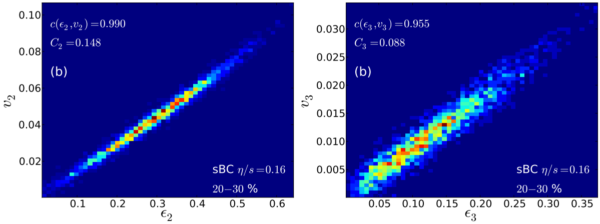



- The random distribution of nucleons can lead to different geometric deformations (elliptic, triangular...) for the same impact parameter. Odd anisotropies (triangular, pentagonal...) can only arise from EBE fluctuations;
- One observes, for *light hadrons*, that $v_n \sim \epsilon_n$ for $n = 2, 3$: anisotropy of particle distribution proportional to geometric eccentricity.

# Event-by-event fluctuations and odd flow-harmonics

The study of odd flow-harmonics $(v_3, v_5)$ in AA collisions requires a modeling of initial-state event-by-event fluctuations. We perform a Glauber-MC sampling of the initial conditions, each one characterized by a *complex eccentricity*

$$s(\mathbf{x}) = \frac{K}{2\pi\sigma^2}\sum_{i=1}^{N_{\rm coll}} \exp\left[-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2\sigma^2}\right] \quad \longrightarrow \quad \epsilon_m e^{im\Psi_m} \equiv -\frac{\left\{r^2 e^{im\phi}\right\}}{\{r^2\}}$$

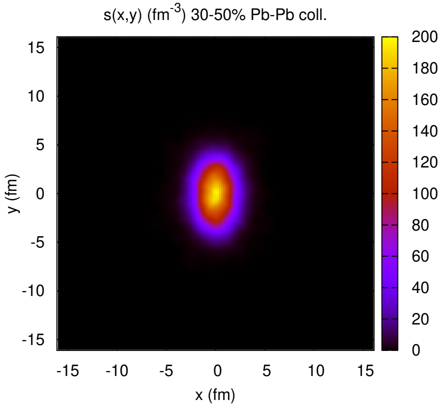
with orientation and modulus given by

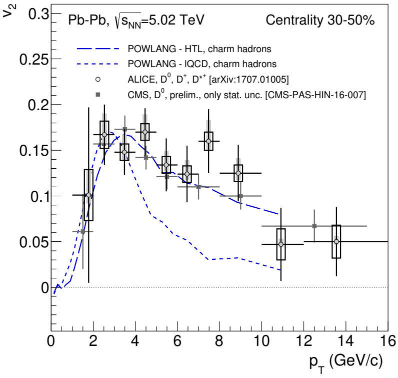
$$\Psi_m = \frac{1}{m}\,\mathrm{atan2}\left(-\{r^2\sin(m\phi)\}, -\{r^2\cos(m\phi)\}\right)$$

$$\epsilon_m = \frac{\sqrt{\{r_\perp^2\cos(m\phi)\}^2 + \{r_\perp^2\sin(m\phi)\}^2}}{\{r_\perp^2\}} = -\frac{\{r^2\cos[m(\phi-\Psi_m)]\}}{\{r^2\}}$$

Exploiting the fact that, on an event-by-event basis, for $m = 2, 3$ $v_m \sim \epsilon_m$ one can again consider an *average background* obtained summing all the events of a given centrality class, each one rotated by its *event-plane* angle $\psi_m$, depending on the harmonic one is considering.

# Event-by-event fluctuations and odd flow-harmonics



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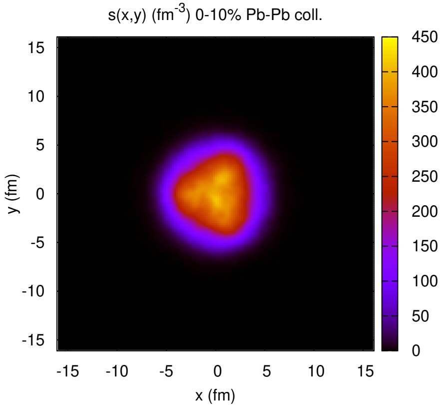


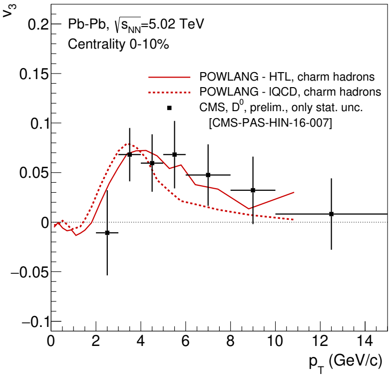

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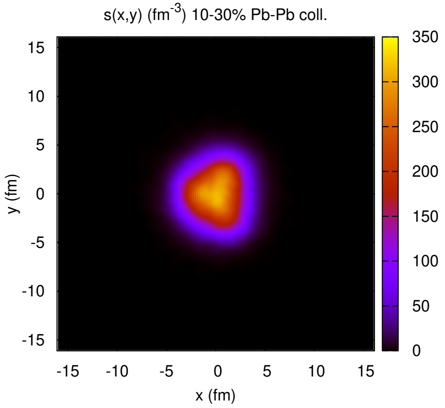


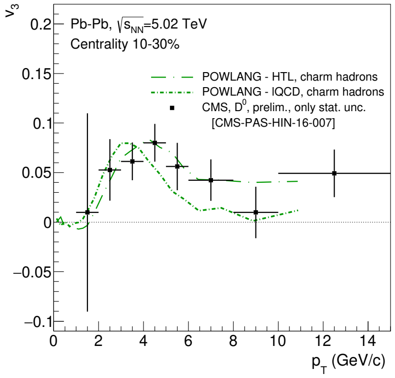

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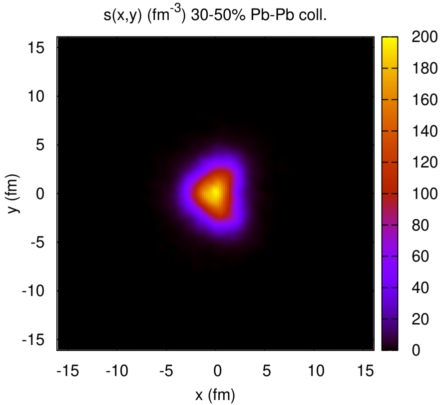


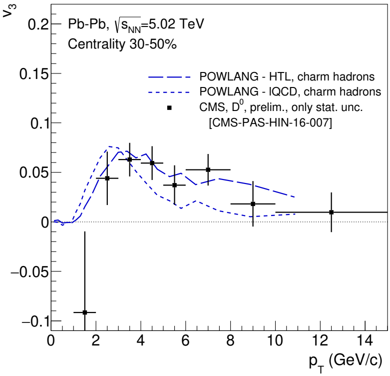

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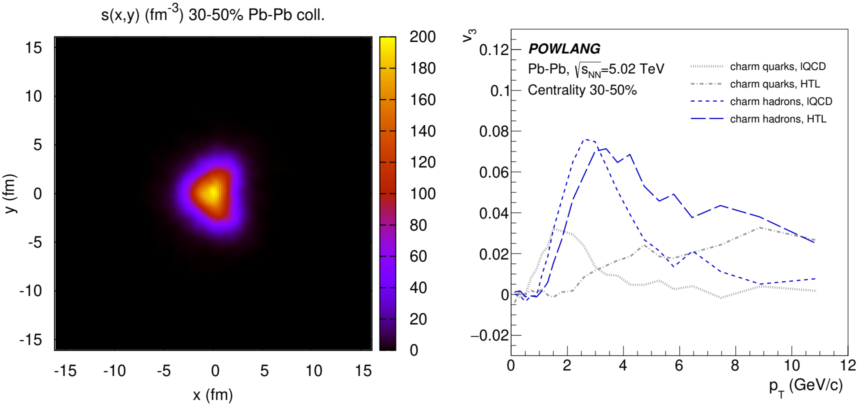
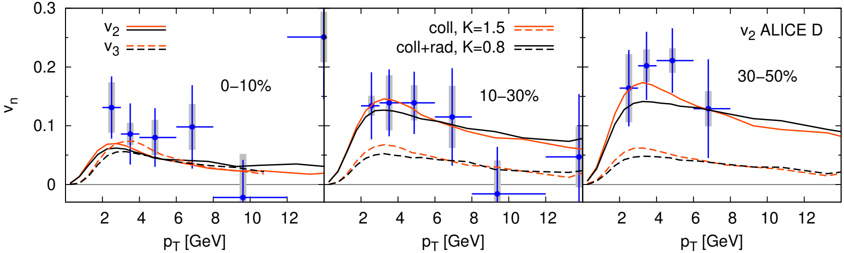



- CMS and ALICE data for *D*-meson $v_{2,3}$ satisfactory described (A.B. *et al.*, JHEP 1802 (2018) 043);
- Recombination with light quarks at hadronization provides a relevant contribution to the *D*-meson $v_n$;

# Event-by-event fluctuations and odd flow-harmonics

Similar analysis for *D* and *B* mesons carried out in M. Nahrgang *et al.*, PRC 91 (2015), 014904 on a full event-by-event basis



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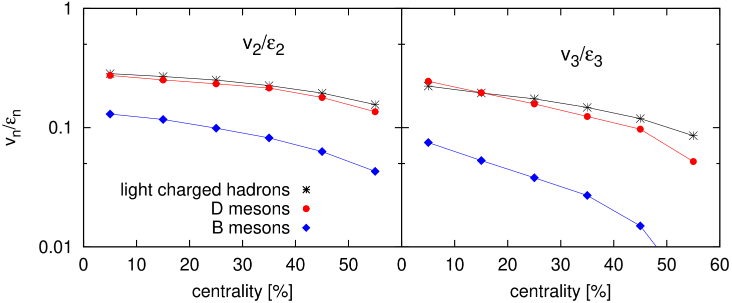
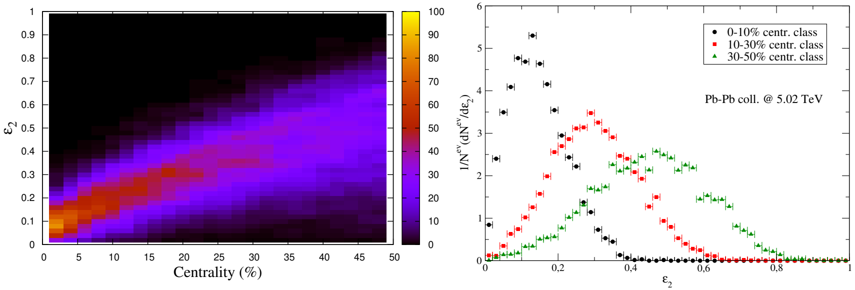



Different response of $D$ and $B$ mesons to the initial eccentricity $\epsilon_n$ looks of interest

# Event-shape-engineering




Very broad eccentricity distribution within a given centrality class!

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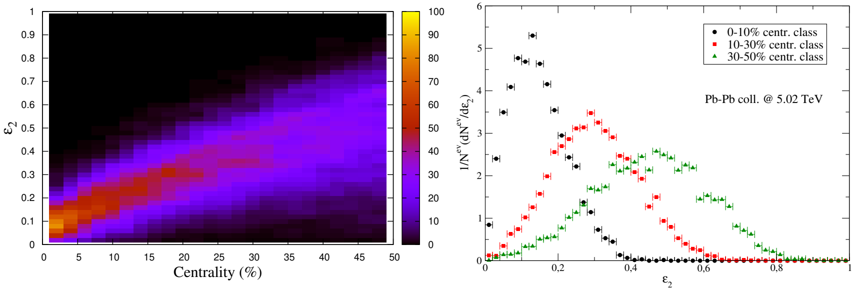


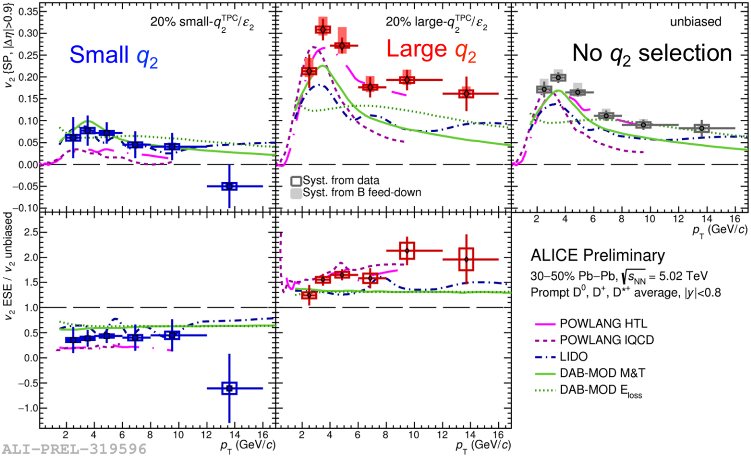

Very broad eccentricity distribution within a given centrality class!

One selects events of similar centrality, but very different initial eccentricity $\epsilon_2$ (th.) or average elliptic flow of light hadrons $q_2$ (exp.)

$$\epsilon_2 = \frac{\sqrt{\{r_\perp^2 \cos(2\phi)\}^2 + \{r_\perp^2 \sin(2\phi)\}^2}}{\{r_\perp^2\}} \qquad \text{Glauber} - \text{MC}$$

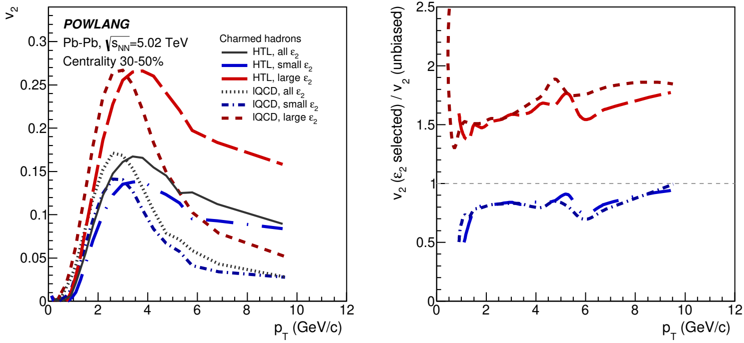
$$q_{2x} = \sum_{i=1}^{M} \cos(2\phi_i)/M \quad q_{2y} = \sum_{i=1}^{M} \sin(2\phi_i)/M \quad \text{detected hadrons}$$

# Event-shape-engineering: theory-to-data comparison



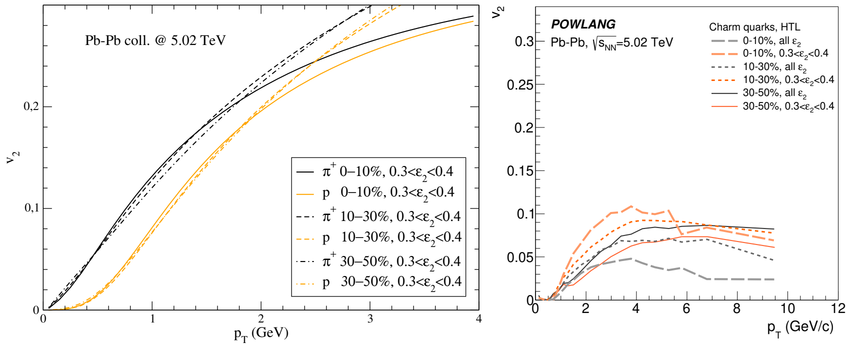

Various transport models reproduce quite well the ratio $v_2^{\mathrm{ESE}}/v_2^{\mathrm{unbiased}}$

# Event-shape-engineering: a deeper insight




Both $v_2^{\mathrm{ESE}}$ and $v_2^{\mathrm{unbiased}}$ are affected by the strength of the HQ-medium interaction, but the ratio $v_2^{\mathrm{ESE}}/v_2^{\mathrm{unbiased}}$ of charm hadrons displays only a mild dependence on the HQ transport coefficients (A.B. *et al.*, Eur.Phys.J. C79 (2019) no.6, 494).

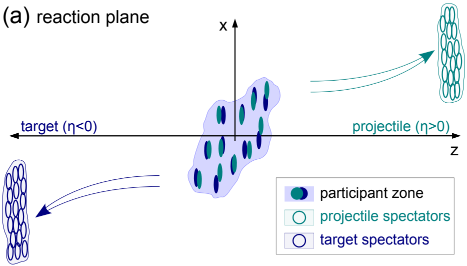
# Event-shape-engineering: a deeper insight




A complementary approach would consist in selecting events of similar eccentricity, but belonging to different centrality class:

- Light hadrons display a very similar flow, independent from centrality;
- The incomplete thermalization of charm quarks leads to lower values of $v_2$ going from more central to more peripheral events

# HF directed flow: initial tilted geometry




- Participant nucleons tend to deposit more energy along the direction of their motion $\longrightarrow$ tilted geometry of the fireball;

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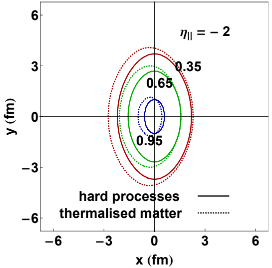


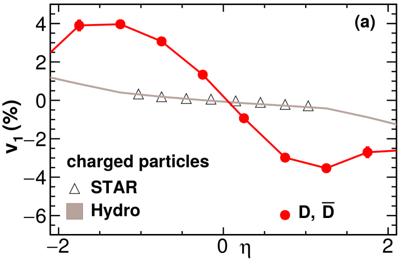


- Participant nucleons tend to deposit more energy along the direction of their motion $\longrightarrow$ tilted geometry of the fireball;
- HQ's on the other hand are distributed according to $n_{\rm coll}(\vec{x}_\perp)$, with no F/B asymmetry, longitudinal position fixed by their initial rapidity

This leads, for non zero rapidity, to a sizable *D*-meson directed flow $v_1$, much larger then the one of light hadrons (S. Chatterjee and P. Bozek, PRL 120 (2018), 192301).

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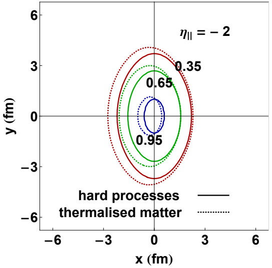


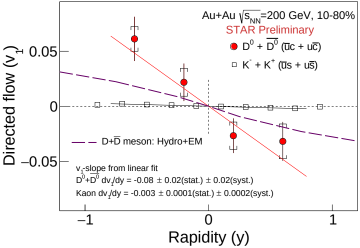


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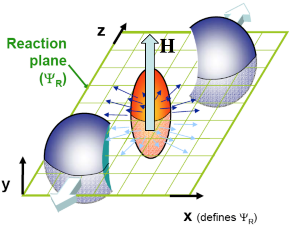
# HF directed flow: signature of the EM field?




Colliding nuclei generate a huge initial magnetic field $B \sim 10^{15}$ T

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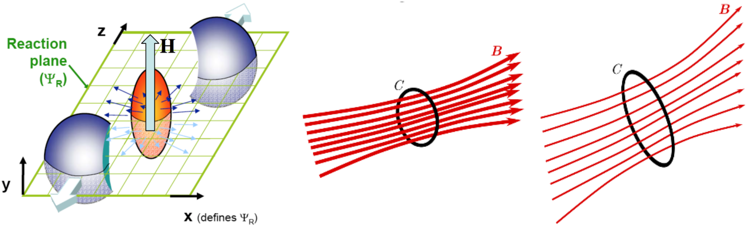


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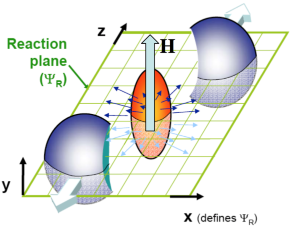


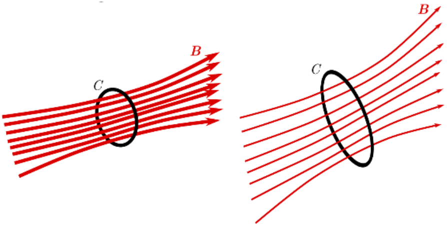


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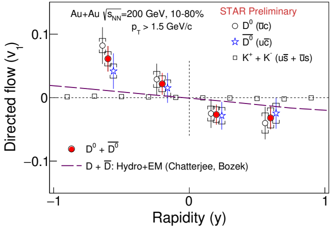


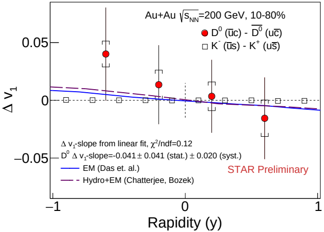


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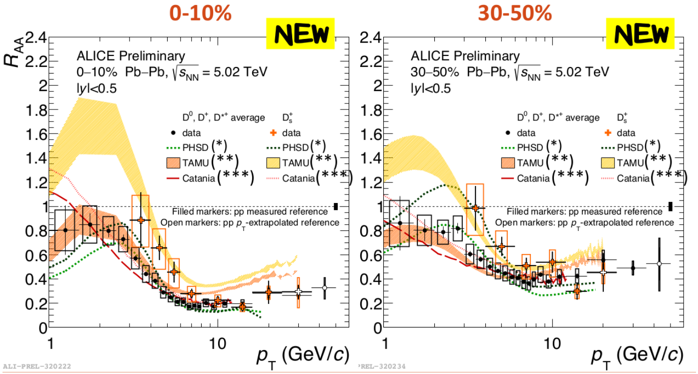
The Langevin equation can be corrected to account for the Lorentz force:

$$\Delta\vec{p}/\Delta t = -\eta_D \vec{p} + \vec{\xi} + Q(\vec{E} + \vec{v} \times \vec{B})$$

This could lead to a different $v_1$ for $D^0$ and $\overline{D}^0$, which could be explained as due to the EM interaction in the QGP phase (S. Chatterjee and P. Bozek arXiv:1804.04893, S.K. Das *et al.*, Phys.Lett. B768 (2017) 260-264)

# Recombination and HF hadrochemistry




In HIC's one expects an enhanced production of $D_s$ mesons and $\Lambda_c$ baryons wrt $D_0$ mesons as compared to p+p collisions. No need to excite $s\bar{s}$ or $qq - \overline{qq}$ pairs from the vacuum: hadronization of charm can occur via recombination with the abundant light quarks and diquarks nearby.

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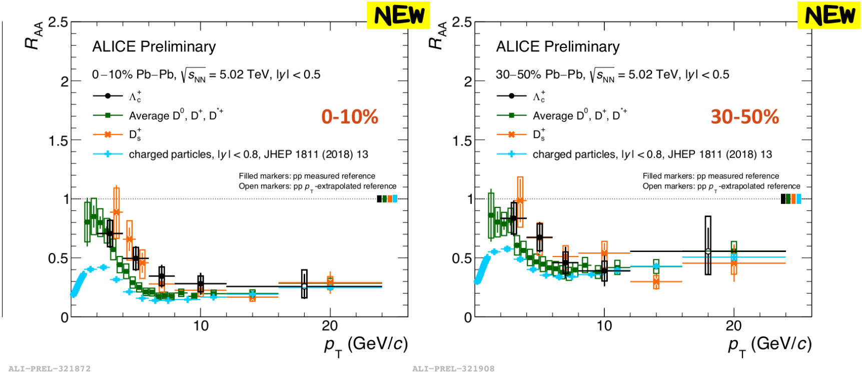


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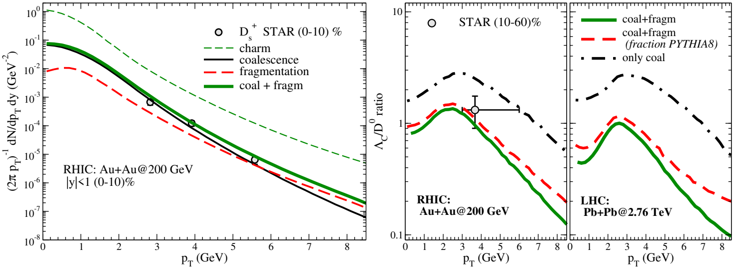
# HF hadronization: coalescence

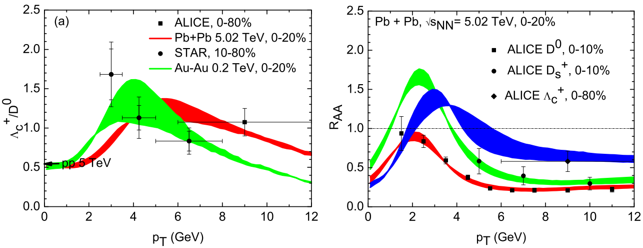



Coalescence is based on a $N \to 1$ mechanism like $Q + \bar{q} \to M$ or $Q + qq \to B$

$$\frac{dN_H}{dyd^2P_T} = g_H \int \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i \; f_{q_i}(x_i, p_i) \underbrace{f_H(x_1 \ldots x_n, p_1 \ldots p_n)}_{\text{Wigner function}} \; \delta^{(2)}\left(P_T - \sum_{i=1}^{n} p_{T,i}\right)$$

The Wigner function expresses the overlap between the hadron wave-function and the one of the coalescing quarks. It is usually taken as a Gaussian in position and momentum: quark must be close in space and have similar velocities to produce a hadron. Data on $D_s$ and $\Lambda_c$ production nicely reproduced (S. Plumari *et al.*, Eur.Phys.J. C78 (2018) no.4, 348)

# HF hadronization: resonant recombination




Resonant Recombination (Rapp *et al.*) $Q + \bar{q} \longleftrightarrow M$ or $Q + qq \longleftrightarrow B$ described by a Boltzmann eq. with loss and gain terms:

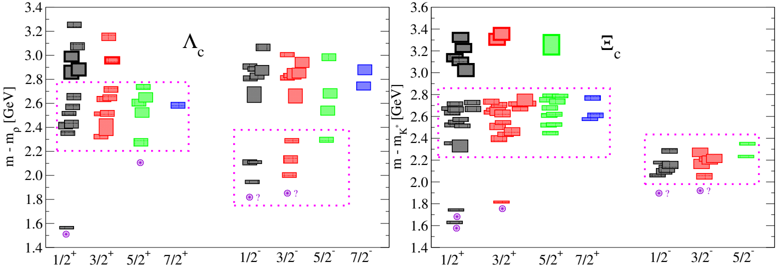
$$\left(\partial_t + \vec{v} \cdot \vec{\nabla}\right) f_M(t, \vec{x}, \vec{p}) = -\underbrace{(\Gamma_M/\gamma_p) f_M(t, \vec{x}, \vec{p})}_{M \to Q+\bar{q}} + \underbrace{\beta(t, \vec{x}, \vec{p})}_{Q+\bar{q} \to M}$$

$$\text{with} \quad \sigma(s) = \frac{4\pi}{k^2} \frac{(\Gamma_M m)^2}{(s - m^2)^2 + (\Gamma_M m)^2}$$

If the rate of resonant processes is much larger than the expansion rate of the fluid, $\Gamma_M \gg \tau_{\text{hadr}}^{-1}$, one can take the equilibrium solution for the hadron PSD

$$f_M(\vec{x}, \vec{p}) = \frac{\gamma_p}{\Gamma_M} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} f_q(\vec{x}, \vec{p}_1) f_{\bar{q}}(\vec{x}, \vec{p}_2) \sigma_M(s) v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

# HF hadrochemistry: uncertainties from spectroscopy




All hadronization models based on recombination with light quarks from the medium are affected by huge systematic uncertainties due the unknown spectrum of excited charm baryons, whose feed-down can contribute to the final $\Lambda_c$ multiplicity. Both relativistic quark models and lattice-QCD calculations[3] predict a much richer spectrum than the one quoted by the PDG

[3] For a review see M. Padmanath *et al.*, 1410.8791

# Summary and outlook

- Strong theoretical progress in the development of a formalism (Open Quantum Systems) allowing a consistent description of HF transport coefficients and in-medium quarkonium evolution;
- Solid first-principle theory calculations still limited to a range of masses ($M \to \infty$) and/or couplings ($g \ll 1$) of limited experimental relevance, although some consistent semi-quantitative information (e.g. for $\kappa$) can be in any case obtained;
- Usual difficulties (*ill-posed problem*) in extracting real-time information from Euclidean lattice-QCD simulations;
- Transport calculations recently quite successfully extended to observables carrying richer and richer information ($v_1$, $v_3$, $v_n^{\mathrm{ESE}}$...);
- If $D^0$ vs $\overline{D}^0$ measurements were confirmed this would open a window on the EM properties (e.g. electric conductivity $\sigma_E$) of the QGP;
- Wait for beauty measurement at low $p_T$ to have a safe framework to extract transport coefficients