# MULTIPARTON SCATTERING AT LHC

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# Outline

1. Double Parton Scattering (DPS) : the quantity of interest is $\sigma_{eff}^{DPS}$

2. Factorization model and geometrical description overlap in

   b-space → b-distribution of partons $\sigma_{eff}^{NPS} \Leftrightarrow T(\mathbf{b})$

3. The eikonal minijet model with soft gluon resummation BN model (Bloch Nordsieck model)

   – Minijets

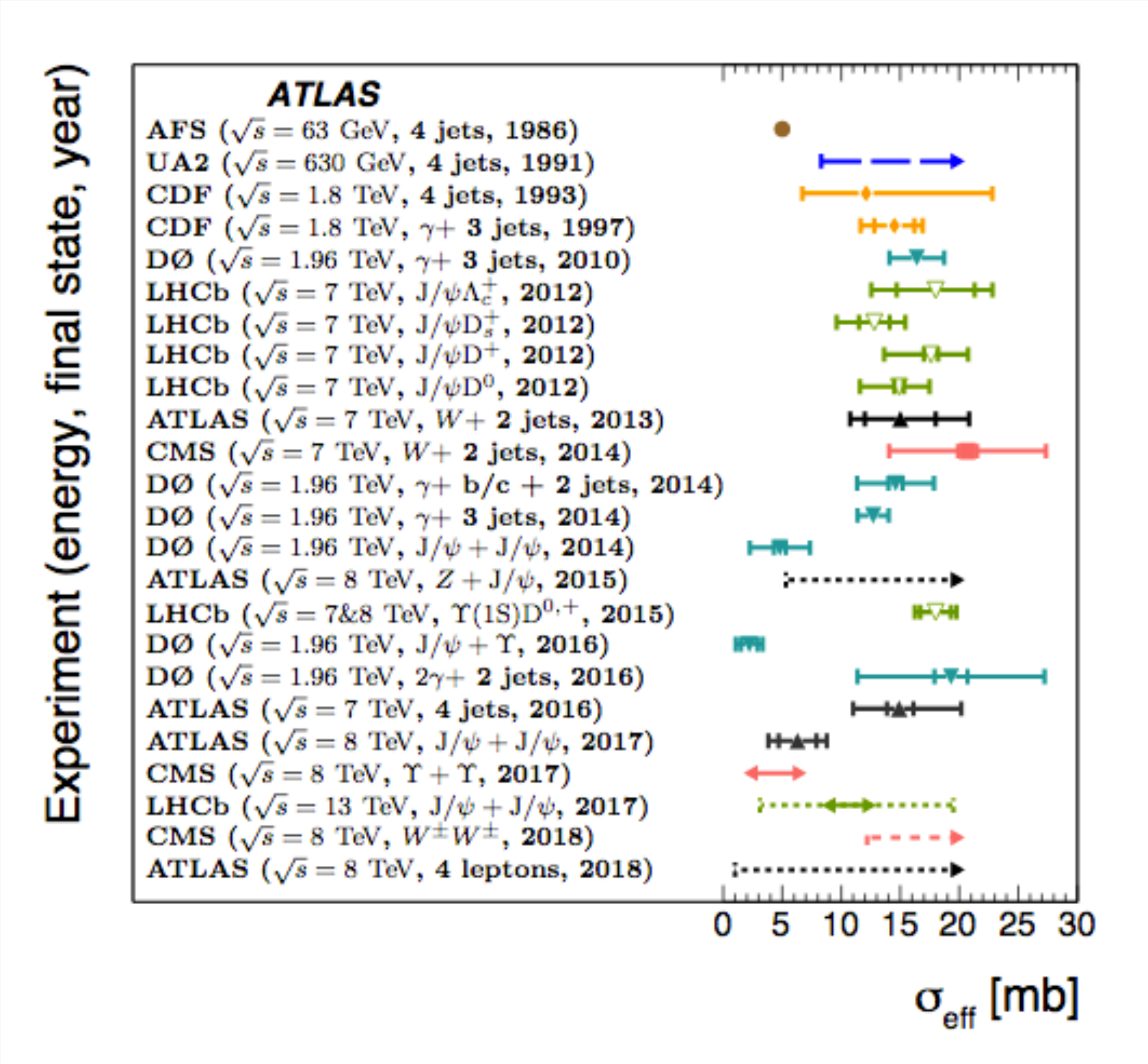
   – Parton impact parameter from soft gluon resummation => an ansatz for $k_t \sim 0$

- Comparison with data and calculation of the effective cross-section from

  – overlap in b-space from BN model

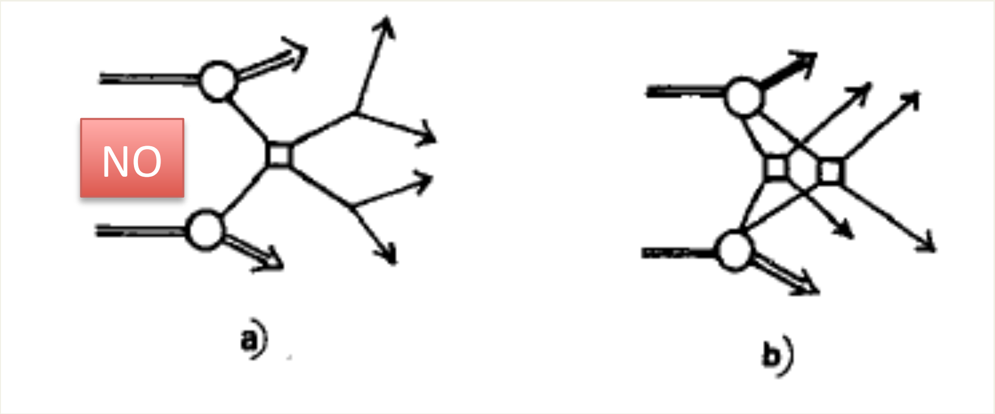
  – Inelastic SND cross-section

# ATLAS DPS compilation Phys. Lett. B790, 595 (2019)



# Interest in DPS soon after jets were measured at SpbarpS

- Paver and Treleani 1982-1986 …double "jets" from disconnected scattering




- For jets: "High" $p_t$ > 4 GeV at ISR -> 100 GeV at LHC
- … How to predict/calculate multiple interactions ?
- → info on inner structure beyond PDF ?

# Double parton scattering (DPS)

$$pp \rightarrow \{A\} + \{B\} + X$$

Experimentally, for instance in case of $pp \rightarrow J/\Psi\ J/\Psi + X$

$$\frac{d\sigma^{J/\Psi}}{dp_t d\eta} \quad \text{vs} \quad \frac{d\sigma^{2J/\Psi}}{dp_{t1} dp_{t2} d\eta_1 \eta_2}$$

Of theoretical or modeling interest

A=B k=1
A≠B k=2

$$\sigma_{DPS}^{AB} = \frac{k}{2} \frac{\sigma_{SPS}^{\{A\}} \sigma_{SPS}^{\{B\}}}{\sigma_{eff}}$$

# NPS(n disconnected parton scattering )

$$\sigma^{NPS}_{h_1h_2\to a_1,a_2,\ldots a_n n} = \left[\frac{m}{\Gamma(n+1)} \frac{\sigma^{SPS}_{h_1h_2\to a_1}\sigma^{SPS}_{h_1h_2\to a_2}\ldots\ldots\sigma^{SPS}_{h_1h_2\to a_n}}{\sigma^{(n-1)}_{eff,NPS}}\right]$$

Rather general

Not so general → under hypothesis of factorization of parallel and transverse momenta, the quantity of interest can be approximated in terms of the normalized parton distribution $T(\mathbf{b})$ inside a hadron in impact parameter space [for reviews M. Diehl, D. D'Enterria 2017]

$$\int (d^2\mathbf{b}) T(\mathbf{b}) = 1;$$

$$\Sigma^{(n)} \equiv \int (d^2\mathbf{b}) T^n(\mathbf{b});$$

$$\sigma^{NPS}_{eff} = [\Sigma^{(n)}]^{-1/(n-1)}$$

$$\sigma^{NPS}_{eff} \longleftrightarrow T(\mathbf{b})$$

# DPS(2 disconnected parton scattering )

$$\sigma^{DPS}_{effective} = [\int d^2\vec{b}\, T^2(b)]^{-1}$$

not so general, but useful for modeling T(b)

- T(b) : a phenomenological choice (MCs) [ D'Enterria&Snigirev 2017]

$$T(b) = N(m, r_p) e^{-(\frac{b}{r_p})^m}$$

m=1 exponential
m=2 Gaussian
Other???

Is $r_p$ energy dependent?

- T(b) : modelling through resummation

$$T(b) \to A_{resum}(b,s) = \frac{e^{-h(b,s)}}{\int d^2\vec{b}\; e^{-h(b,s)}}$$

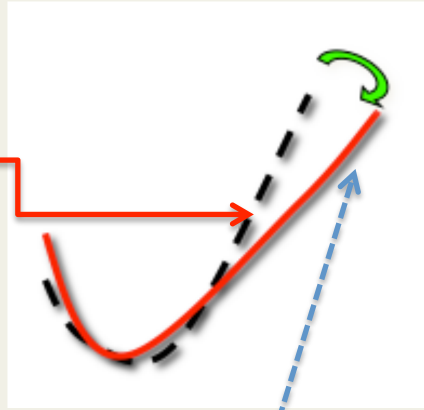
$$\sigma^{resum}_{eff} = \frac{2\pi[\int bdb\; e^{-h(b,s)}]^2}{\int bdb\; e^{-2h(b,s)}}$$

<-- DISCUSSED HERE

The Bloch-Nordsieck (BN) inspired model for the total cross-section provides an energy dependent b-distribution

with A. Grau, R.M. Godbole and Y.N.Srivastava (PRD 1999, 2004)

- Perturbative QCD
  → 1/x gluons → rise with energy



- Soft gluon resummation into the infrared

  +

- Eikonalize → unitarity
  - And resummation of multiple scattering

Tame the rise

# Eikonal mini-jet model for the total cross-section → b-dependence

$$\bar{n}_{coll}(s) = 2\chi_I = A(b,s;p,PDF,p_{tmin})\sigma_{mini-jet}(s;p_{tmin},PDF)$$

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\chi_I(b,s)}]$$

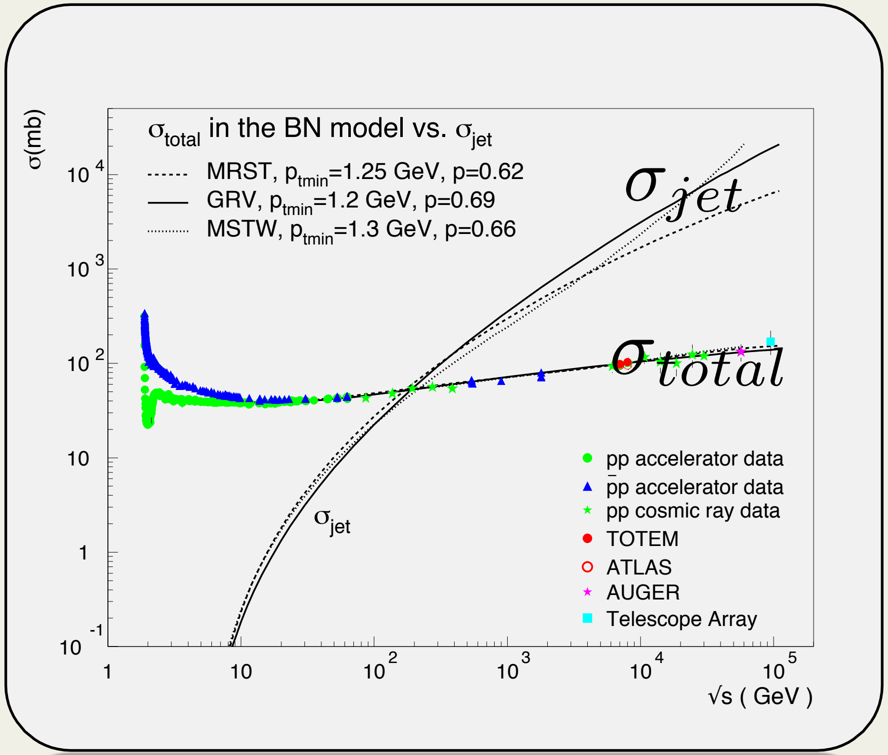
A.Grau,G.P.,Y.N.Srivastava
PRD 60 (1999)

- The **eikonal** function ~ real with scale fixed at low energy through form factor and a constant term
- The **rise** is from pQCD → **minijets** with actual PDFs
- The **taming** (Froissart bound) of minijet rise is from all order resummation of **soft gluons accompanying mini-je**t producing collisions + eikonalization
- scale fixed at low energy through form factor and a constant trm

# PDF driven eikonal-minijet-model: Minijets vs total cross-section

$$\sigma^{AB}_{\rm jet}(s;p_{t_{min}}) = \int_{p_{t_{min}}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i|A}(x_1,p_t^2) f_{j|B}(x_2,p_t^2) \frac{d\hat{\sigma}^{kl}_{ij}(\hat{s})}{dp_t}.$$

Negligible at low energies




But when

with $p_t \geq 1$ GeV → $x \leq 0.1$ $\quad \alpha_{strong}(p_t) \lesssim 1 \to \alpha_{AF}$ → pQCD can be applied

$$\sigma^{AB}_{\rm jet}(s;p_{t_{min}} \simeq 1\ GeV) \sim 10\%\ \sigma^{AB}_{total} \qquad \sqrt{s} \simeq 20 \div 30\ GeV$$

We model the impact parameter distribution for partons → minijets as the Fourier-transform of initial state soft gluon $k_t$ distribution [ and thus obtain a cut-off at large distances → Froissart bound]

$$A_{BN}(b,s) = N \int d^2\mathbf{K}_\perp \; e^{-i\mathbf{K}_\perp\cdot\mathbf{b}} \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\; e^{-h(b,q_{max})}}$$

$$h(b,q_{max}) = \frac{16}{3\pi}\int_0^{q_{max}} \frac{dk_t}{k_t}\alpha_{eff}(k_t)\ln\left(\frac{2q_{max}}{k_t}\right)[1-J_0(bk_t)]$$

**1.** ANSATZ $\alpha_{eff}(k_t \to 0) \sim k_t^{-2p}$ ← ½<p<1

**2.** $q_{max}(\sqrt{s}; p_{tmin}, PDF)$ ? **Calculated** from single gluon emission kinematics

A.Corsetti, A. Grau, G.P., Y.N. Srivastava PLB 1996

# Semi-classical derivation for resummation (B. Touschek 1967)

$$d^2P(\mathbf{K}_t) = \sum_{n_\mathbf{k}} P(\{n_\mathbf{k}\}) d^2\mathbf{K}_t \delta^2(\mathbf{K}_t - \sum_\mathbf{k} \mathbf{k}_t n_\mathbf{k}) =$$

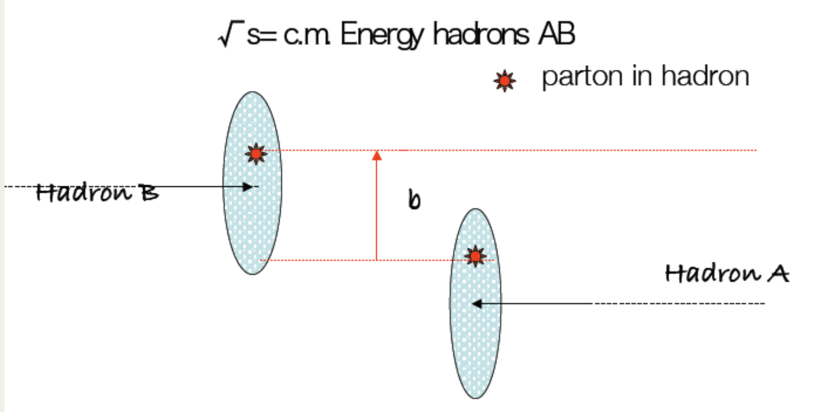
$$\sum_{n_k} \Pi_\mathbf{k} \frac{[\bar{n}_\mathbf{k}]^{n_\mathbf{k}}}{n_\mathbf{k}!} e^{-\bar{n}_\mathbf{k}} d^2\mathbf{K}_t \delta^2(\mathbf{K}_t - \sum_\mathbf{k} \mathbf{k}_t n_\mathbf{k})$$

Exchange Sum with Product →

$$d^2P(\mathbf{K}_t) = \frac{d^2\mathbf{K}_t}{(2\pi)^2} \int d^2\mathbf{b} e^{-i\mathbf{K}_t\cdot\mathbf{b}} exp\{-\sum_\mathbf{k} \bar{n}_\mathbf{k}[1 - e^{i\mathbf{k}_t\cdot\mathbf{b}}]\}$$

✪✁ Continuum limit → $$\frac{d^2\mathbf{K}_t}{(2\pi)^2} \int d^2\mathbf{b} e^{-i\mathbf{K}_t\cdot\mathbf{b}} exp\{-\int d^3\bar{n}_\mathbf{k}[1 - e^{i\mathbf{k}_t\cdot\mathbf{b}}]\}$$

We model the impact parameter distribution for partons →minijets as the Fourier-transform of ISR soft $k_t$ distribution and thus obtain a cut-off at large distances →Froissart bound




$$A_{BN}(b,s) \sim e^{-(b\bar{\Lambda})^{2p}}$$

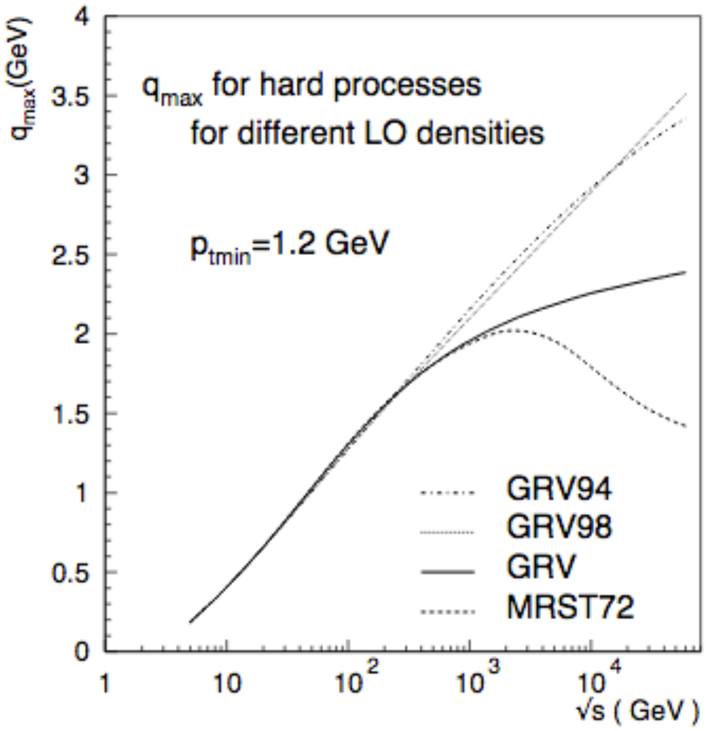
$$\bar{\Lambda} \equiv f(q_{max}, \Lambda_{QCD}, p)$$

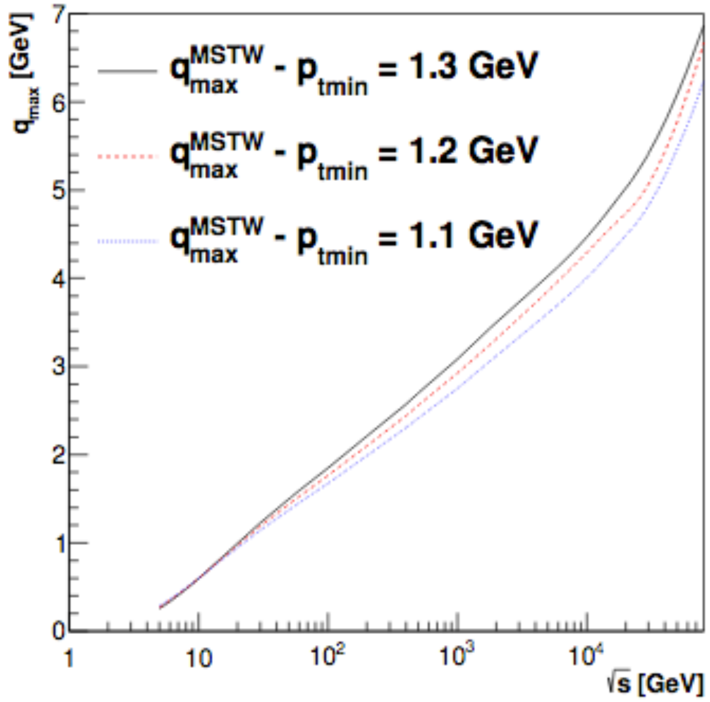
½ < p <1

$$q_{\max}(s;p_{tmin}) = \frac{\sqrt{s}}{2} \frac{\sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{z_{min}}^{1} dz f_i(x_1) f_j(x_2) \sqrt{x_1 x_2}(1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{z_{min}}^{1} dz f_i(x_1) f_j(x_2)}$$

**Calculated** from single gluon emission kinematics and averaged over densities (PRD 1999)

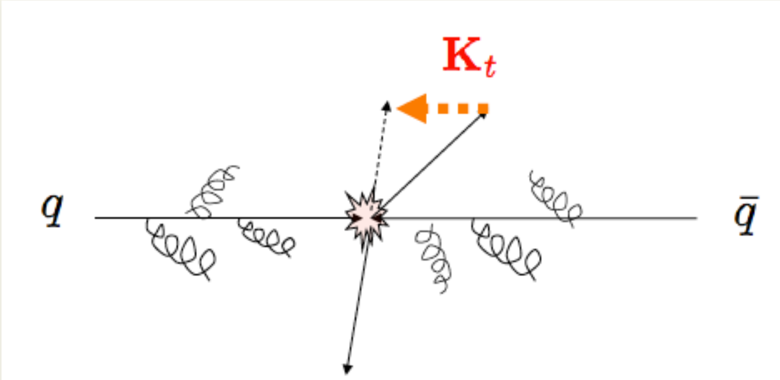
# Energy dependence of $q_{max}$ vs for different PDFs and and $p_{tmin}$



$$h(b, q_{max}) = \frac{16}{3\pi} \int_0^{q_{max}} \frac{dk_t}{k_t} \alpha_{eff}(k_t) \ln(\frac{2q_{max}}{k_t})[1 - J_0(bk_t)]$$

Regularized exponentiated soft gluon spectrum




→ Semiclassical Resummation procedure based on soft gluon Poisson distributions a' la Bloch and Nordsieck+ energy Momentum conservation

→ Needs integrable "effective" quark-gluon coupling constant

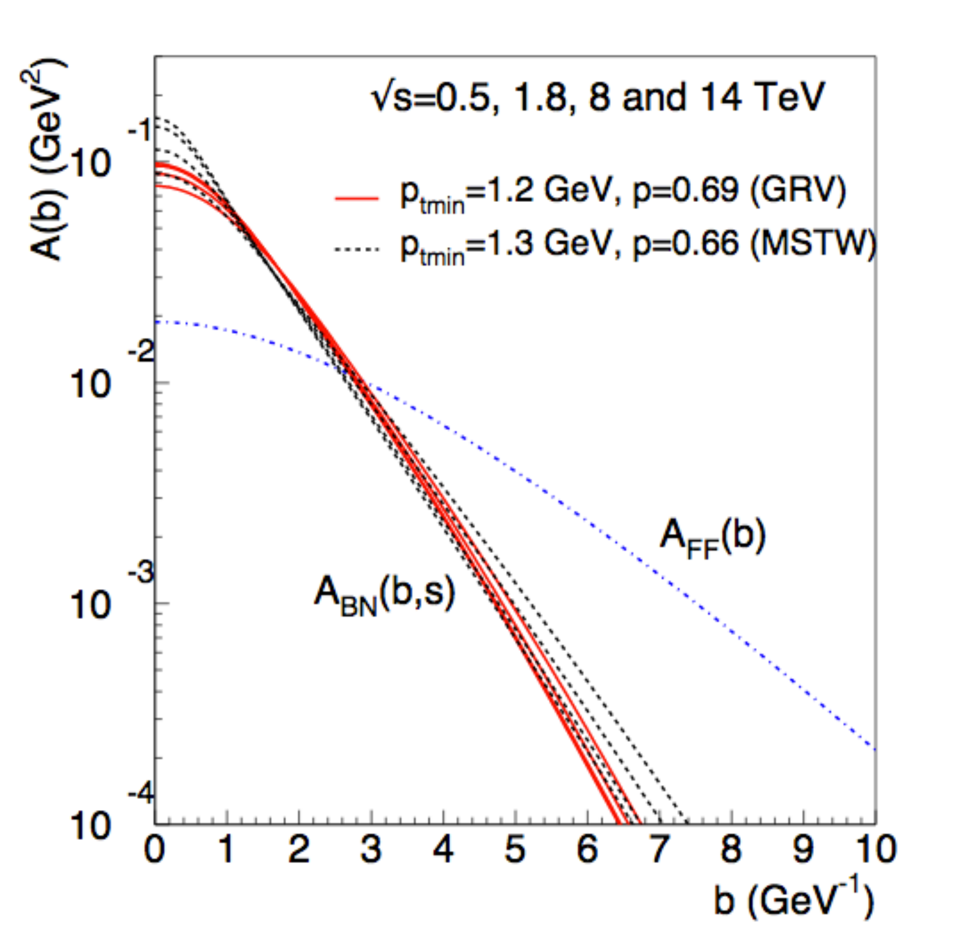
Nakamura, GP, Srivastava 1984

$$\alpha_{eff} = \frac{12\pi}{33 - 2N_f} \frac{p}{\log[1 + p(k_t/\Lambda_{QCD})^{2p}]}$$

Implemented for impact parameter Distribution of partons

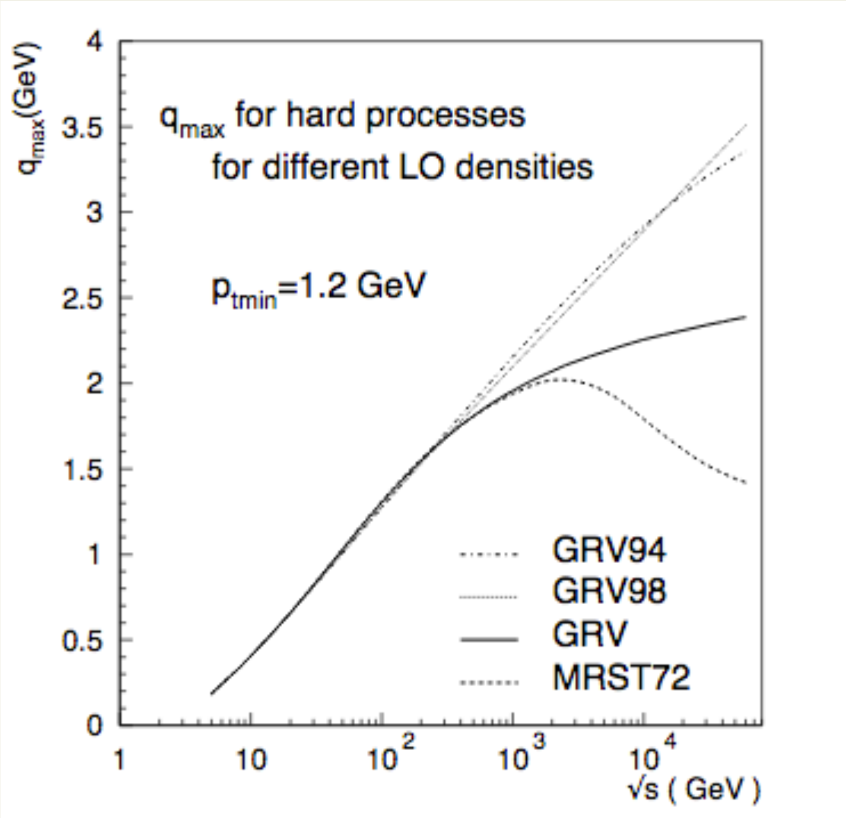
Corsetti, Grau,GP,Srivastava,PLB 1996

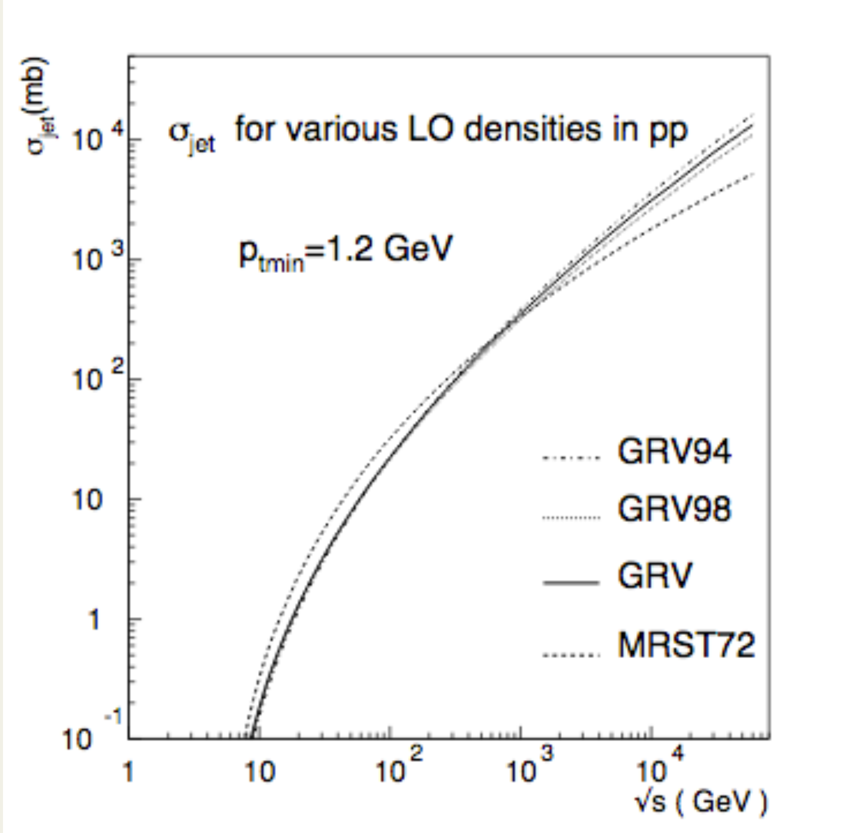
# $A\{b, q_{max}(s, PDF, p_{tmin}); p\}$ (resumming soft gluons) energy dependent




- At large √s resummed soft gluons have stronger b-fall-off for large b than FF
- Different PDFs and $p_{tmin}$-> different A(b)
- A(b) depends on sigularity parameter p

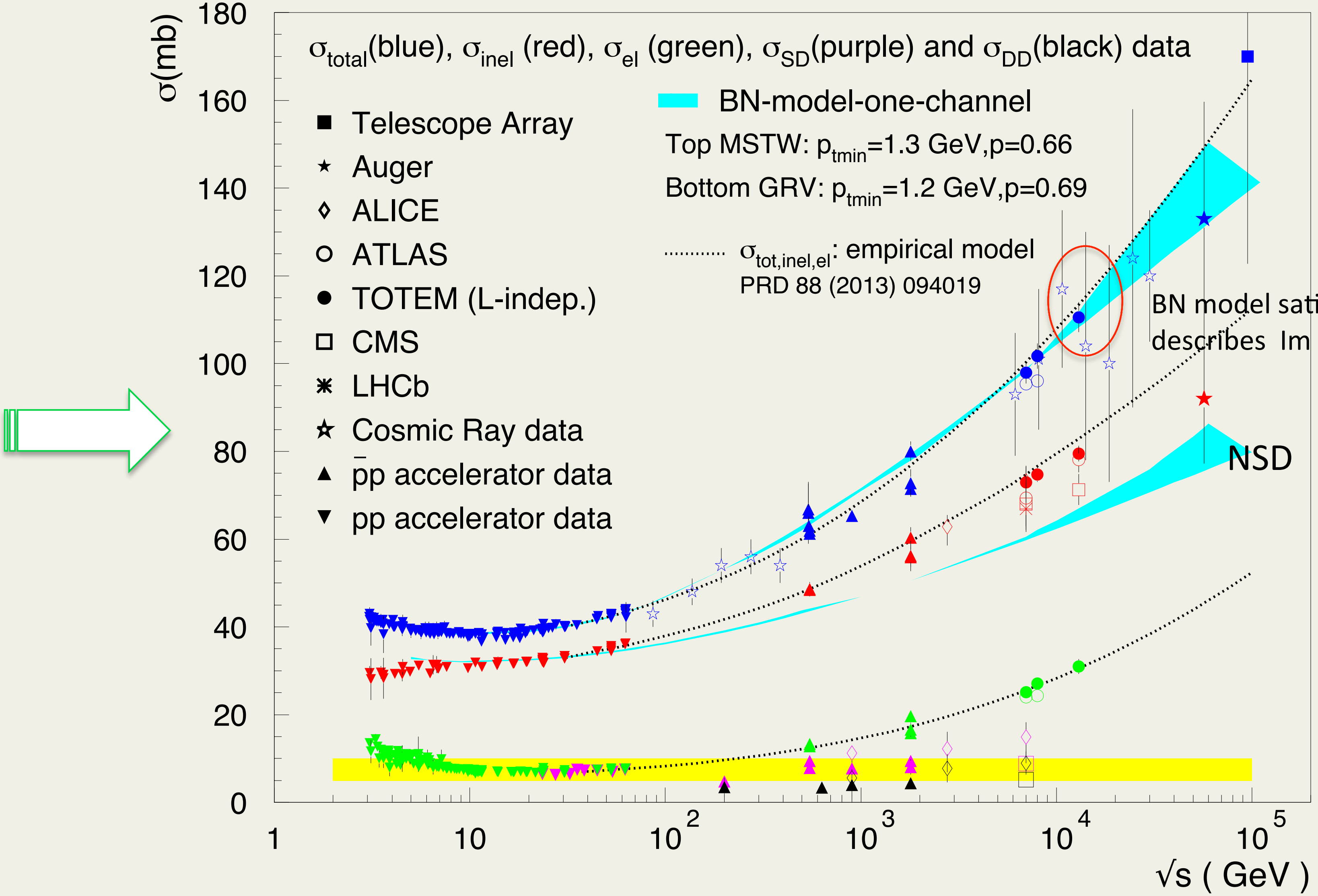
## Calculate Energy dependence of $q_{max}$ (for b-distribution) and $sigma_{minijets}$ for different LO PDF → eikonal function → total x-section




**Phys.Rev. D91 (2015) no.11, 114011**

pp Total, elastic and inelastic cross-sections → 13 TeV updated




BN model satisfactorily describes Im A at t=0

9/10/19

# Energy dependence of $\sigma_{effective}$

$$\sigma_{effective} \equiv \sigma_{eff}^{resum-BN}(s) = \sigma_{eff}^{resum-BN}(q_{max}(s))$$

**IF** $q_{max}(s) \equiv < q_{max}(s, x_1, x_2) >_{densities} \quad \uparrow \sqrt{s}$

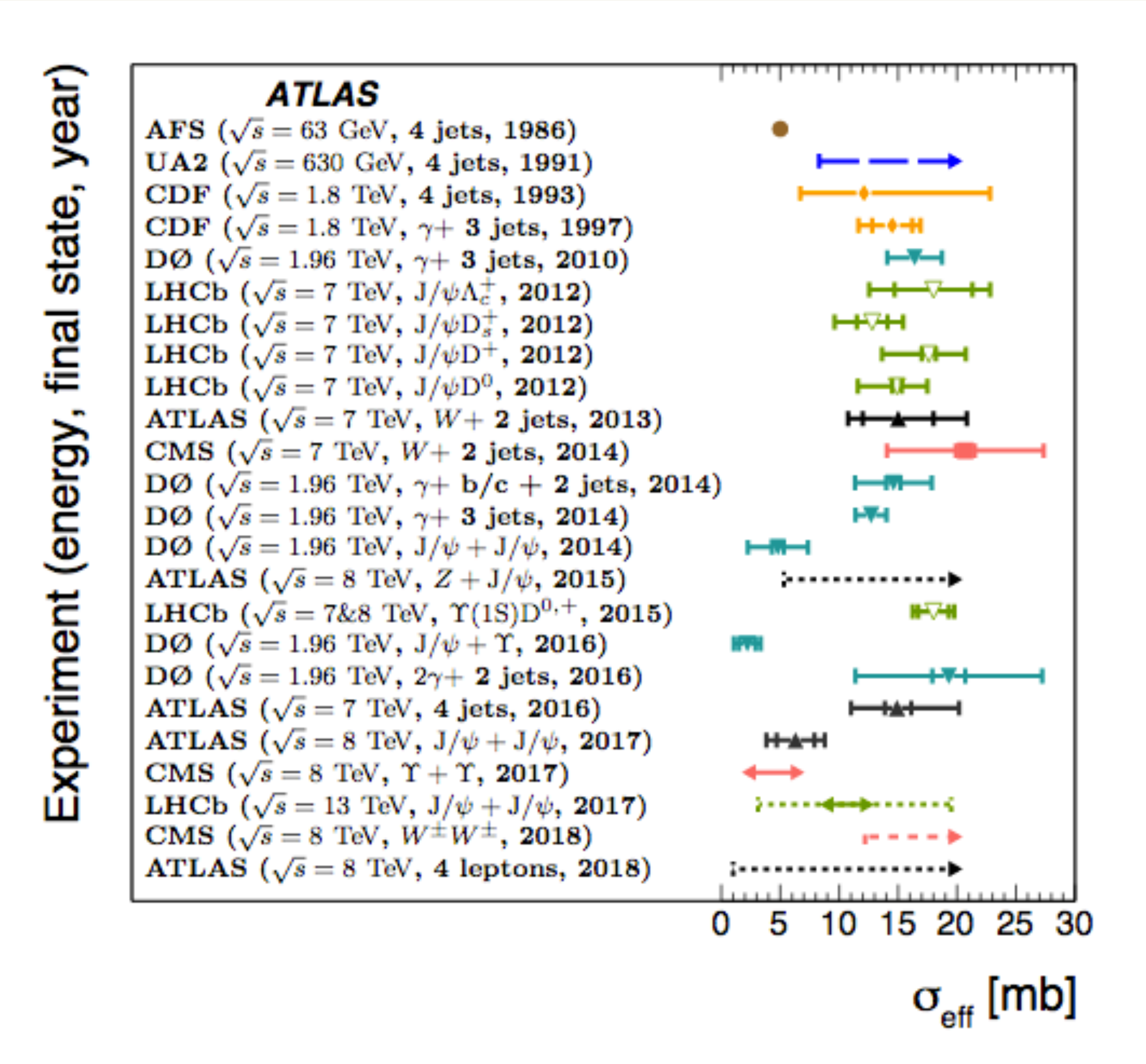
$$h(b,s) \propto q_{max}(s) \quad \uparrow \sqrt{s}$$

$$e^{-h(b,s)} \downarrow \quad \sqrt{s}$$

$$\sigma_{eff} \downarrow \quad \sqrt{s}$$

$$\sigma^{Double\ disconnected\ scattering} \uparrow \quad \sqrt{s}$$

# ATLAS DPS compilation Phys. Lett. B790, 595 (2019




- No error given by AFS for value at 63 GeV
- Eye- average ~ 15 mb
- Double Quarkonia is generally lower

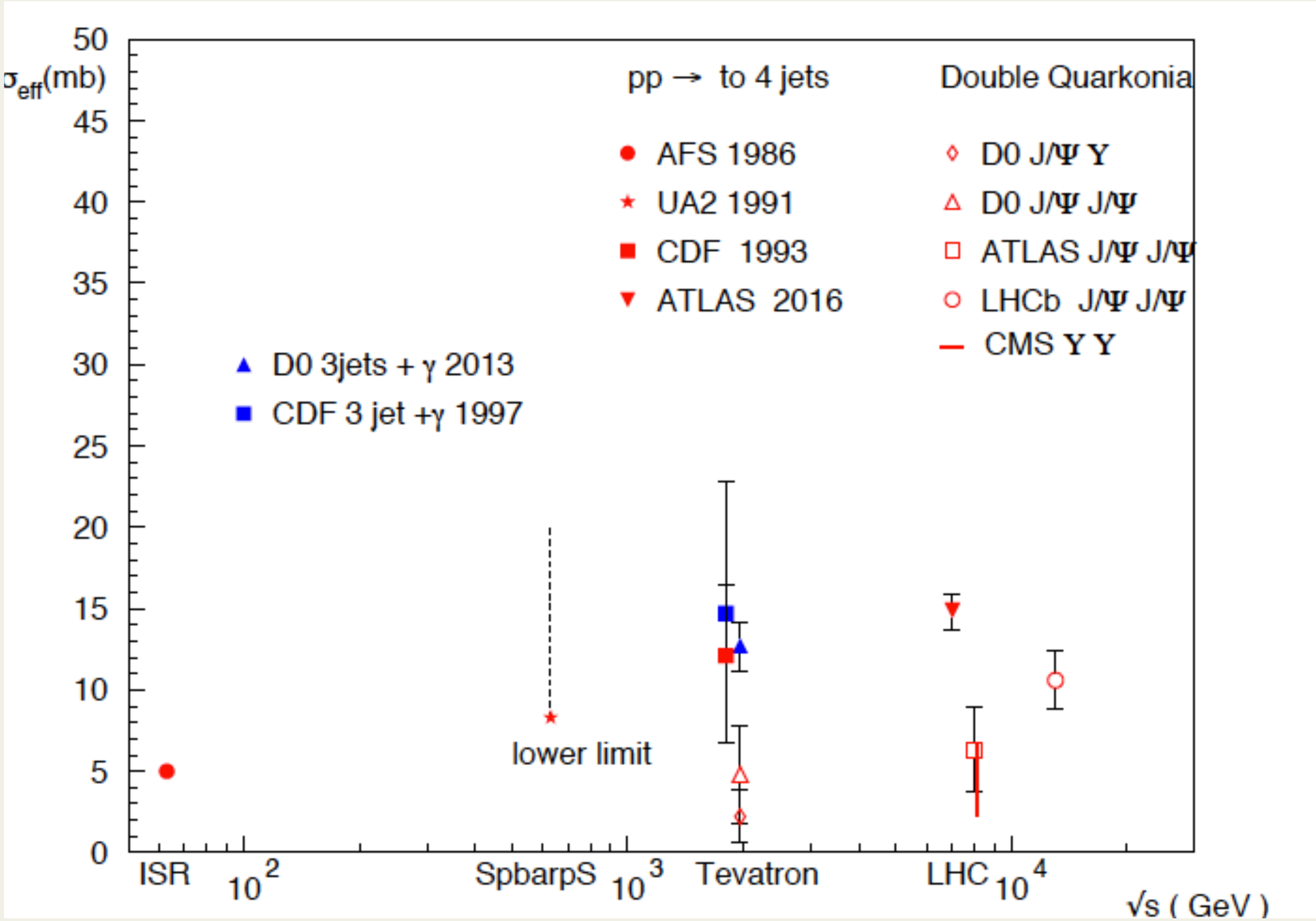
# 1. Identify clear sets of data

First preliminary approach

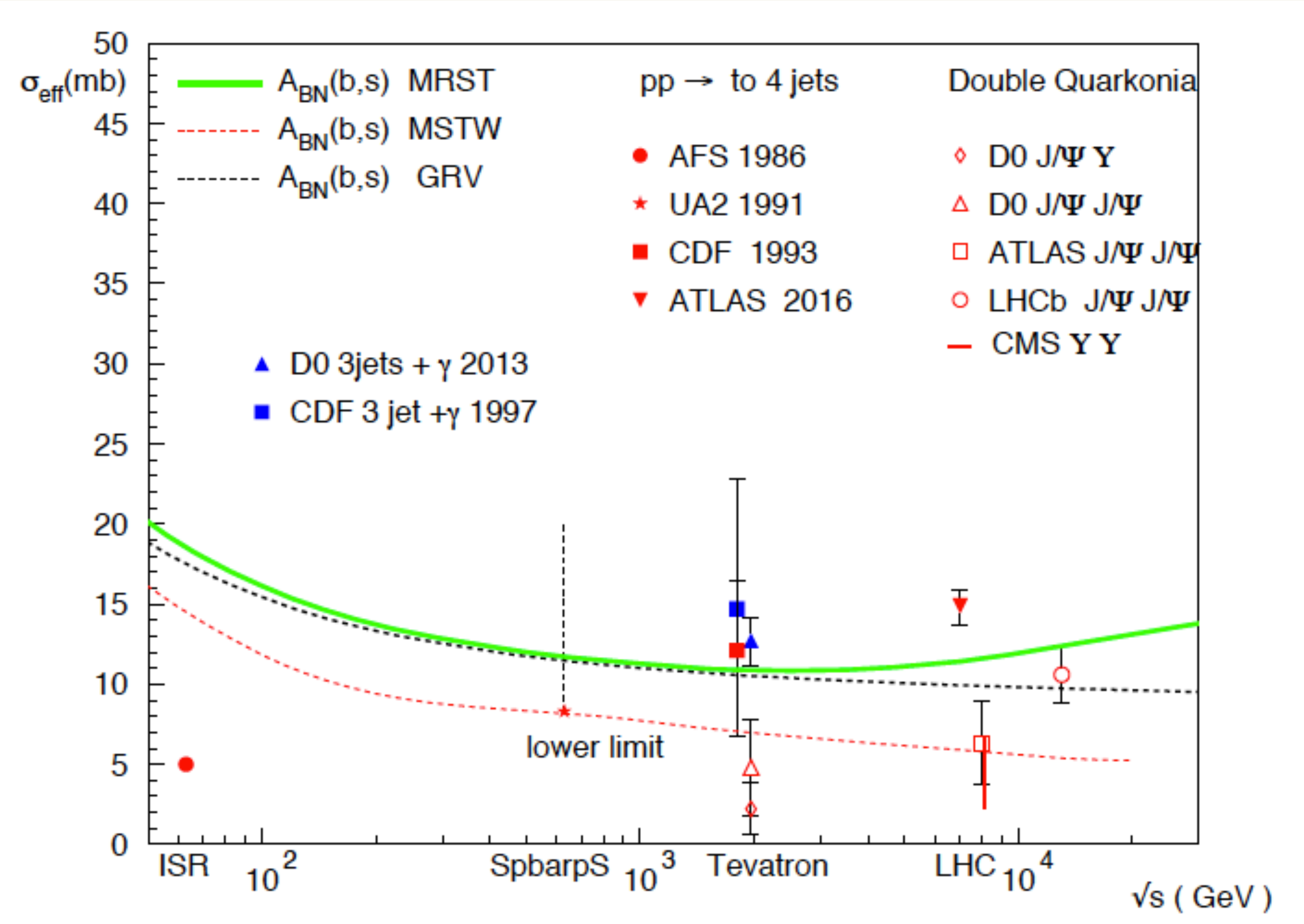
4 jets

Double Quarkonia

3 jets with photon



# 2. Superimpose b-model from resummation




Non trivial result:
Scale in
general
agreement
with most data

~ 15 mb

# $\sigma_{eff}^{DPS}(s)$ in the T(b) model for vs pp → 4 jet, 2 quarkonium data, or 1 gamma + 3jets

Data

- 4 jets or quarkonia → all rise with energy [all gluon gluon]
  - Different scale between 4 jets or quarkonia
- 3 jet + gamma seem to decrease [but valence quarks enter]

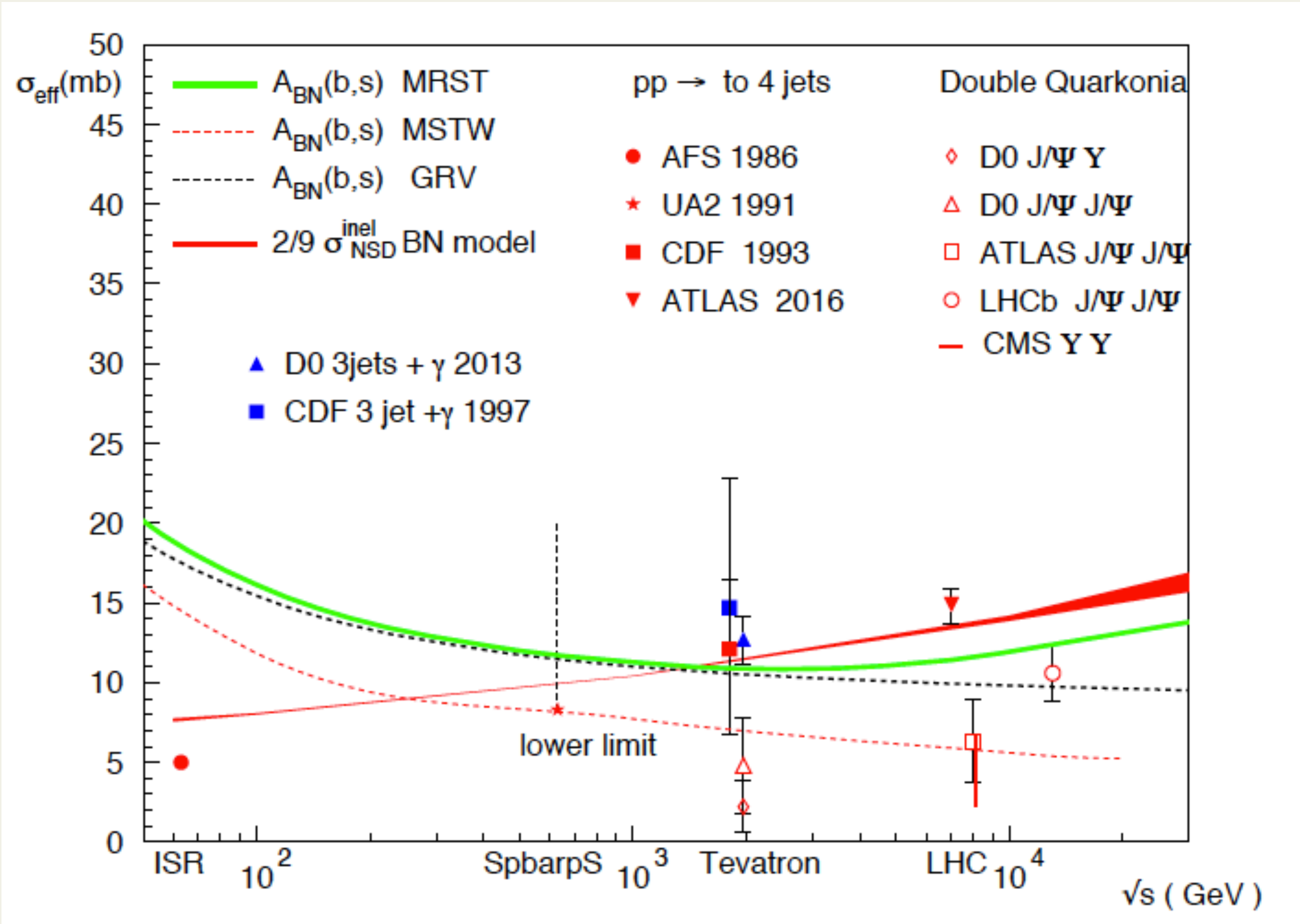
→ Situation is confused ?

Model

- Our model with GRV & MSTW : decrease with energy → constant
- MRST better -> rising?
- Still a different possibility – an old one
  - Paver&Treleani : total cross-section
  - AFS → 13 mb with some "fudge factor" ??? $\sigma_{inel}/2.32$

→ The inelastic cross-section ?

# Comparison of two models with data



# Conclusion

- Data need to be compared with similar final states to check energy behaviour of sigma effective
- A model with b-dependence related to energy dependent soft gluon resummation gives good estimate for scale

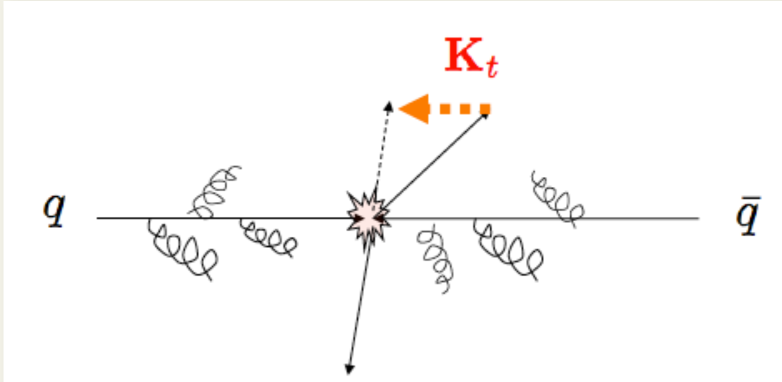
  but is not conclusive, the energy trend may depend on PDFs

- Data for gg initiated processes appear to indicate a rising effective cross-section →but different scale
- Interdependence with mini-jet production as in the inelastic non single diffractive cross-section describes better the energy trend of pp→ g g g g → 4 jets or Quarkonia
- Need to understand the "fudge factor", but inelastic cross-section model may allow better extrapolation to higher energies s

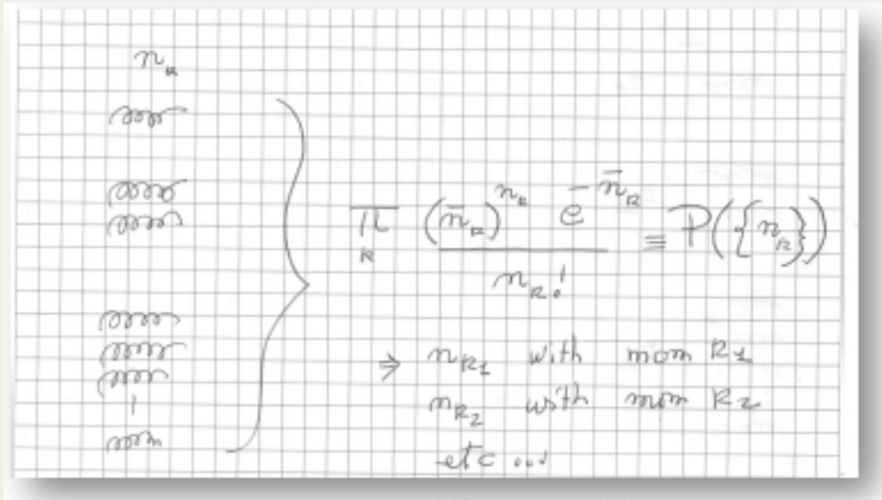
# To check the factorization hypothesis

- Model for T(b) from resummation?
- Compare wih extensive set of DPS from 63 GeV to 13 TeV is available
- Is there a different way to see the energy trends? [answer is yes]

# The BN inspired model for RESUMMING SOFT GLUONS




- Based on a democratic pathway to sum soft quanta – semiclassical approach with the ansatz :

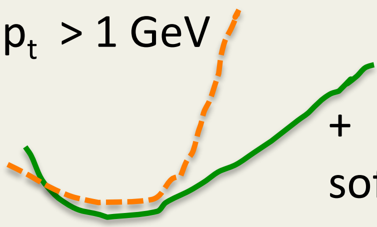


$$\alpha_{eff}(k_t \approx 0) \approx k_t^{-2p}$$

½<p<1

→ $$\sigma_{tot} \lesssim (\ln s)^{1/p}$$

Only minijets $p_t$ > 1 GeV



+ resummed soft gluons $k_t < \Lambda$

Grau, Godbole,GP,Srivastava, **Phys.Lett. B682 (2009)**

# The full eikonal in impact parameter space includes also a "soft" component

$$\bar{n}(b,s) = \bar{n}_{soft}(b,s) + \bar{n}_{mini-jets}(b,s) =$$
$$= A_{FF}(b,s)\sigma_{soft}(s) + A_{BN}(b,s)\sigma_{mini-jets}(s)$$

F-transform of the proton form factor

$$A_{FF}(b) = \frac{\nu^2}{96\pi}(\nu b)^3 K_3(\nu b)$$

ν =0.71 GeV²

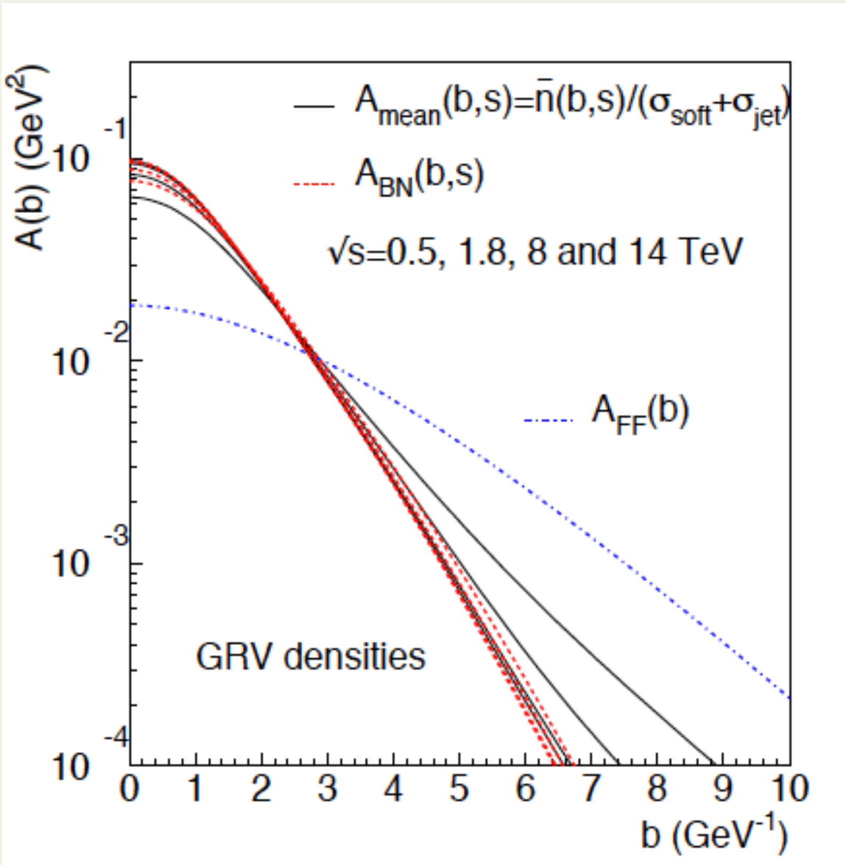
$$\sigma_{soft}(s) = constant \quad or \quad slowly \quad decreasing$$

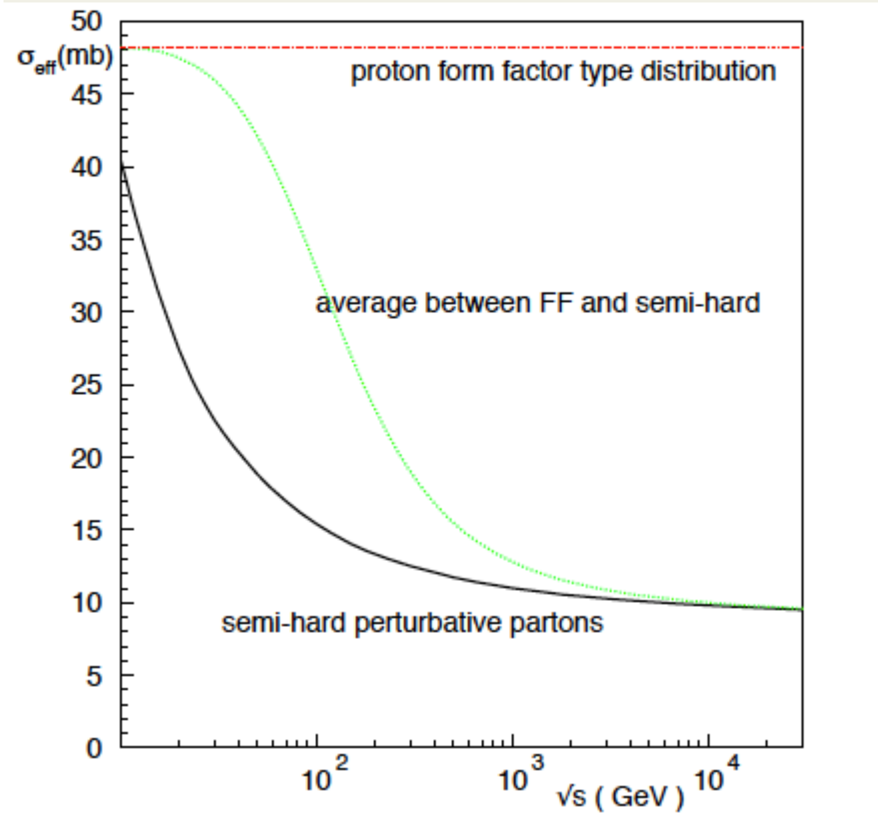
$$A(b,s;p,PDF,p_{min}) \equiv A_{BN}(b,s)$$

BN from resummation of soft gluons → into $k_t \approx 0$ ( Bloch & Nordsieck inspired)

# A(b,s) → $\sigma_{eff}(s)$

$$A_{mean}(b,s) = \frac{A_{FF}(b)\sigma_{soft}(s) + A_{BN}(b,s)\sigma_{mini-jets}(s)}{\sigma_{soft}(s) + \sigma_{mini-jets}(s)}$$



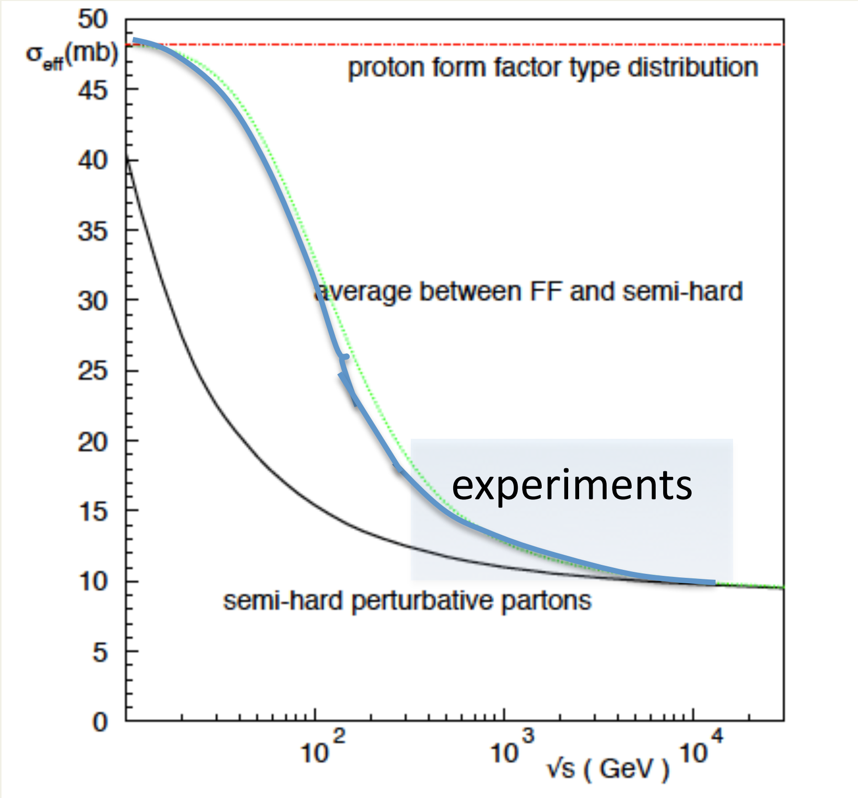



# $\sigma_{eff}(s)$ Averaging over both semi-hard (from resummation) and non-perturbative partons

$$A_{mean}(b,s) = \frac{A_{FF}(b)\sigma_{soft}(s) + A_{BN}(b,s)\sigma_{mini-jets}(s)}{\sigma_{soft}(s) + \sigma_{mini-jets}(s)}$$

- √s ≈ 10 GeV T(b) is dominated by Form Factor type partons
- √s ≈ 10 TeV T(b,s) is dominated by partons which engage with other partons from the other proton
- $\sigma_{eff}^{BN} \approx 10mb$ at √s=13 TeV

  ≈ lower than Strickman,
  ≈ D'Enterria,
  within experimental limit



# Similar procedure leads to $K_t$ resummation in QCD but…

$$h^{(PP)}(b,s) = \frac{4}{3\pi^2}\int_{M^2}^{Q^2} d^2k_\perp [1 - e^{i\mathbf{k}_\perp \cdot \mathbf{b}}]\alpha_s(k_\perp^2)\frac{\ln(Q^2/k_\perp^2)}{k_\perp^2}$$

G.Parisi R.Petronzio 1979
With Asymptotic Freedom

Our Proposal (ZPC 1984)

$$M^2 \to 0$$

$$\alpha_{strong}(k_t \le \Lambda) \to \alpha_{eff}(k_t) \to [\frac{\Lambda}{k_t}]^{2p}$$

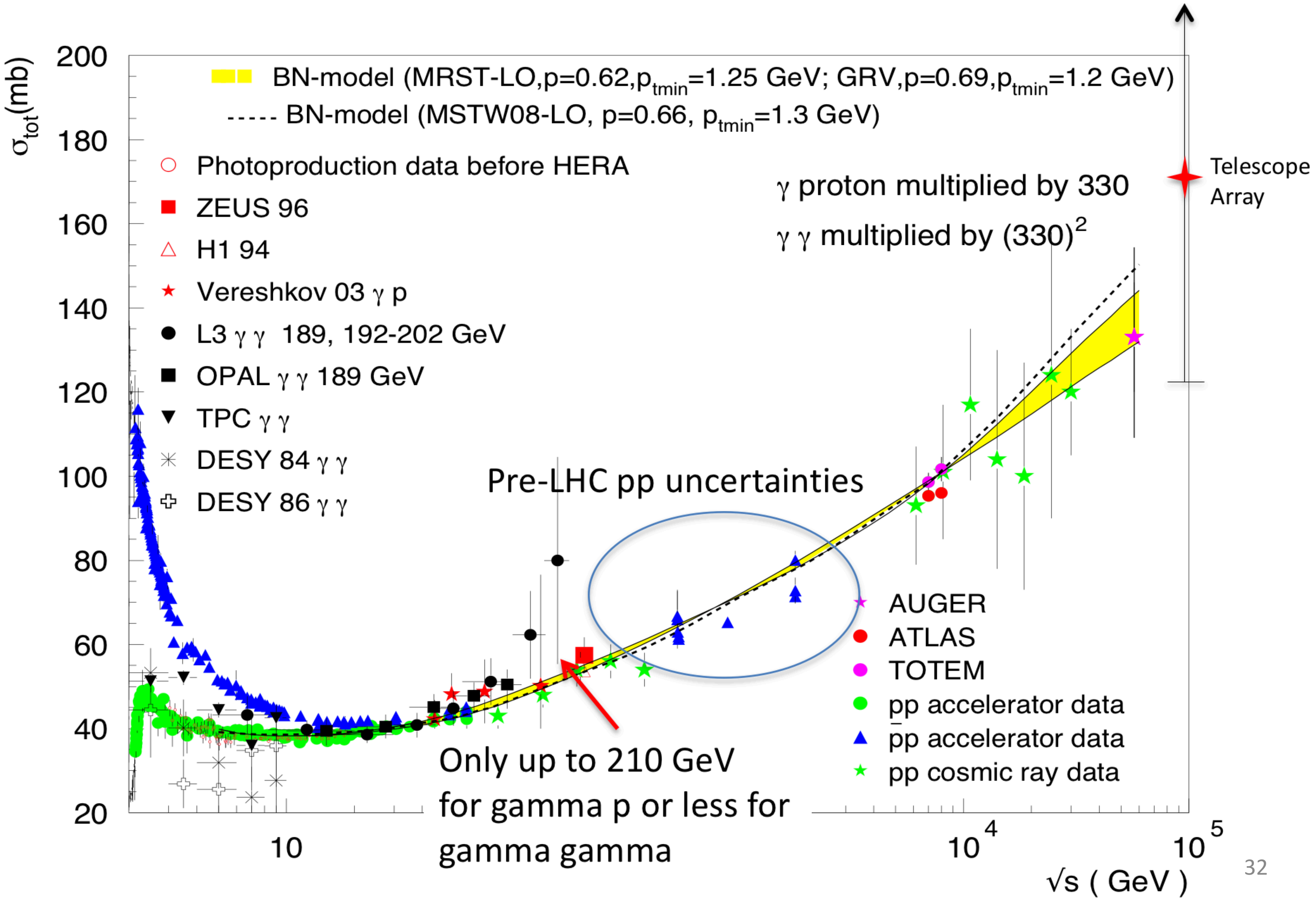
For integrability
And rising potential:

1/2<p < 1

# Total hadronic cross-sections

post-LHC update [before 13 TeV data]

from R.M. Godbole, A. Grau, G. Pancheri, Y.N. Srivastava, Eur.Phys.J. C63 (2009) 69-85



## pp Total, elastic and inelastic cross-sections → 13 TeV