LOW/HIGH ENERGY TRANSFER REGIONS IN VALENCIA MODEL

Joanna Sobczyk
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ENERGY RANGE

supernovae/solar neutrinos

energy transfer: few/tens MeV

accelerator neutrinos

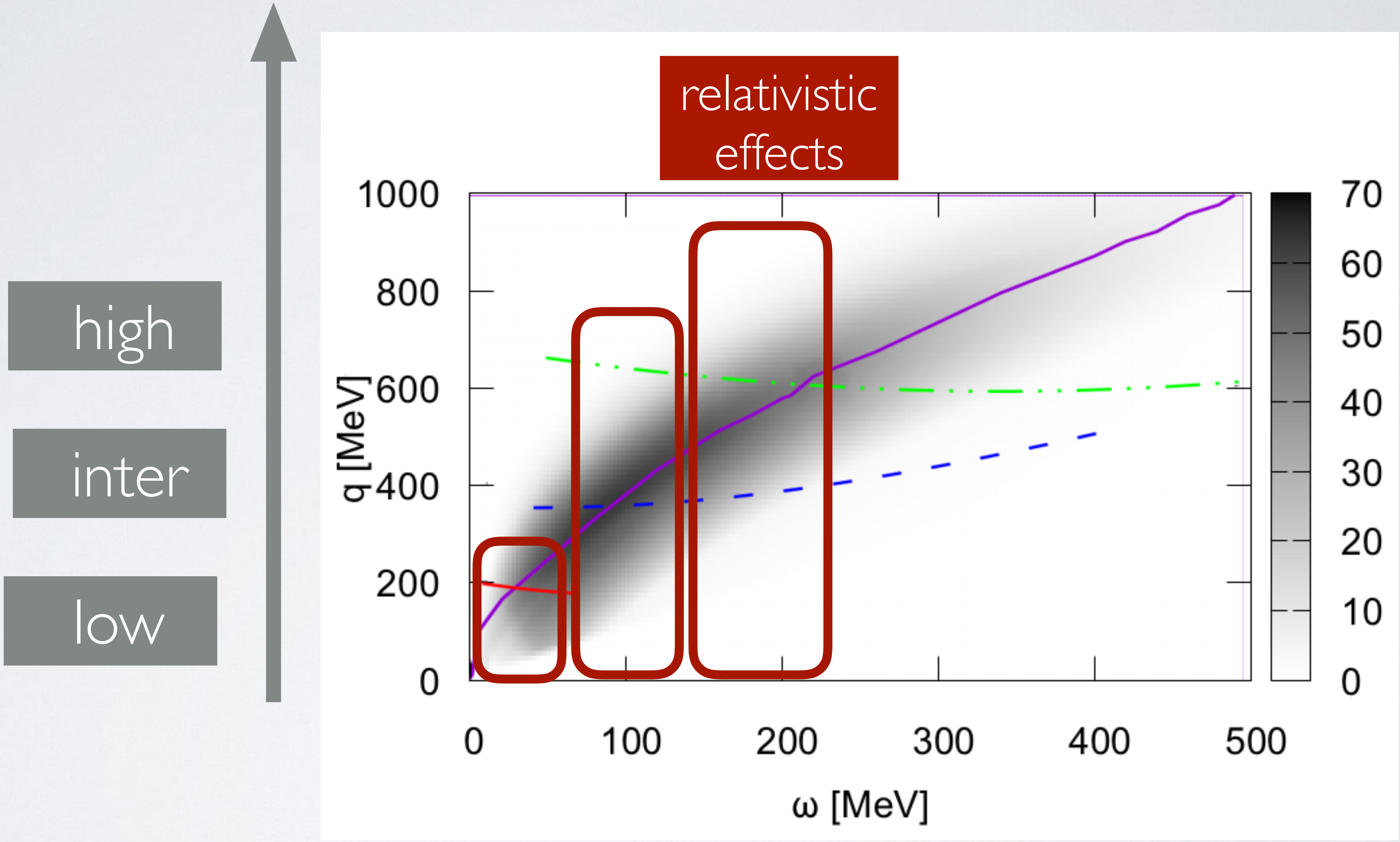
energy transfer: MeV - GeV

What kind of physics is needed in each case?

SUPERNOVAE/SOLAR NEUTRINOS

- S. Gardiner talk
- Available models:
  - CRPA
  - ab-initio calculations

ACCELERATOR NEUTRINOS



high

inter

low

relativistic effects

$$\left.\frac{d\sigma}{dq\,d\omega}\right|^{\text{osc}}_{\text{T2K}} = \frac{1}{\mathcal{W}}\int dE \left.\frac{d\sigma}{dq\,d\omega}\right|^{\text{CCQE}} \mathcal{F}(E)\mathcal{P}_{\nu_\mu\to\nu_e}(E)$$

J.E.S; PRC **96**, 045501 (2017)

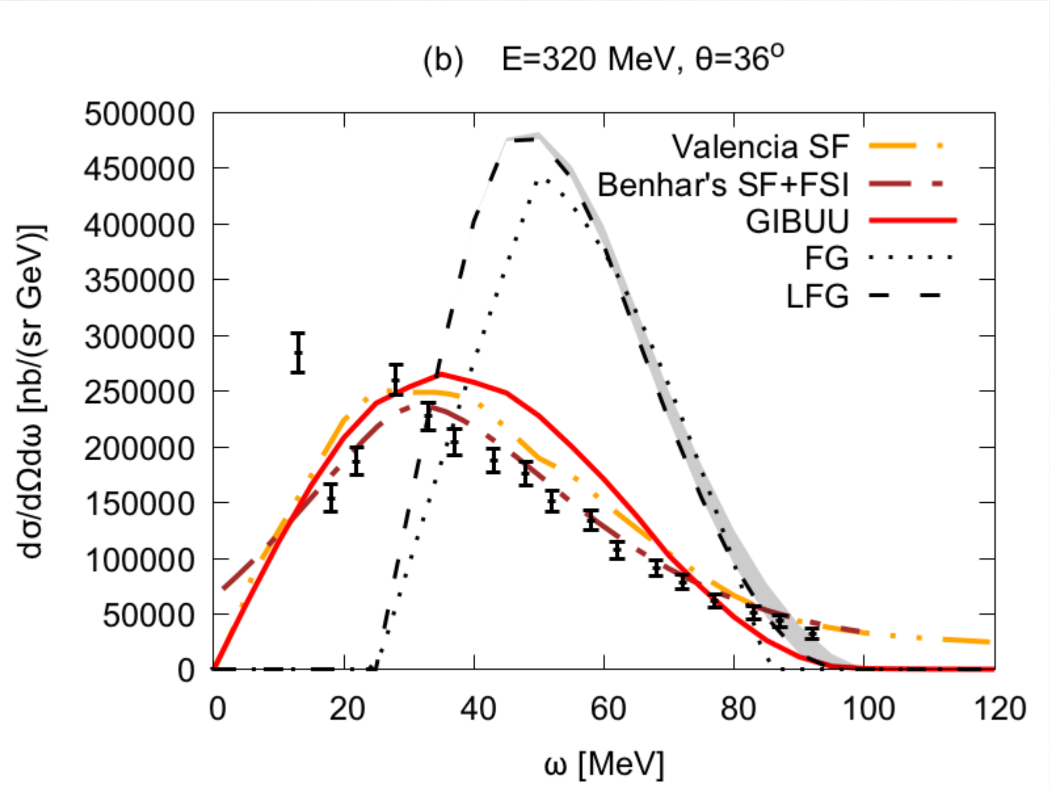
EXISTING DESCRIPTIONS

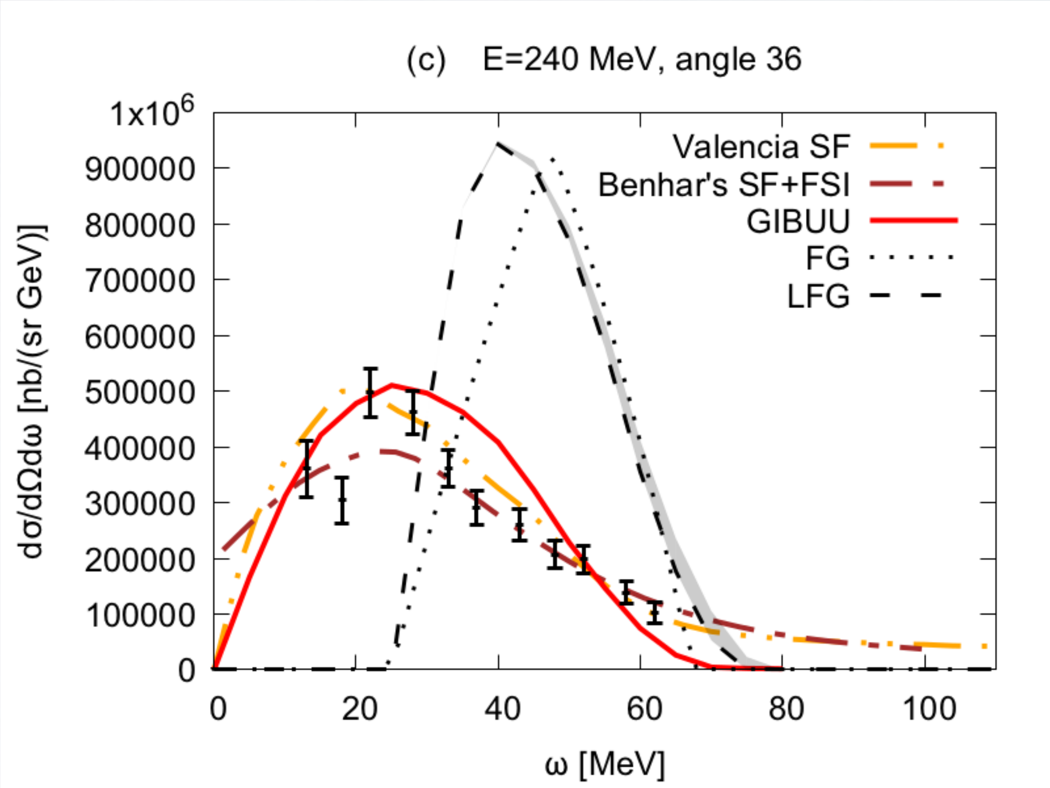
- LDA + potential (GiBUU)
- SF + Optical potential (Rome)
- Hartree-Fock +CRPA (Ghent)
- LDA + SF (Valencia)
- SuSAv2
- ab-initio (GFMC)

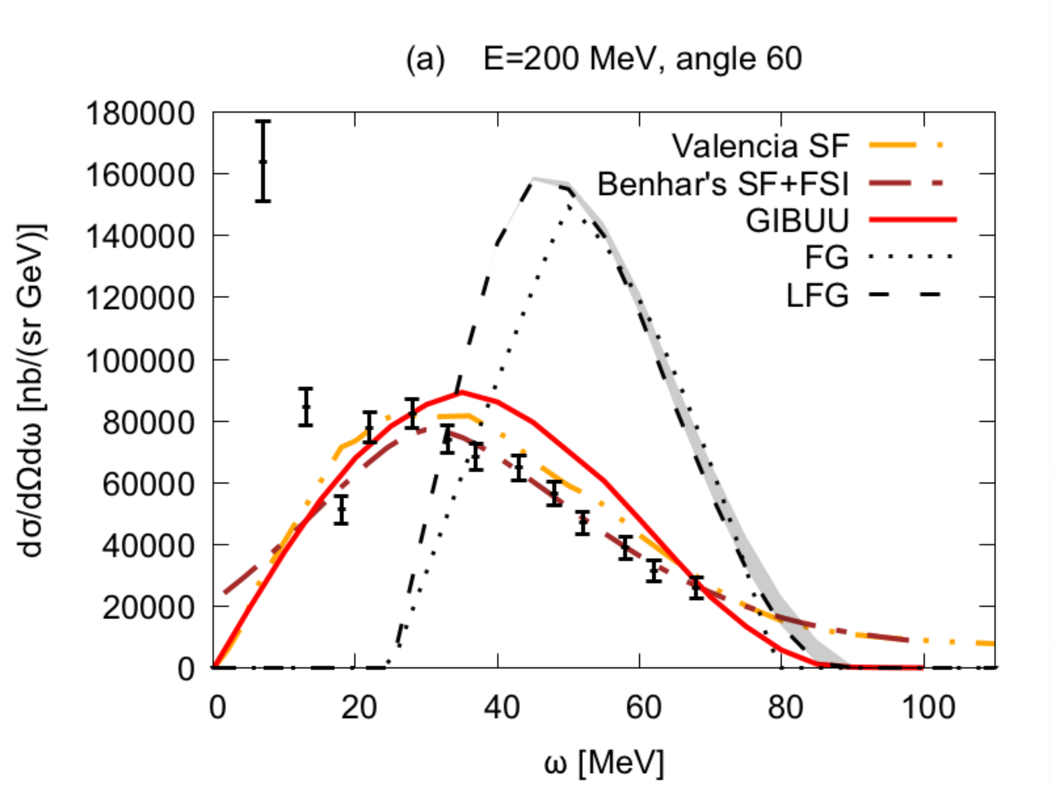
How low/high can we get with these models?

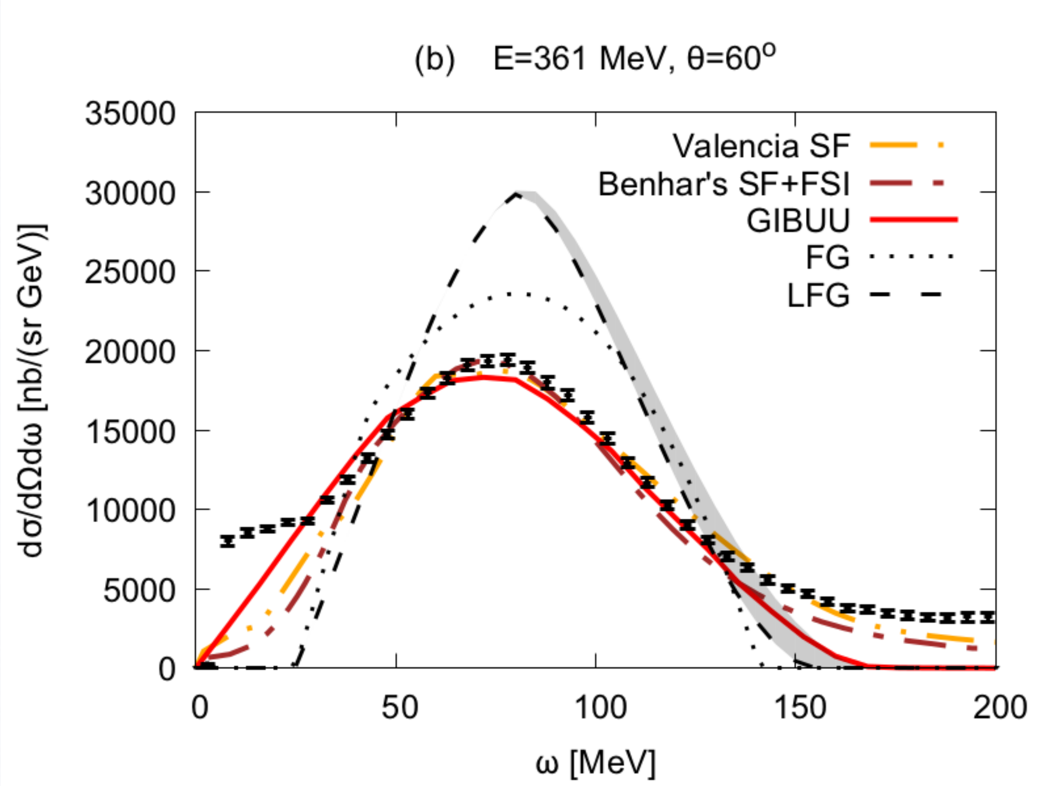
For inclusive QE data they seem to work decently well*

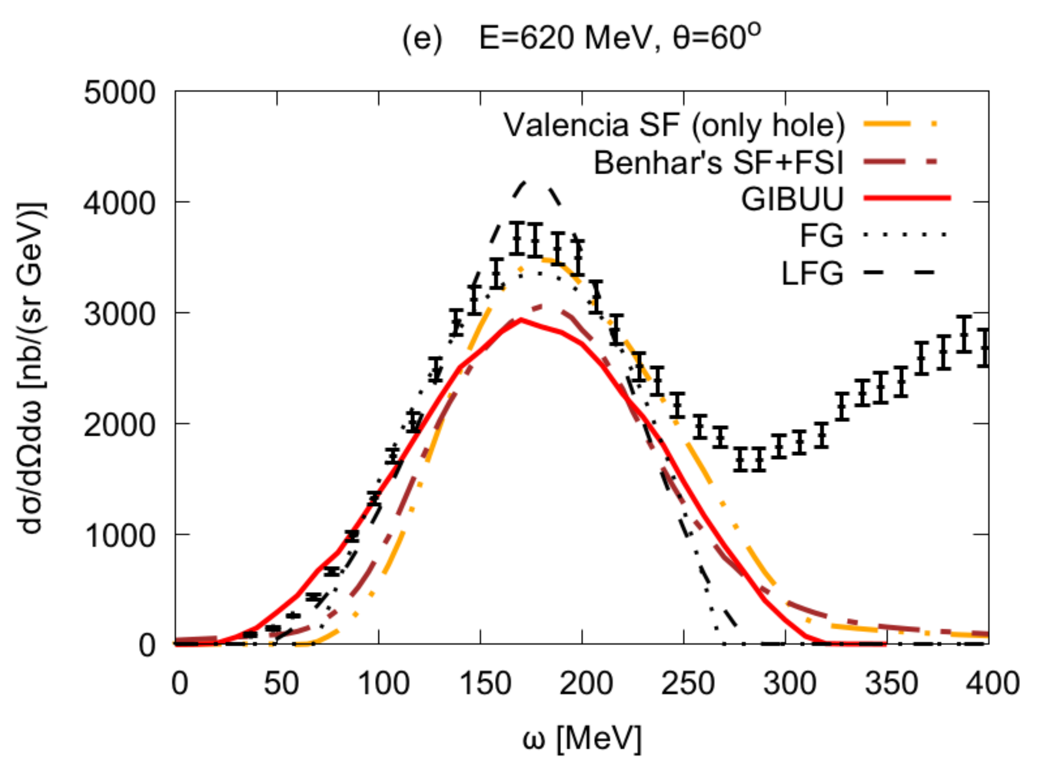
**check when 2p2h and pion production are included*

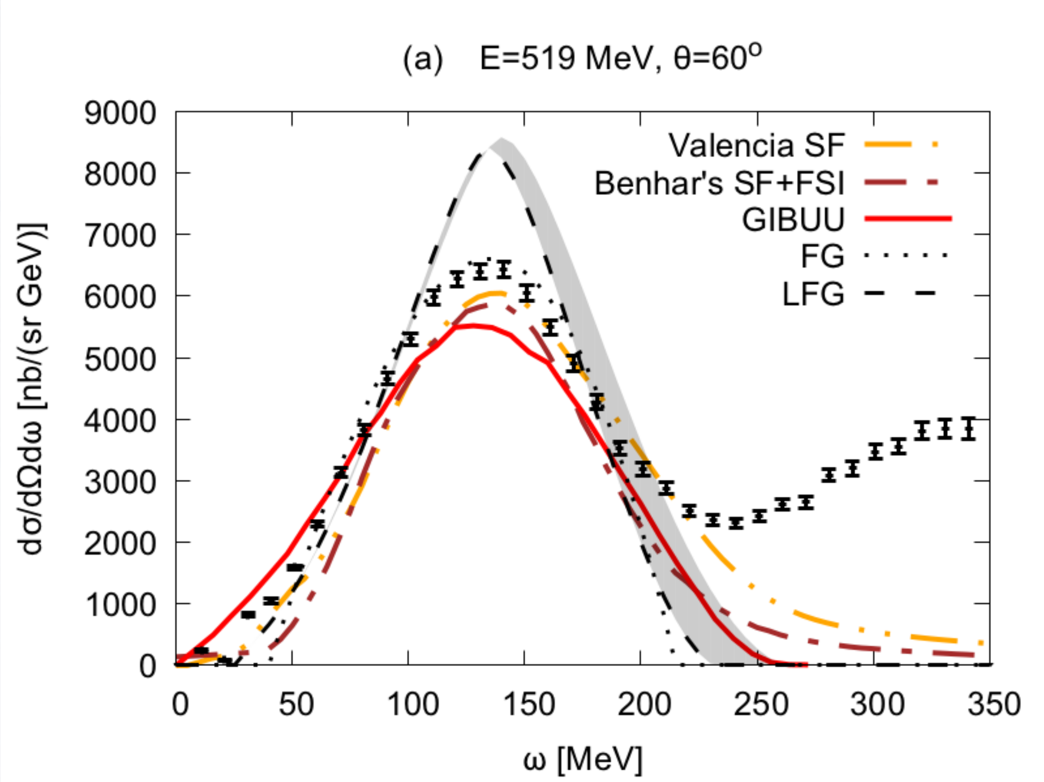

(b) E=320 MeV, θ=36°
dσ/dΩdω [nb/(sr GeV)]
500000
450000
400000
350000
300000
250000
200000
150000
100000
50000
0
0
20
40
60
80
100
120
ω [MeV]
Valencia SF
Benhar's SF+FSI
GIBUU
FG
LFG

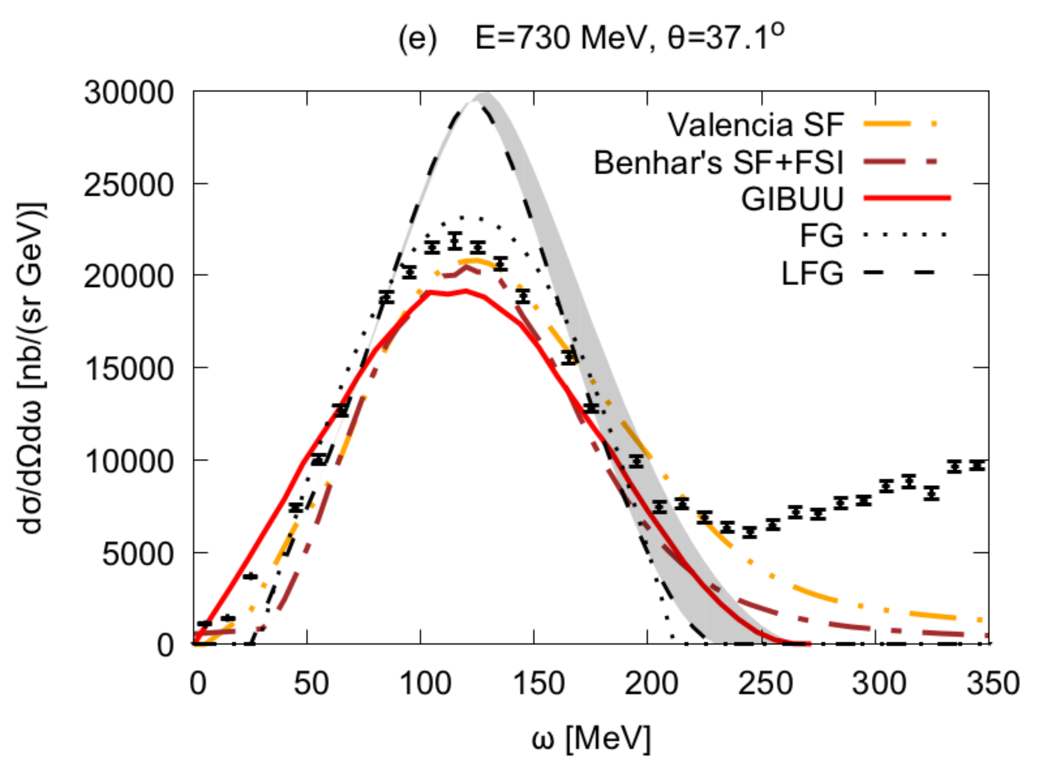

(c) E=240 MeV, angle 36
dσ/dΩdω [nb/(sr GeV)]
1x10^6
900000
800000
700000
600000
500000
400000
300000
200000
100000
0
0
20
40
60
80
100
ω [MeV]
Valencia SF
Benhar's SF+FSI
GIBUU
FG
LFG

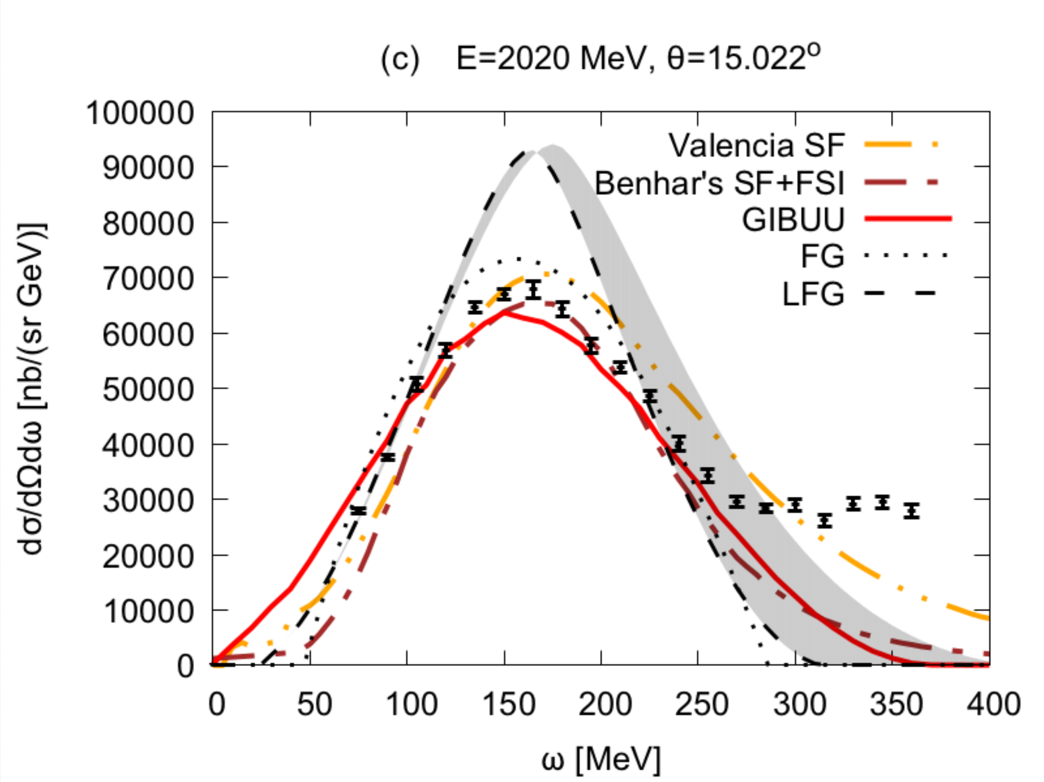

(a) E=200 MeV, angle 60
dσ/dΩdω [nb/(sr GeV)]
180000
160000
140000
120000
100000
80000
60000
40000
20000
0
0
20
40
60
80
100
ω [MeV]
Valencia SF
Benhar's SF+FSI
GIBUU
FG
LFG


(b) E=361 MeV, θ=60°
dσ/dΩdω [nb/(sr GeV)]
35000
30000
25000
20000
15000
10000
5000
0
0
50
100
150
200
ω [MeV]
Valencia SF
Benhar's SF+FSI
GIBUU
FG
LFG


(e) E=620 MeV, θ=60°


(a) E=519 MeV, θ=60°

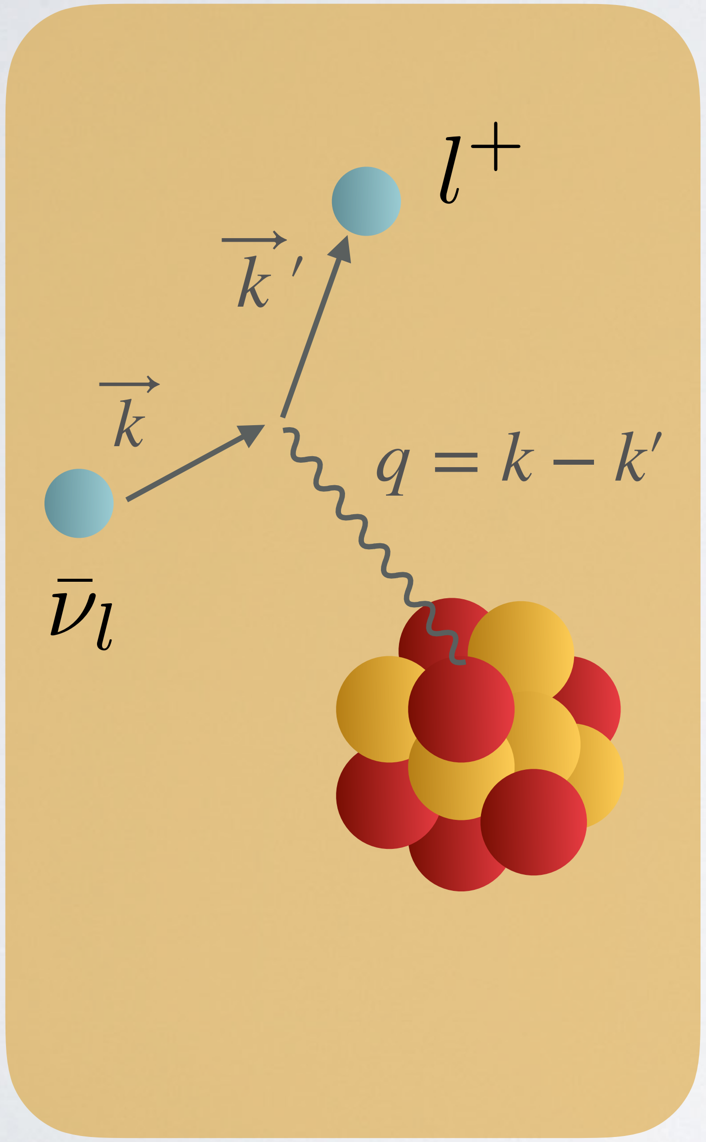

(e) E=730 MeV, θ=37.1°


(c) E=2020 MeV, θ=15.022°

VALENCIA MODEL: QE MECHANISM

- LDA (Local Density Approximation)
- 2 “ingredients” to describe the initial nucleus:
    - RPA (~M. Martini et al. model)
    - Spectral function
- The outgoing nucleon (and its possible interaction with the residual system):
    - Particle spectral function
    - Plane wave

GROUND STATE



spectral function model:

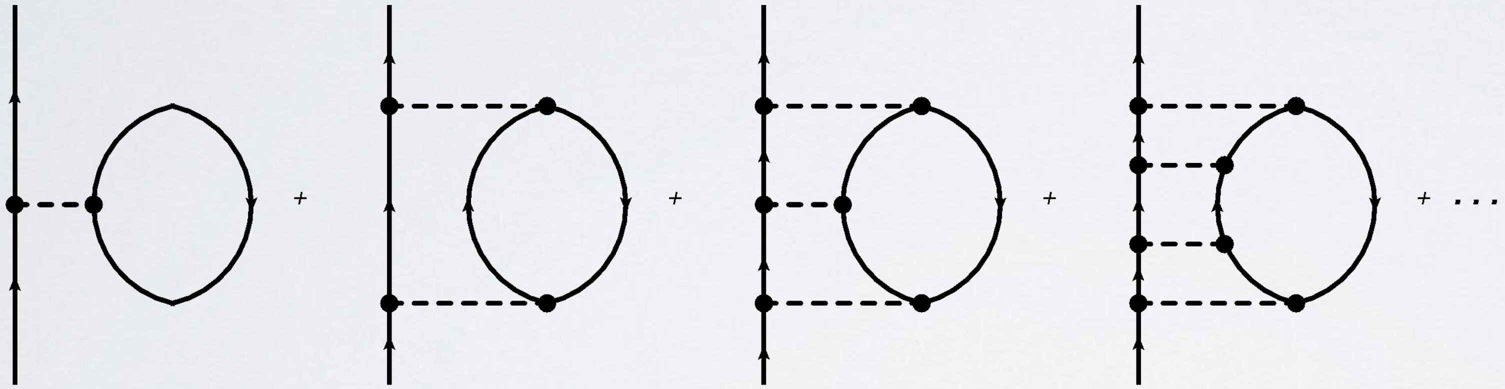
P. Fernandez de Cordoba, E. Oset
Phys. Rev. C46 (1992) 1697-1709

$$\frac{d\sigma}{dE_{k'}d\Omega(\hat{k}')} = \frac{G_F^2 \sin^2\theta_C}{4\pi^2}\frac{|\vec{k}\,'|}{|\vec{k}\,|}L^{(\bar{\nu})}_{\mu\sigma}(k,k')W^{\mu\sigma}(q)$$

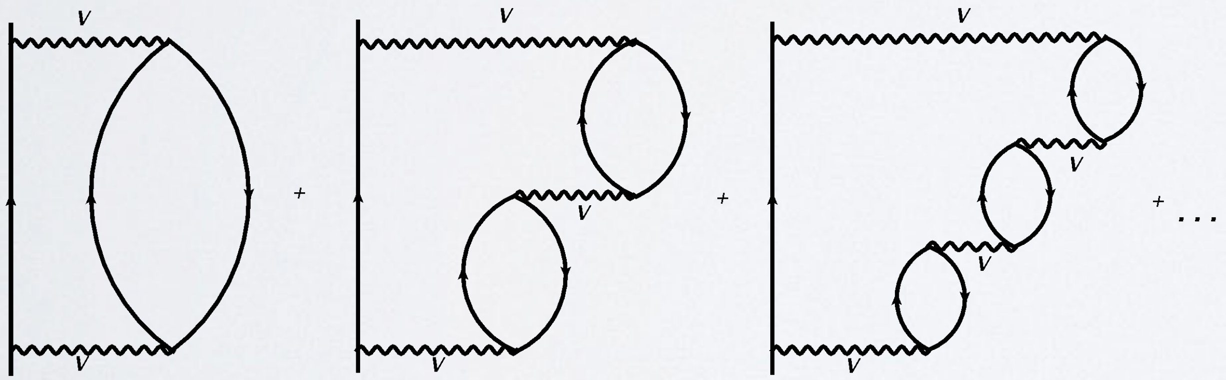
$$L^{(\bar{\nu})}_{\mu\sigma}(k,k') = k_\mu k'_\sigma + k'_\mu k_\sigma - g_{\mu\sigma}k\cdot k' - i\epsilon_{\mu\sigma\alpha\beta}k'^{\alpha}k^{\beta}$$

GROUND STATE (I)

- Semi-phenomenological model for nucleon self-energy $\Sigma(p, E)$ in nuclear medium, satisfying low density theorem.



NN interaction taken from scattering data



polarisation effects using empirical spin-isospin interaction

$$\mathrm{Re}\Sigma(p,E) = -\frac{1}{\pi}\mathscr{P}\int_{E_F}^{\infty} dE' \frac{\mathrm{Im}\Sigma(p,E')}{E-E'} + \frac{1}{\pi}\mathscr{P}\int_{-\infty}^{E_F} dE' \frac{\mathrm{Im}\Sigma(p,E')}{E-E'}$$

P. Fernandez de Cordoba, E. Oset
Phys. Rev. C46 (1992) 1697-1709

GROUND STATE (I)

- Spectral functions:

$$S_{p,h}^{\mathrm{LDA}}(\mathbf{p},E) = \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\mathbf{p},E)}{\left(E-\mathbf{p}^2/2m-\mathrm{Re}\Sigma(\mathbf{p},E)\right)^2+\mathrm{Im}\Sigma(\mathbf{p},E)^2}$$

- Local density approximation (LDA)

$$W_{LDA}^{\mu\nu}(q) = 2\int d^3r\int\frac{d^3p}{(2\pi)^3}\int dE\, S_h^{\mathrm{LDA}}(E,\mathbf{p},\rho)\frac{M}{E_p}\frac{M_Y}{E_{p+q}^Y}\delta(E+q^0-E_{p+q}^Y(\rho))A^{\mu\nu}(p,q)$$

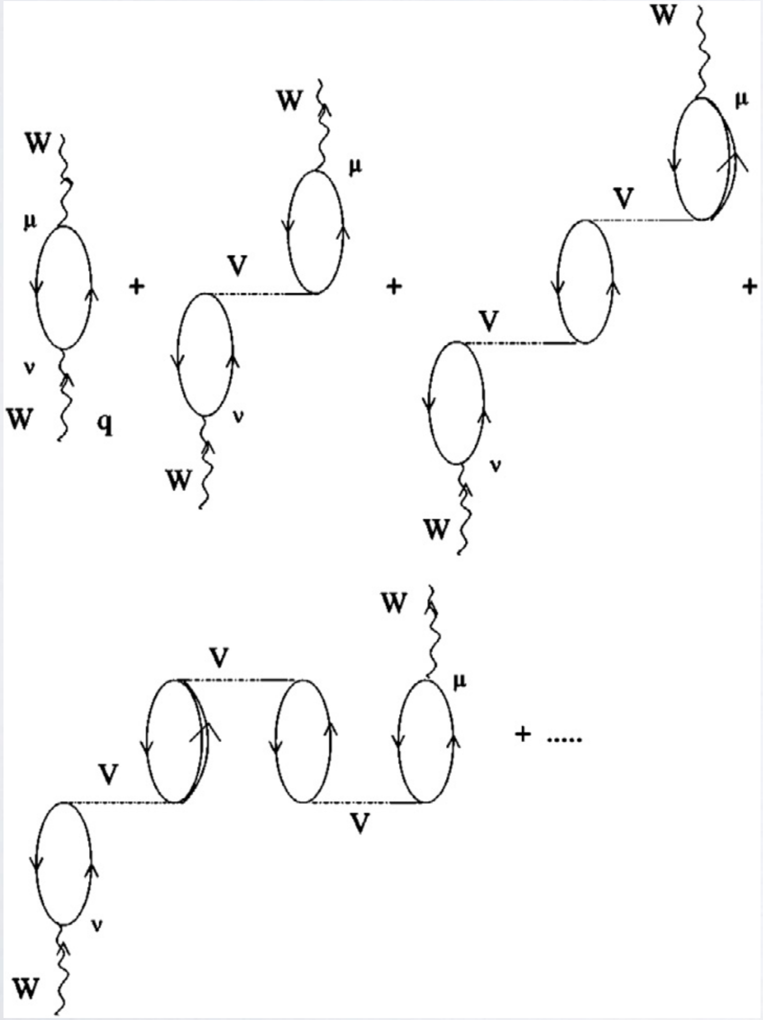
$$A^{\mu\nu}(p,q) = \langle\mathbf{p}\,|\,(j_{cc}^{\mu})^\dagger\,|\,\mathbf{p}+\mathbf{q}\rangle\langle\mathbf{p}+\mathbf{q}\,|\,j_{cc}^{\nu}\,|\,\mathbf{p}\rangle$$

matrix element for a single nucleon

RPA EFFECTS

Landau-Migdal potential

$$V = c_0 \left\{ f_0(\rho) + f_0'(\rho)\vec{\tau}_1 \cdot \vec{\tau}_2 + g_0(\rho)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + g_0'(\rho)\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2)\,(\vec{\tau}_1 \cdot \vec{\tau}_2) \right\}$$



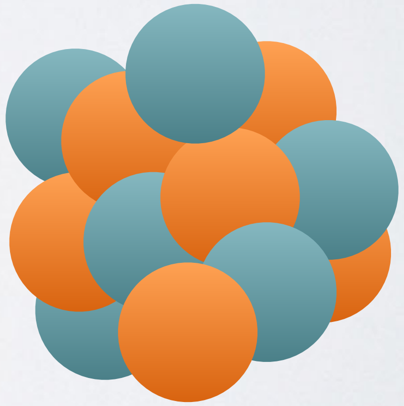
$$V_l(q) = \frac{f^2}{m_\pi^2} \left\{ \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2 \frac{\vec{q}^{\,2}}{q^2 - m_\pi^2} + g' \right\}, \qquad f^2/4\pi = 0.08, \qquad \Lambda_\pi = 1200\,\mathrm{MeV}$$

$$V_t(q) = \frac{f^2}{m_\pi^2} \left\{ C_\rho \left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2} \right)^2 \frac{\vec{q}^{\,2}}{q^2 - m_\rho^2} + g' \right\}, \qquad C_\rho = 2, \qquad \Lambda_\rho = 2500\,\mathrm{MeV}$$

What is the relation between CRPA and RPA?

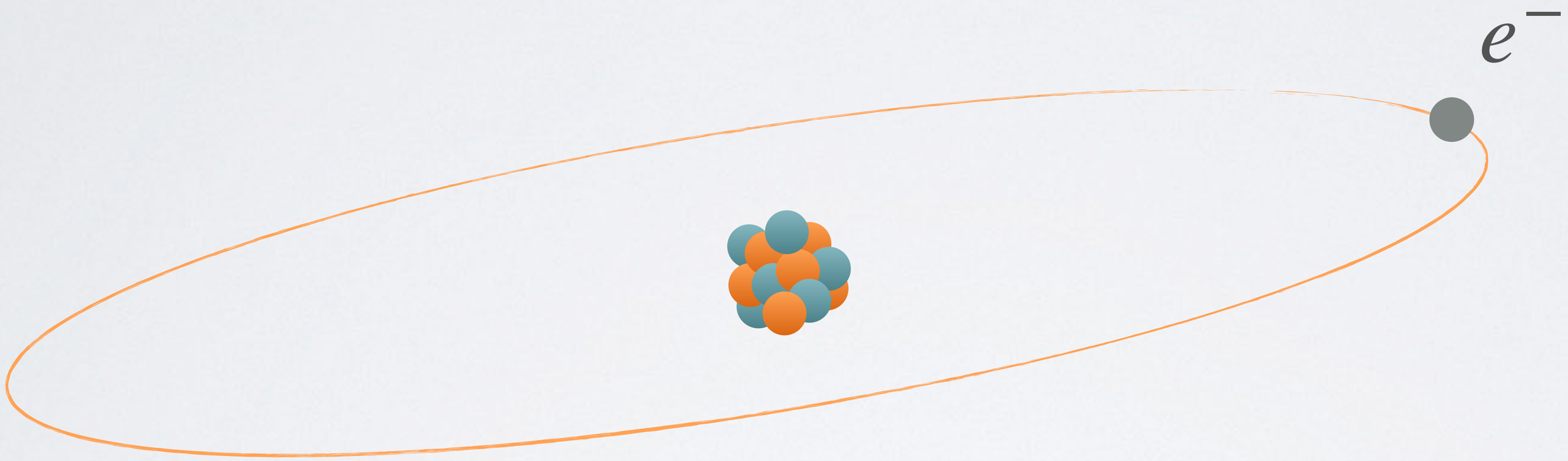
HOW LOW CAN WE GET?

using spectral function
for both particle and hole



LOW ENERGY PROCESSES

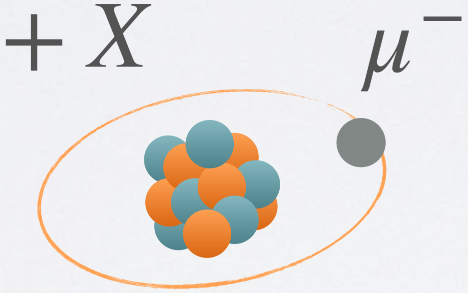
muon capture



LOW ENERGY PROCESSES

muon capture

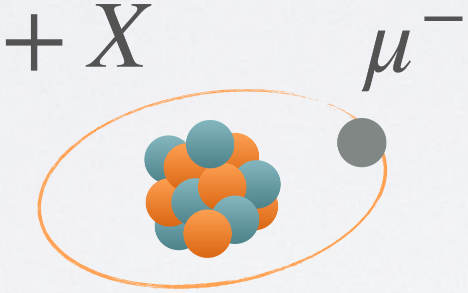
$$(A - \mu^-)^{1s}_{\text{bound}} \to \bar{\nu} + X$$



LOW ENERGY PROCESSES

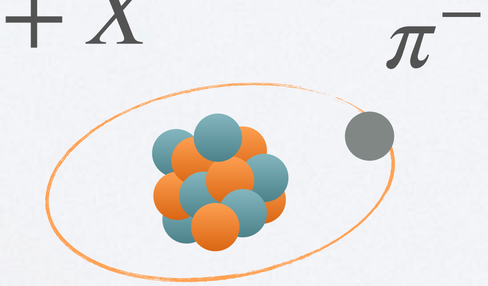
muon capture

$$(A - \mu^-)^{1s}_{\text{bound}} \rightarrow \bar{\nu} + X$$



pion radiative capture

$$(A - \pi^-)_{\text{bound}} \rightarrow \gamma + X$$



muons are 300 times heavier than electrons

↓

the wave function overlaps with nucleus

↓

the system is unstable (interaction with nucleus)

governed by the same CC interaction

KINEMATICS

bound muon/pion are at rest

$$(q_0, \overrightarrow{q}) = (m_{\mu/\pi} - |\overrightarrow{k}|, \overrightarrow{k})$$

maximal energy transferred

$$m_\mu = 105 \text{ MeV}$$

$$m_\pi = 135 \text{ MeV}$$

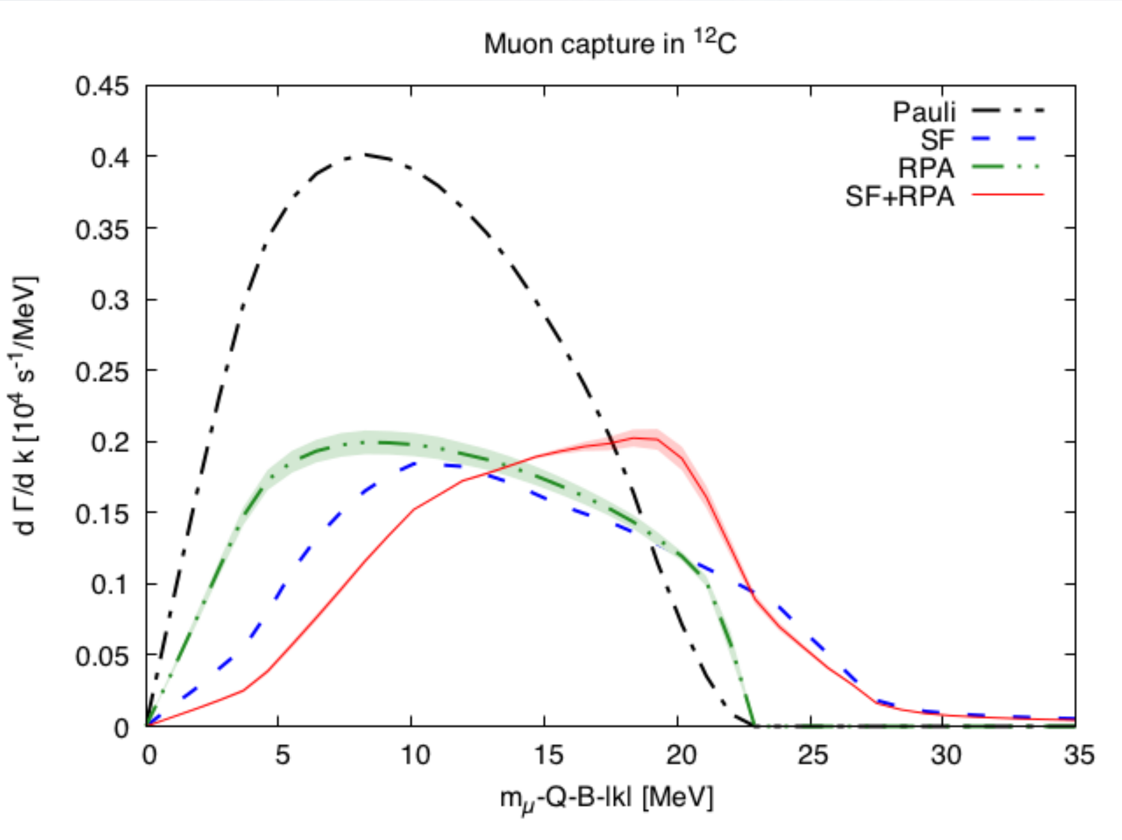
the position of the quasi-elastic peak

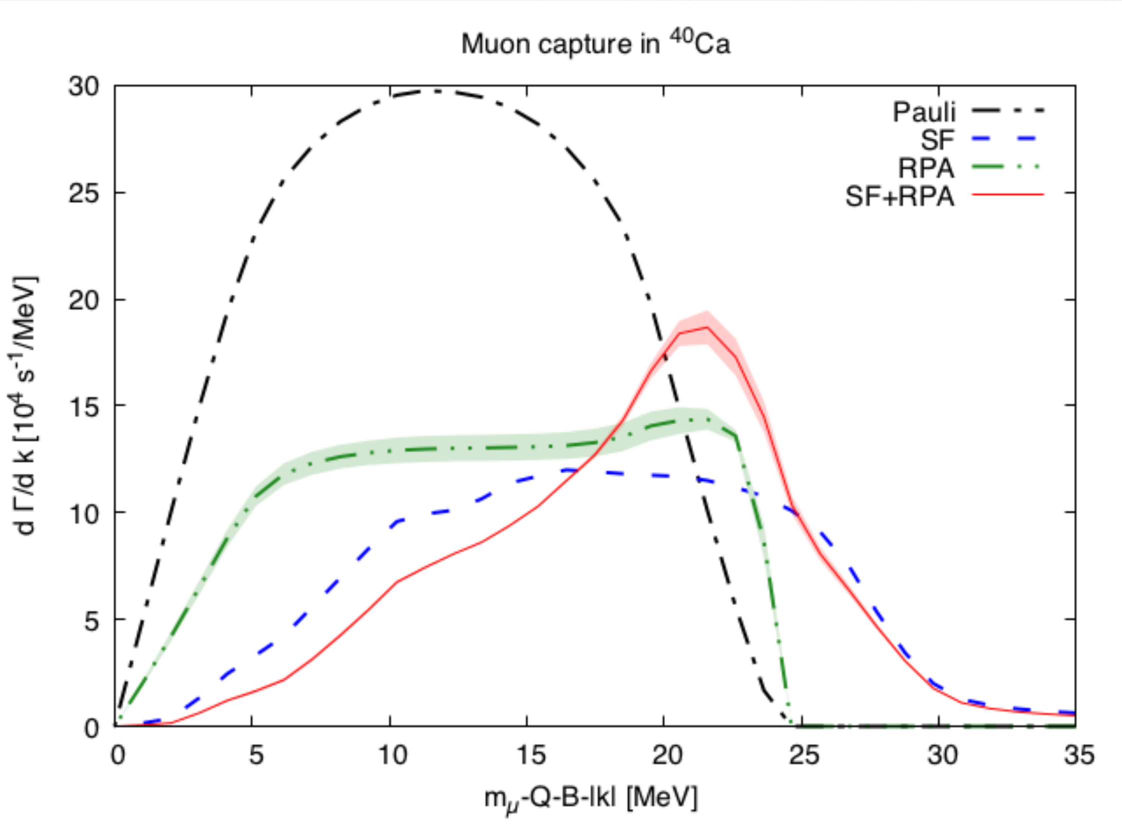
$$E_{QE} \approx \frac{m^2_{\mu/\pi}}{2M}$$

$$\approx 10 \text{ MeV}$$

MUON CAPTURE

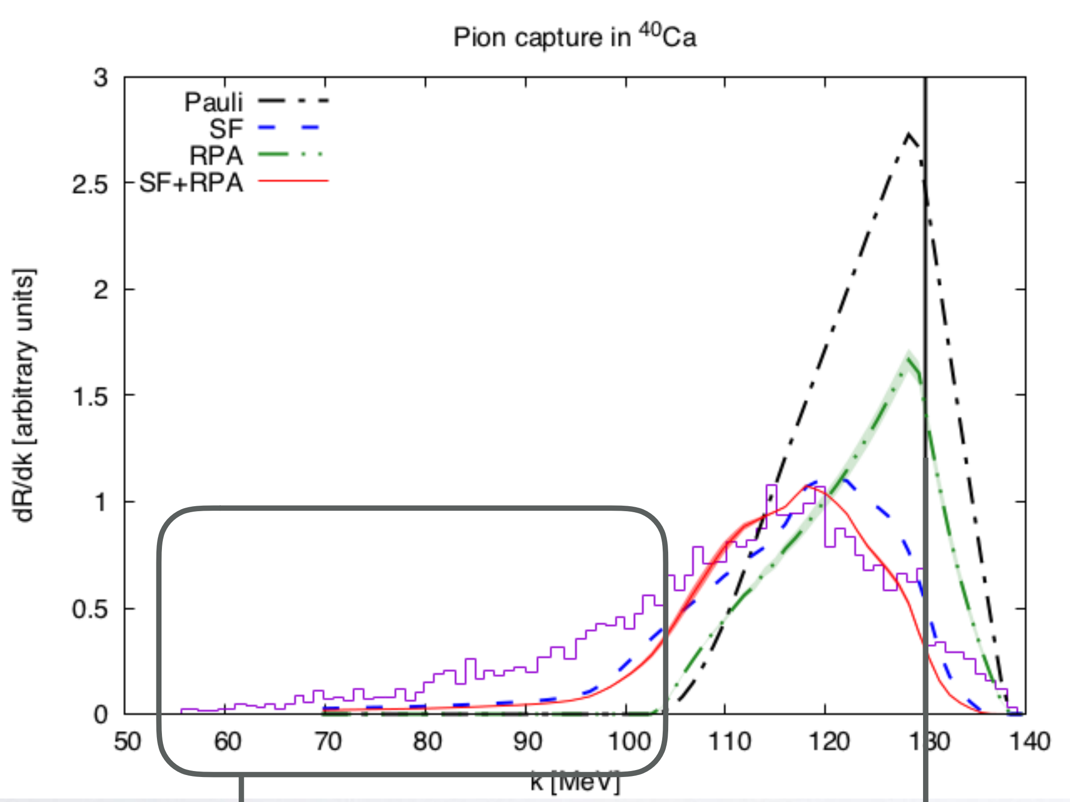
(shape shows the general trend)





Nucleus	Pauli (10^4 s^{-1})	RPA (10^4 s^{-1})	SF (10^4 s^{-1})	SF+RPA (10^4 s^{-1})	Exp. (10^4 s^{-1})
^{12}C	5.76	3.37 ± 0.16	3.22	3.19 ± 0.06	3.79 ± 0.03
^{16}O	18.7	10.9 ± 0.4	10.6	10.3 ± 0.2	10.24 ± 0.06
^{18}O	13.8	8.2 ± 0.4	7.0	8.7 ± 0.1	8.80 ± 0.15
^{23}Na	64.5	37.0 ± 1.5	30.9	34.3 ± 0.4	37.73 ± 0.14
^{40}Ca	498	272 ± 11	242	242 ± 6	252.5 ± 0.6

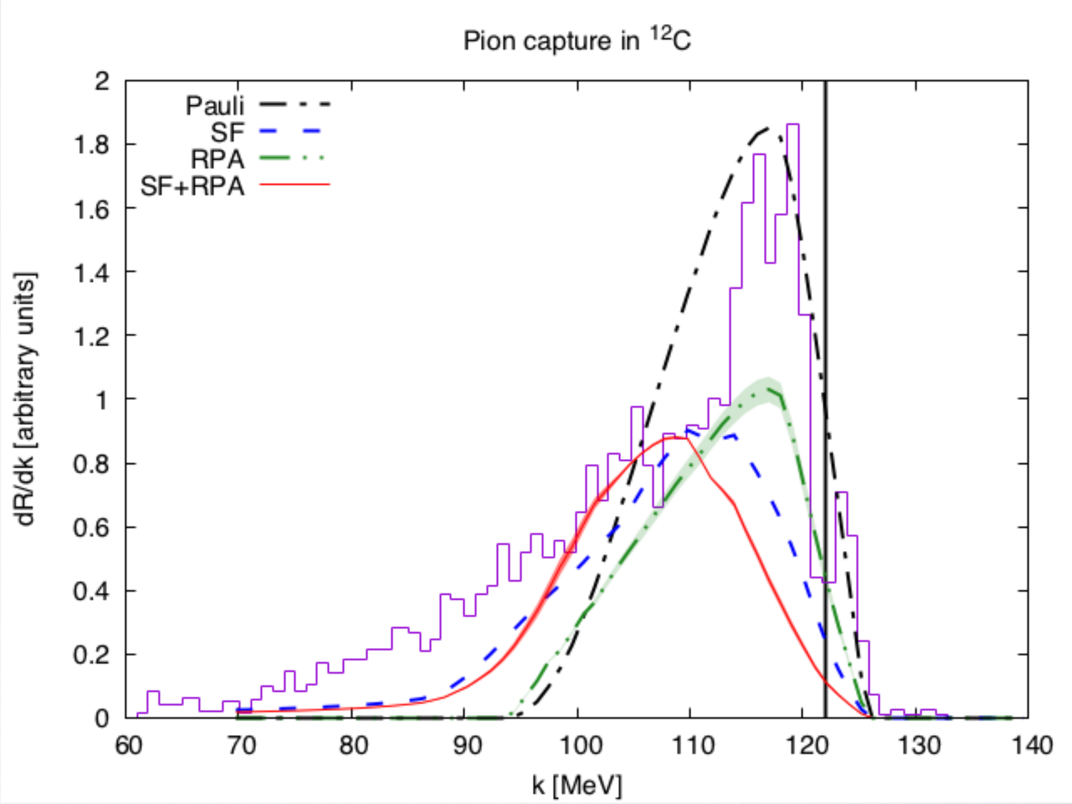
PION RADIATIVE CAPTURE



maximum photon energy
(final nucleus in the ground state)

$$(A_Z - \pi^-)_{\text{bound}} \to \gamma + n + (A-1)_Z$$

contributions beyond 1p1h



discrete transitions are more important

$$\frac{dR^{(\gamma)}}{d|\vec{k}\,|} = \sum_{nl} \frac{w_{nl}}{\Gamma_{nl}^{abs}} \frac{d\Gamma_{nl}^{(\gamma)}}{d|\vec{k}\,|}$$

pionic levels

NEUTRINO SCATTERING DATA

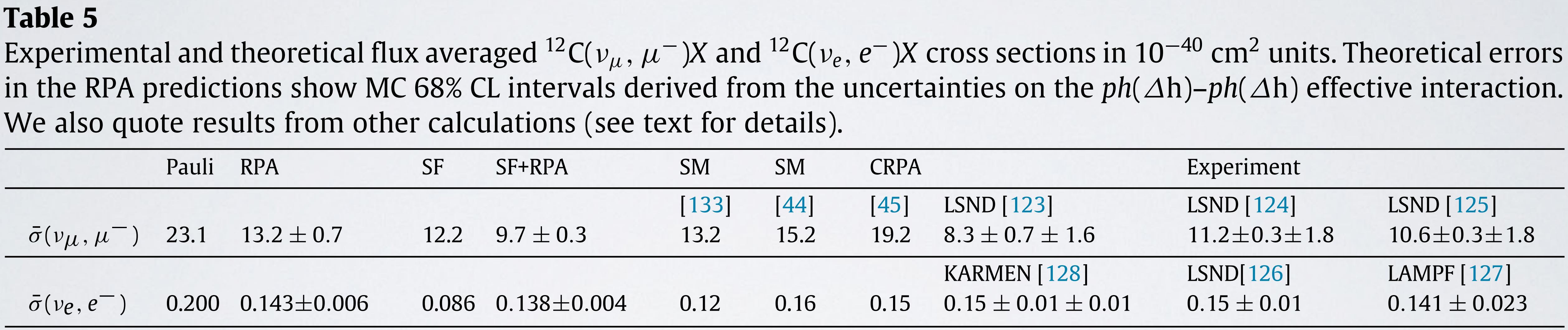


Table 5

Experimental and theoretical flux averaged $^{12}\mathrm{C}(\nu_\mu, \mu^-)X$ and $^{12}\mathrm{C}(\nu_e, e^-)X$ cross sections in 10^{-40} cm^2 units. Theoretical errors in the RPA predictions show MC 68% CL intervals derived from the uncertainties on the *ph*(Δh)–*ph*(Δh) effective interaction. We also quote results from other calculations (see text for details).

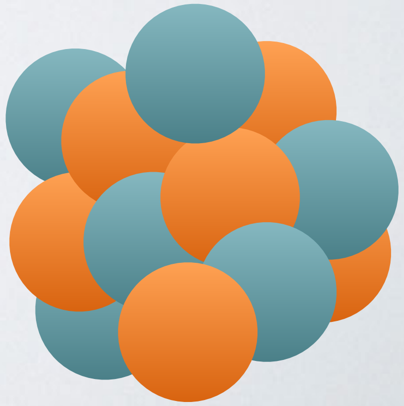
	Pauli	RPA	SF	SF+RPA	SM	SM	CRPA		Experiment	
					[133]	[44]	[45]	LSND [123]	LSND [124]	LSND [125]
$\bar{\sigma}(\nu_\mu, \mu^-)$	23.1	13.2 ± 0.7	12.2	9.7 ± 0.3	13.2	15.2	19.2	8.3 ± 0.7 ± 1.6	11.2±0.3±1.8	10.6±0.3±1.8
								KARMEN [128]	LSND[126]	LAMPF [127]
$\bar{\sigma}(\nu_e, e^-)$	0.200	0.143±0.006	0.086	0.138±0.004	0.12	0.16	0.15	0.15 ± 0.01 ± 0.01	0.15 ± 0.01	0.141 ± 0.023

What are the CRPA Ghent results?

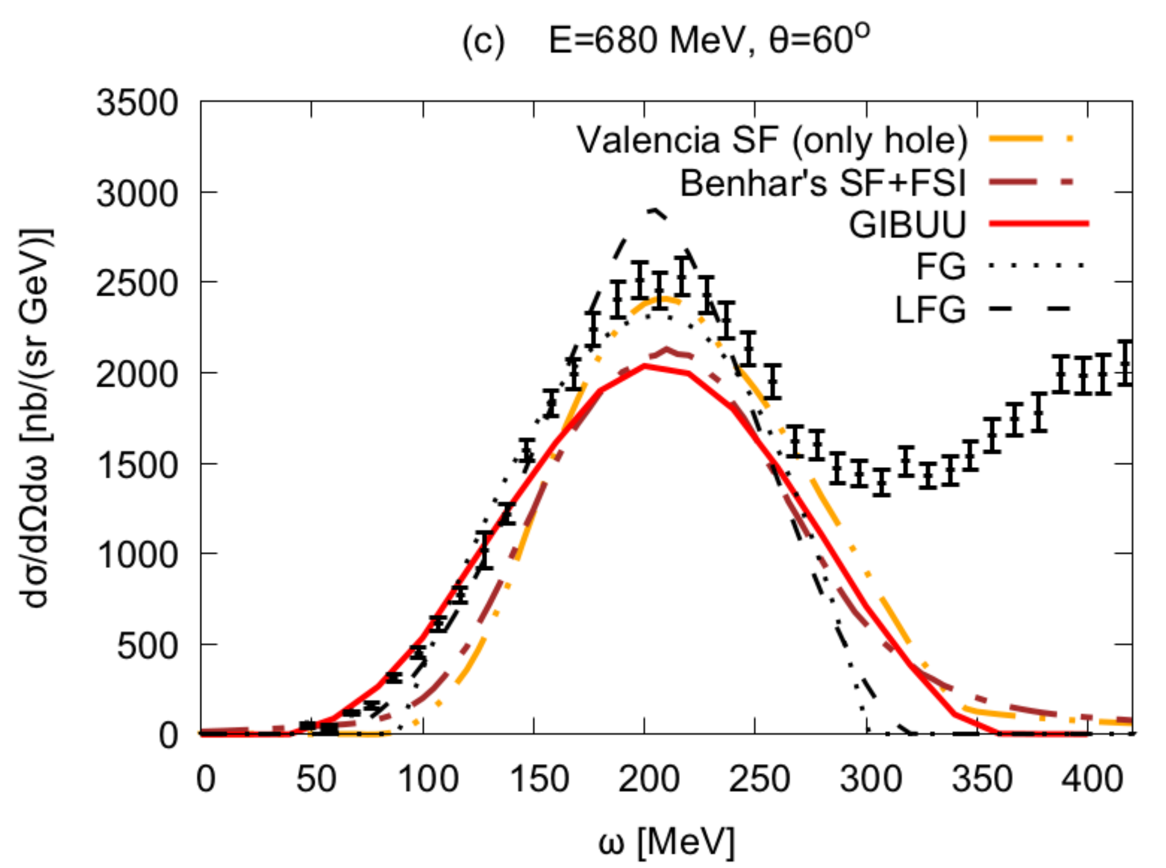
HOW HIGH CAN WE GET?

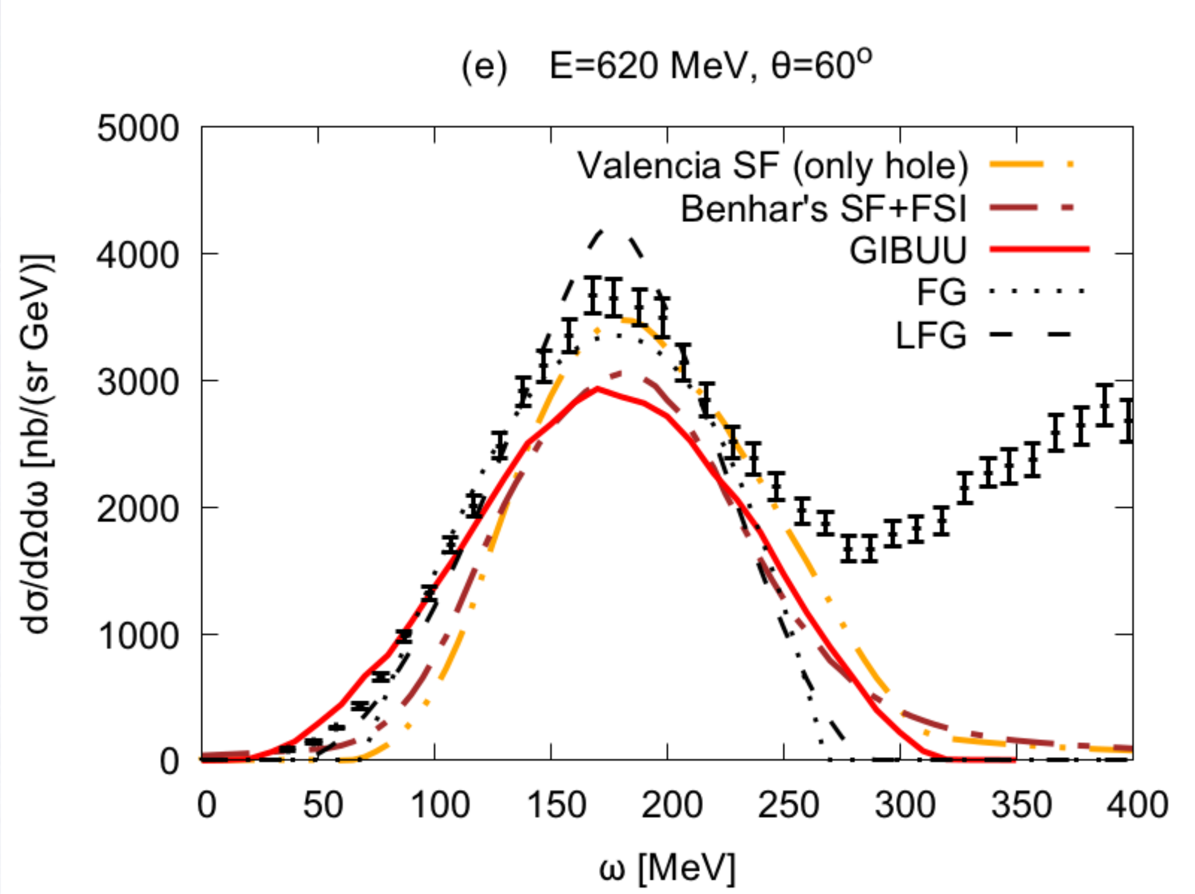
using spectral function
only for the hole

using spectral function
for both particle and hole



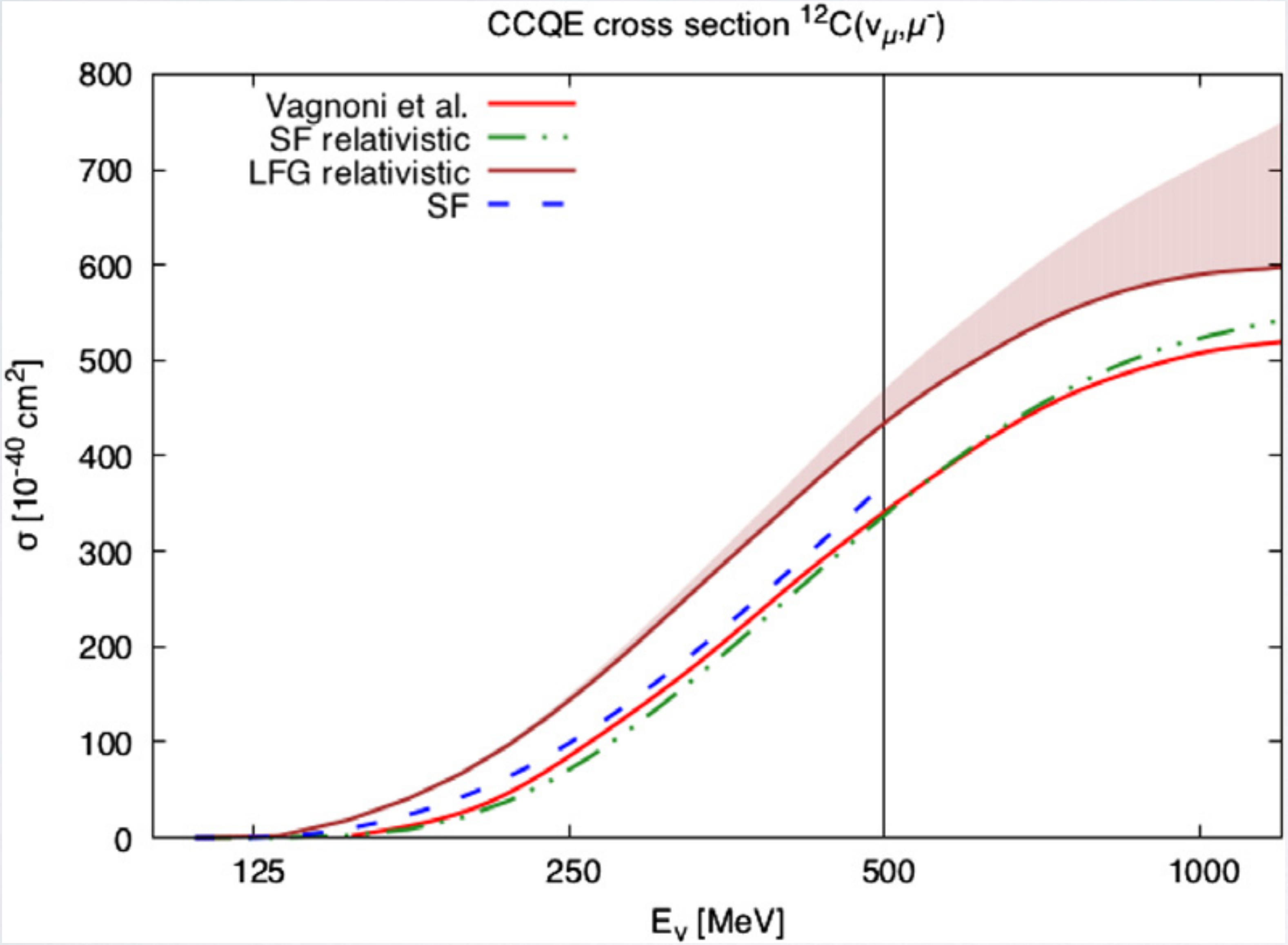
ONLY HOLE SF





FSI: quenches the response and enhances the high-energy tail

HIGHER ENERGIES: COMPARISON



FSI: do they change the total cross section or just redistribute strength?

POSITIVE SIDES OF THIS APPROACH

- It is based on the LDA: direct possibility to generate the primary vertex
- Possibility to use of various targets
- Simplicity
- Contains both RPA and SF
- works decently well for electrons
- for low energy transfers description of the hole and particle states within the same formalism

DEFICIENCIES OF THIS APPROACH

- For low energies describes the total strength of RPA but not the spectrum
- Simplicity: no insight into details of the nuclear structure
- For high momentum transfer we can use only plane wave (now we do not use any model for FSI)