SuSAv2 model implementation in GENIE and tests of the factorisation approximations

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DE LA RECHERCHE À L'INDUSTRIE

Physique des 2 Infinis et des Origines

Overview

Based on: arXiv:1905.08556

- Recently implemented the SuSAv2 1p1h and 2p2h models in GENIE using hadron tensors.
- Based on implementations of the Valencia 2p2h (NEUT/GENIE)
- Exactly reproduces the *inclusive* predictions of the models
- (Semi-)exclusive predictions are obtained using ad-hoc "*factorisation*" approximations common to most model implementations

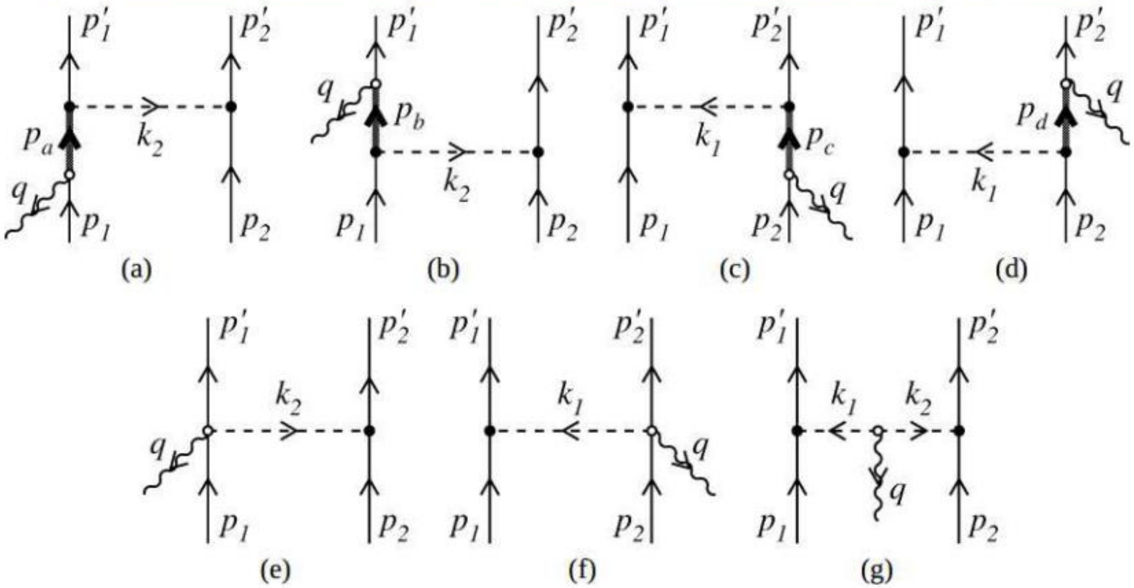
This talk:

- SuSAv2-MEC and comparison to Valencia model
- GENIEv3 implementations
- Generating hadron kinematics – the ugly truth
- Future improvements

SuSAv2 2p2h

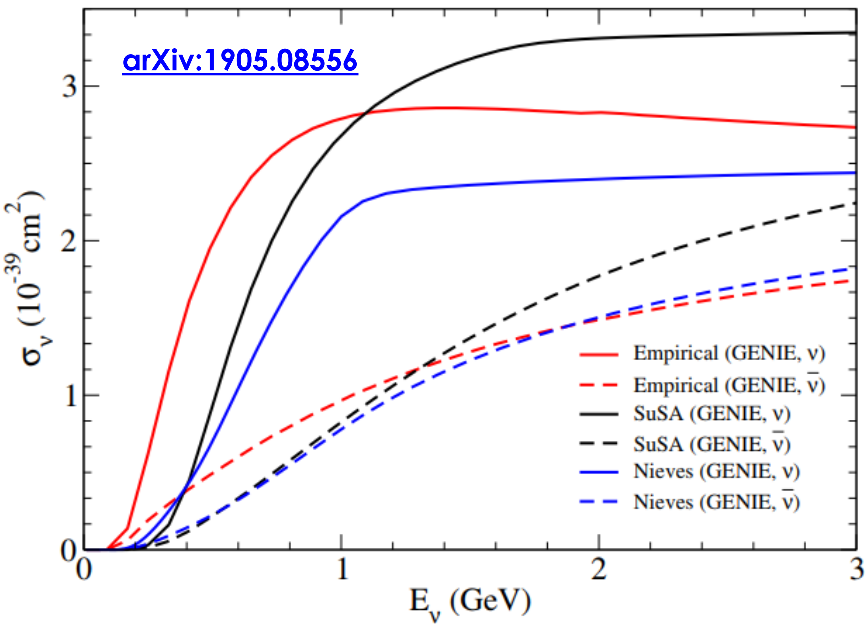
$$W^{\mu\nu}_{2p-2h} = \frac{V}{(2\pi)^9}\int d^3p'_1 d^3h_1 d^3h_2 \frac{M^4}{E_1E_2E'_1E'_2}\Theta(p'_1,p'_2,h_1,h_2) \; r^{\mu\nu}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{h}_1,\mathbf{h}_2)\delta(E'_1+E'_2-E_1-E_2-\omega)\,,$$

Over 100,000 terms are involved in the calculation, with seven-dimensional integrations



- Based on the calculation performed by De Pace et al., (2003) for (e, e') scattering and extended to the weak sector by Amaro, Ruiz Simo et al.
  [PRD 90, 033012 (2014); PRD 90, 053010 (2014); JPG 44, 065105 (2017); PLB 762, 124 (2016)]
- Performed within an RFG nuclear model (like Nieves), SuSAv2-MEC is fully relativistic – no approximations
- HUGE calculation, takes a long time to calculate a full cross section
  - Normally a parameterisation is used

Comparison to Valencia 2p2h



- Valencia model makes some non-relativistic approximations limiting validity above 1.2 GeV, SuSAv2-MEC does not.

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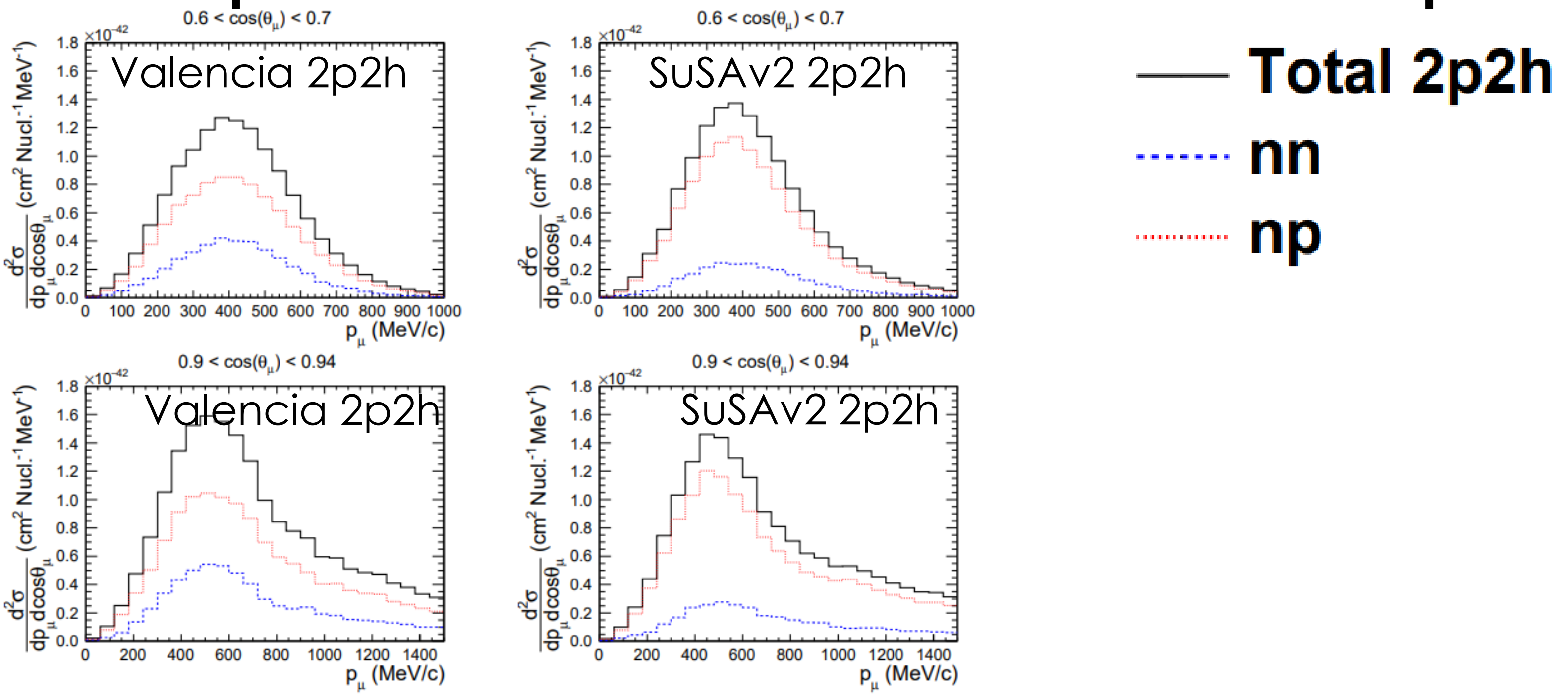

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- Valencia model rejects direct/exchange interference terms, SuSAv2-MEC does not – Valencia predicts relatively less np initial states

10.1016/j.physletb.2016.09.021

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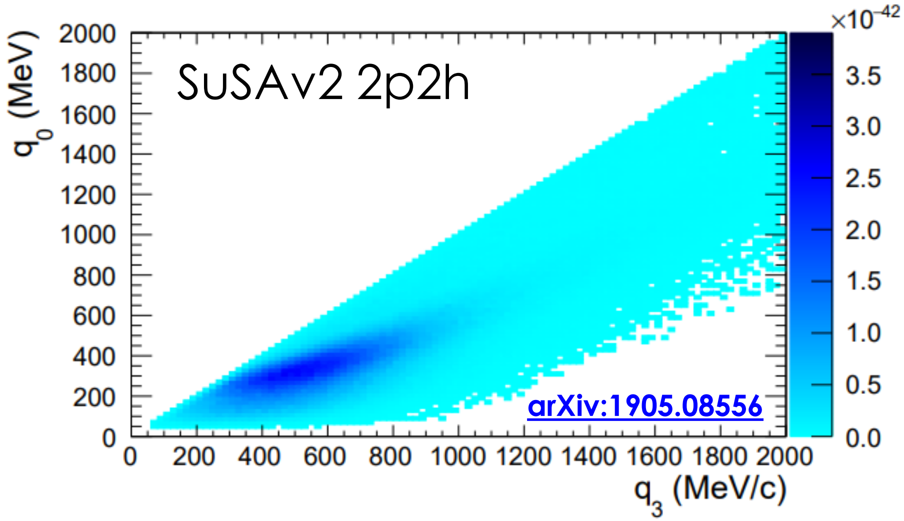
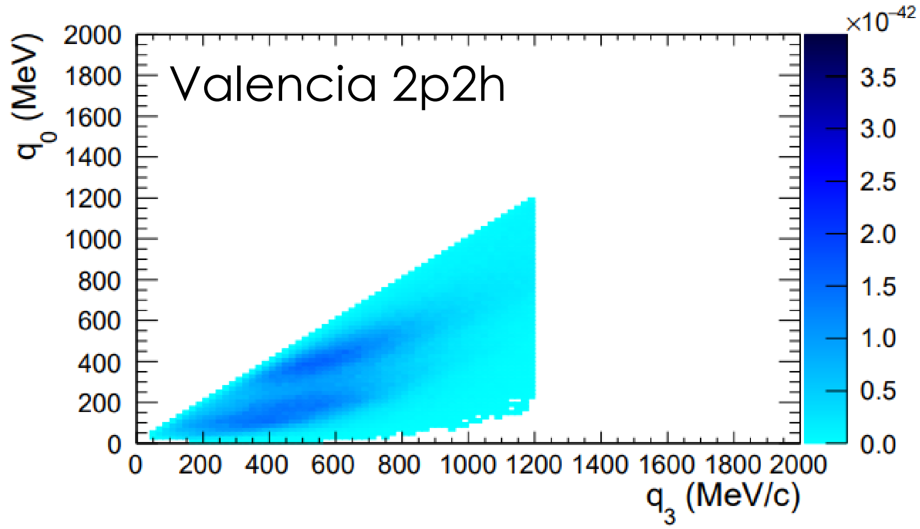




- Valencia model makes some non-relativistic approximations limiting validity above 1.2 GeV, SuSAv2-MEC does not.
- Valencia model rejects direct/exchange interference terms, SuSAv2-MEC does not – Valencia predicts relatively less pp final states
- Valencia model includes a different set of diagrams (some from imaginary part of the W)

SuSAv2 1p1h – very brief theory!

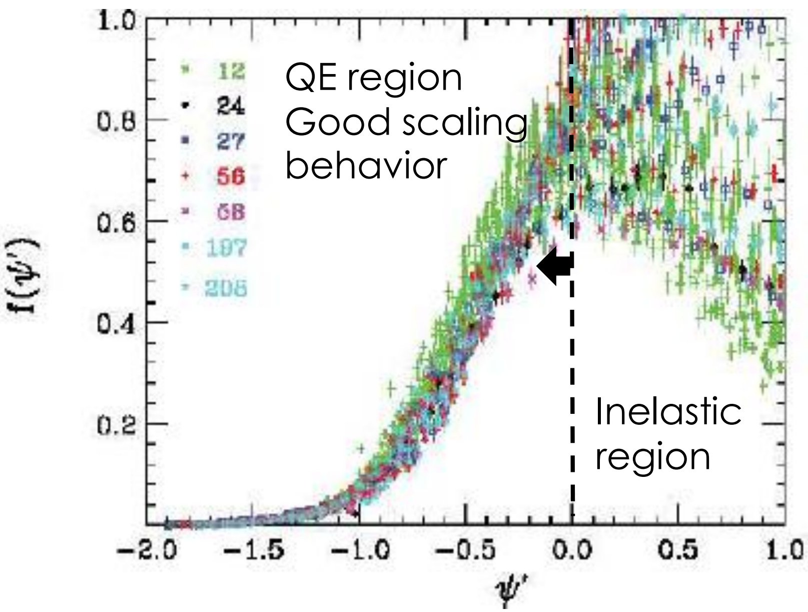
Basic idea: use the scaling function encode nuclear dynamics

$$f(\psi) \equiv f(q,\omega) \sim \frac{\sigma_{QE}(\text{nuclear effects})}{\sigma_{\text{single nucleon}}(\text{no nuclear effects})} \quad ; \quad \psi\text{-scaling variable}$$

In inclusive QE scattering we can observe:

- ☆ Scaling of 1st kind (independence on q)
- ☆ Scaling of 2nd kind (independence on Z)

$\Longrightarrow$ SuperScaling



SuSA: extract scaling function from e, e' data and then assume $f_L = f_T$

- In reality not quite true ($f_T^{ee'} > f_L^{ee'}$)

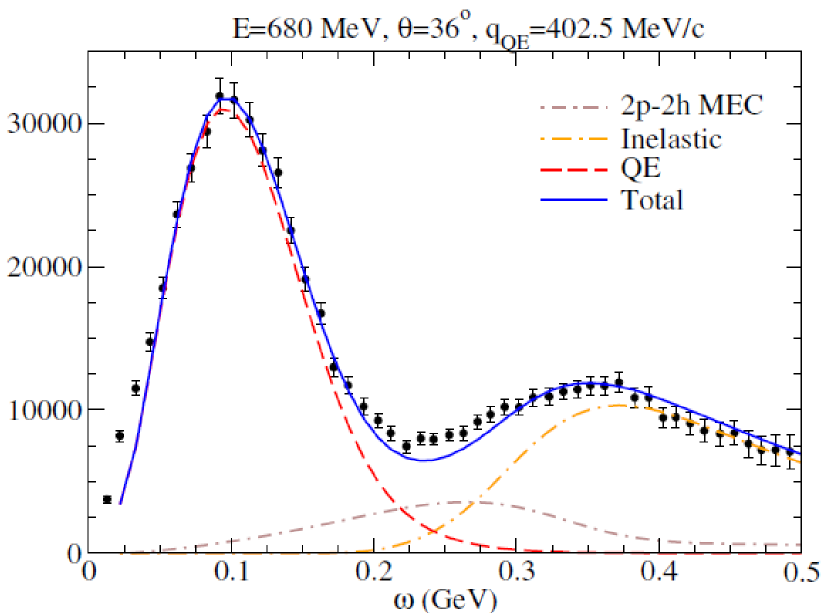
(see G.D. Megias' Thesis for details)

SuSAv2: build scaling function from microscopic model – **Relativistic Mean Field** (RMF) theory

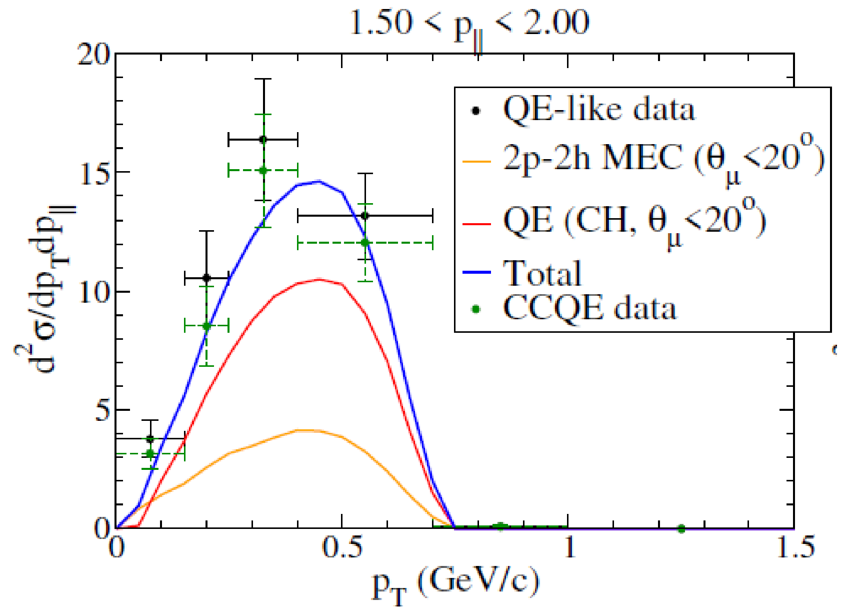
- Excellent description of QE e, e' data
- A quick way of getting RMF predictions!

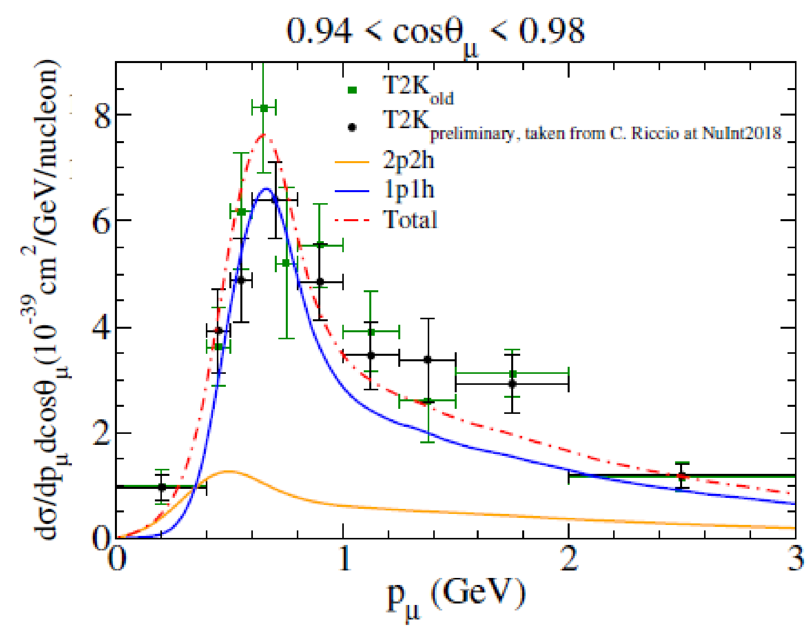
PRC90, 035501 (2014) PRD94, 013012 (2016)

SuSAv2-MEC



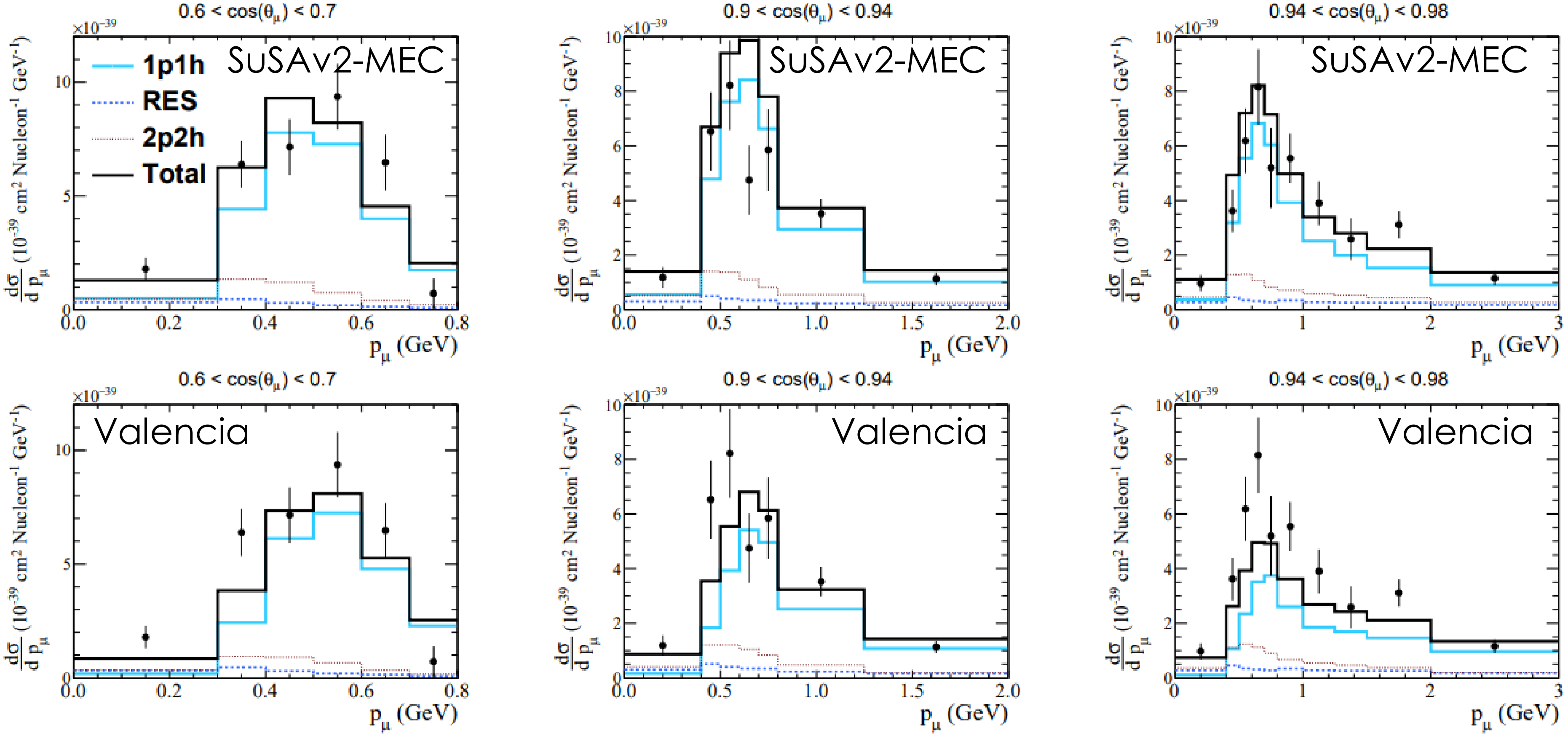
- Based on sound microscopic model calculations
- **Well validated on electron scattering data**
- Is able to describe neutrino scattering data





arXiv:1905.08556

Comparison to Valencia model



Provides a significantly different predictions to the Valencia model

Complimentary addition to the generators

Implementation – lepton part

Like the Valencia model, SuSAv2-MEC is able to predict only the outgoing lepton kinematics. The double differential cross section can be written:

$$\frac{d^2\sigma}{dq_0 q_3} = \sigma_0 \eta_{ij}(q_0, q_3) W_{ij}(q_0, q_3)$$

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- The hadron tensor elements are stored in tables which specify q0 and q3 in bins of 5 MeV between 0 and 2 GeV – unique SuSAv2-MEC tensors
  - Use a GENIE's bilinear interpolation function to evaluate specific q0,q3
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  - Hadron tensors will be provided for a few select targets (C and O so far, may add others). Can scale to other nuclei.
- GENIE implements the hadron tensor from the full microscopic calculation, but this is not what is often used by the SuSA group (they use a parameterization)
  - GENIE's implementation is slightly more true to the full calculation (most of the time)

So far, so good

- So we can re-produce inclusive model predictions in the generators, no trouble!
- Works for all neutrinos and electrons – can even separate 2p2h initial state pairs.
- But neutrino oscillation analyses need hadron kinematics too.
- As you heard yesterday, this is where things get tricky…

Factorisation Approximation (FA)

https://indico.ectstar.eu/event/19/contributions/221/

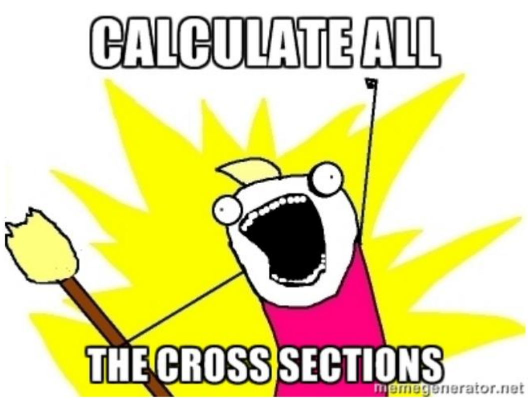
The inclusive models we use only have the hadron tensor elements needed to calculate lepton kinematics

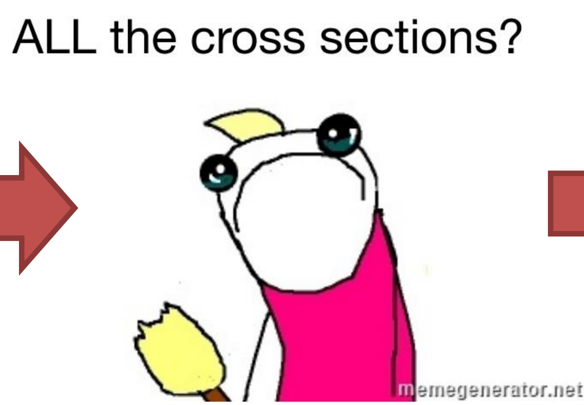
$$\sigma \sim \eta_{\mu\nu} W^{\mu\nu}$$

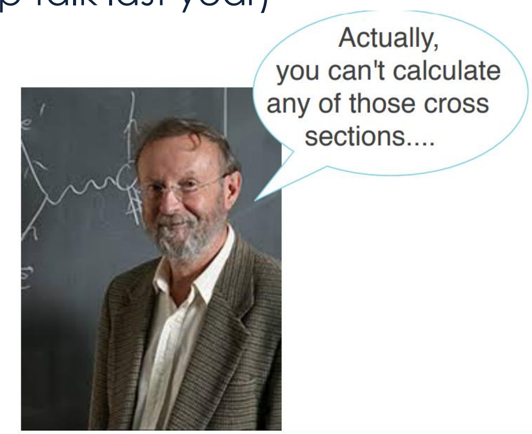
$$\begin{aligned}
\eta^{s}_{\mu\nu} W^{\mu\nu}_{s} \quad \sim \quad & \widehat{V}_{CC} W^{CC}_{semi} + \widehat{V}_{CL} W^{CL}_{semi} + \widehat{V}_{LL} W^{LL}_{semi} \\
& + \widehat{V}_{T} W^{T}_{semi} + \cancel{\widehat{V}_{TT} W^{TT}_{semi}} + \cancel{\widehat{V}_{TC} W^{TC}_{semi}} + \cancel{\widehat{V}_{TL} W^{TL}_{semi}} \\
\eta^{a}_{\mu\nu} W^{\mu\nu}_{a} \sim \; & \widehat{V}_{T'} W^{T'}_{semi} + \cancel{\widehat{V}_{TC'} W^{TC'}_{semi}} + \cancel{\widehat{V}_{TL'} W^{TL'}_{semi}}
\end{aligned}$$

The terms needed to calculate hadrons **simply are not there**, everything we do to predict them is *ad-hoc* and should be treated with caution

Progression of the workshop (from G. Perdue's GENIE ECT* workshop talk last year)

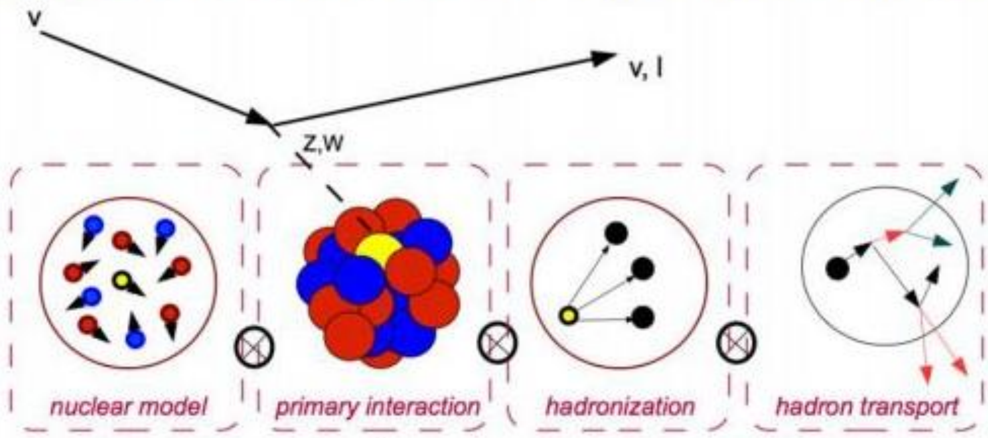






Factorisation Approximation (FA)

In the generators we factorise interactions



We use an inclusive ($\frac{d\sigma_{CCQE}}{dp_\mu dcos\theta_\mu}$) model for this bit. Only ad-hoc nuclear / FSI models then allow us to predict hadron kinematics

Theorists seem unconvinced …

- There is no guarantee of getting a result similar to a full semi-inclusive calculation:

$$\frac{d\sigma_{CCQE}}{dp_\mu d\theta_\mu dp_p d\theta_p d\theta_{\mu p}}$$

All models in GENIE (including the new SuSAv2) are **only valid for inclusive predictions**. When we try and use them to make semi-inclusive predictions we do **a lot of questionable physics**.

The ugly truth

Xsec calculation:

- Perform inclusive calculation (q_0, q_3) using SuSAv2 hadron tensor

Hadronic side: more complicated …
Bold/*italics* **bits represent very**/*slightly* **questionable physics** in the FA, but this is no worse than in most other model implementations.

- Draw target nucleon from chosen nuclear model **irrespective of q_0, q_3**
- Get removal energy from RMF-like treatment, *re-throw from nuclear model if nucleon is Pauli blocked*
- Transfer all of ω, q to nucleon, **none to remnant**
- Subtract removal energy, **put proton on-shell with adjustment of p** (only needed for 1p1h) **then conserve momentum by adjusting remnant kinematics**
- *Do FSI cascade and rest of interaction using standard GENIE methods*

The ugly truth

Assumptions to get hadron kinematics from inclusive models:

- *Inclusive interaction kinematics are independent of the initial-state nucleon kinematics and binding energy*
- *Semi-classical FSI-cascade, unrelated to the inclusive model's nuclear dynamics*
- *Simplistic treatment of energy(/momentum) transfer to the nuclear remnant (treatment depends on generator)*

More general than just the impulse approximation

But without more exclusive model predictions, we can't do much better…

Why do I care (oscillations)

Inclusive interaction kinematics are independent of the initial-state nucleon kinematics and binding energy

Simple example of what I mean:

- Consider a model where deeper nucleons have a larger maximum momentum p_{max} and larger E_b (LFG or a shell model)
- In reality we might expect higher q_0 would allow interactions with deeper nucleons. So the p_{max} and E_b will depend on q_0.
- But our choice of p in generators is entirely factorized from q_0

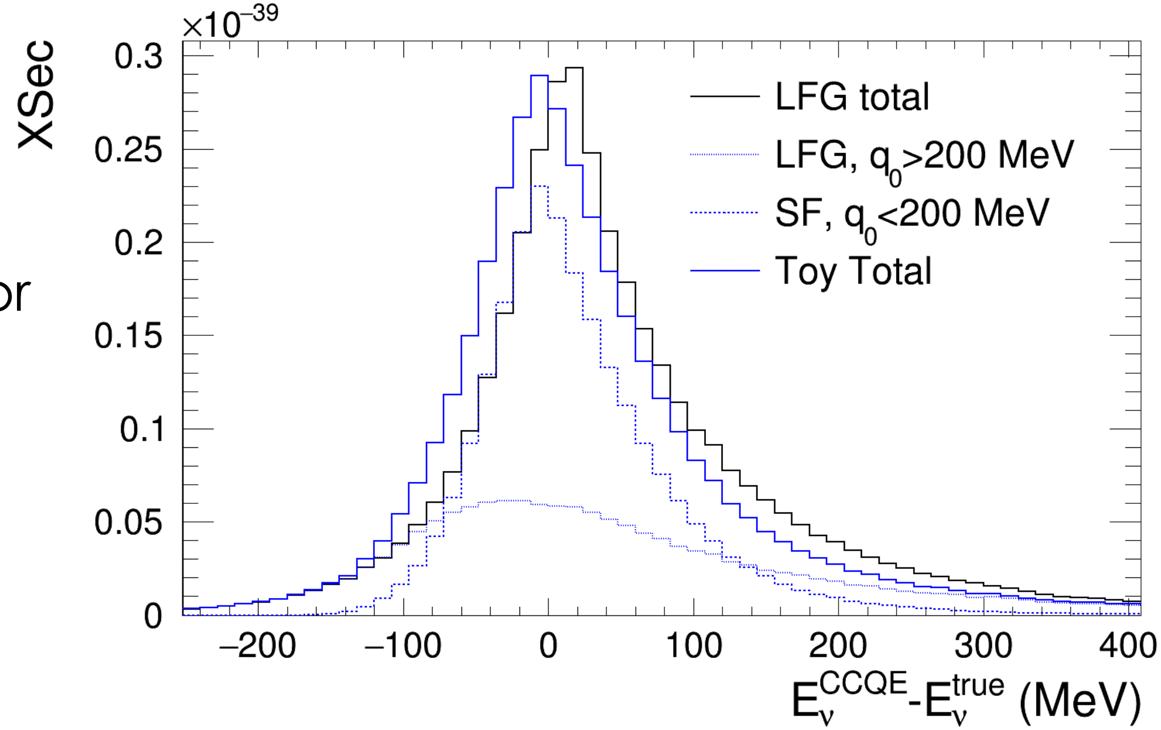
- If the Fermi motion and binding energy sampled depends on the initial-state nucleon kinematics then reconstruction of the neutrino energy will be affected → impact on neutrino oscillation analyses

Why do I care (oscillations)

Inclusive interaction kinematics are independent of the initial-state nucleon kinematics and binding energy

For T2K/HK we can mock this effect up with a simple toy:

- Let's say (arbitrarily) the real Fermi motion/E_b behaves more like an SF for low q_0 and like an LFG at higher q_0
- We can compare the $E_\nu^{CCQE} - E_\nu^{true}$ for this mixed model with what we get for a pure LFG



Testing the FA using RMF

(See yesterdays talk)

The plan to test FA:

- Compute exclusive results using theory, compare it to the same theory implemented in a generator
- Relativistic mean field theory (the base model of SuSAv2) allows this (the current neutrino version can compute $|p_p|$ but not θ_p)
- Will do this test calculating ν_μ 1p1h contribution for T2K flux with (exclusive) and without (inclusive) a restriction on the momentum of the outgoing proton (500 MeV/c) as was measured in Phys. Rev. D 98, 032003 (2018)

Caveats:

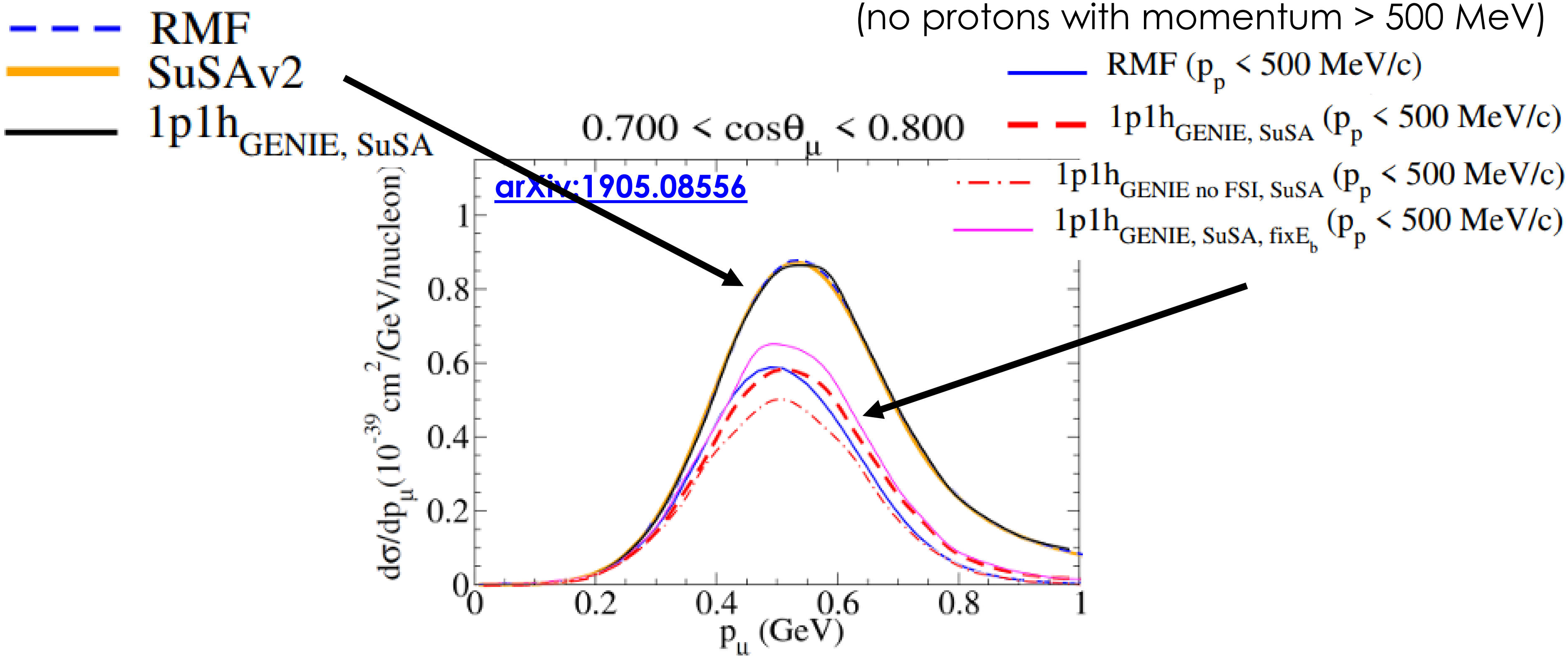
- Even for the inclusive case, SuSAv2 and RMF are not quite identical at very high and low kinematics – will stick to a good kinematic region
- For the FA, will use LFG rather than the real RMF spectral function (work in progress)

A first test of the FA

(See yesterdays talk)

These lines show the ***inclusive*** 1p1h prediction (no proton constraints)

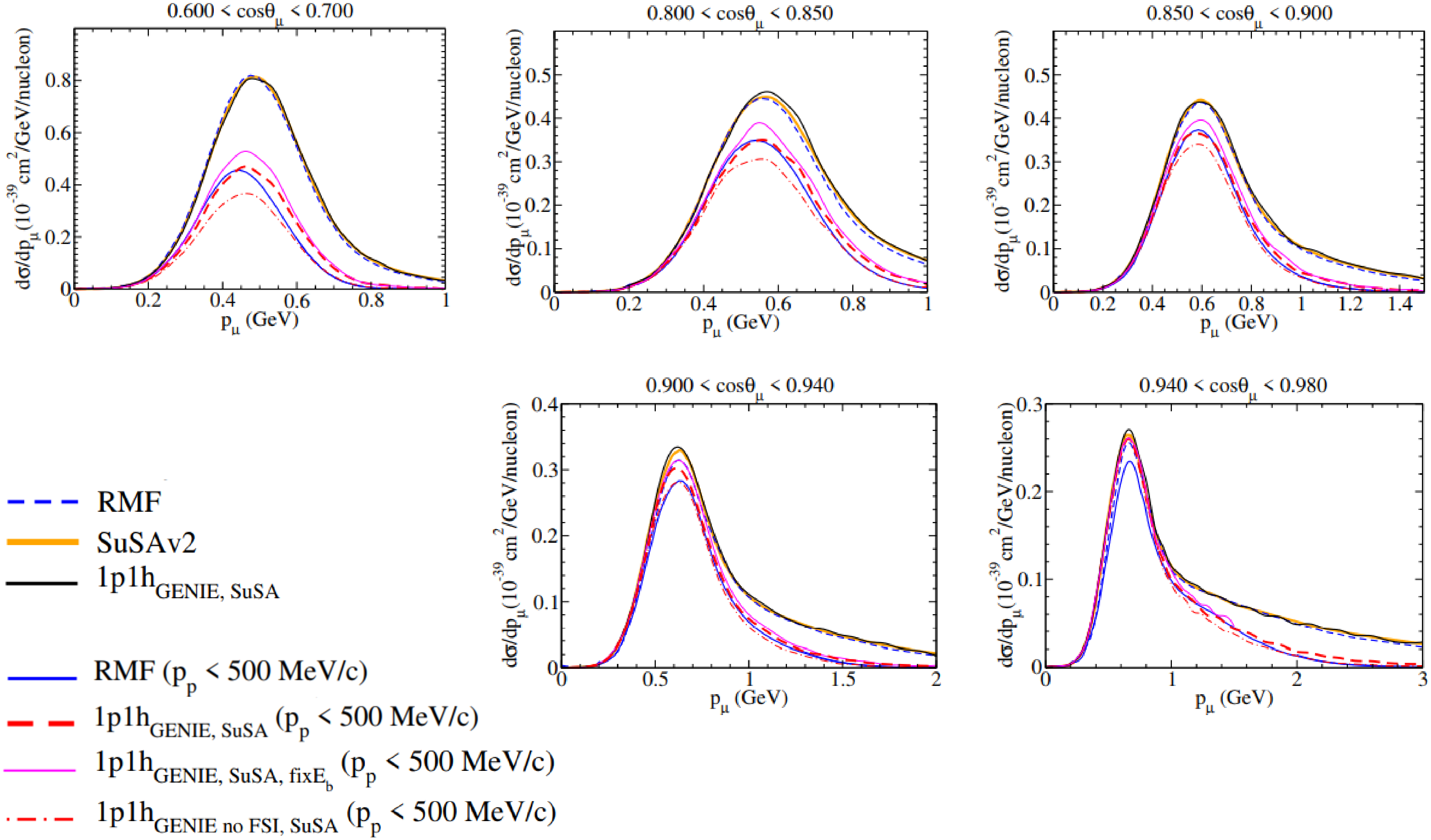
These lines show the ***exclusive*** 1p1h prediction
(no protons with momentum > 500 MeV)



- For *inclusive* calculations the microscopic base model (RMF), the inclusive theory (SuSAv2) and the implementation (in GENIE) all agree.
- *Exclusive* GENIE calculations do not match RMF. Varying the ingredients to the FA leads to quite different predictions.

A first test of the FA

arXiv:1905.08556



Lots of scope for improvement!

Implementation of SuSAv2-MEC hadronic part (FA):

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Avoid by sampling full exclusive xsec?

Mitigate by making nuclear model q_0, q_3 dependant?

Different model for each shell?

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Removal energy should also depend on chosen initial nucleon momentum which should depend on inclusive kinematics …

The remnant should take *some* momentum … how much?

This is just bad, we're working on this. Have some ideas.

How much can we improve FSI? Better motivated cascades? GiBUU-like hadron transport?

Lots of scope for improvement!

Avoid by sampling full exclusive xsec?

Mitigate by making nuclear model q_0, q_3 dependant?

Different model for each shell?

Im… …2-MEC hadronic part (FA):

- Dr… …hosen nuclear model **irrespe…**
- Get removal e… …ent, *re-throw from nuclear mode…*
- Transfer all of ω, q to nucleon…
- Subtract removal energy, **put proton… adjustment of p** (only needed for 1p1h) **…momentum by adjusting remnant kinematics**
- *Do FSI cascade and rest of interaction using standard GENIE methods*

Removal energy should also depend on chosen initial nucleon momentum which should depend on inclusive kinematics …

The remnant should … …some momentum …uch?

so…

How much can we improve FSI? Better motivated cascades? GiBUU-like hadron transport?

Any substantial improvement requires more semi-inclusive inputs from theory!

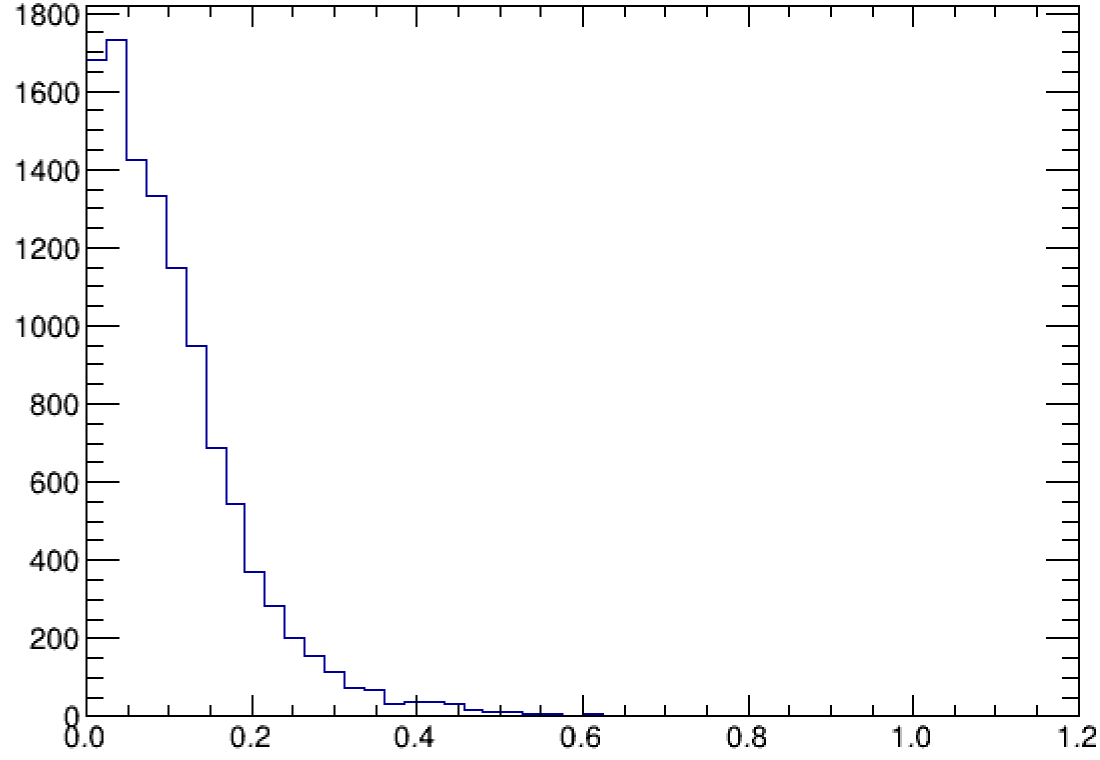
Very preliminary

$E_f = E_i + q_0$

$p_f = p_i + q_3$

$E_f = \sqrt{p_f^2 + m_f^2}$

$E_i = \sqrt{p_i^2 + m_i^2} - E_b$



Inferred Initial nucleon momentum

Summary

- We now have a well established framework to implement new models in GENIE (and beyond) using hadron tensors
- Exactly reproduces *inclusive* input model predictions
- Hadron kinematic predictions are made using “factorisation” approximations (FA) – ad-hoc and probably unreliable
- Showed some very simple tests of FA – need more detailed analysis to better assess validity
- Lot’s of scope for improvement, but I think any serious progress needs more exclusive inputs from theory

Discussion topics

Hadron tensor implementations

- What does the calculation of an xsec using a hadron tensor look like?
- How should this be implemented in the generators?
- Is this the same for 1p1h, 2p2h and pion production?
- What choices do have for making semi-inclusive predictions in the generators? How do we currently make these choices?

Factorization approximations

- Can we quantify the impact? Develop uncertainties to cover the difference?
- What are the possible biases from this for neutrino oscillation analyses?
- What can we learn about its validity from electron scattering data? (E.g. to what extent does the missing energy and momentum depend on the kinematics?)
- What can we measure in neutrino scattering to test this (transverse imbalance as a function of lepton kinematics?)

Factorization mitigation

- Can we simply implement full semi-inclusive calculations directly?
  - Would probably require a new paradigm for event generation
  - 15 vs 5 nuclear responses – is this too hard or too slow?
  - Did we already do this for electron scattering? Were models for e,e'p fully exclusive?
- Even if we do this, how should we treat FSI?
- SF models are a bit different – are they immune to factorisation issues?
- Can we use some information from semi-inclusive predictions to make better choices in the factorisation scheme?
- Can we implement separate hadron tensors and spectral functions for each shell?

Bonus topic: What can we learn from LHC experiences? Can they tell us how far we can go in complexity in our MC generators and what tricks that we can use to do so?