



# IMPACT OF THE RECOIL SCHEME ON THE ACCURACY OF ANGULAR-ORDERED PARTON SHOWERS

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**LFC19:** Strong dynamics for physics within and beyond  
the Standard Model at LHC and Future Colliders

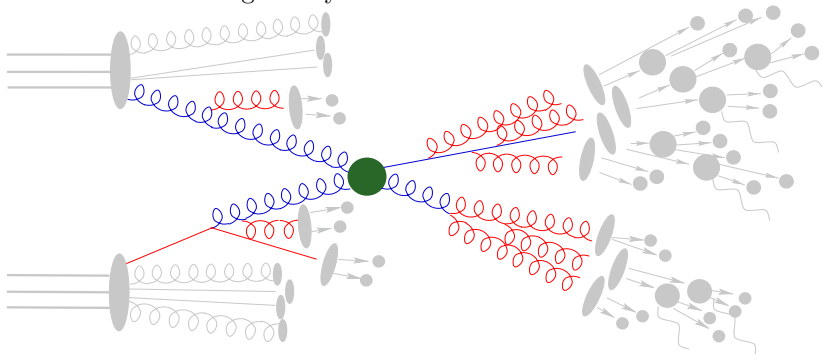
Based on Bewick, S.F.R., Richardson and Seymour [[arxiv:1904.11866](https://arxiv.org/abs/1904.11866)]

# Introduction: Shower Monte Carlo generators

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**Logarithmic accuracy of parton showers: a fixed-order study**, by Dasgupta, Dreyer, Hamilton, Monni and Salam, introduced approach for assessing the logarithmic accuracy of PS algorithms based on the ability to reproduce:

- 1 the singularity structure of multi-parton matrix elements
- 2 logarithmic resummation results

# Logarithmic accuracy of parton showers: a fixed-order study (I)

- Case of study: double gluon emission, well separated in rapidity, in  $e^+e^- \rightarrow q\bar{q}$ :

$$dP_2 = \frac{C_F^2}{2!} \prod_{i=1}^2 \frac{2\alpha_s(p_{T,i})}{\pi} \frac{dp_{T,i}}{p_{T,i}} d\eta_i$$

$$\text{where } \eta_i = -\log\left(\tan\left(\frac{\theta}{2}\right)\right)$$

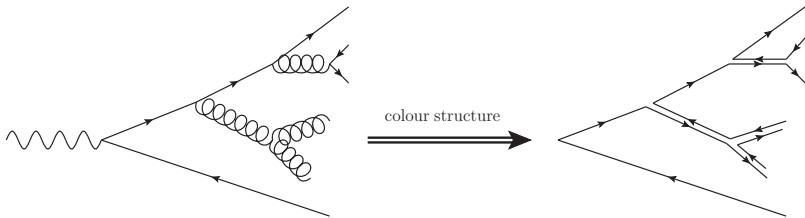
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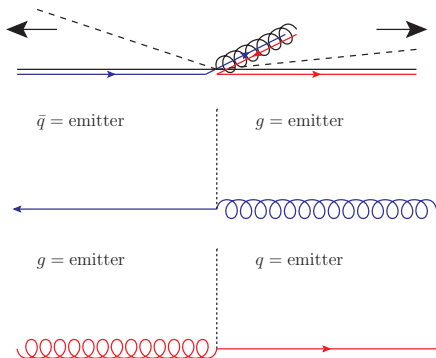
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- **Dipole showers** implemented in the Pythia8 and Sherpa generators were considered.



# Logarithmic accuracy of parton showers: a fixed-order study (II)

## Issue with dipole showers

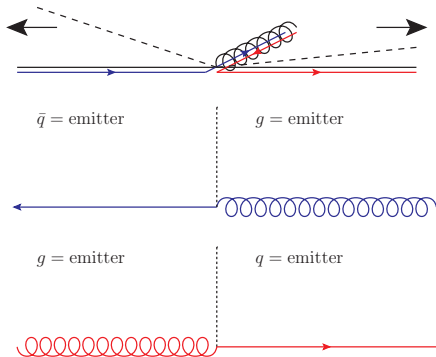


- Dipole frame: there are regions where the second gluon looks closer to the first gluon
- When the gluon is identified as emitter:
  - 1  $\vec{p}_{T,1} \rightarrow \vec{p}_{T,1} - \vec{p}_{T,2}$
  - 2 Wrong colour  $C_A$  instead of  $2C_F$  (subleading  $N_c$ ).



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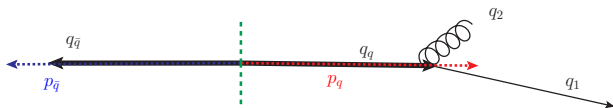
What happens for the **angular-ordered** shower implemented in Herwig7???

# Herwig7 angular-ordered parton shower

- The (anti-)quark is identified as **shower progenitor** and the anti-quark (quark) is its colour partner: each shower progenitor is showered independently in the frame where it is anti-collinear with the colour partner.

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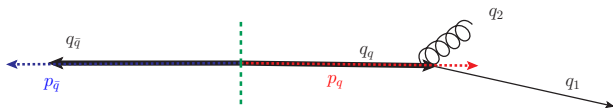


$$\begin{cases} q_1 &= z p_q + \beta_1 p_{\bar{q}} + p_T \\ q_2 &= (1-z) p_q + \beta_2 p_{\bar{q}} - p_T \\ q_q &= p_q + (\beta_1 + \beta_2) p_{\bar{q}} \end{cases}$$

$$\begin{aligned} \tilde{q}^2 &= \frac{q_q^2}{z(1-z)} = \frac{2q_1 \cdot q_2}{z(1-z)} = \frac{p_T^2}{z^2(1-z)^2} \\ &\sim E_q^2 \theta^2 \end{aligned}$$

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- Soft limit:**  $1 - z \equiv \epsilon \rightarrow 0$ ,

$$p_T \rightarrow \epsilon \tilde{q}, \quad \eta \rightarrow \log \left( \frac{Q}{\tilde{q}} \right)$$

$$dP^{\text{herwig}} \rightarrow 2C_F \frac{\alpha_s(\epsilon \tilde{q})}{\pi} \frac{d\tilde{q}}{\tilde{q}} \frac{d\epsilon}{\epsilon} = 2C_F \frac{\alpha_s(p_T)}{\pi} \frac{dp_T}{p_T} d\eta$$

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2  $\eta_2 > 0, |\eta_1 - \eta_2| \gg 1$ :

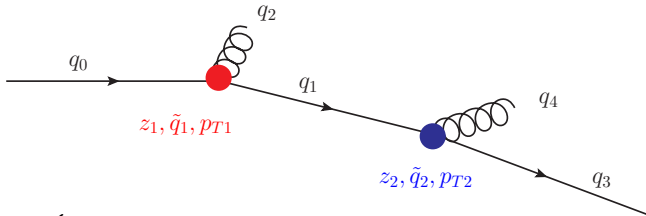


$|\eta_1 - \eta_2| \gg 1$ : this suppress the gluon splitting;  
angular ordering  $\mathbf{z}_1^2 \tilde{\mathbf{q}}_1^2 > \tilde{\mathbf{q}}_2^2$  imposes that the one with smallest rapidity comes first;

To do

We achieve the correct **colour factor**, we need to check the **recoil**

# Herwig7 angular-ordered parton shower



$$\left\{ \begin{array}{l} q_0 = p_q + (\beta_2 + \beta_3 + \beta_4)p_{\bar{q}} \\ q_1 = z_1 p_q + (\beta_3 + \beta_4)p_{\bar{q}} + p_{T1} \\ q_2 = (1 - z_1)p_q + \beta_2 p_{\bar{q}} - p_{T1} \\ q_3 = z_2 z_1 p_q + \beta_3 p_{\bar{q}} + z_2 p_{T1} + p_{T2} \\ q_4 = (1 - z_2)z_1 p_q + \beta_4 p_{\bar{q}} + (1 - z_2)p_{T1} - p_{T2} \end{array} \right.$$

$$\boxed{q_0^2 = \frac{p_{T1}^2}{z_1(1-z_1)} + \frac{q_1^2}{z_1}} \Rightarrow \text{Impossible to preserve simultaneously } q_q^2 \text{ and } p_{T1}^2$$

The choice of the preserved quantity determines the **recoil scheme**

The original (and simplest) choice of [hep-ph/0310083](#) (Gieseke, Stephens and Webber) is to preserve the **transverse momentum**:

$$\tilde{q}_i^2 = \frac{p_{Ti}^2}{z_i^2(1-z_i)^2}$$

- $p_{Ti}^2 = z_i^2(1-z_i)^2 \tilde{q}_i^2 \rightarrow \epsilon_i^2 \tilde{q}_i^2$        $\eta_i \rightarrow \log\left(\frac{Q}{\tilde{q}_i}\right)$

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- $q_0^2 = z_1(1-z_1)\tilde{q}_1^2 + \frac{z_2(1-z_2)\tilde{q}_2^2}{z_1}$

$\Rightarrow$  too much hard radiation in the parton shower as there is no compensation between the transverse momentum of the branching and the virtualities of the partons produced in the branching

In Ref. [1708.01491 \(Reichelt, Richardson and Siodmok\)](#) the **virtuality**-preserving scheme is introduced:

$$\tilde{q}_i^2 = \frac{q_i^2}{z_i(1-z_i)}$$

- The transverse momentum of the first emission is reduced

$$\boxed{p_{T1}^2} = \max [0, (1-z_1) [z_1^2(1-z_1)\tilde{q}_1^2 - z_2(1-z_2)\tilde{q}_2^2]]$$

$$\rightarrow \max [0, \epsilon_1 [\epsilon_1 \tilde{q}_1^2 - \epsilon_2 \tilde{q}_2^2]]$$

$$\boxed{\eta_1 \rightarrow \frac{1}{2} \log \left( \frac{Q^2}{\tilde{q}_1^2 - \frac{\epsilon_2}{\epsilon_1} \tilde{q}_2^2} \right)}$$

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- Better description of the tail of the distributions and in general better agreement with data

# Dot-product preserving scheme

In [1904.11866](#) we suggested something with intermediate properties

$$\tilde{q}^2 = \frac{2q_1 \cdot q_2}{z_i(1 - z_i)}$$

- The transverse momentum of the first emission is reduced by subsequent emissions

$$p_{T1}^2 = (1 - z_1)^2 \left[ z_1^2 \tilde{q}_1^2 - \sum_{i=2}^n z_i(1 - z_i) \tilde{q}_i^2 \right]$$

but  $\tilde{q}_{i+1} < z_i \tilde{q}_i$  implied  $p_{T1} > 0$  even for infinite emissions;  
the **double-soft** limit is correct

$$p_{T1}^2 \rightarrow \epsilon_1^2 [\tilde{q}_1^2 - \epsilon_2 \tilde{q}_2^2] \rightarrow \epsilon_1^2 \tilde{q}_1^2,$$

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- However, the virtuality still increases ...

$$q_0^2 = z_1(1 - z_1) \tilde{q}_1^2 + z_2(1 - z_2) \tilde{q}_2^2$$

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$$\frac{d\Phi_n(q, \bar{q}, \dots)}{d\Phi_2(q, \bar{q})} = \lambda\left(1, \frac{q_q^2}{s}, \frac{q_{\bar{q}}^2}{s}\right) \prod_{i=1}^n \frac{d\tilde{q}_i^2}{(4\pi)^2} z_i(1-z_i) dz_i$$

where  $\lambda(1, a, b) = \sqrt{1 - 2(a + b) + (a^2 - b^2)^2}$ .

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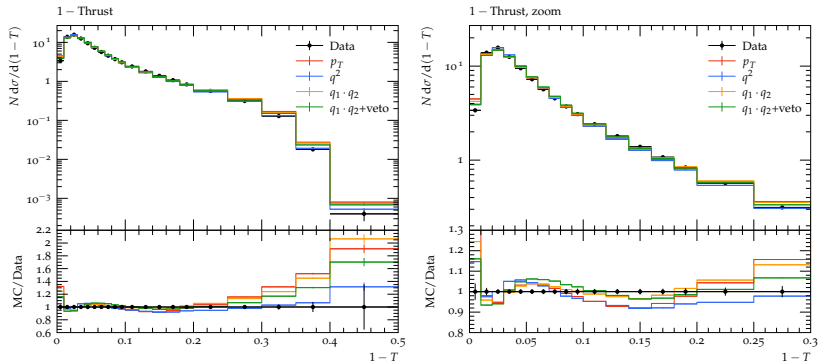
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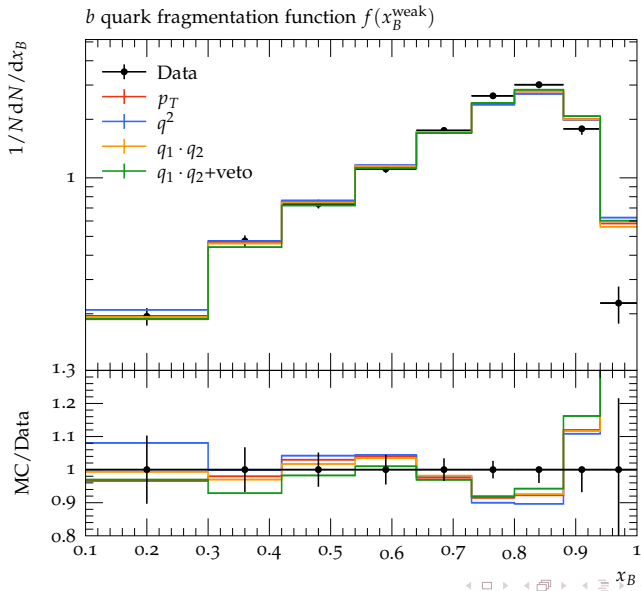
- $\lambda \approx 1$  if the emissions are all soft or collinear and is far from 1 in the hard region of the spectrum.
- We can accept the event with probability  $\lambda$  to improve the description of the tail of the distributions, (large virtualities) without spoiling the soft-collinear region (small virtualities).

## Thrust, DELPHI 1996



$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

## Energy distribution of weakly-decaying $b$ hadrons from DELPHI 2011



# Summary and Outlook

- We need a recoil scheme for final-state radiation able to describe multiple soft-collinear emissions that does not overpopulate the hard region of the spectrum;
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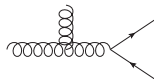
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- Open issues:



- ordering condition in case of massive recoilers:  
 $p_T$  can become negative also in the dot-scheme;

- $g \rightarrow q\bar{q}$ : argument of  $\alpha_s$  and ordering condition for massive  $q$ ;



- $b$ -quark fragmentation (does it depend on the PS or on the hadronization model?);

# BACKUP

- Prior the shower  $p_i = \{\sqrt{m_i^2 + |\vec{q}_i|}, \vec{p}_i\}$  that satisfy

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$$q_i \xrightarrow{\beta_i} q'_i = \{\sqrt{q_i^2 + \lambda^2 |\vec{p}_i|^2}, \lambda \vec{p}_i\} \Rightarrow \sum_i \vec{q}'_i = \lambda \sum_i \vec{p}_i = \vec{0}$$

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If only soft-emissions take place  $q_i^2 = m_i^2 + \mathcal{O}(\epsilon)$ , thus this boost gives subleading contributions

# Double soft-emission kinematics

Summary of Lund variables:

Preserved quantity	$p_T^2$	$q^2$	$q_1 \cdot q_2$
$p_{T1}^2$	$\epsilon_1^2 \tilde{q}_1^2$	$\epsilon_1 [\epsilon_1 \tilde{q}_1^2 - \epsilon_2 \tilde{q}_2^2]$	$\epsilon_1^2 \tilde{q}_1^2$
$\eta_1$	$\frac{1}{2} \log \left( \frac{Q^2}{\tilde{q}_1^2} \right)$	$\frac{1}{2} \log \left( \frac{Q^2}{\tilde{q}_1^2 - \frac{\epsilon_2}{\epsilon_1} \tilde{q}_2^2} \right)$	$\frac{1}{2} \log \left( \frac{Q^2}{\tilde{q}_1^2} \right)$
$p_{T2}^2$		$\epsilon_2^2 \tilde{q}_2^2$	
$\eta_2$		$\frac{1}{2} \log \left( \frac{Q^2}{\tilde{q}_2^2} \right)$	

- The kinematics of the second emission is always correct;
- The kinematic of the first emission is correct in the  $p_T$  and dot-product preserving schemes.

# Double soft-emission kinematics

Emission of two-soft gluons with Lund variables  $k_{T_a}^2, \eta_a$  and  $k_{T_b}^2, \eta_b$ .

$$dP_2^{\text{exact}} = \frac{C_F^2}{2!} \frac{\alpha_s^2}{\pi^2} \left[ \frac{dk_{T_a}^2}{k_{T_a}^2} d\eta_a \right] \left[ \frac{dk_{T_b}^2}{k_{T_b}^2} d\eta_b \right]$$

$$\frac{dP_2^{\text{herwig}}}{dk_{T_a}^2 d\eta_a dk_{T_b}^2 d\eta_b} = \int \frac{C_F^2}{2!} \frac{\alpha_s^2}{\pi^2} \prod_{i=1}^2 \left[ \frac{d\tilde{q}_i^2}{\tilde{q}_i^2} \frac{d\epsilon_i}{\epsilon_i} \right] \Theta(\tilde{q}_1^2 - \tilde{q}_2^2)$$

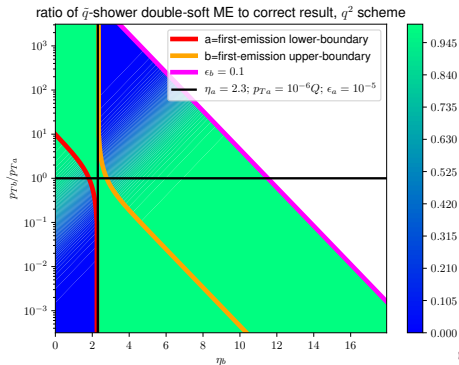
$$\times [\delta(\eta_1 - \eta_a) \delta(k_{T_1}^2 - k_{T_a}^2) \delta(\eta_2 - \eta_b) \delta(k_{T_2}^2 - k_{T_b}^2) + a \leftrightarrow b]$$

- The  $p_T^2$  and  $q_1 \cdot q_2$  preserving schemes yield the correct double soft limit;
- For the  $q^2$  scheme:

$$R = \frac{1}{1 + \frac{k_{T_b}}{k_{T_a}} e^{\eta_a - \eta_b}}$$

$$\times \Theta\left(\frac{k_{T_b}}{k_{T_a}} - 2 \sinh(\eta_a - \eta_b)\right)$$

$$+ a \leftrightarrow b$$



Each modification of the PS requires a new tuning of the hadronization parameters: interplay perturbative & non perturbative

Preserved	$p_T$	$q^2$	$q_i \cdot q_j$	$q_i \cdot q_j + \text{veto}$
<b>Light-quark hadronization and shower parameters</b>				
AlphaMZ ( $\alpha_s^{\text{CMW}}(M_Z)$ )	<b>0.1074</b>	<b>0.1244</b>	<b>0.1136</b>	<b>0.1186</b>
pTmin	0.900	1.136	0.924	0.958
ClMaxLight	4.204	3.141	3.653	3.649
ClPowLight	3.000	1.353	2.000	2.780
PSplitLight	0.914	0.831	0.935	0.899
PwtSquark	0.647	0.737	0.650	0.700
PwtDIquark	0.236	0.383	0.306	0.298
<b>Bottom hadronization parameters</b>				
ClMaxBottom	5.757	2.900	6.000	3.757
ClPowBottom	0.672	0.518	0.680	0.547
PSplitBottom	0.557	0.365	0.550	0.625
ClSmrBottom	0.117	0.070	0.105	0.078
SingleHadronLimitBottom	0.000	0.000	0.000	0.000
<b>Charm hadronization parameters</b>				
ClMaxCharm	4.204	3.564	3.796	3.950
ClPowCharm	3.000	2.089	2.235	2.559
PSplitCharm	1.060	0.928	0.990	0.994
ClSmrCharm	0.098	0.141	0.139	0.163
SingleHadronLimitCharm	0.000	0.011	0.000	0.000