Higher order QCD corrections to Higgs boson transverse-momentum distribution

Chris Wever (TUM)

In collaboration with: F. Caola, K. Kudashkin, J. Lindert , K. Melnikov, P. Monni, L. Tancredi

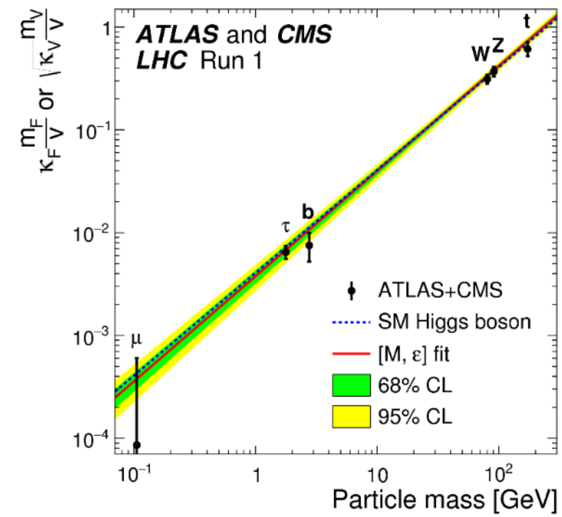
TECHNISCHE UNIVERSITÄT MÜNCHEN

LFC19, Trento, 9-13 September, 2019

Introduction (1/6): Higgs couplings as probe to New Physics

- *Questions:* is the Higgs the SM Higgs? Is it composite? Does it couple to other particles outside the SM picture or can we use it as a probe of BSM?

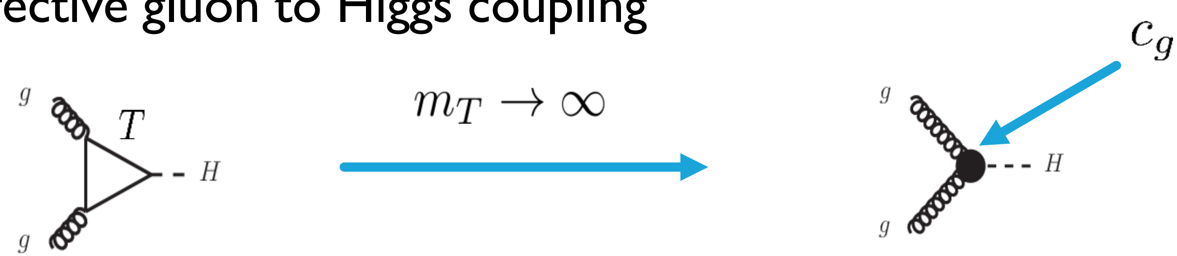
- To test: measure the couplings to other SM particles and search for deviations from theory



[arXiv:1606.02266]

- Many beyond the SM (BSM) models lead to different Higgs couplings

- For example models with extra massive top partners T contribute to direct effective gluon to Higgs coupling

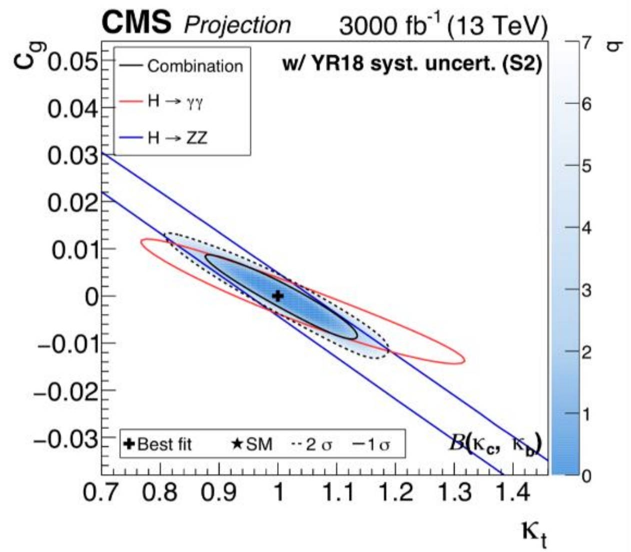


Introduction (2/6): New Physics probe

- In practice the Higgs couplings deviations studied in SMEFT framework, where all non-SM particles are integrated out and most general Lagrangian consistent with SM symmetries left
- The SMEFT Lagrangian extends the SM Lagrangian to include also higher dimension operators, e.g.

$$\frac{m_t}{v}\bar{t}tH \rightarrow -c_g \frac{\alpha_s}{12\pi v} G^a_{\mu\nu} G^{\mu\nu,a} H + \kappa_t \frac{m_t}{v}\bar{t}tH$$

- In the SM, $c_g = 0$ but in SMEFT it arises effectively after integrating out massive top-partners
- Varying the couplings and comparing with measured cross sections gives constraints on the effective couplings in SMEFT

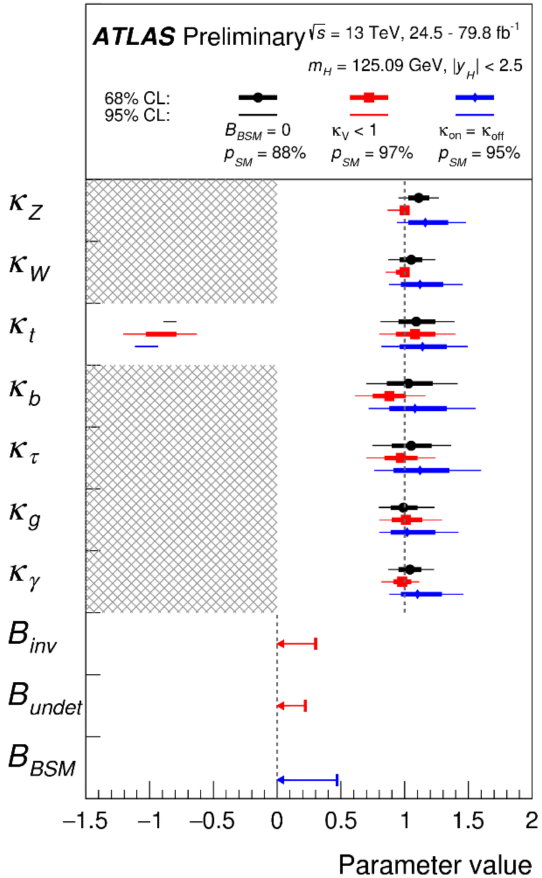


[arXiv:1902.00134]

Introduction (3/6): Higgs couplings

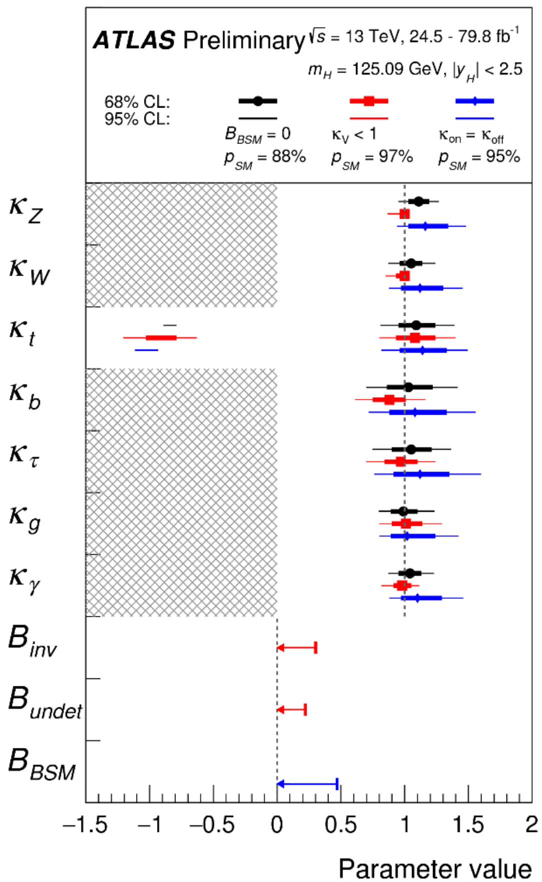
Present

[arXiv:1902.00134]



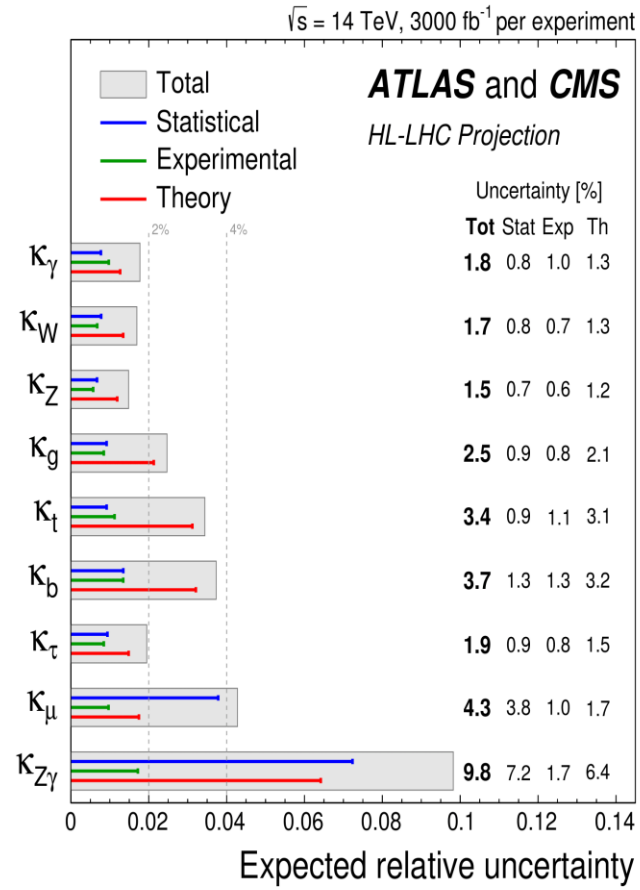
Introduction (3/6): Higgs couplings

Present



Future

[arXiv:1902.00134]

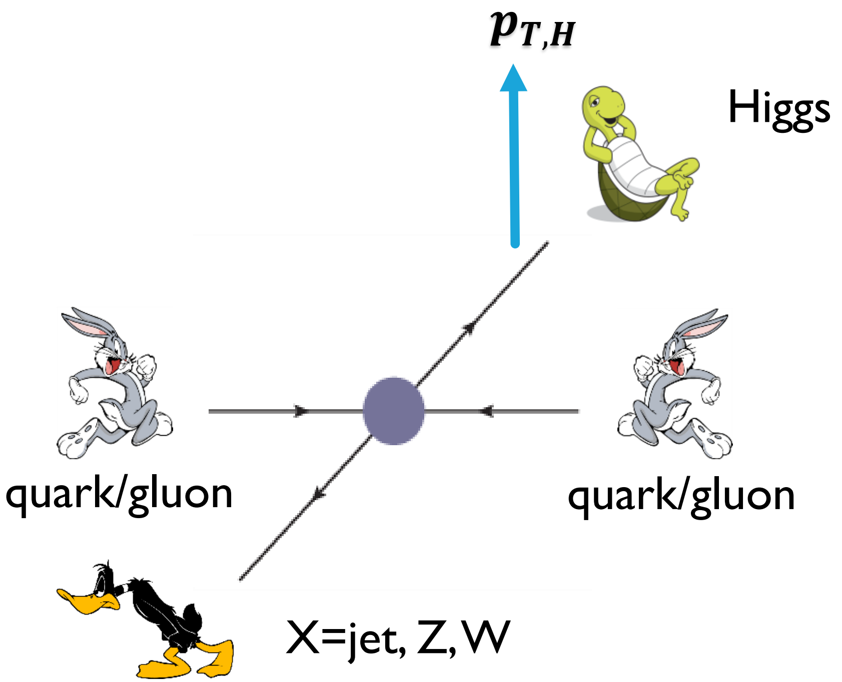


No New Physics found at LHC as of yet → **Entering the era of precision (%) Higgs physics**

Introduction (4/6): Higgs transverse momentum distribution

- If the Higgs recoils against another particle X, it acquires a transverse momentum ($p_{T,H}$)

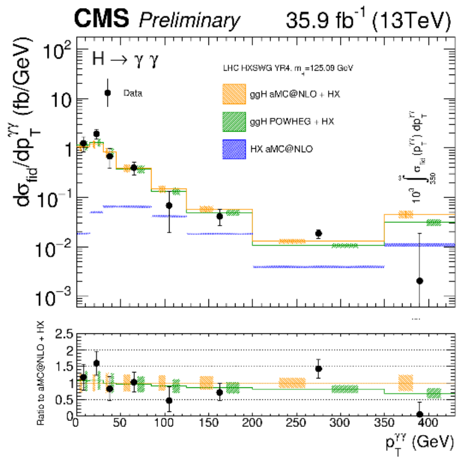
- The transverse momentum distribution of the Higgs contains much more information than full inclusive cross section



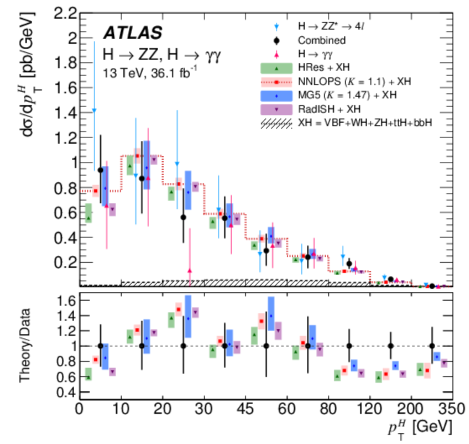
- Largest contribution comes from recoil to quarks and gluons (jet)

Introduction (5/6): Transverse momentum distribution measurement

- Atlas and CMS have started measuring the Higgs transverse momentum ($p_{T,H}$), with errors currently in the range of 20-40%, but the error will decrease



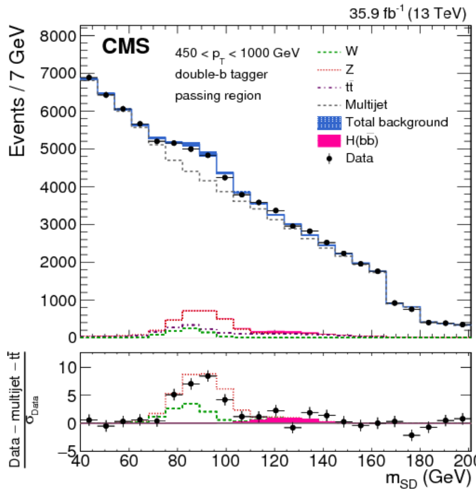
[CMS-PAS-HIG-17-015]



[CERN-EP-2018-080]

- Recently CMS and Atlas started probing high $p_{T,H}$ values (tail) of the Higgs through decay to bottoms, but the error is still ~100%

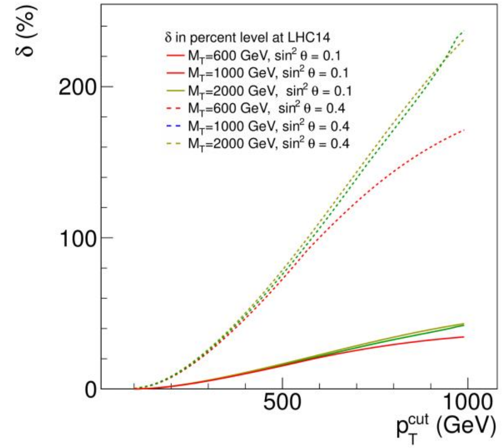
- Alternative method to measure the Higgs to bottom decay channel



[CMS-HIG-17-010-003]

Introduction (6/6): Higgs tail

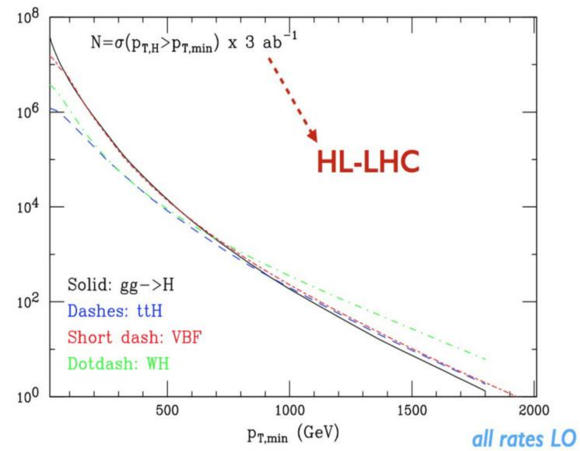
- The tail (i.e. large values) of the $p_{T,H}$ distribution is important for probing the effective gluon-Higgs coupling c_g, since at large $p_{T,H}$, corrections from top-partners get enhanced compared to top-contribution



[Banfi, Martin, Sanz, arXiv:1308.4771]

- Can distinguish the gluon-Higgs c_g from top-Yukawa coupling κ_t

- At future HL-LHC there will be enough events to probe tail of the Higgs transverse momentum distribution with good accuracy ~10% error

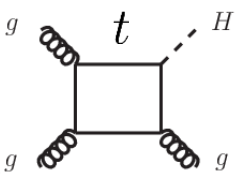


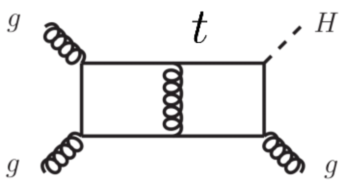
[Mangano talk at Higgs Couplings 2016]

- For comparison we will need accurate SM theory predictions of the tail (focus of talk)

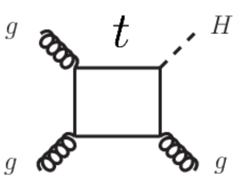
Recoiled Higgs Feynman diagrams at NLO

- To study Higgs transverse momentum distribution (coming from recoil off jet) we consider an extra quark/gluon in the final state

LO: 

NLO, not all computed: 

- Feynman integrals at LO:



$$\sim \int \frac{d^d k}{(k^2 - m_t^2)((k+p_1)^2 - m_t^2)((k+p_{12})^2 - m_t^2)((k+p_{123})^2 - m_t^2)}$$

- Either one evaluates integrals numerically (advantage: algorithmic, disadvantage: precision) or analytically (advantage: good precision, disadvantage: not always algorithmic)

- To overcome analytic difficulty, notice that at the tail, $m_t/p_{T,H} \ll 1$, so we may expand in small top mass m_t!

Expansion in small parameters

- Integrals with massive quark loops are complicated at NLO

$$\begin{aligned}
&\log\left(x_3x_1^2 - x_1^2 + x_2x_1 - 4x_3x_1 + R_1(x_1)R_2(x_1)R_7(x)\right),\\
&\log\left(-x_2^2 + x_1x_2 - x_1x_3x_2 + 2x_3x_2 + 2x_1x_3 + R_1(x_2)R_2(x_2)R_7(x)\right),\\
&\log\left(-x_3^2x_1^2 + 3x_3x_1^2 + 4x_3^2x_1 - 4x_2x_3x_1 + R_1(x_3)R_5(x)R_6(x)x_1\right),\\
&\log\left(x_3R_1(x_2)R_2(x_2) + x_2R_1(x_3)R_2(x_3)\right),\\
&\log\left(x_1R_1(x_2)R_2(x_2) + x_2R_1(x_1)R_2(x_1)\right),\\
&\log\left(x_1R_1(x_3)R_2(x_3) - R_1(x_1)R_1(x_3)R_5(x)\right),\\
&\log\left(x_3R_1(x_1)R_2(x_1) - R_1(x_1)R_1(x_3)R_5(x)\right),\\
&\log\left(-x_2R_1(x_1)R_2(x_1) + x_3R_1(x_1)R_2(x_1) + x_1R_3(x_3)R_4(x_3)\right),\\
&\log\left(-x_2R_1(x_2)R_2(x_2) + x_3R_1(x_2)R_2(x_2) + x_2R_3(x_3)R_4(x_3)\right),\\
&\log\left(-x_2R_1(x_3)R_2(x_3) + x_1R_1(x_3)R_2(x_3) + x_3R_3(x_1)R_4(x_1)\right),\\
&\log\left(-x_2R_1(x_2)R_2(x_2) + x_1R_1(x_2)R_2(x_2) + x_2R_3(x_1)R_4(x_1)\right),\\
&\log\left(-x_3^2x_1^2 + 3x_3x_1^2 + 4x_3^2x_1 - 3x_2x_3x_1 + R_1(x_1)R_1(x_3)R_5(x)R_7(x)\right),\\
&\log\left(x_2R_1(x_1)R_1(x_3)R_5(x) - x_1x_3R_1(x_2)R_2(x_2)\right),\\
&\log\left(-x_2x_3 + x_1x_3 + R_1(x_2)R_2(x_2)x_3 - R_1(x_1)R_1(x_3)R_5(x)\right).
\end{aligned}$$

$$\begin{aligned}
&R_1(x_1) = \sqrt{-x_1}\,,\ R_1(x_3) = \sqrt{-x_3}\,,\ R_1(x_2) = \sqrt{-x_2}\,,\\
&R_2(x_1) = \sqrt{4-x_1}\,,\ R_2(x_3) = \sqrt{4-x_3}\,,\ R_2(x_2) = \sqrt{4-x_2}\,,\\
&R_3(x_1) = \sqrt{x_2-x_1}\,,\ R_3(x_3) = \sqrt{x_2-x_3}\,,\\
&R_4(x_1) = \sqrt{x_2-x_1-4}\,,\ R_4(x_3) = \sqrt{x_2-x_3-4}\,,\\
&R_5(x) = \sqrt{4x_2 + x_1x_3 - 4(x_1+x_3)}\,,\\
&R_6(x) = \sqrt{2x_3(-2x_2+x_1+2x_3) - x_1x_3^2 - x_1}\,,\\
&R_7(x) = \sqrt{2x_1x_3(x_2-x_1) + (x_2-x_1)^2 + (x_1-4)x_1x_3^2}\,.
\end{aligned}$$

[Bonciani et al '16]

- As perturbation has thought us, expanding in small parameters useful

$$I_{abc}(m_t) := \int {}_2F_1\left(a,b,c;m_tx\right)\sqrt{1+x^2}dx = \frac{1}{2}\left(x\sqrt{x^2+1}+\sinh^{-1}(x)\right) + \frac{ab\left(x^2+1\right)^{3/2}}{3c}m_t + \mathcal{O}(m_t^2)$$

- There exist methods for expanding under the integral signs of Feynman integrals (*expansion by regions*) but they can be complicated and are not always algorithmic
- Easier is to use differential equations

Differential equations

- Taking derivative w.r.t. m_t of previous example

$$\frac{d}{dm_t} I_{abc}(m_t) = \int \frac{abx\sqrt{x^2+1}\,{}_2F_1(a+1,b+1;c+1;m_t x)}{c} dx$$

- Hypergeometric functions with shifted indices are related

$${}_2F_1(a,b;c;z) = \frac{(z(a-b-1)+2b-c+2)\,{}_2F_1(a,b+1;c;z)}{b-c+1} + \frac{(b+1)(z-1)\,{}_2F_1(a,b+2;c;z)}{b-c+1}$$

- If one considers the derivative of full *class* of functions with integer indices $\{a, b, c\}$, the system of derivatives sometimes closes onto itself

- This happens with Feynman integrals by using so-called integration by parts identities (IBP)

$$I_{abcd} = \int \frac{d^d k}{(k^2-m_t^2)^a((k+p_1)^2-m_t^2)^b((k+p_{12})^2-m_t^2)^c((k+p_{123})^2-m_t^2)^d} \stackrel{\text{IBP}}{=} c_1 I_{1111}(m_t) + c_2 I_{1110}(m_t) + \cdots$$

- Because of these identities, taking a derivative w.r.t. the mass m_t, a closed system of DE for so-called *Master Integrals* can be derived

Expanding with differential equations

- System of linear differential equations (DE) in m_t with IBP relations

$$\frac{\partial}{\partial m_t}\vec{I}^{MI}(m_t,\epsilon) \stackrel{\text{IBP}}{=} \text{Matrix}(m_t,\epsilon)\cdot\vec{I}^{MI}(m_t,\epsilon)$$

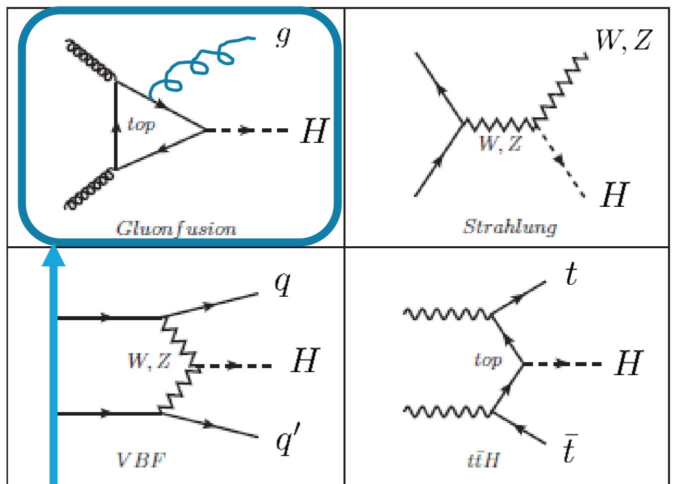
- Solve DE in m_t with following ansatz

$$I_i^{MI}(m_t^2,s,t,m_h^2,\epsilon) = \sum_{ijkn} c_{ijkn}(s,t,m_h^2,\epsilon)\left(\frac{m_t^2}{s}\right)^{j-k\epsilon}\log^n\left(\frac{m_t^2}{s}\right)$$

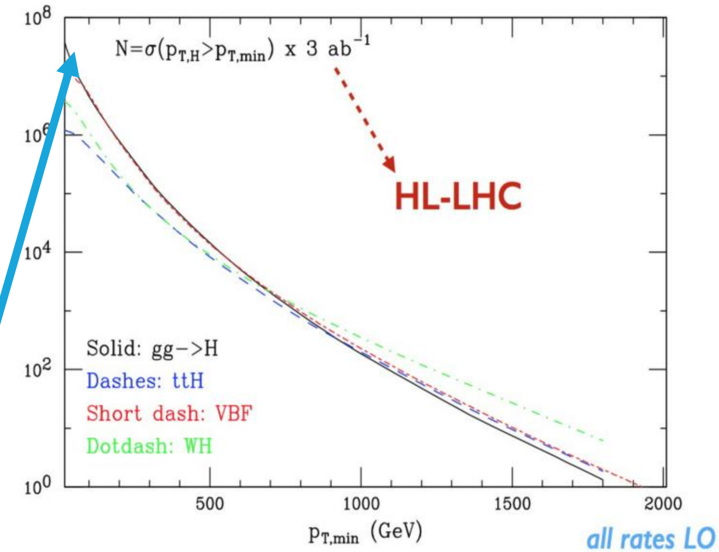
- The appearance of logarithms in m_t indicates that we could not have just expanded in small m_t under integral sign

- The coefficients c_{ijkl} are typically much easier to compute, both analytically and to evaluate numerically. This way we find a perturbative expression for the cross section in small top mass m_t (i.e. $m_t/p_{T,H} \ll 1$)

Recoiled Higgs production at the LHC (1/2)

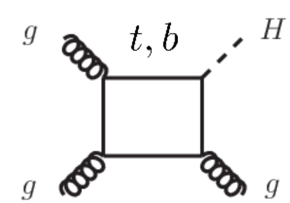


Main channels



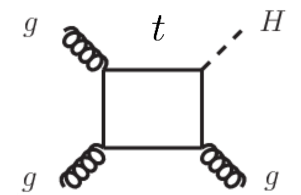
[Mangano talk at Higgs Couplings 2016]

- Largest contribution: gluon fusion through quark loop



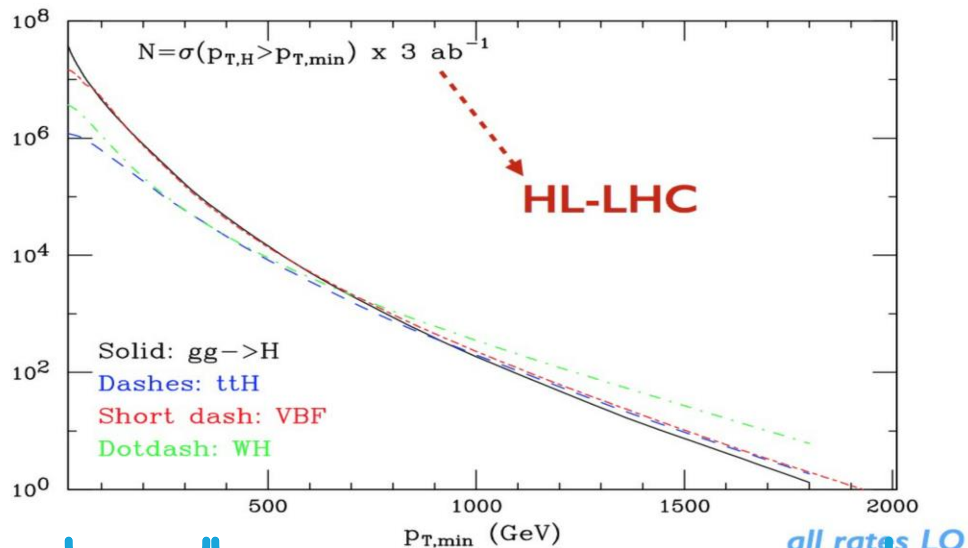
- Top quark loop ~ 55% and bottom loop ~ 1-5%
- Other diagrams (VBF, Strahlung and ttH) contribute about ~ 40%

Recoiled Higgs production at the LHC (2/2)



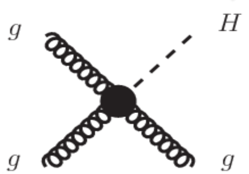
- We focus on gluon fusion through top quark
- $p_{T,H}$ distribution split into two regions: $p_{T,H}$ <350 GeV and $p_{T,H}$ >350 GeV

[Mangano talk at Higgs Couplings 2016]



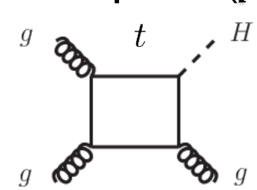
Below top thr. ($p_{T,H}$ <350 GeV)

HEFT



$m_t \to \infty$

Above top thr. ($p_{T,H}$ >350 GeV)

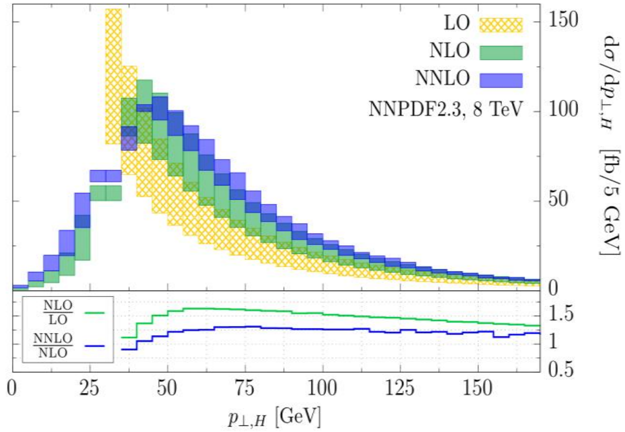


$\frac{4m_t^2}{p_\perp^2} \ll 1$

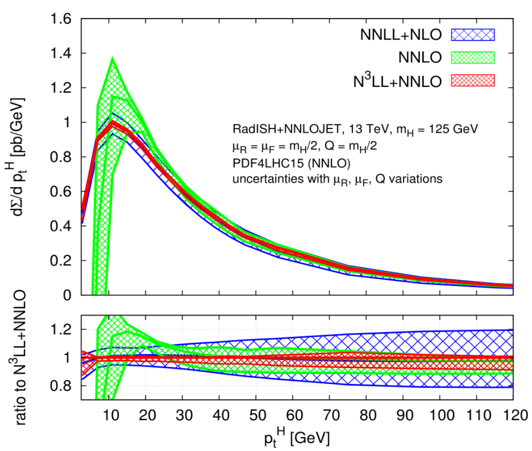
Higgs tail

Recent gg-fusion theory progress

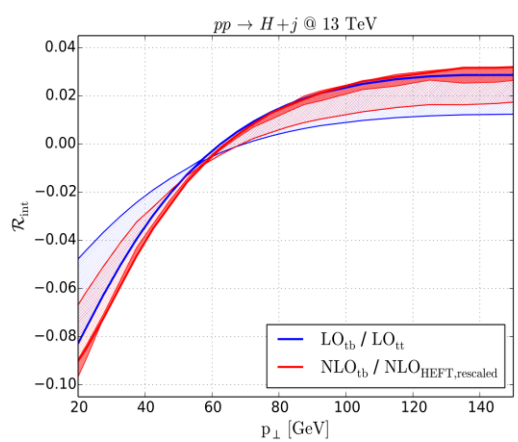
- Fixed order at NNLO QCD in HEFT
- Low $p_{T,H}$ resummation at N3LL+NNLO QCD in HEFT
- Bottom mass corrections at NLO QCD
- High (tail) $p_{T,H}$ region at NLO QCD with full top mass



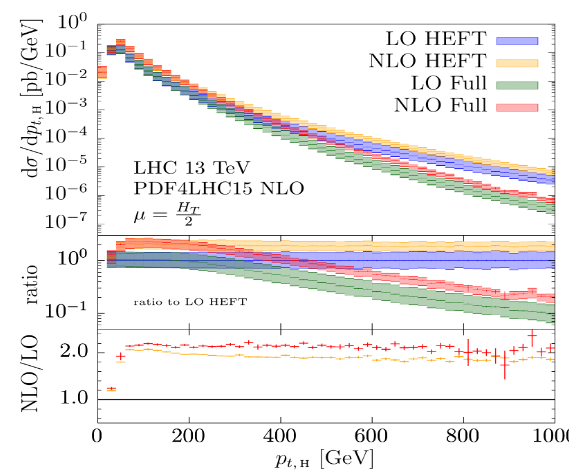
[Boughezal, Caola et al., arXiv: 1504.07922]



[Bizon, Chen et al., arXiv: 1805.0591]



[Lindert et al., arXiv: 1703.03886]



[Jones et al., arXiv: 1802.00349]

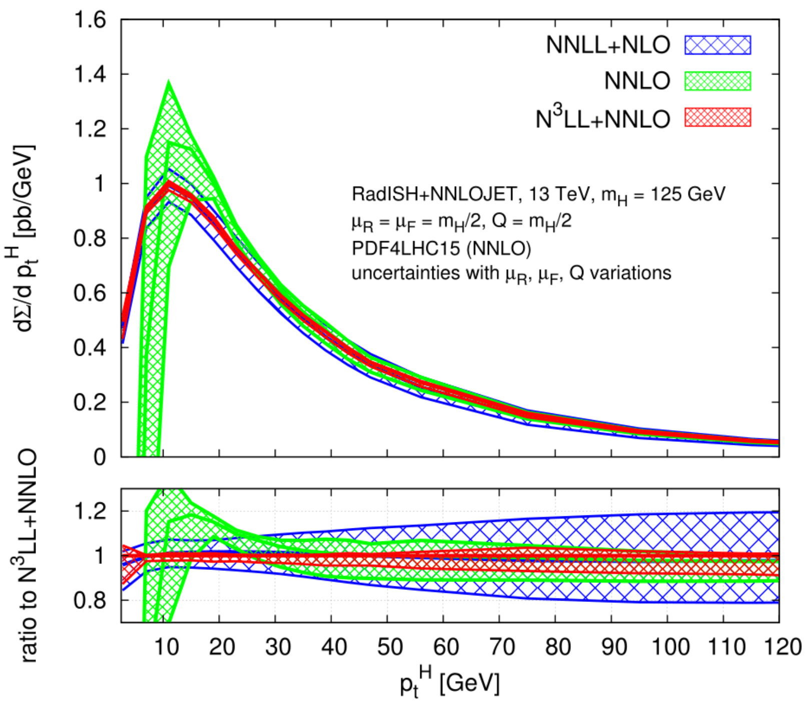
Below top threshold $p_{T,H} \leq 350$ GeV

- Infinite top mass approximation valid

- Large Sudakov logarithms at very low $p_{T,H} \leq 30$ GeV

$$\frac{d\sigma}{dp_{T,H}} \sim \exp\{\alpha_s \log^2\left(\frac{p_{T,H}}{m_h}\right) + \alpha_s \log\left(\frac{p_{T,H}}{m_h}\right) + \cdots\}$$

- Higgs distribution at low $p_{T,H} \leq 30$ GeV requires resumming these logarithms. Perturbative expansion good at higher $p_{T,H} > 30$ GeV



[Bizon, Chen et al., arXiv: 1805.0591]

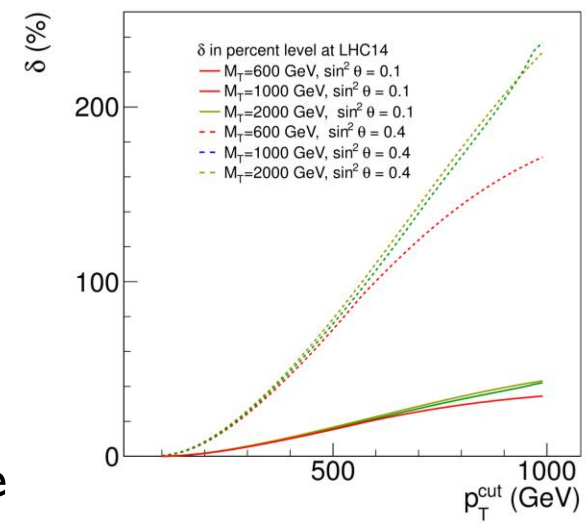
- Resummation reduces scale error: top contribution now understood well to within few percent error

Tail of the Higgs: $p_{T,H} \geq 350$ GeV

- CMS and ATLAS have begun probing tail of the Higgs
- At HL-LHC enough statistics for differential at $p_{T,H} \geq 350$ GeV
- Higgs couplings to top-partners induce effective gluon-Higgs coupling

$$\frac{m_t}{v}\bar{t}tH \to -c_g \frac{\alpha_s}{12\pi v} G^a_{\mu\nu} G^{\mu\nu,a} H + \kappa_t \frac{m_t}{v}\bar{t}tH$$

- Inclusive rate only constrains sum $c_g + \kappa_t$, while tail of Higgs distribution can disentangle the two contributions



[Banfi, Martin, Sanz, arXiv:1308.4771]

- Theoretical complication: infinite top mass approximation breaks down at large $p_{T,H}$ and top mass corrections cannot be neglected

High $p_{T,H}$: boosted regime

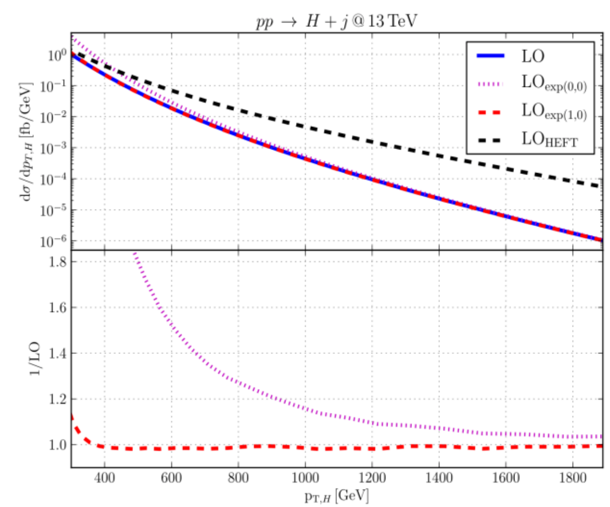
- Amplitude contains enhanced Sudakov-like logarithms above top threshold

$$p_{\mathrm{T,H}} > 2m_t \sim 350\,\mathrm{GeV}\ : \qquad \mathcal{A}^{\text{top-loop}}_{gg\to Hg} = \frac{y_t m_t}{p_{\mathrm{T,H}}}\left\{\log^2\left(\frac{4m_t^2}{p_{\mathrm{T,H}}^2}\right) + \mathcal{O}\left(\frac{4m_t^2}{p_{\mathrm{T,H}}^2}\right)\right\}$$

- Use scale hierarchy, $p_{T,H} > 2m_t$ to expand result in "small" top mass

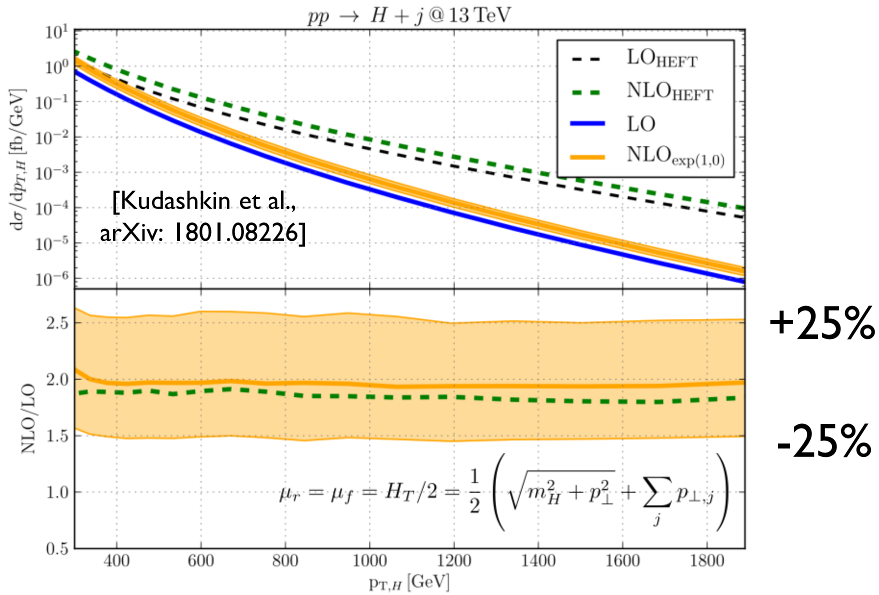
- Expansion in Higgs and top mass converges quickly

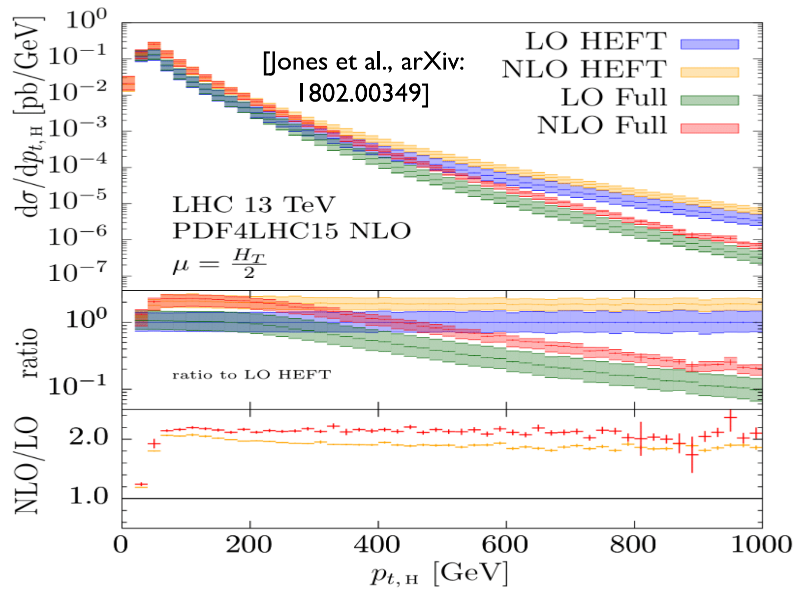
- In practice first top-mass correction is enough for approximating exact result within 1%



[Kudashkin et al., arXiv: 1801.08226]

High $p_{T,H}$: NLO results





Feynman integrals: expansion in top mass

Feynman integrals: numerical evaluation

- The top mass expansion and numerical predictions agree very well
- Comparison with CMS

$$\sigma^{\text{theory,NLO}}_{p_{T,H}\geq 450\,\text{GeV}}(gg \to H(\to b\bar{b})) \sim 7\,\text{fb} \pm 25\%$$

$$\sigma^{\text{CMS}}_{p_{T,H}\geq 450\,\text{GeV}}(gg \to H(\to b\bar{b})) \sim 74 \pm 48(\text{stat}) \pm 17(\text{syst})\,\text{fb}$$

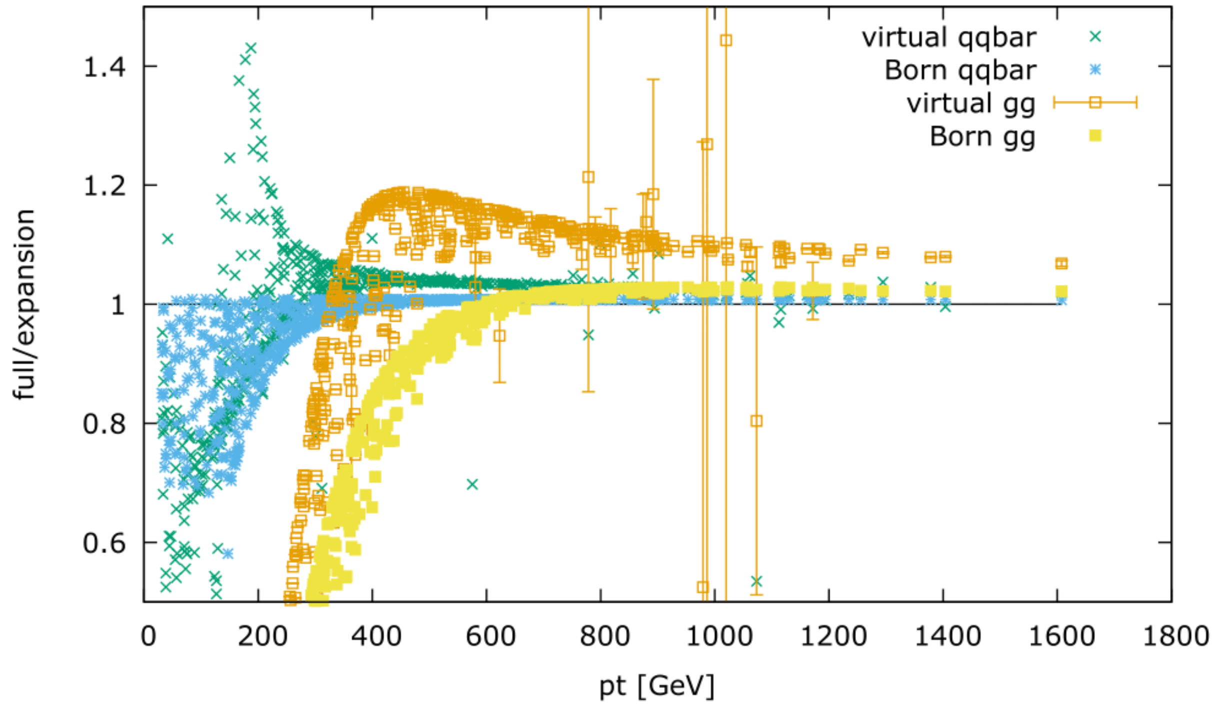
Summary and Outlook

- Higgs is an important probe to New Physics
- No deviations from SM predictions found → precision era
- As luminosity increases at the LHC, we will have access to Higgs transverse momentum distribution with improving precision
- Tail of the Higgs $p_{T,H}$ distribution expected to be probed to ~ 10% error at end of HL-LHC (puts constraints on various BSM models)
- Best theory prediction for high-$p_{T,H}$ predictions including top mass: NLO with ~25% error

→ For comparison with future HL-LHC, we will require NNLO contributions of H+jet plus contributions from other channels

Backup slides

High-pT expansion comparison at NLO



[Plot from Matthias Kerner '18]

- Comparison of full (Secdec) and high-pT expanded virtual contributions

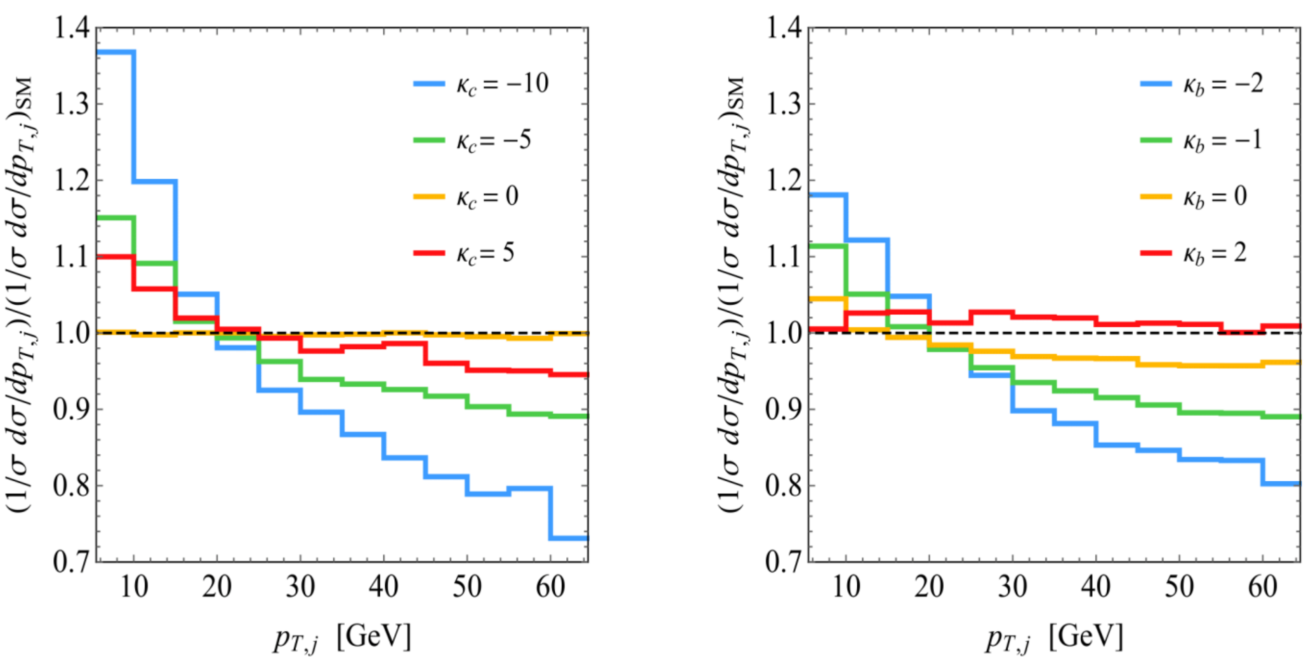
[Kudashkin et al, Jones et al '18]

- Agreement is good, within 20% difference down to 400 GeV
- Virtual piece contributes ~10-20%. Dominant real can be computed exactly w. Openloops

Below top threshold

- <u>Constrain bottom- and charm-quark Yukawa couplings</u>
- Light quark contributions appear pre-dominantly through interference with top. However relative contribution of direct $q\bar{q} \to Hg, qg \to Hq$ contribution increases with light Yukawa coupling →
- Shape of $p_{T,H}$ distribution may put strong constraints on light-quark Yukawa couplings

[Bishara, Monni et al '16; Soreq et al '16]



$$\kappa_j = y_j / y_{j,SM}$$

- Bounds expected from HL-LHC $\kappa_c \in [-0.6, 3.0]$ $\kappa_b \in [0.7, 1.6]$

[Bishara, Monni et al '16]

Below top threshold $p_{T,H} \leq 350$ GeV: including bottom

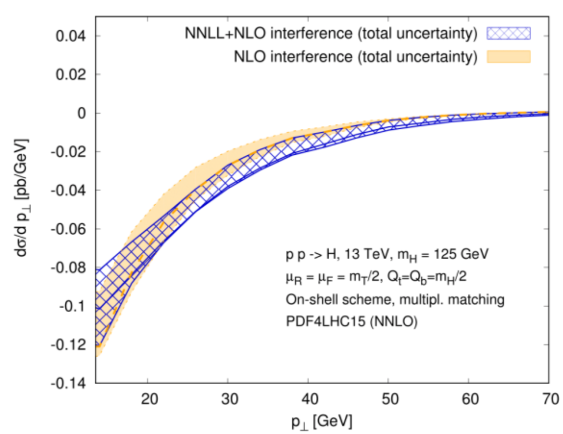
- <u>Theoretical complication</u>: $p_{T,H}$ above bottom threshold and thus bottom loop ***<u>does not factorize</u>***

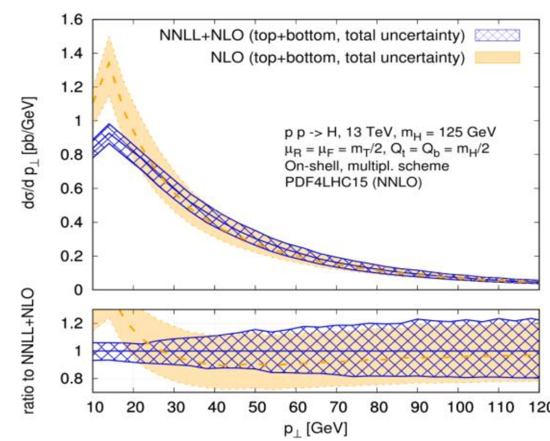
$$p_{\mathrm{T,H}} > 2m_b \sim 10\,\mathrm{GeV} \; : \qquad \mathcal{A}^{\text{bottom-loop}}_{gg\to Hg} \sim \frac{y_b m_b}{p_{\mathrm{T,H}}} \log^2\left(\frac{4m_b^2}{p_{\mathrm{T,H}}^2}\right)$$

- Bottom contribution to $p_{T,H}$ computed recently at NLO [Lindert et al '17]
- Previous N2LL resummed predictions can now be matched to full NLO with bottom [Caola et al. '18]
- Resummation of Sudakov-logarithms $\log\left(p_{\mathrm{T,H}}/m_h\right)$ only possible when quark loop factorizes. At small $p_{T,H} \sim 10$ GeV logs still large so best we can do is to resum and gauge error of different resummation scales and schemes

[Caola et al., ArXiv: 1804.07632]

- Interference contribution error~20%, translates to ~1-2% error on total
- Largest uncertainty of the top-bottom interference contribution from bottom mass scheme choice





- Open question: can we resum the bottom mass logarithms $\log\left(\frac{4m_b^2}{p_{\mathrm{T,H}}^2}\right)$? [Penin, Melnikov '16]

Virtual bottom amplitudes

[Melnikov, Tancredi, CW ’16-’17]

$$\mathcal{I}_{\text{top}}(a_1, a_2, ..., a_8, a_9) = \int \frac{\mathfrak{D}^d k \mathfrak{D}^d l}{[1]^{a_1}[2]^{a_2}[3]^{a_3}[4]^{a_4}[5]^{a_5}[6]^{a_6}[7]^{a_7}[8]^{a_8}[9]^{a_9}}$$

- All scalar integrals appear in three topologies (sets of propagators)

Prop.	Topology PL1	Topology PL2	Topology NPL
[1]	k^2	$k^2 - m_b^2$	$k^2 - m_b^2$
[2]	$(k-p_1)^2$	$(k-p_1)^2 - m_b^2$	$(k+p_1)^2 - m_b^2$
[3]	$(k-p_1-p_2)^2$	$(k-p_1-p_2)^2 - m_b^2$	$(k-p_2-p_3)^2 - m_b^2$
[4]	$(k-p_1-p_2-p_3)^2$	$(k-p_1-p_2-p_3)^2 - m_b^2$	$l^2 - m_b^2$
[5]	$l^2 - m_b^2$	$l^2 - m_b^2$	$(l+p_1)^2 - m_b^2$
[6]	$(l-p_1)^2 - m_b^2$	$(l-p_1)^2 - m_b^2$	$(l-p_3)^2 - m_b^2$
[7]	$(l-p_1-p_2)^2 - m_b^2$	$(l-p_1-p_2)^2 - m_b^2$	$(k-l)^2$
[8]	$(l-p_1-p_2-p_3)^2 - m_b^2$	$(l-p_1-p_2-p_3)^2 - m_b^2$	$(k-l-p_2)^2$
[9]	$(k-l)^2 - m_b^2$	$(k-l)^2$	$(k-l-p_2-p_3)^2$

- *Integration by parts* (IBP) identities

$$\int \Big(\prod_i d^d k_i\Big) \frac{\partial}{\partial k_j^\mu}\Big(v^\mu I\Big) = \text{Boundary term} \overset{DR}{=} 0 \qquad v \in \{k_1, \cdots, k_n, \text{external momenta}\}$$

- Popular public programs Reduze and FIRE5 can solve these IBP identities and reduce to set of *Master Integrals* (MI)

$$\mathcal{I}_{a_1\cdots a_n}(s) = \sum_{(b_1\cdots b_n)\in\text{Master Integrals}} \text{Rational}^{b_1\cdots b_n}_{a_1\cdots a_n}(s,d)\text{MI}_{b_1\cdots b_n}(s)$$

Real corrections with Openloops

- Channels for real contribution to Higgs plus jet at NLO

$$gg \to Hgg,\ gg \to Hq\bar{q},\ qg \to Hqg,\ q\bar{q} \to Hgg,\ \cdots$$

- Receives contributions from kinematical regions where one parton become soft or collinear to another parton
- This requires a delicate approach of these regions in phase space integral
- Openloops algorithm is publicly available program which is capable of dealing with these singular regions in a numerically stable way
- Crucial ingredient is tensor integral reduction performed via expansions in small Gram determinants: Collier

[Cascioli et al '12, Denner et al '03-'17]

- Exact top and bottom mass dependence kept throughout for one-loop computations