Recent progress in Transverse Momentum Dependent Parton Distribution Functions

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in collaboration with
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Introduction

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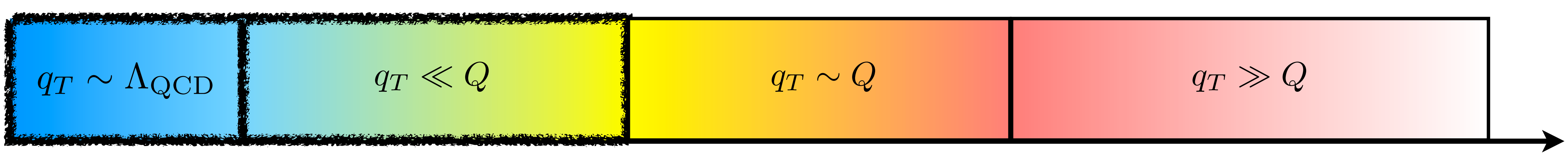

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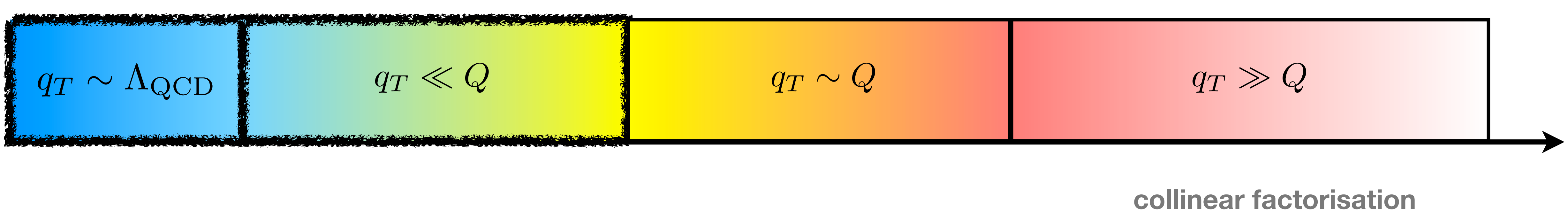

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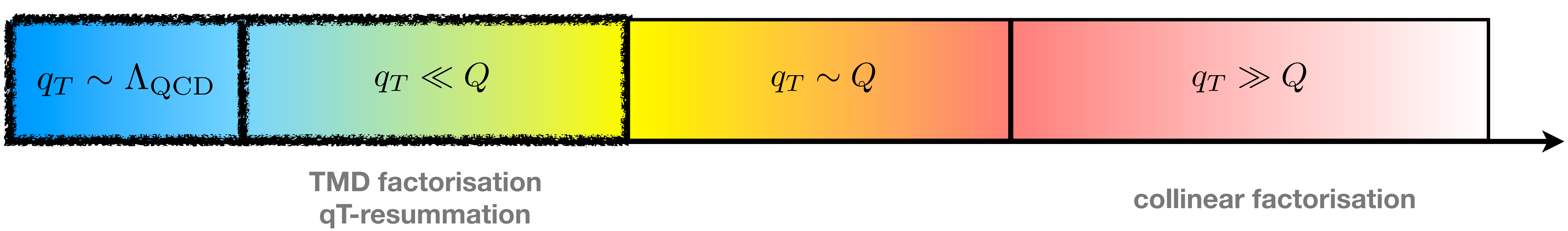

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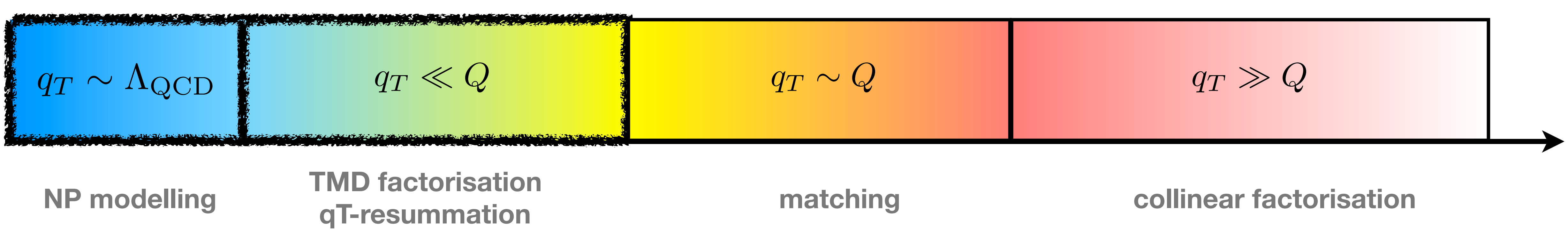

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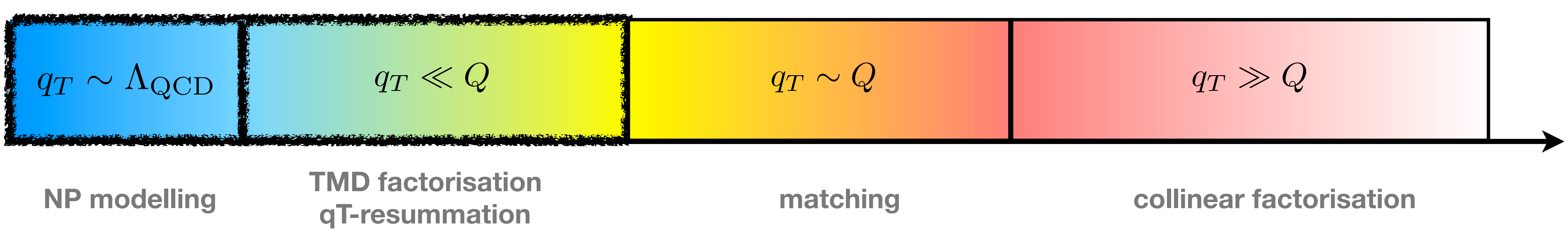

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  - the **renormalisation scale μ**, originating from the UV renormalisation,
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- Anomalous dims. and matching funcs. **perturbatively** computable.

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- The single TMD distributions are then given by:

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_T; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \qquad : A$$

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- CS and RGE evolution,
- evolution to large b_T,
- perturbative.

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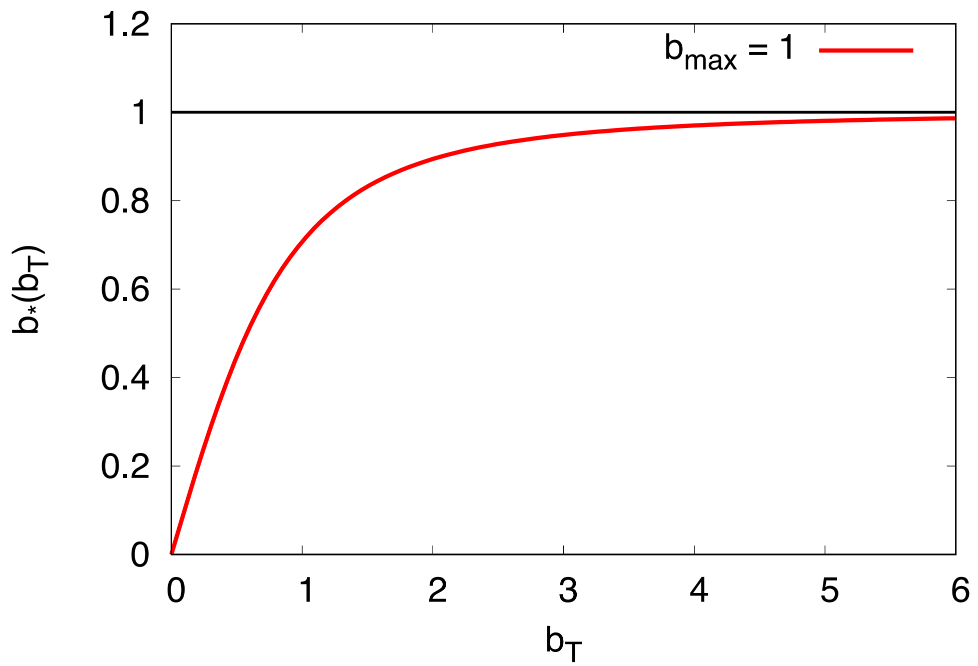

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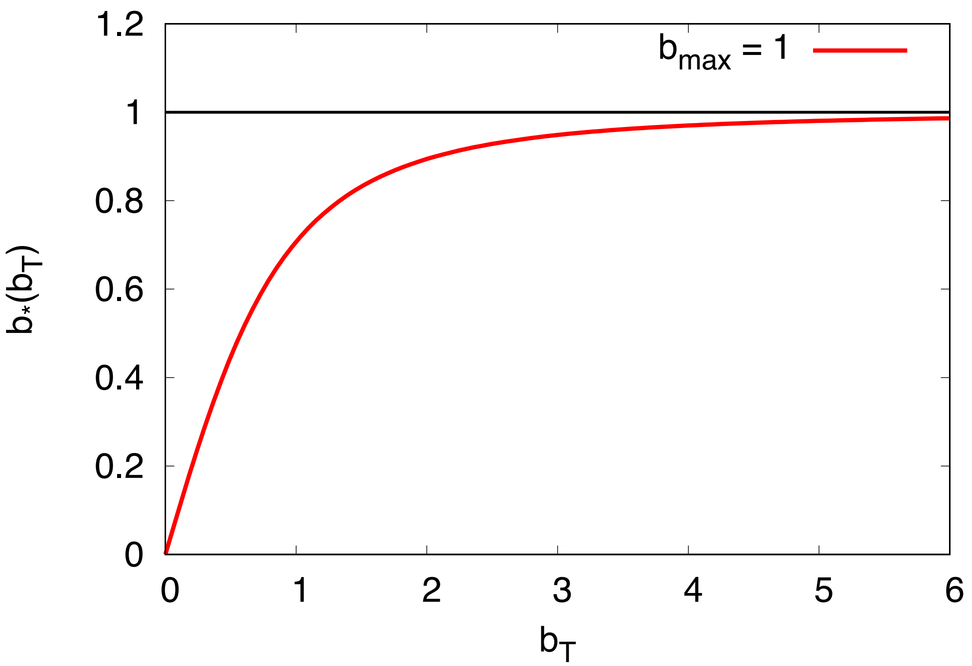

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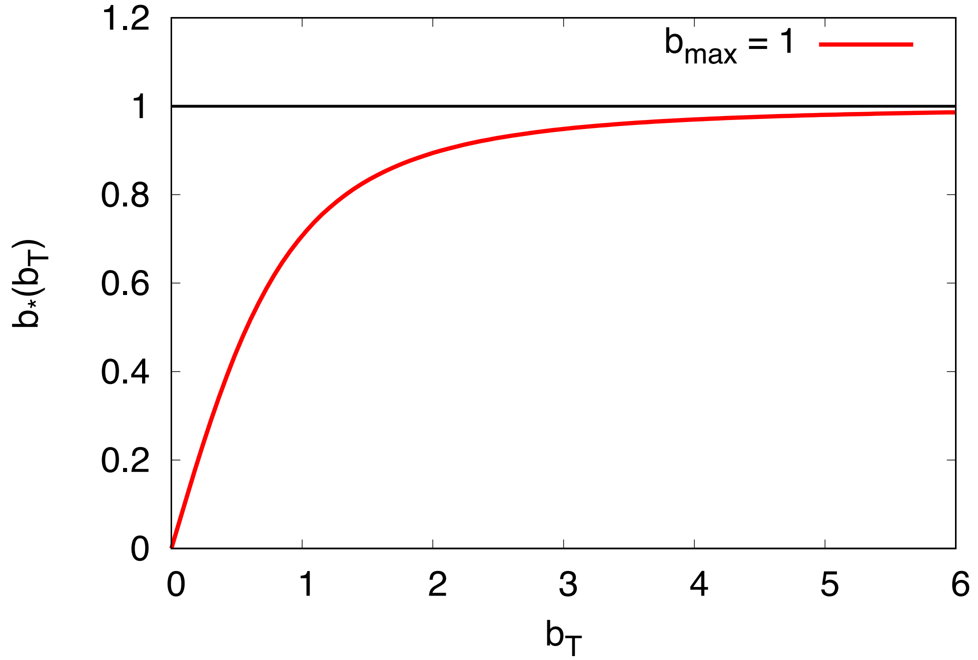

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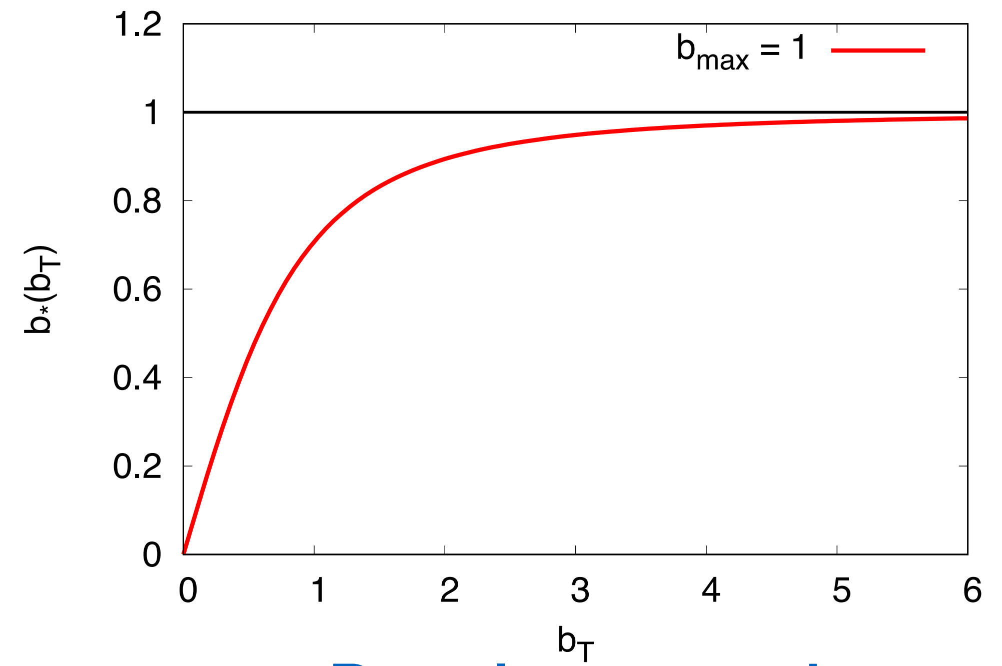

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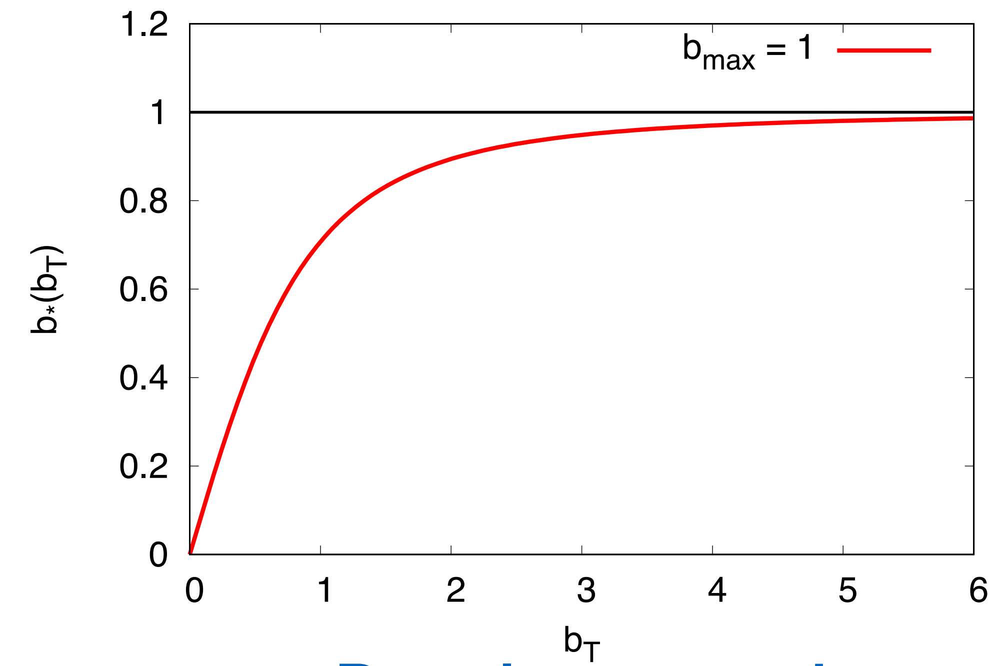

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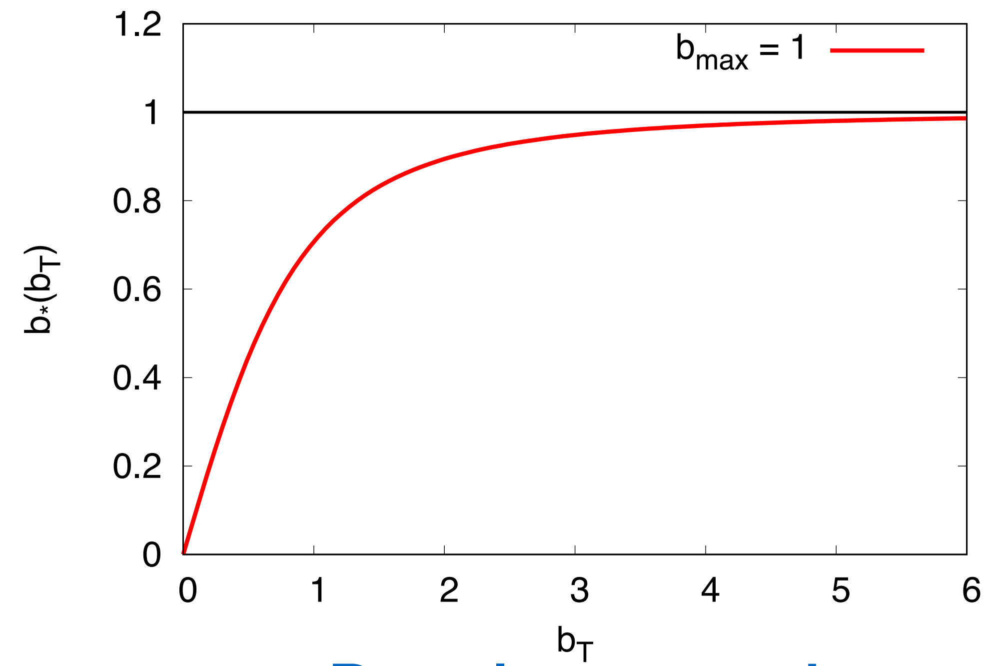

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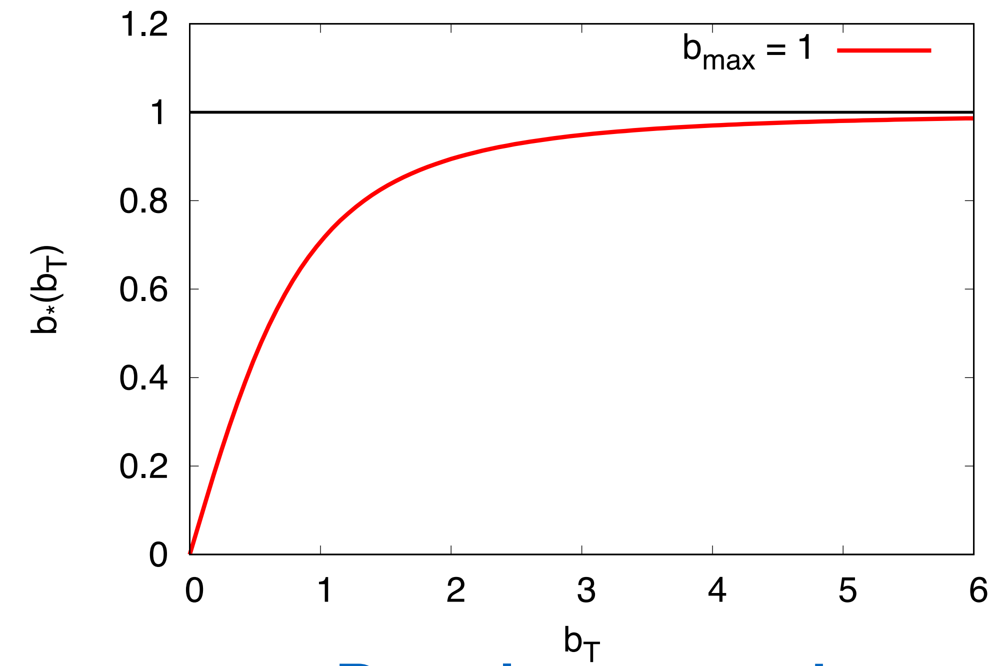

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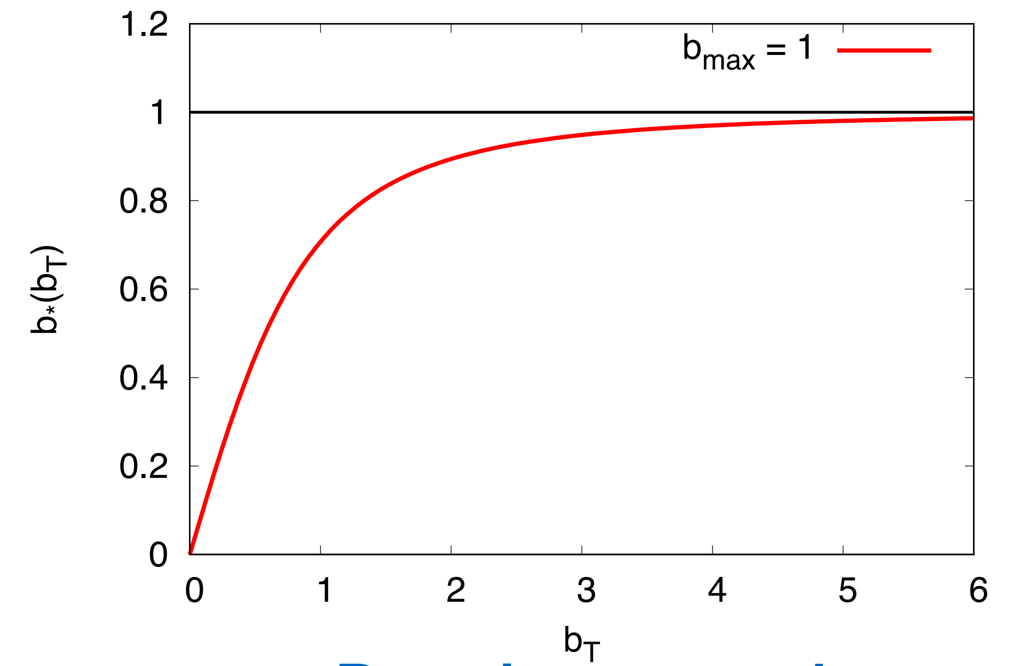

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- Bottom line: avoidance of the non-perturbative region upon integration in b_T implies the presence of **both** b_*-prescription and $f_{\rm NP}$.

TMD factorisation

- Final expression:

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \qquad : A$$

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\begin{aligned}
F_{f/P}(x,\mathbf{b}_T;\mu,\zeta) \quad &= \sum_j C_{f/j}(x,b_*;\mu_b,\zeta_F)\otimes f_{j/P}(x,\mu_b) && :A \\
&\times \quad \exp\left\{K(b_*;\mu_b)\ln\frac{\sqrt{\zeta_F}}{\mu_b}+\int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F-\gamma_K\ln\frac{\sqrt{\zeta_F}}{\mu'}\right]\right\} && :B \\
&\times \quad \exp\left\{\underline{g_{j/P}(x,b_T)}+g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}\right\} && :C
\end{aligned}
$$

- matching to the collinear region at $b_\mathrm{T} \ll 1/\Lambda_\mathrm{QCD}$,
- factorises as *hard* (perturbative) and *longitudinal* (*i.e.* collinear, non-perturbative).

TMD factorisation

- Final expression:

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- matching to the collinear region at $b_T \ll 1/\Lambda_{QCD}$,
- factorises as *hard* (perturbative) and *longitudinal* (*i.e.* collinear, non-perturbative).

- avoid the Landau pole,
- f_{NP} accounts for the introduction of b_*,
- f_{NP} is non-perturbative thus **fit** to data.

- CS and RGE evolution,
- evolution to large b_T,
- perturbative.

Logarithmic counting

Logarithmic counting

- TMD factorisation provides **resummation** of large logs $L = \log(q_T/Q)$ implemented through the **Sudakov** form factor

$$\exp\left\{K(b_*;\mu_b)\ln\frac{\sqrt{\zeta_F}}{\mu_b}+\int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F-\gamma_K\ln\frac{\sqrt{\zeta_F}}{\mu'}\right]\right\}$$

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$\alpha_s L^2$	$\alpha_s L$	$\ldots$	$\ldots$	$\mathcal{O}(\alpha_s)$	(LO)
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\mathcal{O}(\alpha_s^2)$	(NLO)
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\ldots$	$\mathcal{O}(\alpha_s^n)$	(N^nLO)
LL	NLL	NNLL	$\ldots$	$\ldots$	

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...	...	...	...	...	...
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- A **perturbative expansion** of the Sudakov at LL, NLL, NNLL, … would (roughly) give the terms in the 1st, 2nd, 3rd, … columns

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- Multiplying it by a power p of α_s would generate N^{n+2p} terms

Logarithmic counting

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...	...	...	...	...	...
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	...	$\mathcal{O}(\alpha_s^n)$	$(N^n LO)$
LL	NLL	NNLL	...	...	

- A **perturbative expansion** of the Sudakov at LL, NLL, NNLL, … would (roughly) give the terms in the 1st, 2nd, 3rd, … columns

- Multiplying it by a power p of α_s would generate N^{n+2p} terms

- Bottom line: any additional power of α_s causes a shift of **two units** in the logarithmic ordering *in the observable*.

Logarithmic counting

Accuracy	γ_K	γ_F	K	$C_{f/j}$	H
LL	α_s	-	-	1	1
NLL	α_s^2	α_s	α_s	1	1
NLL’	α_s^2	α_s	α_s	α_s	α_s
N^{2}LL	α_s^3	α_s^2	α_s^2	α_s	α_s
N^{2}LL’	α_s^3	α_s^2	α_s^2	α_s^2	α_s^2
N^{3}LL	α_s^4	α_s^3	α_s^3	α_s^2	α_s^2

N.B. if matching is performed, ***primed*** quantities are mandatory (NLL’+LO, NNLL’+NLO, …)

Factorising processes

- Processes for which leading-power TMD factorisation has been **proven**:

Factorising processes

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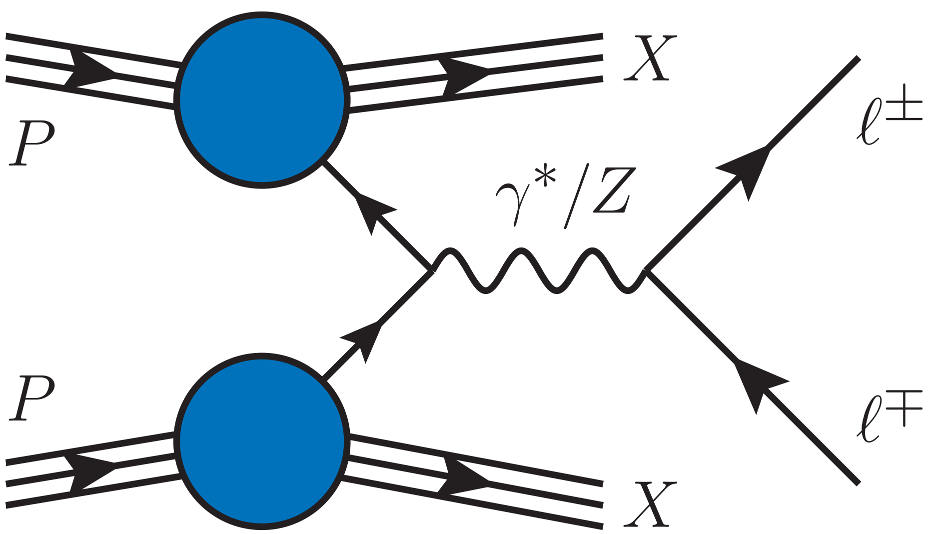
Drell-Yan



$PP \longrightarrow \ell^{\pm}\ell^{\mp}\ X$

- **Two** TMD **PDFs**:
- Lots of data:
  - low-energy: FNAL,
  - mid-energy: RHIC,
  - high-energy: Tevatron, LHC.

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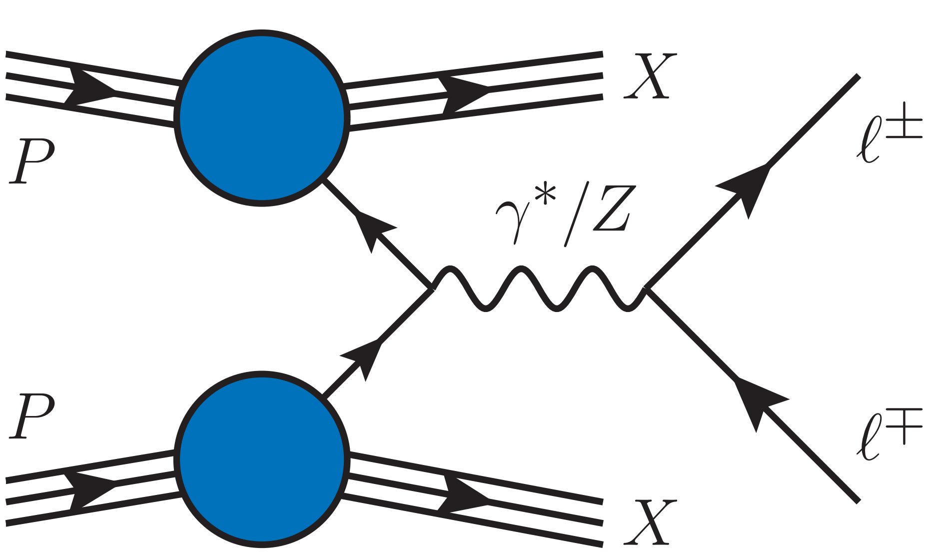
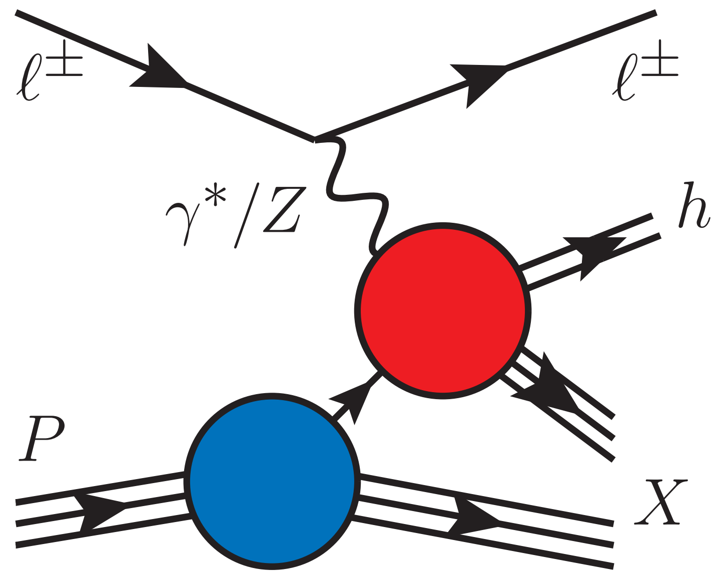


$PP \longrightarrow \ell^\pm \ell^\mp X$

Semi-inclusive DIS



$P\ell^\pm \longrightarrow \ell^\pm h X$

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- many precise data points:
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  - COMPASS at CERN.

Factorising processes

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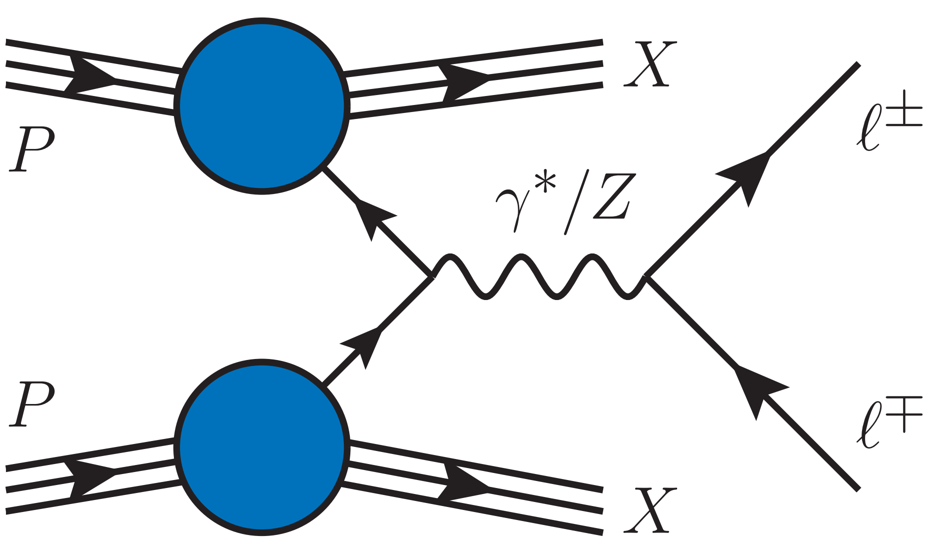

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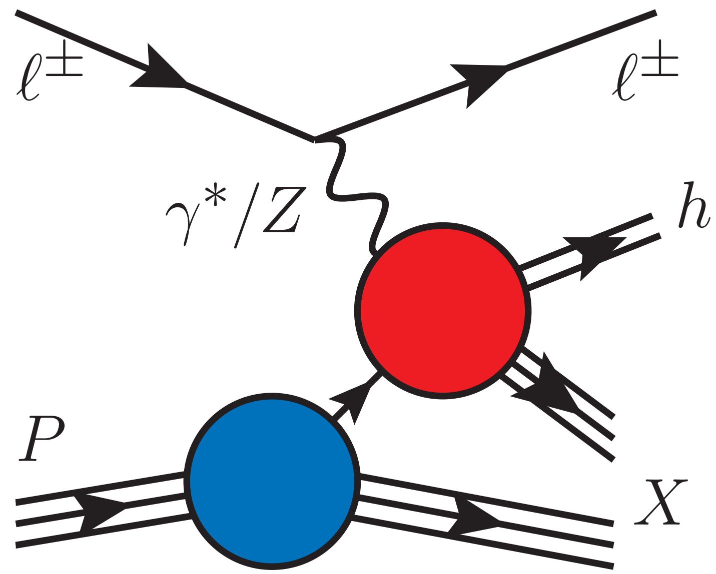
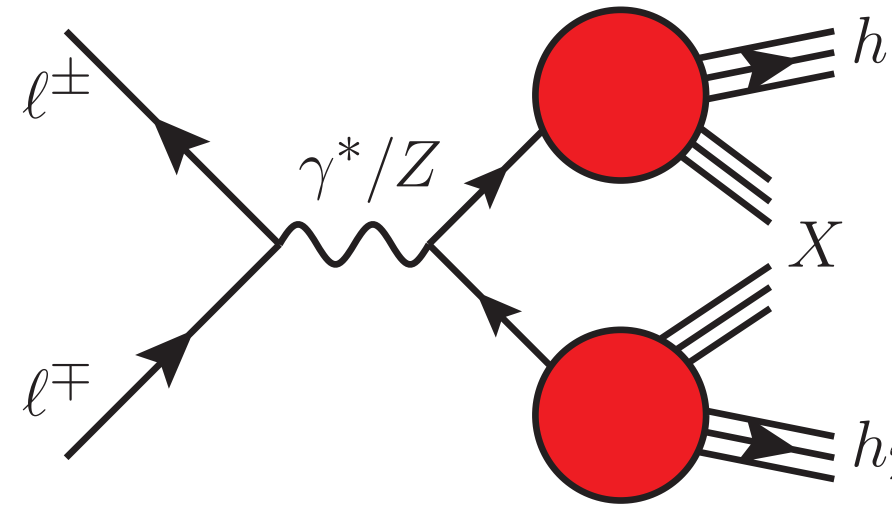


$P\ell^\pm \longrightarrow \ell^\pm h\ X$

e^+e^- annihilation



$\ell^\pm \ell^\mp \to h_1 h_2\ X$

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- **Two** TMD **FFs**:
- di-hadron prod. from:
  - BELLE at KEK,
  - BABAR at SLAC.

Unpolarised TMD extractions

A selection of results

	Accuracy	HERMES	COMPASS	Low-energy DY	Z production	N. of points
KN 2006 hep-ph/0506225	NLL	✘	✘	✔	✔	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evolution	✔	✘	✘	✘	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evolution	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NNLL	✘	✘	✔	✔	223
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLL(')	✘	✘	✔	✔ (LHC)	309
BSV 2019 arXiv:1902.08474	NNLL(')	✘	✘	✔	✔ (LHC)	457
Pavia 2019	up to N^3LL	✘ (✔)	✘ (✔)	✔	✔ (LHC)	$O(400)$

Pavia 2017
The dataset

- Semi-Inclusive DIS data:

	HERMES $D \to \pi^+$	HERMES $D \to \pi^-$	HERMES $D \to K^+$	HERMES $D \to K^-$	COMPASS $D \to h^+$	COMPASS $D \to h^-$
Reference	[74]				[75]	
Cuts	$Q^2 > 1.4$ GeV^2 $0.20 < z < 0.74$ $P_{hT} <$ Min$[0.2\ Q, 0.7\ Qz] + 0.5$ GeV					
Points	190	190	189	189	3125	3127
Max. Q^2	9.2 GeV^2				10 GeV^2	
x range	$0.04 < x < 0.4$				$0.005 < x < 0.12$	
Notes					Observable: $m_{\rm norm}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$, Eq. (41)	

	HERMES $p \to \pi^+$	HERMES $p \to \pi^-$	HERMES $p \to K^+$	HERMES $p \to K^-$
Reference	[74]			
Cuts	$Q^2 > 1.4$ GeV^2 $0.20 < z < 0.74$ $P_{hT} <$ Min$[0.2\ Q, 0.7\ Qz] + 0.5$ GeV			
Points	190	190	189	187
Max. Q^2	9.2 GeV^2			
x range	$0.04 < x < 0.4$			

- Low-energy Drell-Yan production data:

	E288 200	E288 300	E288 400	E605
Reference	[79]	[79]	[79]	[80]
Cuts	$q_T < 0.2\ Q + 0.5$ GeV			
Points	45	45	78	35
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	38.8 GeV
Q range	4-9 GeV	4-9 GeV	5-9, 11-14 GeV	7-9, 10.5-11.5 GeV
Kin. var.	η=0.40	η=0.21	η=0.03	$x_F = 0.1$

- High-energy Drell-Yan production data at the Z peak:

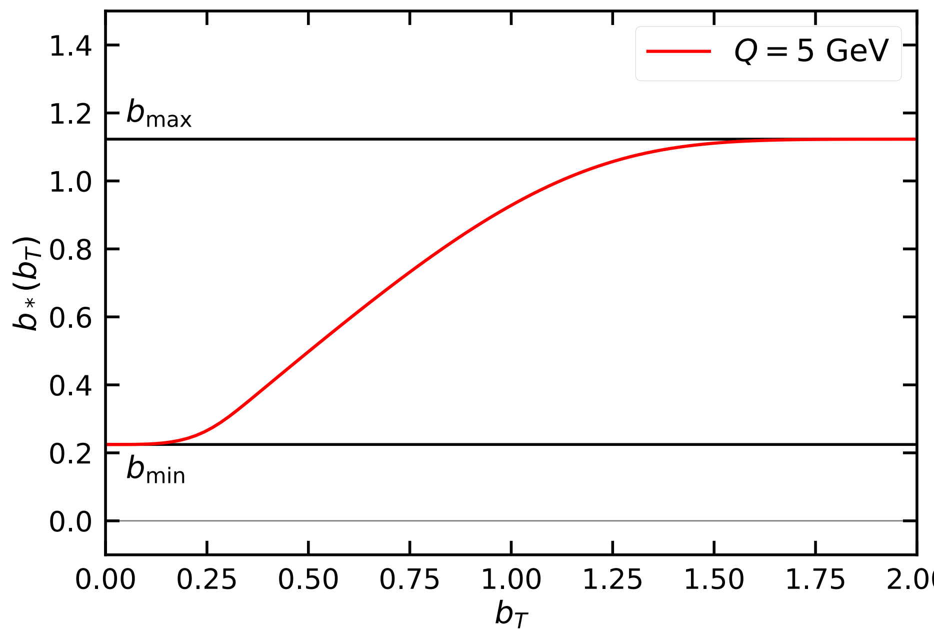
	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Reference	[81]	[82]	[83]	[84]
Cuts	$q_T < 0.2\ Q + 0.5$ GeV $= 18.7$ GeV			
Points	31	14	37	8
$\sqrt{s}$	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
Normalization	1.114	0.992	1.049	1.048

Pavia 2017

The settings

- b_* prescription:

$$b_*(b_T) = b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right)^{1/4} \quad \text{with} \quad \begin{cases} b_{\max} = 2e^{-\gamma_E} \\ b_{\min} = b_{\max}/Q \end{cases}$$



Pavia 2017

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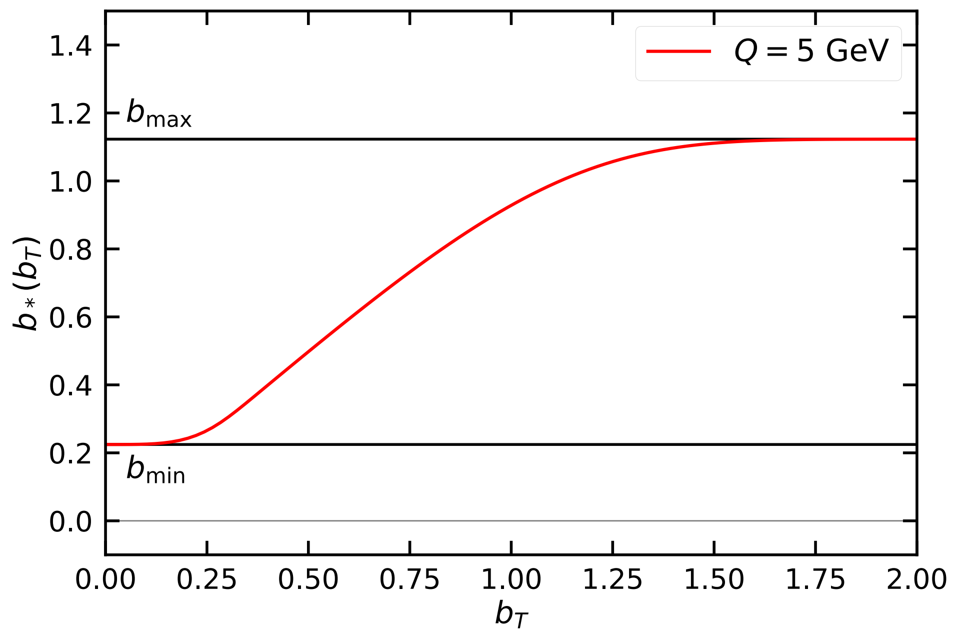

- Non-perturbative function f_{NP}:
  - evolution:

$$g_K(b_T) = -\frac{1}{2} g_2 b_T^2$$

  - PDFs:

$$\widetilde{f}_{1NP}^{a}(x, \xi_T^2) = \frac{1}{2\pi} e^{-g_{1a}\frac{\xi_T^2}{4}}\left(1 - \frac{\lambda g_{1a}^2}{1+\lambda g_{1a}}\frac{\xi_T^2}{4}\right)$$

  - FFs:

$$\widetilde{D}_{1NP}^{a\to h}(z, \xi_T^2) = \frac{g_{3a\to h}\ e^{-g_{3a\to h}\frac{\xi_T^2}{4z^2}} + (\lambda_F/z^2) g_{4a\to h}^2 \left(1 - g_{4a\to h}\frac{\xi_T^2}{4z^2}\right)\ e^{-g_{4a\to h}^2\frac{\xi_T^2}{4z^2}}}{2\pi z^2\left(g_{3a\to h} + (\lambda_F/z^2) g_{4a\to h}^2\right)}$$

Pavia 2017

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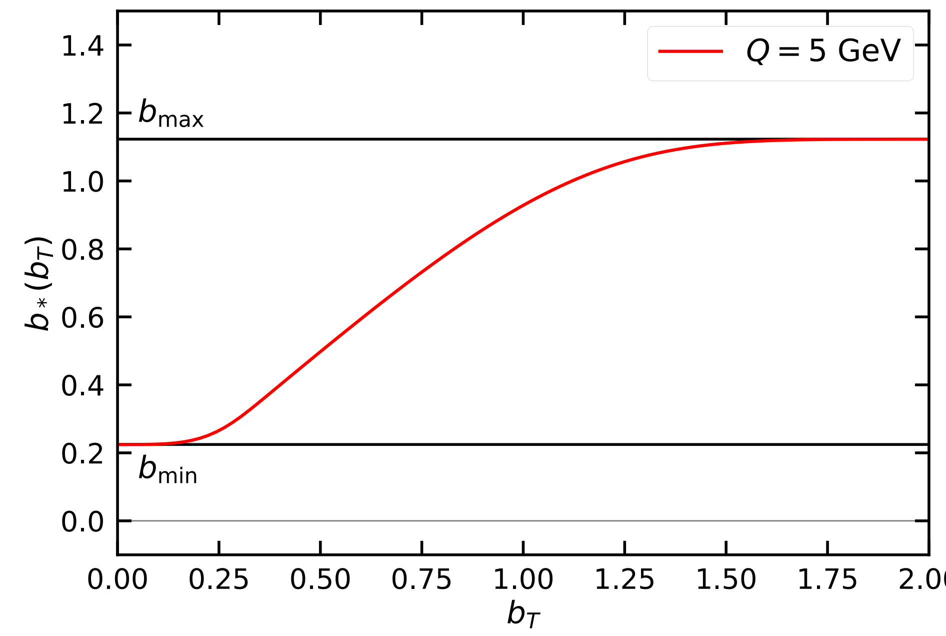

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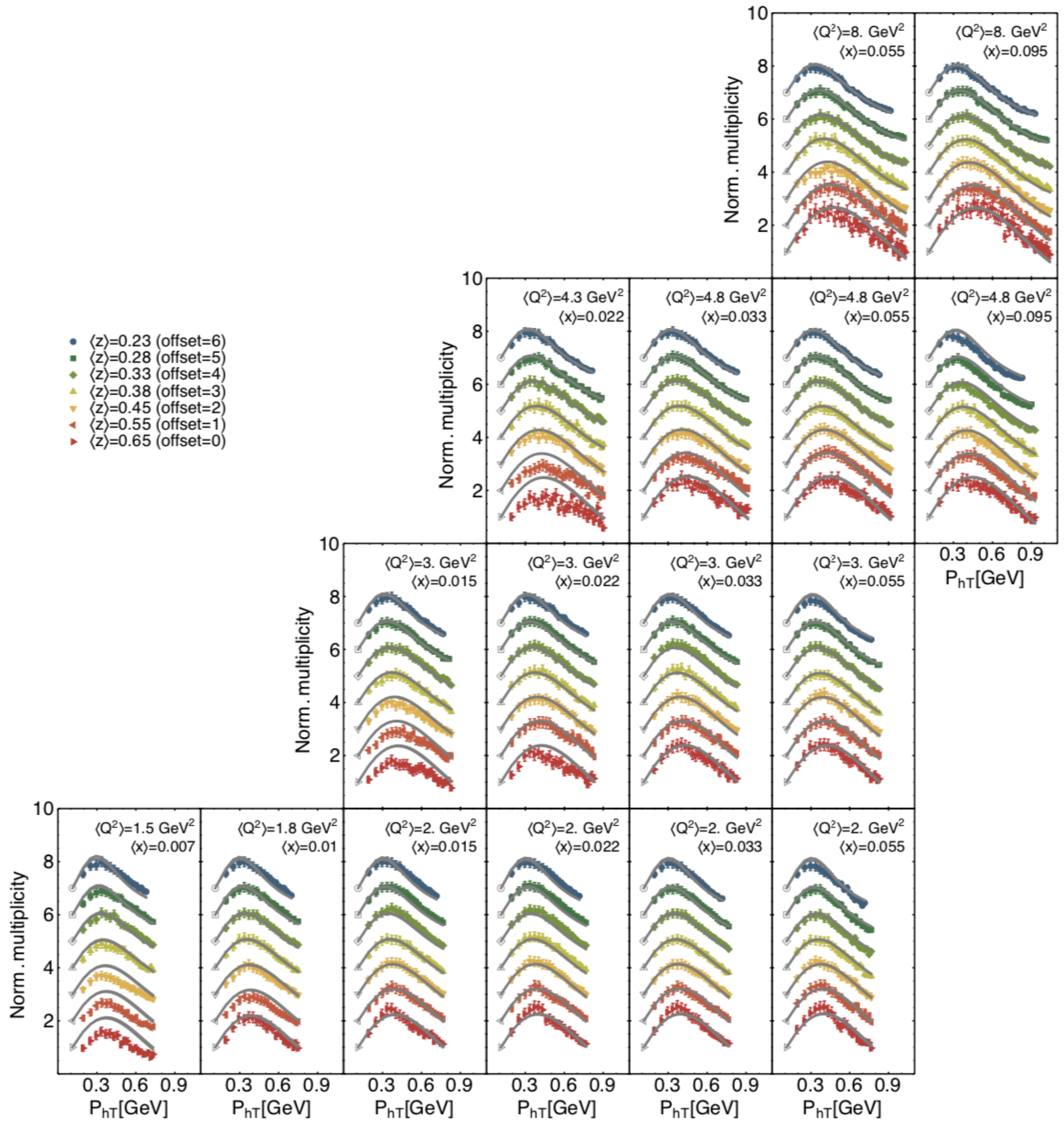
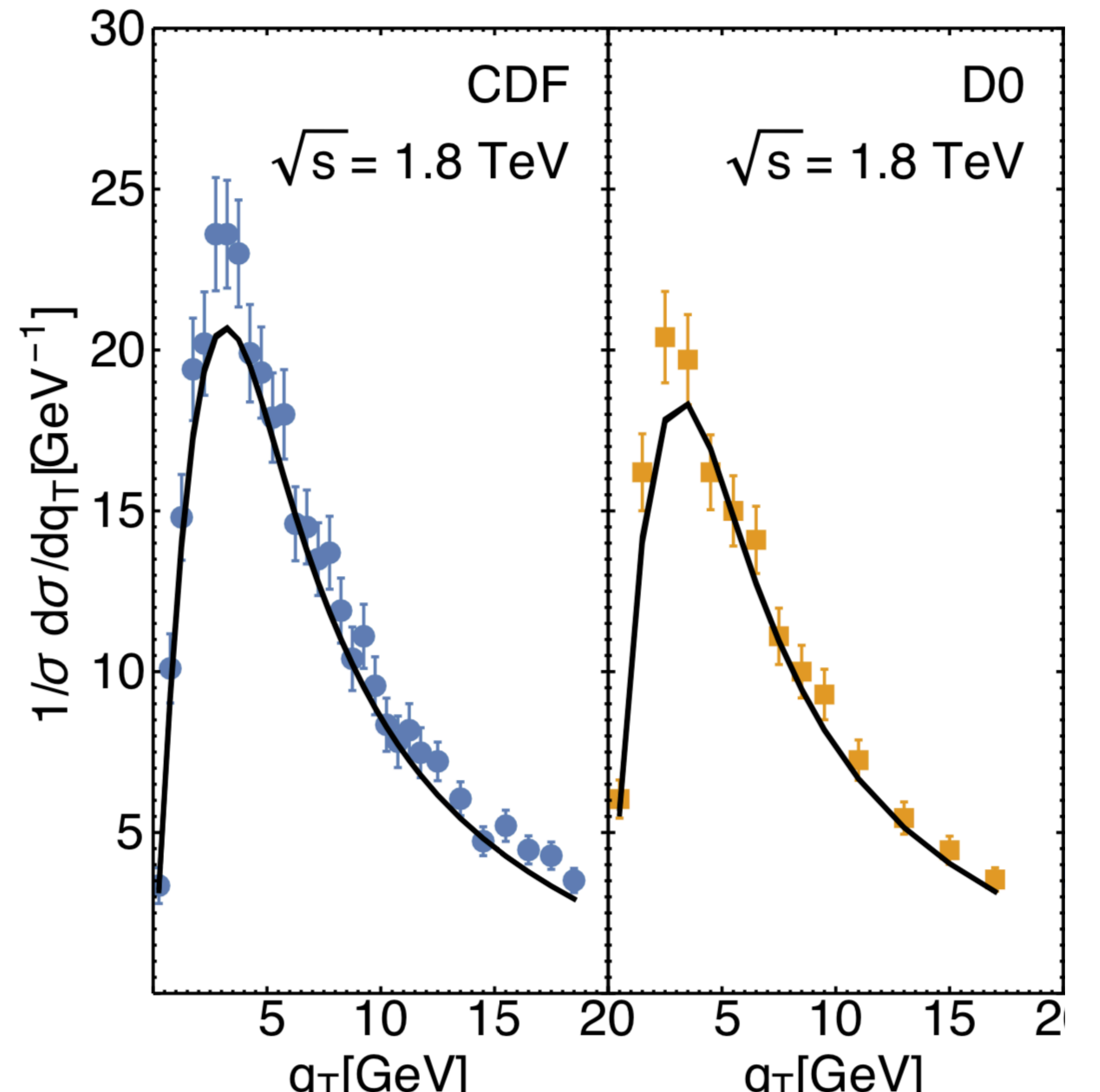
- **11 free parameters** to fit to data.
- Perturbative accuracy: **NLL**
- **Monte Carlo** method for the experimental error propagation.

Pavia 2017

Fit quality

Points	Parameters	χ^2	χ^2/d.o.f.
8059	11	12629 ± 363	1.55 ± 0.05





	HERMES $D \to \pi^+$	HERMES $D \to \pi^-$	HERMES $D \to K^+$	HERMES $D \to K^-$	COMPASS $D \to h^+$	COMPASS $D \to h^-$
Points	190	190	189	189	3125	3127
χ^2/points	3.46 ± 0.32	2.00 ± 0.17	1.31 ± 0.26	2.54 ± 0.57	1.11 ± 0.03	1.61 ± 0.04

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Points	31	14	37	8
χ^2/points	1.36 ± 0.00	1.11 ± 0.02	2.00 ± 0.02	1.73 ± 0.01

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An assessment

Pavia 2017
An assessment

- **PROs:**
  - almost a **global fit** of quark unpolarised TMDs,
  - includes TMD **evolution**
  - Monte Carlo (replica) method,
  - **kinematic dependence** of the intrinsic q_T,
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Pavia 2017

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- **CONs:**
  - no **flavour dependence**,
  - **theoretical accuracy** not the state of the art,
  - no **LHC** data,
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Pavia 2017
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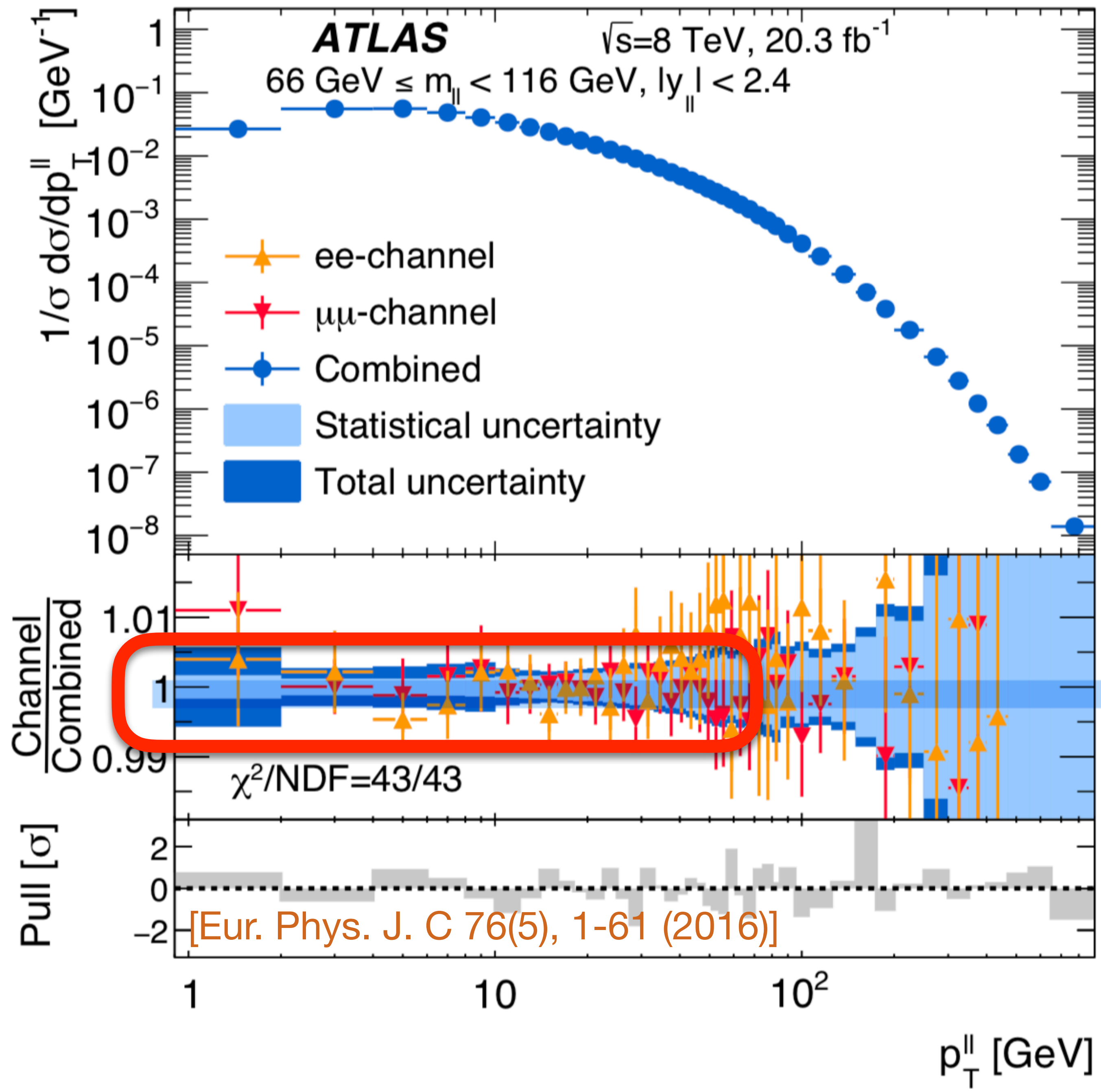
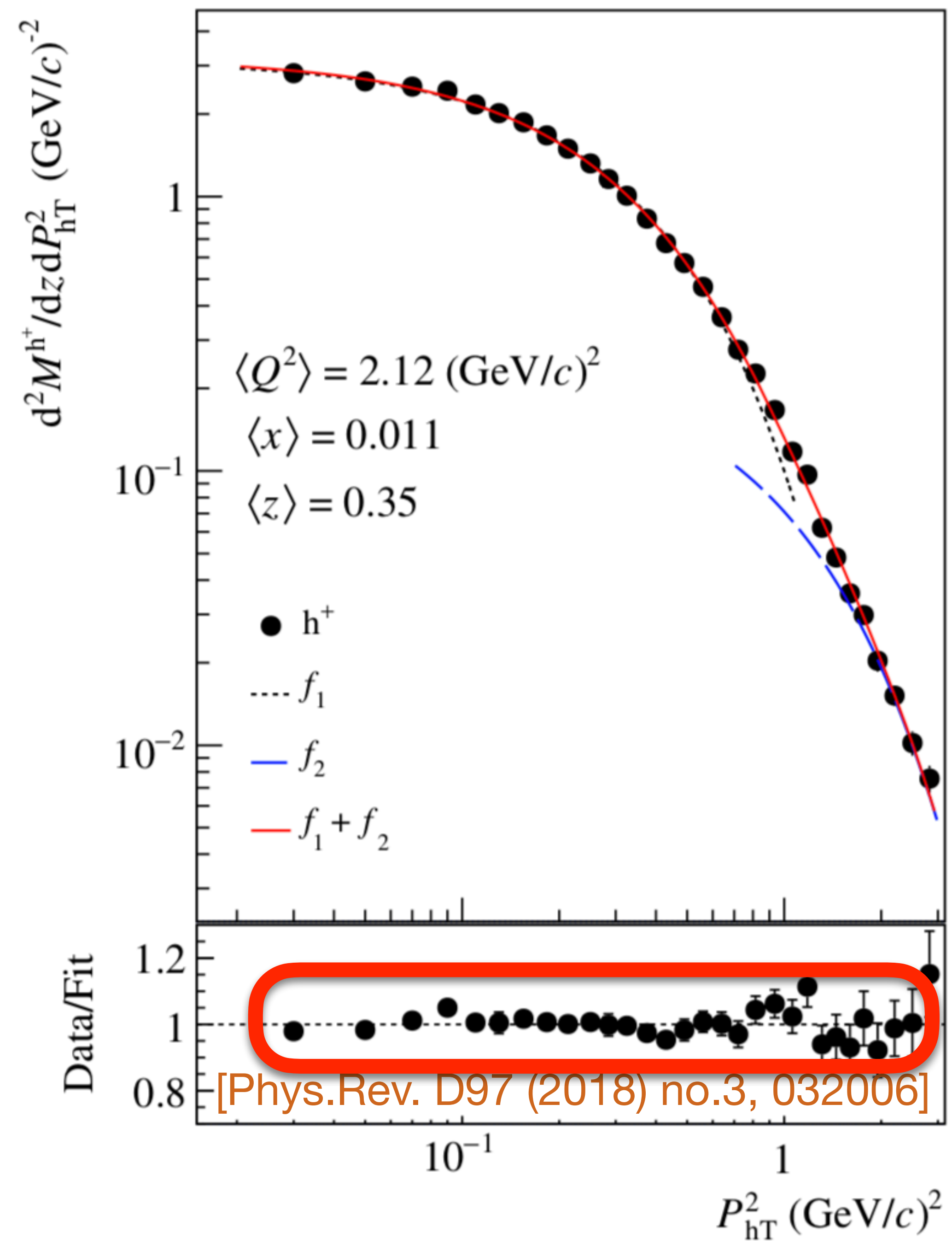
- Actively working to improve on the downsides.

Pavia 2019
Higher-order corrections

- Measurements of q_T distributions have reached the **percent level** uncs.:



[Eur. Phys. J. C 76(5), 1-61 (2016)]



[Phys.Rev. D97 (2018) no.3, 032006]

Pavia 2019
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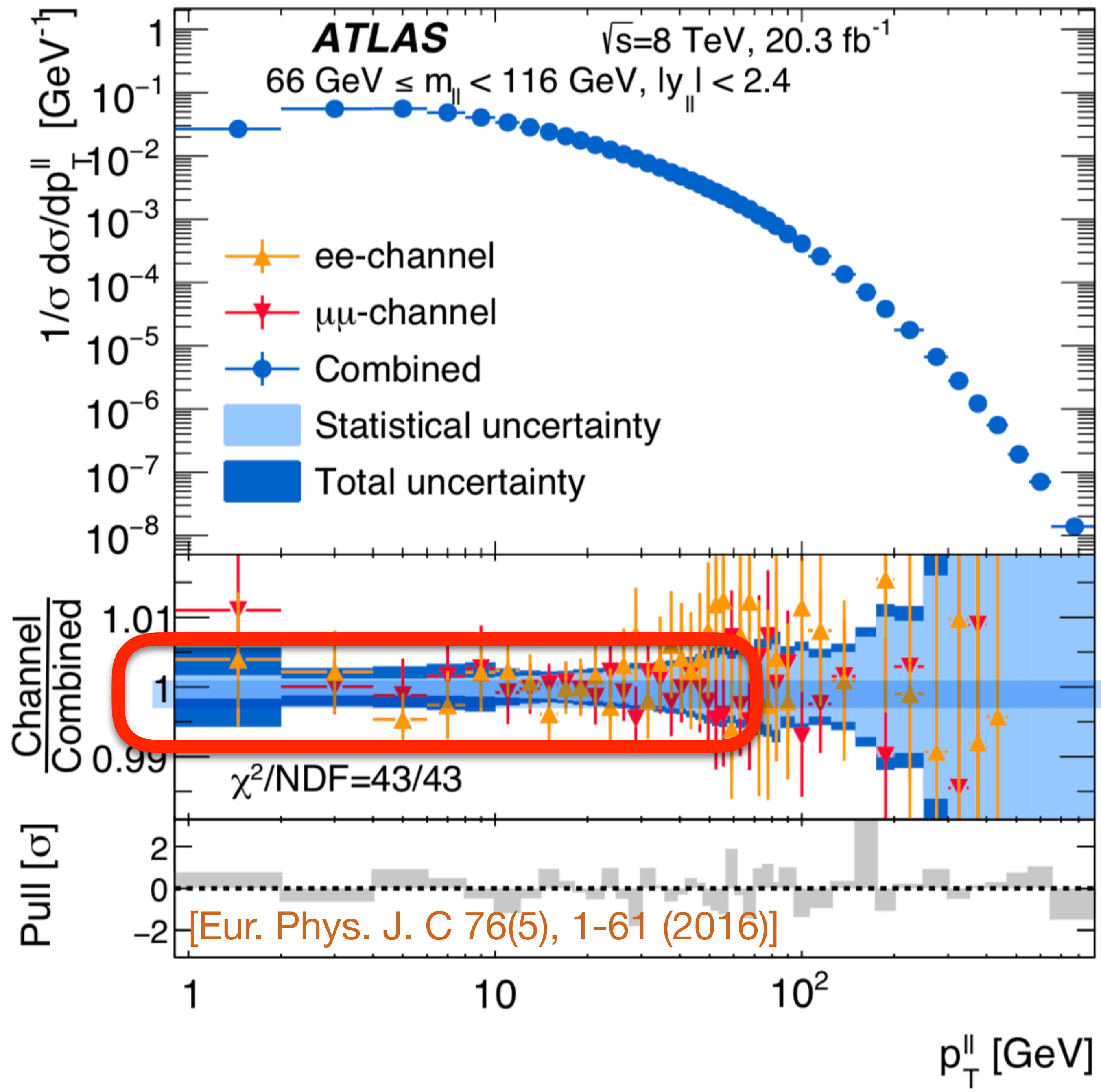
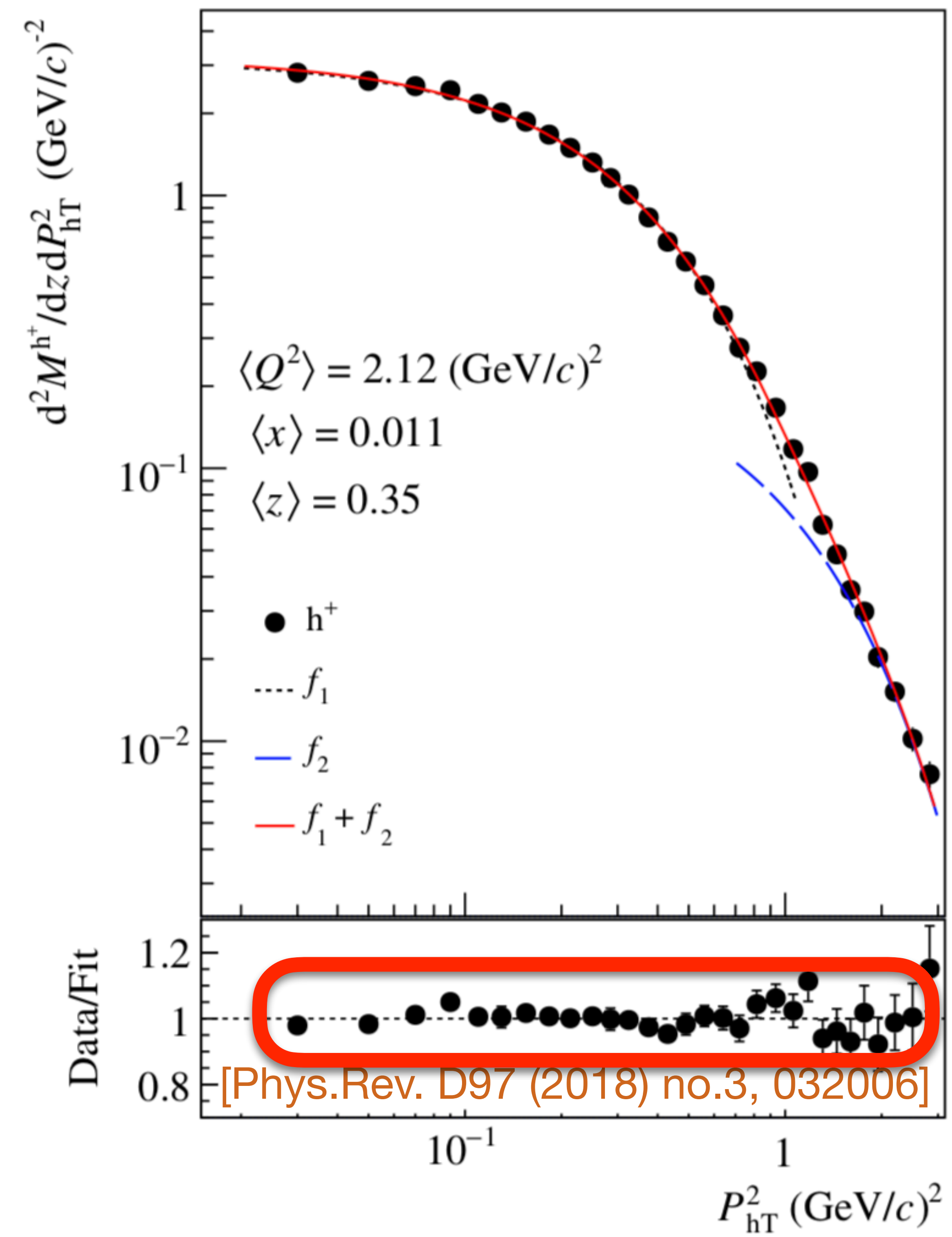




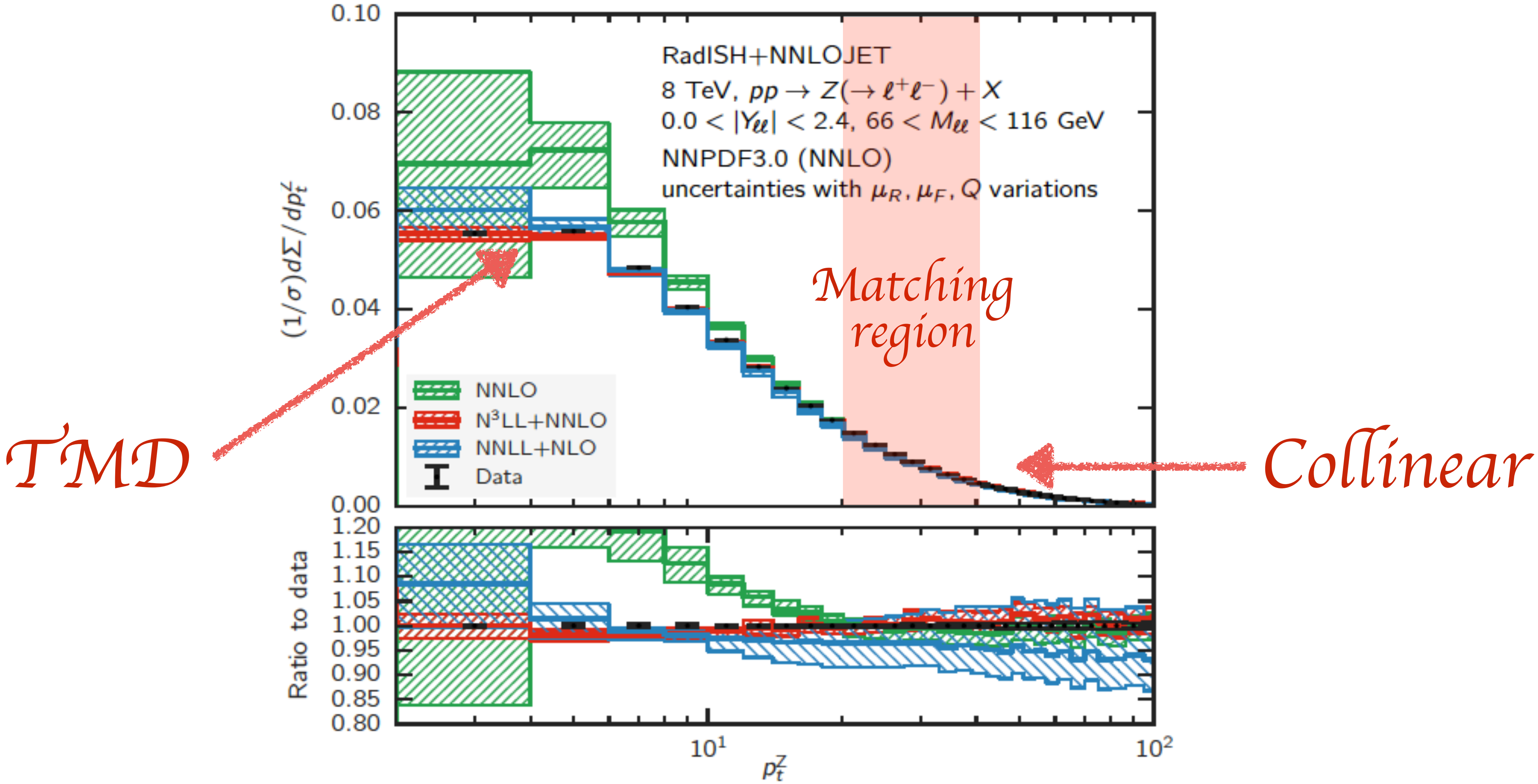
- **State-of-the-art** calculations are thus necessary to hope to describe this data:
  - **higher-order** corrections and possibly **matching** between **TMD** and **collinear**.

Pavia 2019

Higher-order corrections

- Current state-of-the-art: **N³LL + NNLO**:

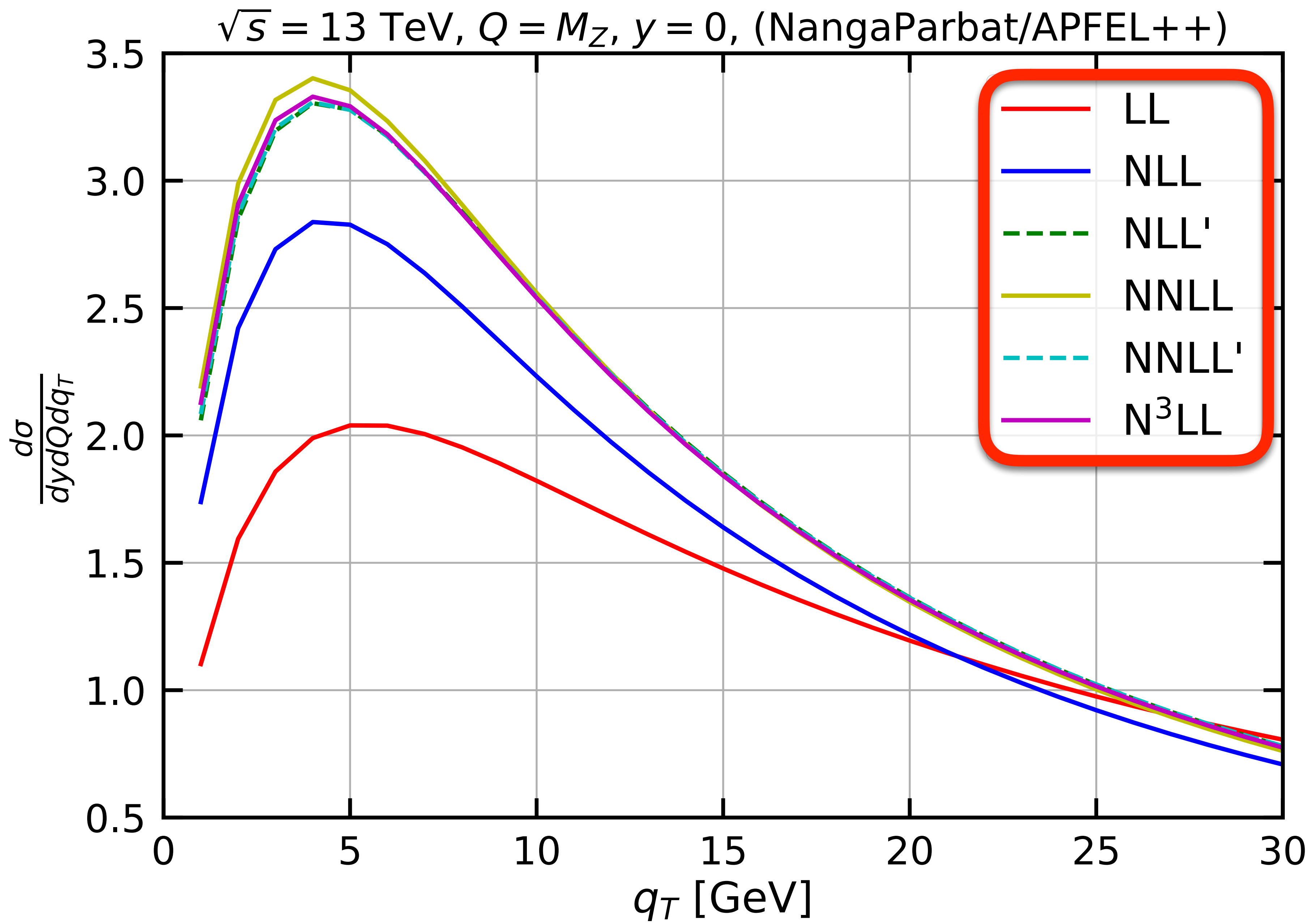
[10.1007/JHEP12(2018)132]



- required to describe the precise ATLAS Z-production data.
- This data can be used to determine the non-pert. component.

Pavia 2019
Higher-order corrections

- In Pavia, we are actively working to reach the "state-of-the-art" accuracy:
  - in fact, in the TMD region we already got there!



- A fast computation of this observable(s) is implemented in a dedicated framework conceived to extract TMD distributions: **NangaParbat**.

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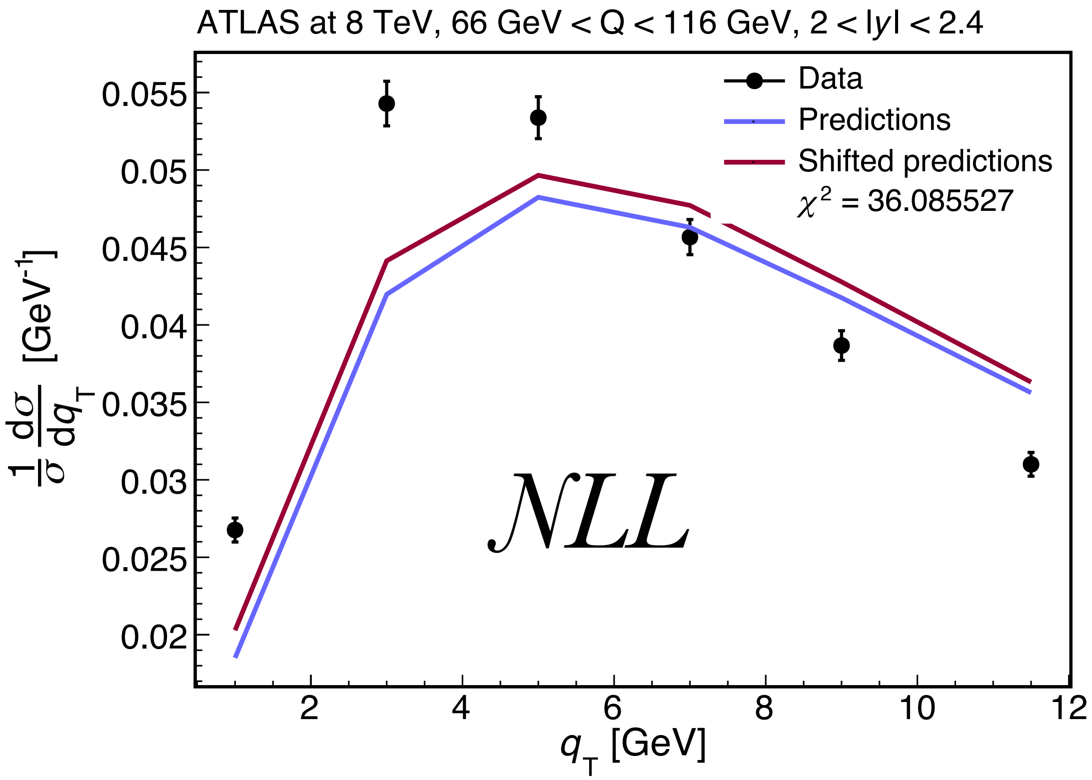
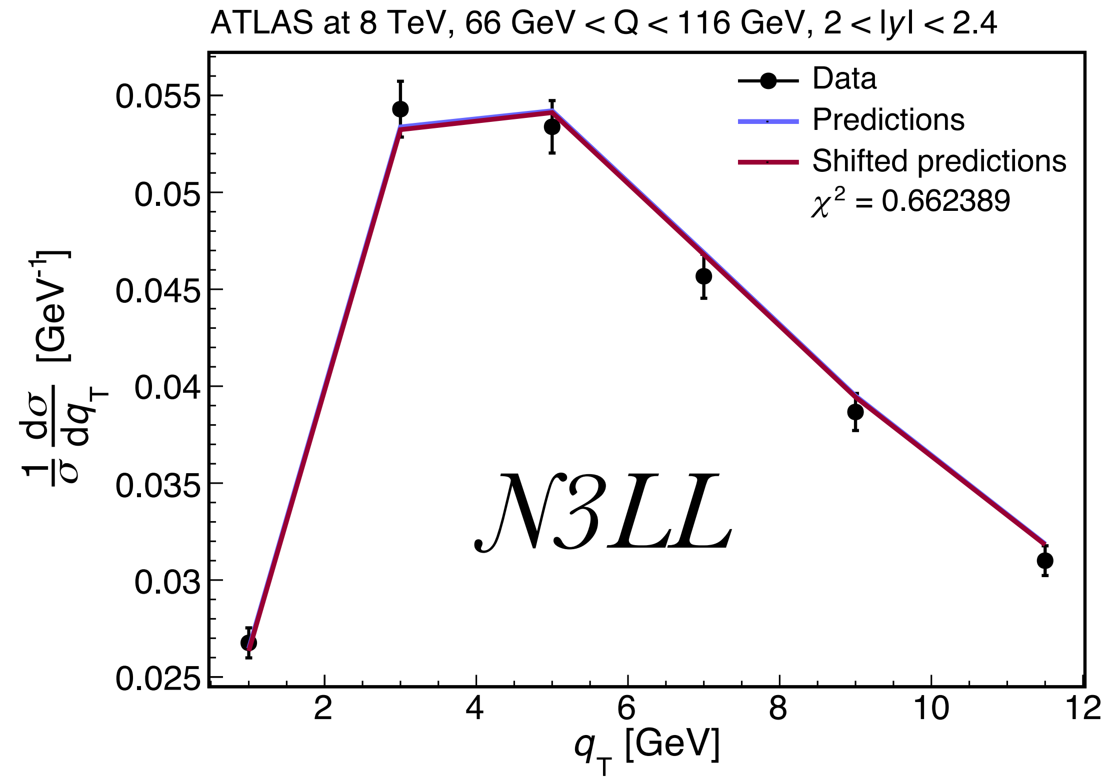
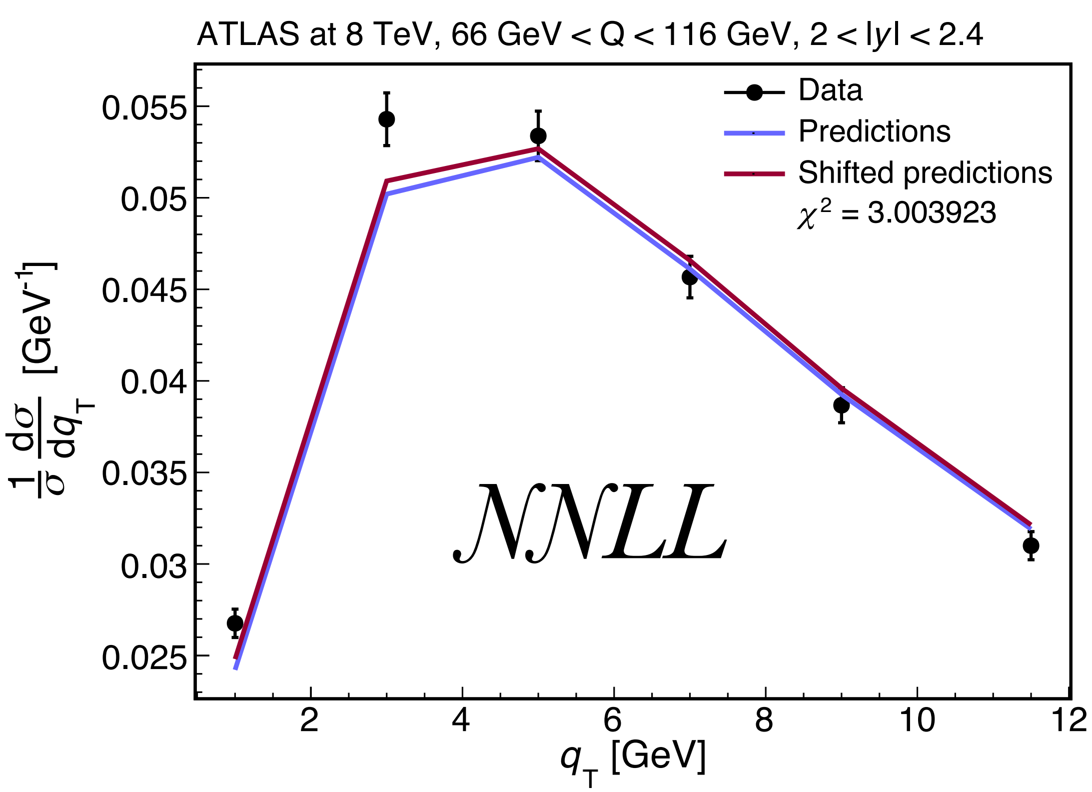
Preliminary: N3LL fit of LHC data







- Chi-square (all LHC data):
  - NLL: 30.82
  - NNLL: 2.40
  - N3LL: 1.32

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SIDIS studies: q_T-integrated multiplicities

- Let us start considering ***q_T*-integrated** SIDIS multiplicities:

$$M^h(x,z,Q^2) = \frac{d^3\sigma^h/dxdzdQ^2}{d^2\sigma/dxdQ^2}$$

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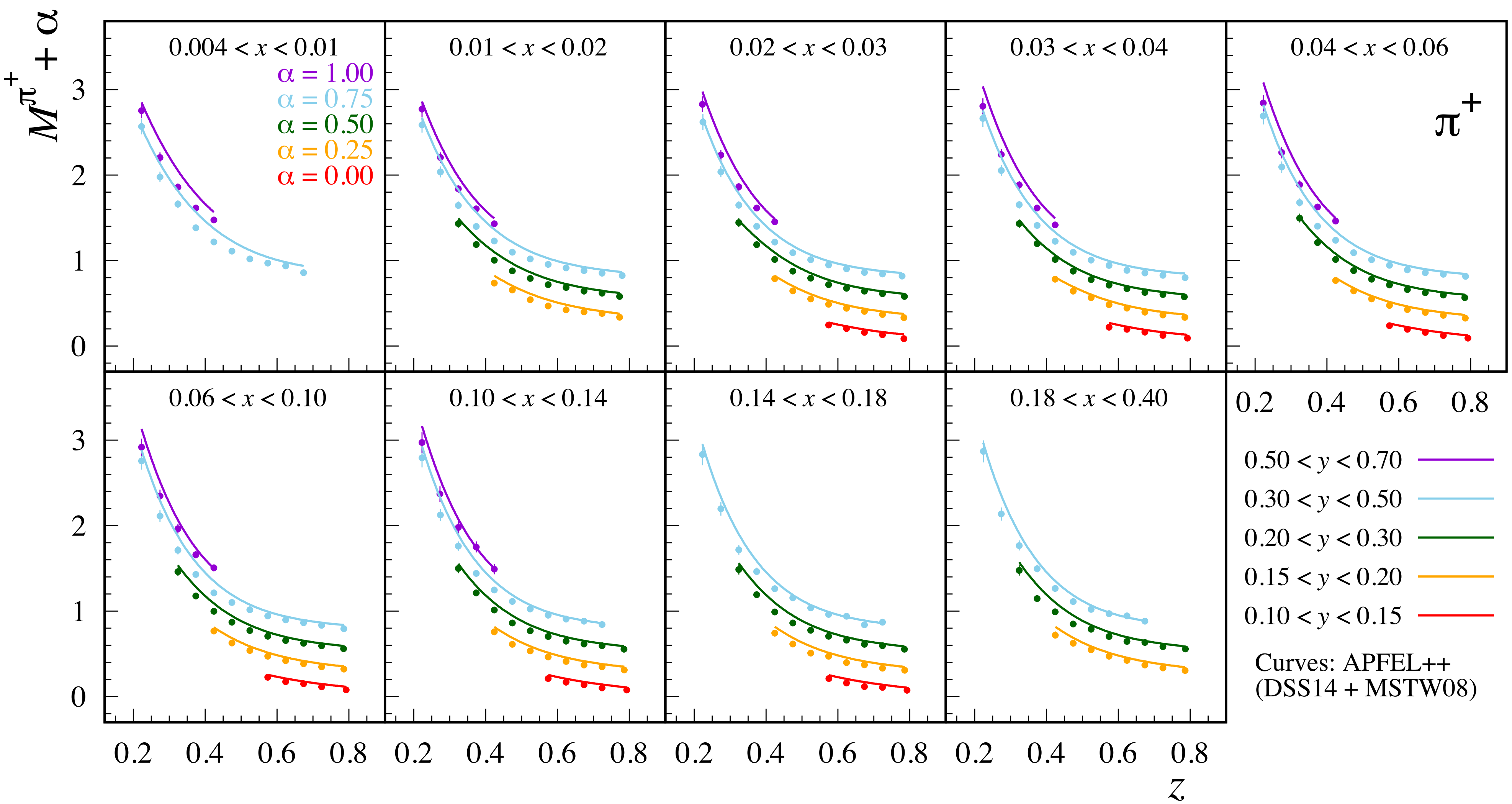
COMPASS data



[PoS DIS2017 (2018) 201]

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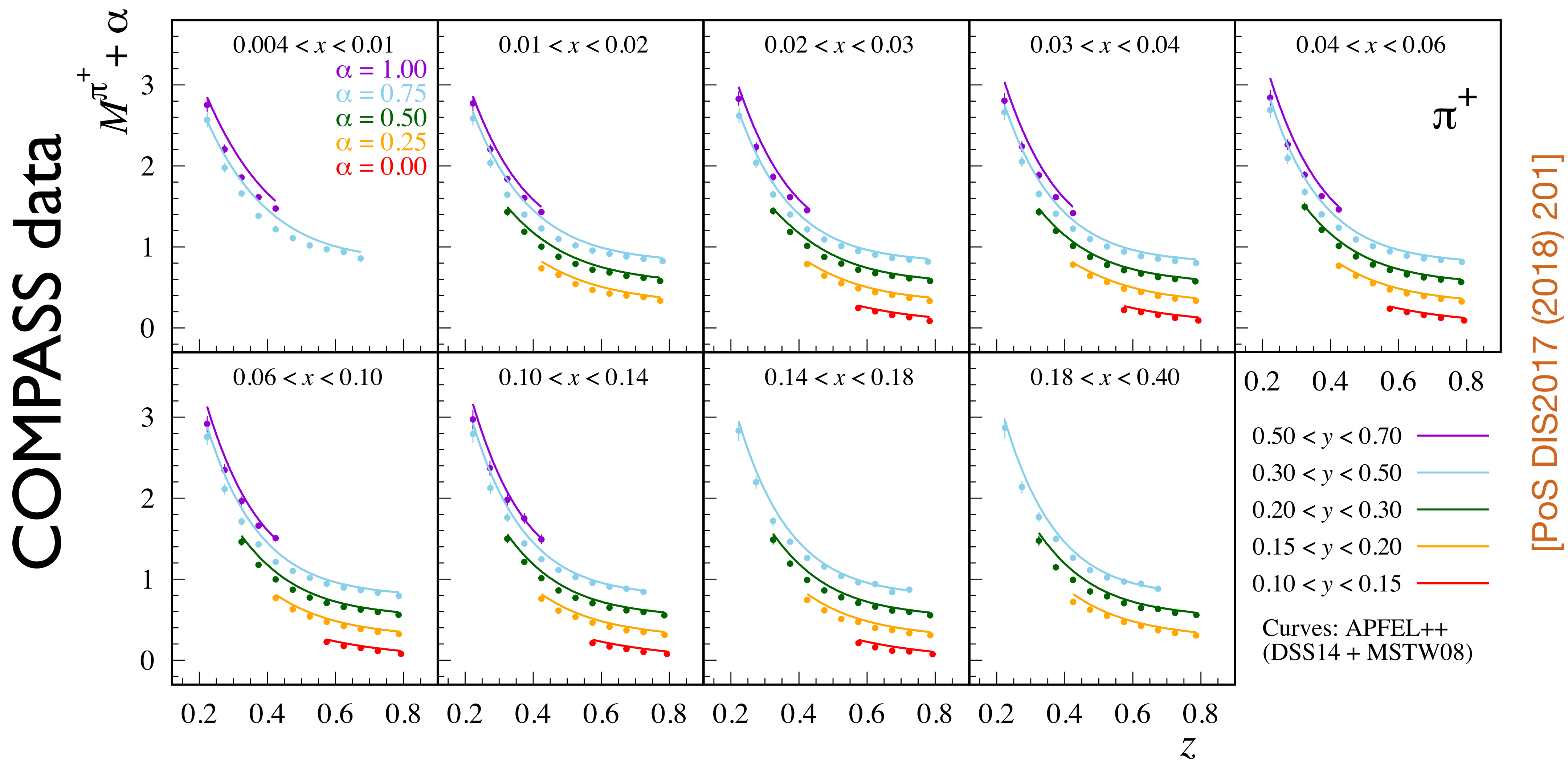

- This works pretty nicely.
- This data has actually be included in the DSS14 fit of collinear FFs.

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SIDIS studies: q_T-differential multiplicities

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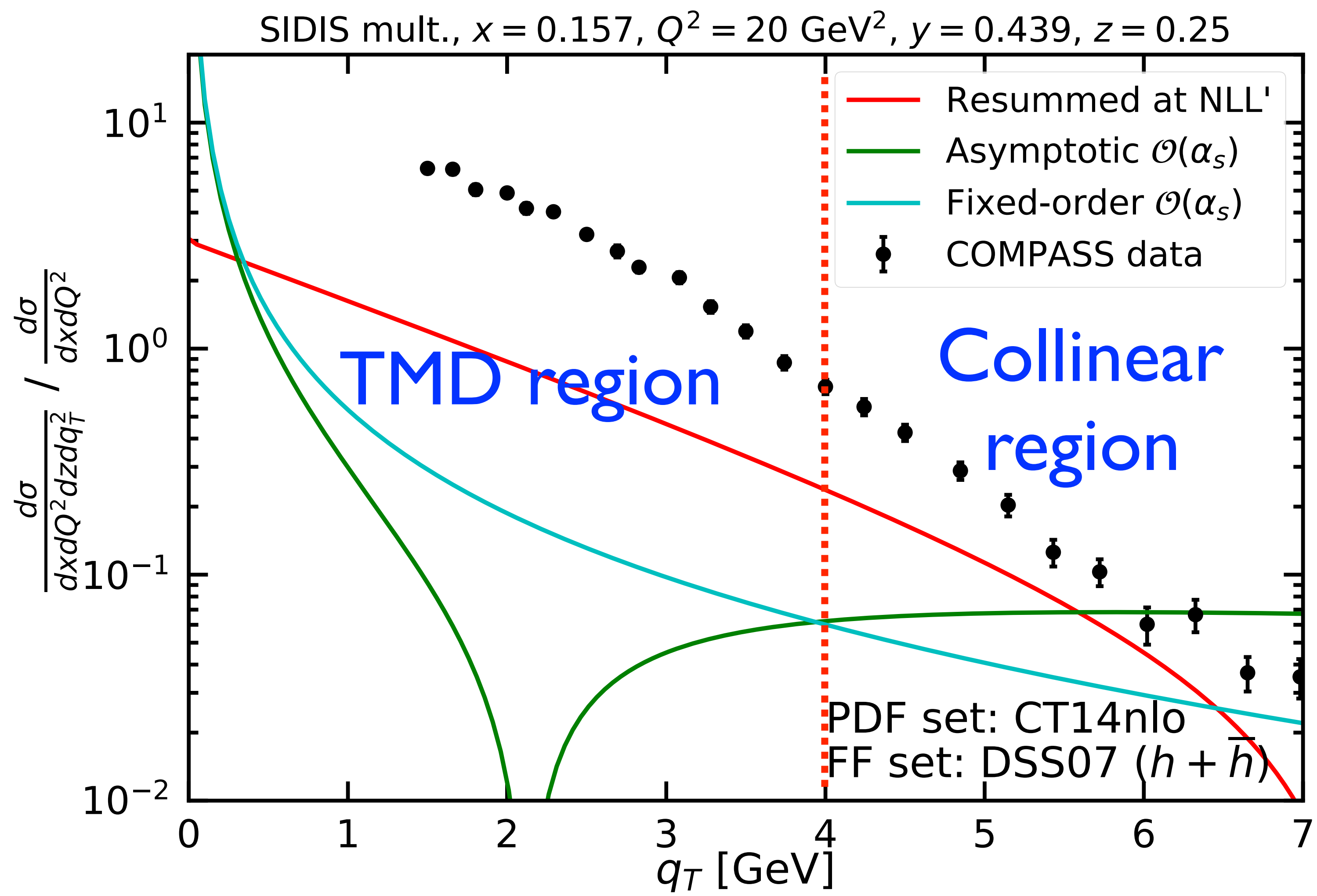

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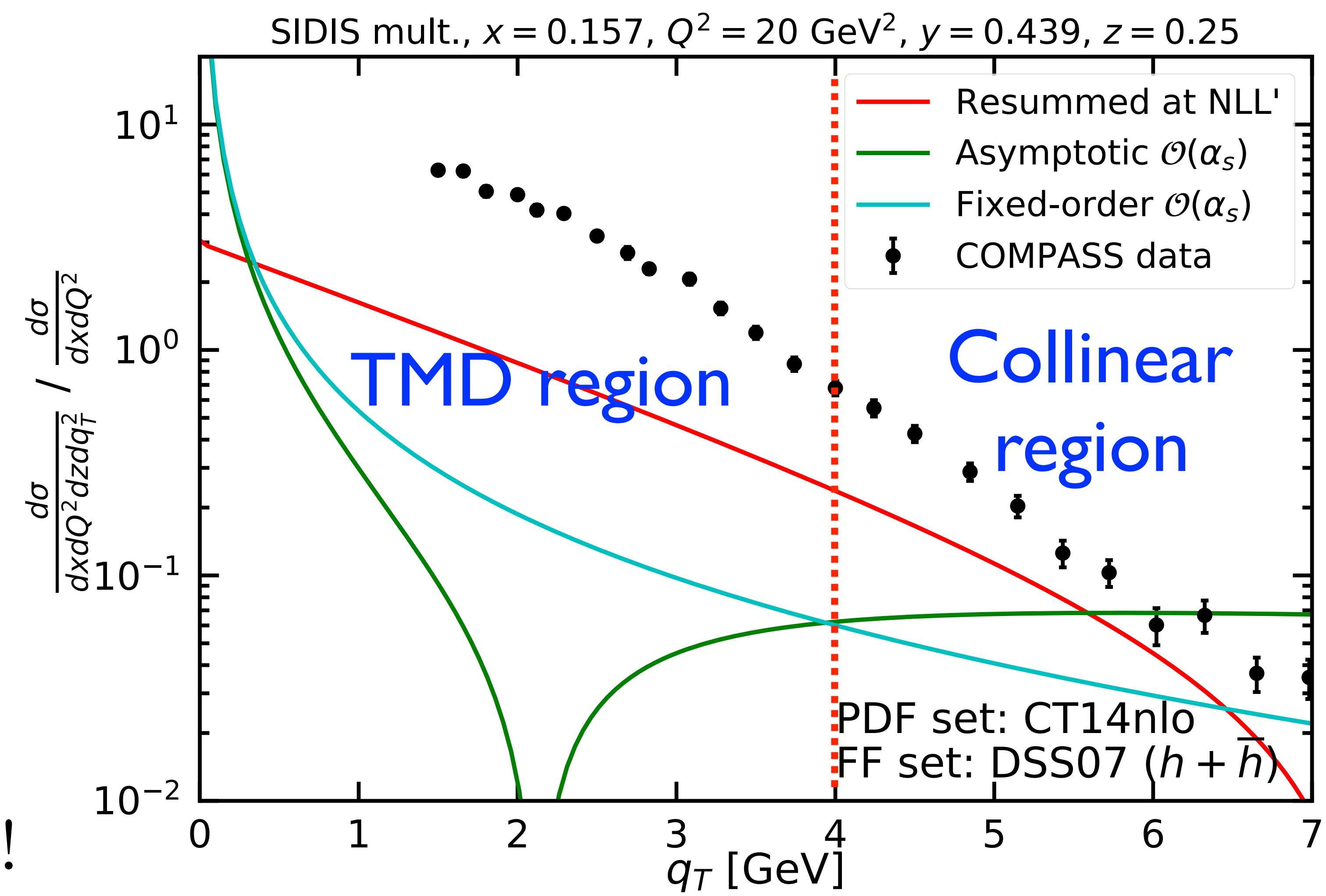

- Utterly off!
- Unlikely that non-perturbative effects can accommodate such differences.
- How comes that q_T-integrated works and q_T-differential does not?

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SIDIS studies: q_T-differential multiplicities

- One may try to integrate analytically the $O(\alpha_s)$ fixed-order q_T-diff:

$$\int dq_T^2 \frac{d^3\sigma^h}{dx\,dz\,dQ^2\,dq_T^2} = \frac{d^3\sigma^h}{dx\,dz\,dQ^2}$$

- This should give the q_T-integrated cross section that we know to work.

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    - this reproduces the $O(1)$ term, but still not enough
    - **threshold-enhanced terms** are still missing from the $O(\alpha_s)$ corrections
    - *preliminary*: **soft-gluon** (threshold) **resummation** possibly crucial!

Conclusions

- **TMD factorisation** provides a valuable tool to descrive $q_{\rm T}$ distributions at small values of $q_{\rm T}$ (resummation of large logs),
  - written in terms of **TMD distributions,**
- Non-perturbative component of TMDs to be determined from **data**
- A lot of effort is being invested on the extraction of TMD PDFs and FFs:
  - wide and precise **datasets** (COMPASS, HERMES, LHC and Tevatron exps.),
  - state-of-the-art **theoretical computation** ($\rm N^3LL$ at small $q_{\rm T}$),
- SIDIS multiplicities from COMPASS and HERMES are challenging:
  - **neither TMD nor collinear** factorisations seem to describe them,
  - more corrections needed (*e.g.* **soft-gluon (threshold) resummation**)
  - find the optimal matching prescription