Coulomb and Optical Highlights

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Reference: Removal Energies and Final State Interaction in Lepton Nucleus Scattering
Arie Bodek, Tejin Cai 10.1140/epjc/s10052-019-6750-3
https://arxiv.org/abs/1801.07975 published in Eur. Phys. J. C. (2019) 79: 293

Discussion topic 1.

While we are waiting for MC implementations of better models, we should adjust the parameters in the models that we use e.g**. RFG, local Fermi gas (LFG), spectral function(SF)** as follows.

1. Use the correct **AVERAGE removal energy** (for RFG, LFG)

2. Account for the **optical potential in the final state** (RFG, LFG, **SF**)

3. Account for the **Coulomb potential** (RFG, LFG, **SF**)

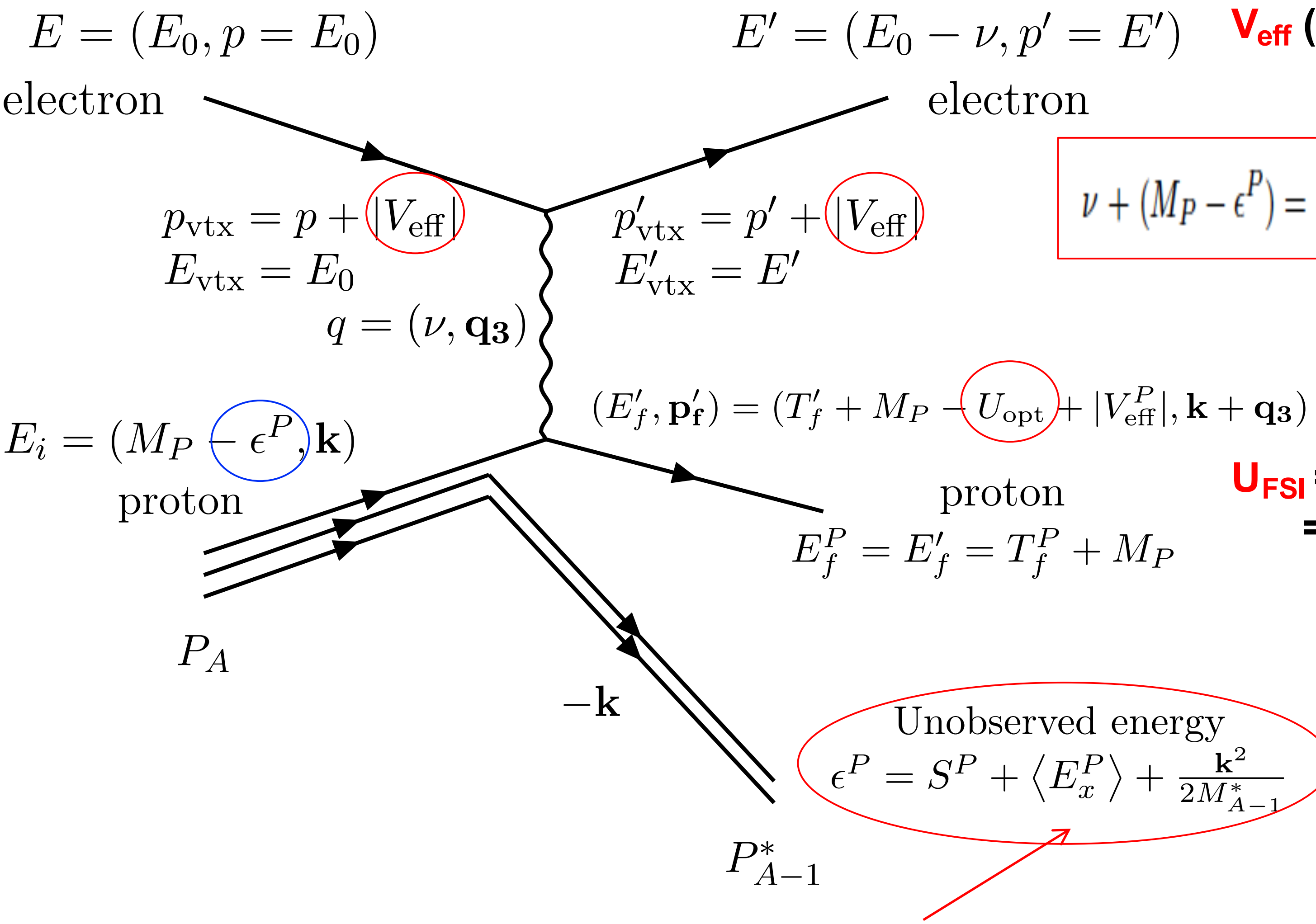
4. Be careful about what is called **"binding/removal energy**" in RFG, LFG. **Best to use the same definition of <Em> as in spectral function measurements** -then RFG, LFG are cases for which spectral function is a delta function at <Em>.

Note: Optical and Coulomb potential are **accounted for in Mean Field calculations. They need to be explicitly added ti RFG, LFG and Spectral Function**

Electron QE Scattering on bound protons in a Coulomb and Nuclear potential

Energy (sun of kinetic and potential energy) is conserved at every step. Momentum changes and momentum conservation is taken care of by the spectator nucleus (with negligible energy)

Electron scattering on proton (effective momentum approximation)



$E = (E_0, p = E_0)$

electron

$E' = (E_0 - \nu, p' = E')$

electron

$p_{\mathrm{vtx}} = p + |V_{\mathrm{eff}}|$

$E_{\mathrm{vtx}} = E_0$

$q = (\nu, \mathbf{q_3})$

$p'_{\mathrm{vtx}} = p' + |V_{\mathrm{eff}}|$

$E'_{\mathrm{vtx}} = E'$

$(E'_f, \mathbf{p'_f}) = (T'_f + M_P - U_{\mathrm{opt}} + |V^P_{\mathrm{eff}}|, \mathbf{k} + \mathbf{q_3})$

$E_i = (M_P - \epsilon^P, \mathbf{k})$

proton

proton

$E^P_f = E'_f = T^P_f + M_P$

P_A

$-\mathbf{k}$

P^*_{A-1}

Unobserved energy

$\epsilon^P = S^P + \langle E^P_x \rangle + \frac{\mathbf{k}^2}{2M^*_{A-1}}$

V_{eff} (3 to 20 MeV)

$$\nu + (M_P - \epsilon^P) = \sqrt{(\mathbf{k} + \mathbf{q}_3)^2 + M_P^2} - |U_{FSI}| + |V^P_{eff}|$$

U_{FSI} = $-U_{opt}$

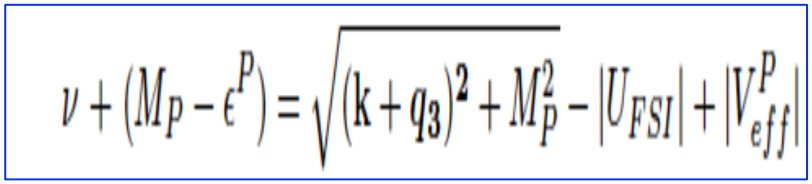
=(-20 to -30 MeV) low Q2

Use average Ex If a spectral function is not used

Neutrino QE Scattering on bound neutrons in a Coulomb and Nuclear potential

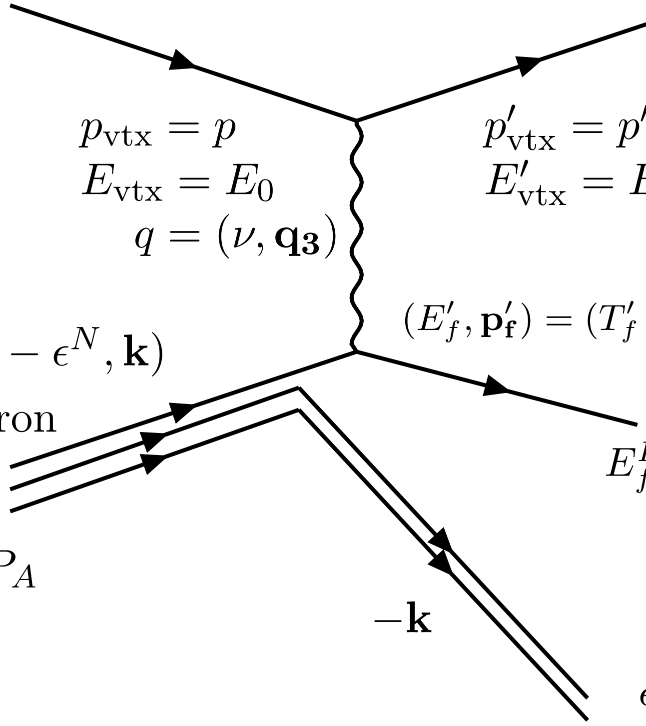
Energy (sun of kinetic and potential energy) is conserved at every step. Momentum changes by momentum conservation is taken care of by the spectator nucleus (with negligible energy)

(effective momentum approximation)



$$\nu + (M_P - \epsilon^P) = \sqrt{(k+q_3)^2 + M_P^2} - |U_{FSI}| + |V_{eff}^P|$$

Neutrino scattering on neutron



$E = (E_0, p = E_0)$

ν_μ

$E' = (E_0 - \nu, p' = E')$

μ^-

$p_{\rm vtx} = p$

$E_{\rm vtx} = E_0$

$q = (\nu, \mathbf{q_3})$

$p'_{\rm vtx} = p' + |V_{\rm eff}|$

$E'_{\rm vtx} = E'$

$(E'_f, \mathbf{p'_f}) = (T'_f + M_P - U_{\rm opt} + |V_{\rm eff}|, \mathbf{k} +$

$E_i = (M_N - \epsilon^N, \mathbf{k})$

neutron

proton

$E_f^P = E'_f = T_f^P + M_P$

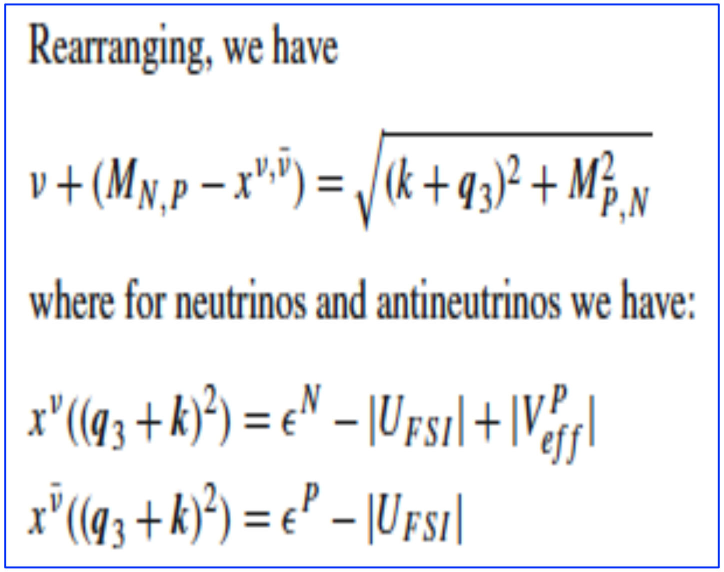
P_A

$-\mathbf{k}$

Unobserved energy

$\epsilon^N = S^N + \langle E_x^N \rangle + \frac{\mathbf{k}^2}{2M^*_{A-1}}$

P^*_{A-1}



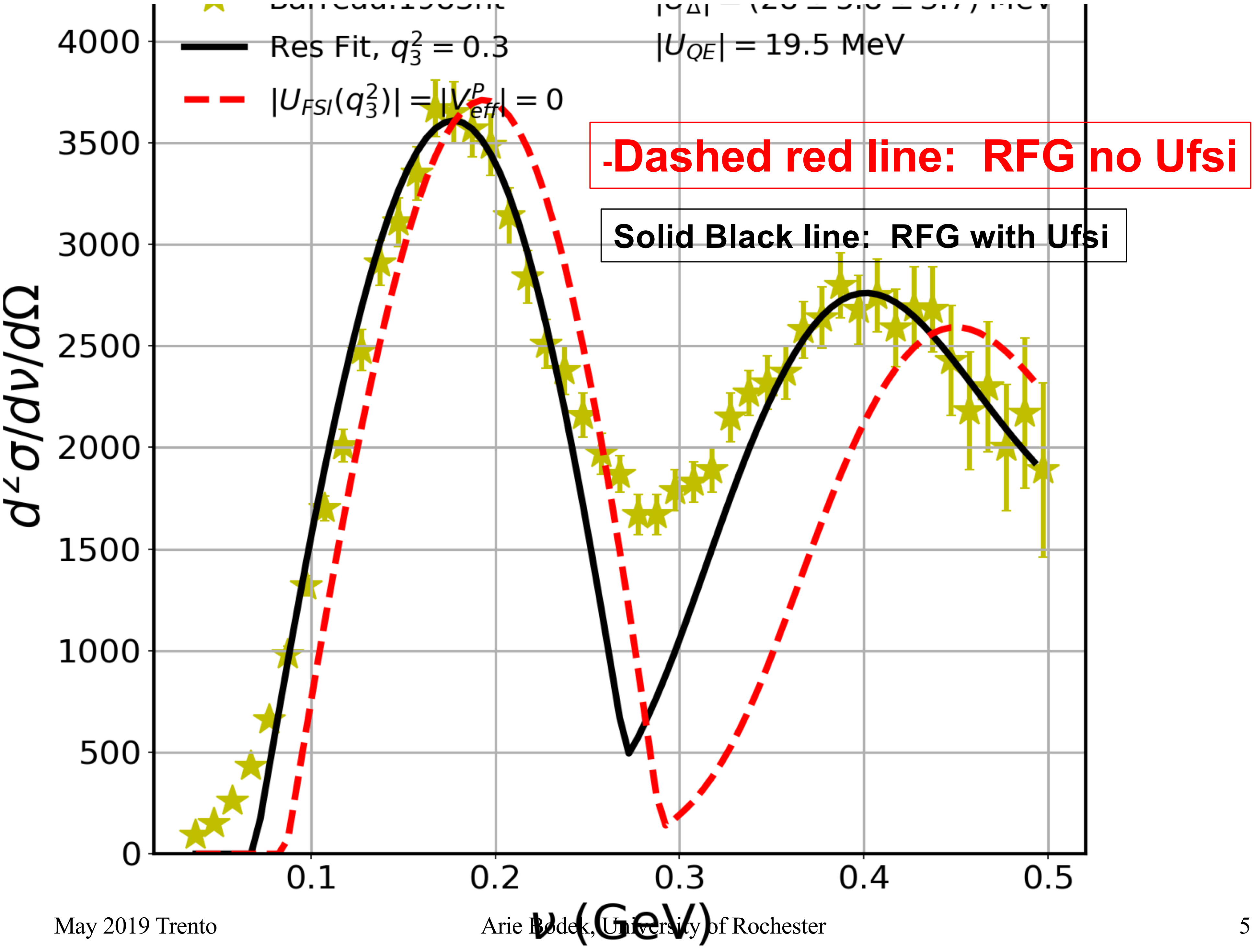
Rearranging, we have

$$\nu + (M_{N,P} - x^{\nu,\bar\nu}) = \sqrt{(k+q_3)^2 + M_{P,N}^2}$$

where for neutrinos and antineutrinos we have:

$$x^\nu((q_3+k)^2) = \epsilon^N - |U_{FSI}| + |V_{eff}^P|$$

$$x^{\bar\nu}((q_3+k)^2) = \epsilon^P - |U_{FSI}|$$



-Dashed red line: RFG no Ufsi

Solid Black line: RFG with Ufsi

- - - r =0 U_{opt}

— Average U_{opt}

Cooper 2009 calculated by
Jose Manuel Udias

PRC 80, 034605 (2009) Cooper et al

Average U_{opt} —

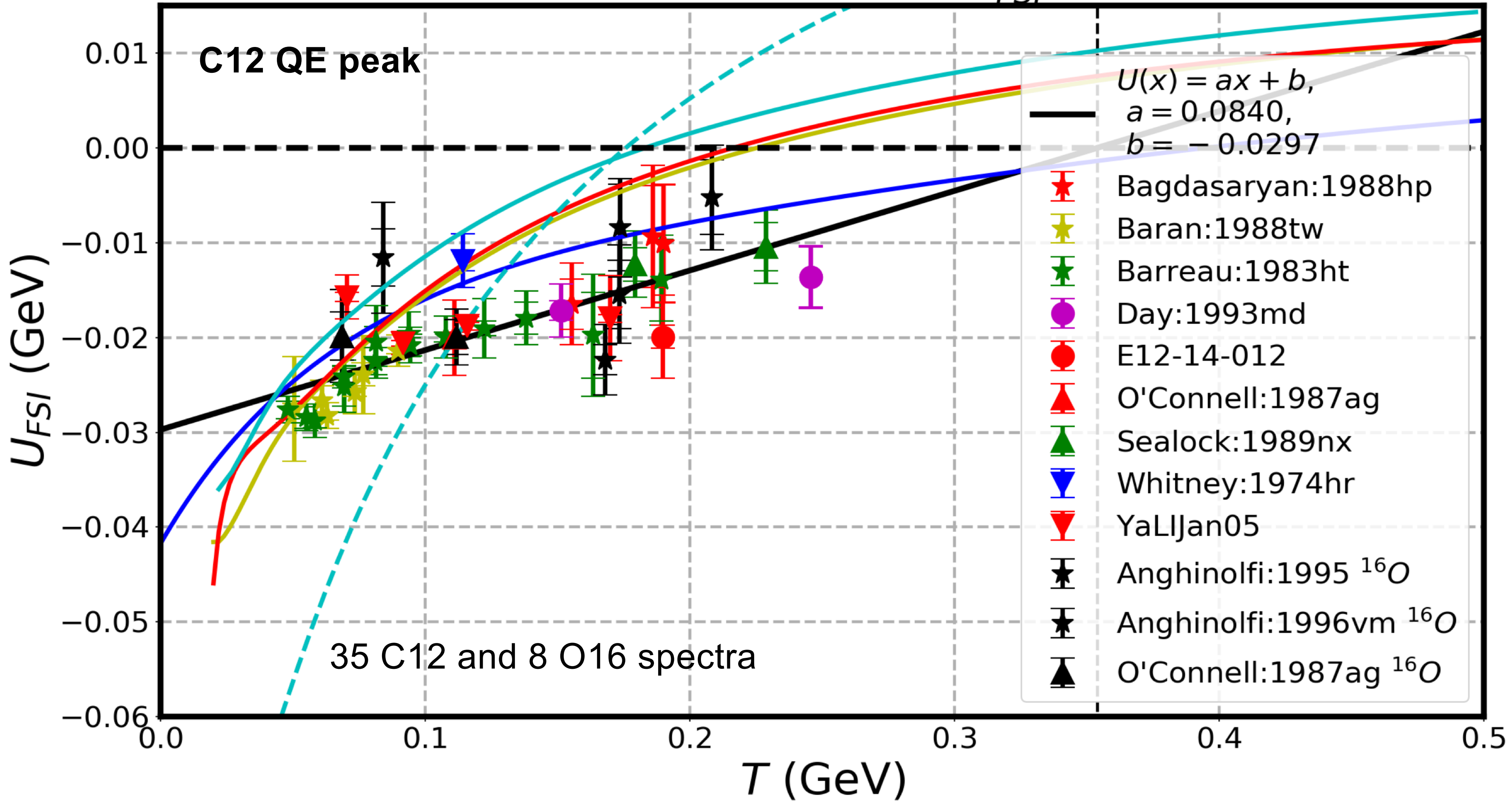
Cooper 2009 calculated by Artur Ankowski

Average U_{opt} —

Cooper 1993 calculated by Artur Ankowski

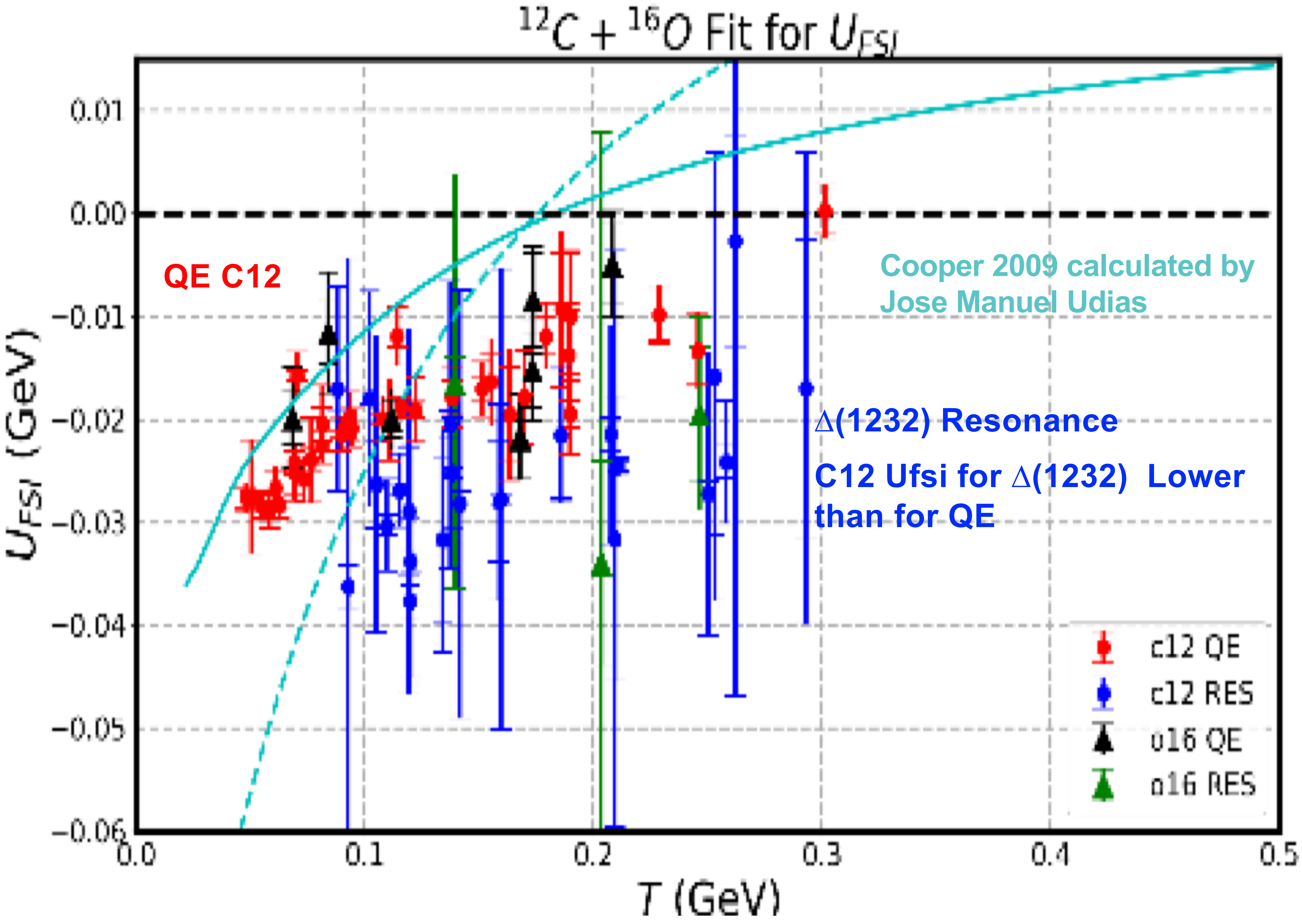
Cooper-et al PRC 47, 297(1993)

$^{12}C + ^{16}O$ Fit for U_{FSI}



Gibuu uses same Ufsi for everything except Delta for which they multiply by 2/3. It does not look like we get this

Compare QE to Δ(1232)



QE C12

Cooper 2009 calculated by
Jose Manuel Udias

Δ(1232) Resonance

C12 Ufsi for Δ(1232) Lower than for QE

29 CA 40 spectra and 2 Ar40 spectra

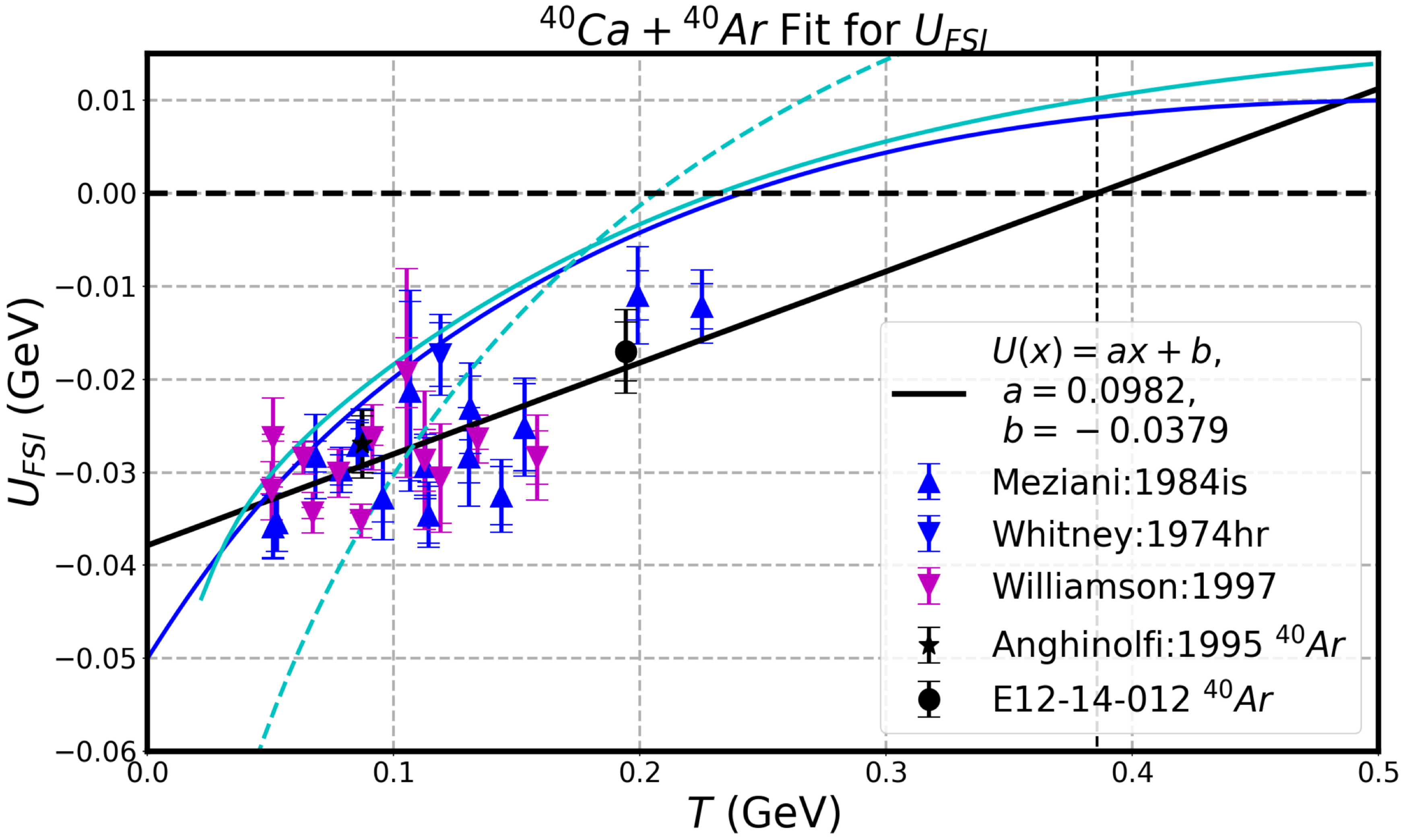
r =0 U_{opt}

Average U_{opt}

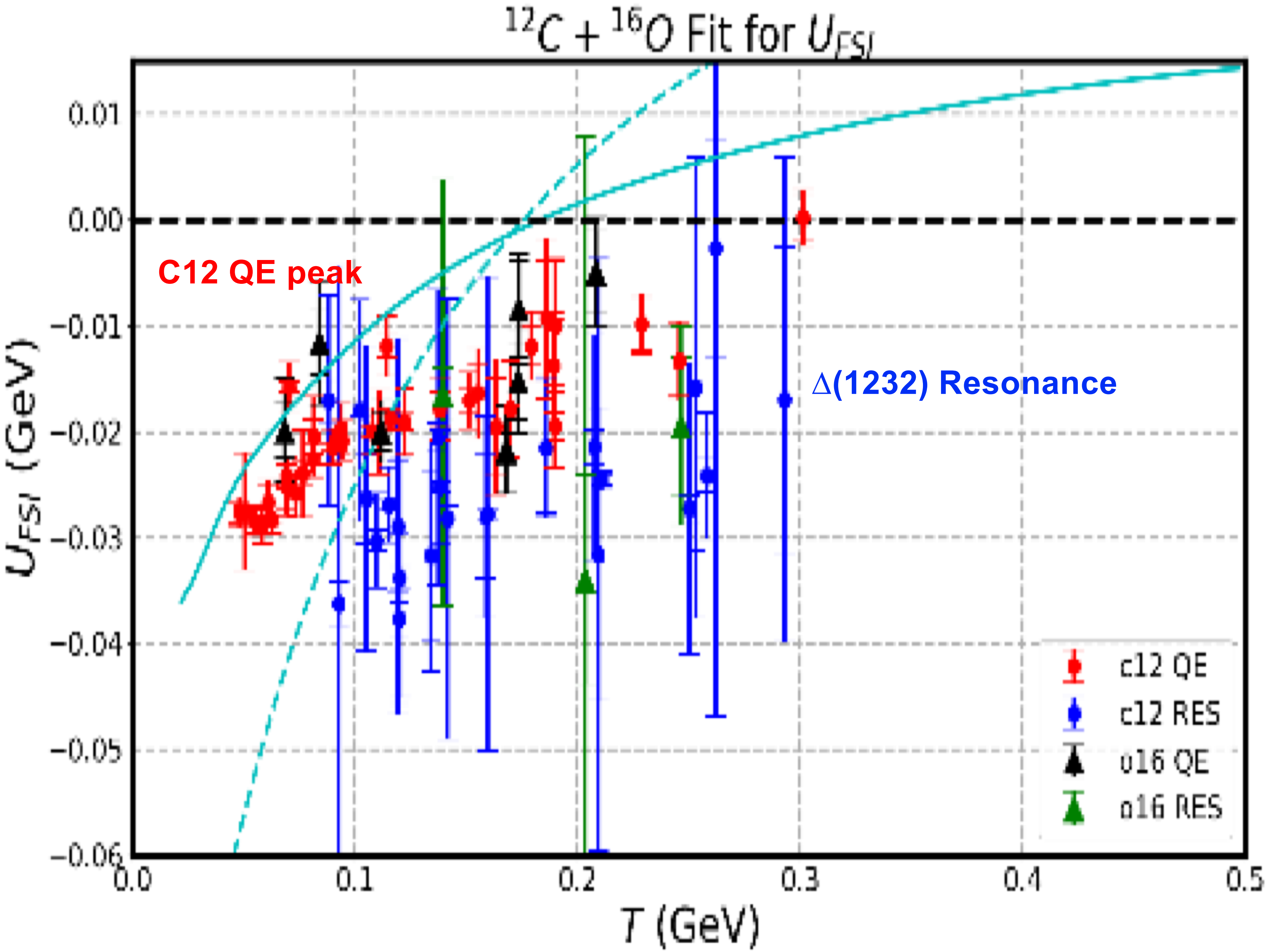
Cooper 2009 calculated by Jose Manuel Udias

Average U_{opt}

Cooper 2009 calculated by Artur Ankowski



Compare QE to Δ(1232)



C12 QE peak

Δ(1232) Resonance

Plan

Re-extract Ufsi using super-scaling for QE and Delta and include 2p2h.

Discussion topic 2

If you have a new MC model (e.g. super-scaling in GENIE) which only has a prediction for the inclusive cross section, you can add the hadronic part by reweighting spectral function MC events.

Example, Psi scaling: take the super scaling variable distribution in spectral function MC, and reweight each event according to it’s super-scaling variable value by the ratio of the distributions of the super-scaling variable in the two models.

It has the following advantages:

1. It is not sensitive to the statistics in two MC samples.
2. No code for the final state needs to written for the new model.
3. If another model comes along, the same thing may be done.

Discussion topic 3.

It is good to see that comparisons with electron scattering data will be done for the all neutrino MC generators.

1. If there is an electron scattering mode in the MC we are all set. But, make sure to account for the effect of Coulomb and Optical potentials.

2. If there is no electrons-scattering option then generate NC neutrino events as follows.

(a) set MA=100000 GeV
(b)Set incident energy and angle to be the same as electron scattering spectra (but add **Veff** to the energies of the incident and final state neutrinos to mimic the Coulomb effects in the electron scattering case).

Appendix

$$\nu + (M_P - \epsilon^P) = \sqrt{(\mathbf{k} + q_3)^2 + M_P^2} + \boxed{U_{FSI}} + \boxed{|V^P_{eff}}$$

Veff (Coulomb) From comparison of inclusive e+ A and e- A

$$\boxed{\epsilon^P = S^P + \langle E^P_x \rangle + \frac{\langle k^2 \rangle}{2M^*_{A-1}}}$$

PHYSICAL REVIEW C, VOLUME 60, 044308

Coulomb distortion measurements by comparing electron and positron quasielastic scattering off ^{12}C and ^{208}Pb

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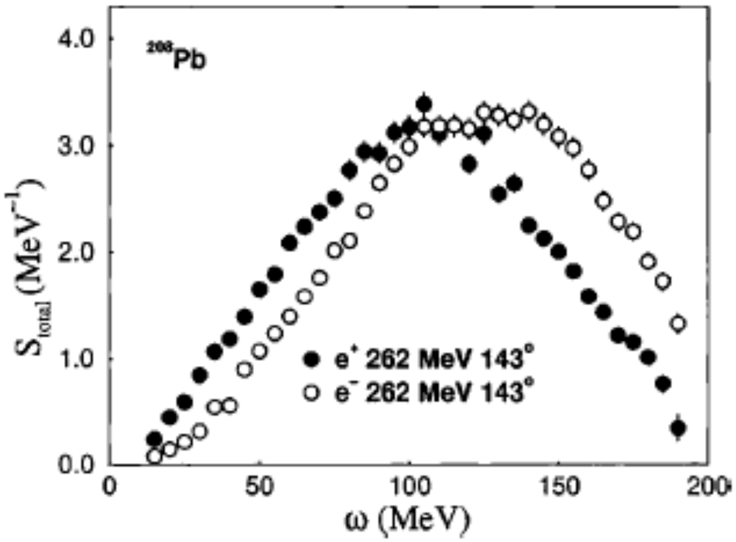

FIG. 6. Positron and electron response functions for the kinematics ^{208}Pb 262 MeV-143°.

TABLE II. Coulomb potential energies of several nuclei evaluated using the experimental charge densities of Ref. [26]. Both the Coulomb potential at the origin $|V_C(0)|$ and its averaged value $|V_C|$ from Eq. (10) are shown. The values of the fit of Eq. (9) are also shown together with the experimental charge mean-square radii.

Nucleus	$\langle r^2 \rangle^{1/2}_{exp}$ (fm)	$\lvert V_C(0) \rvert$ (MeV)	$\lvert V_C \rvert$ (MeV)	$\lvert V_C \rvert_{fit}$ (MeV)
^{12}C	2.464	4.6	3.3	3.1±0.25
^{40}Ca	3.450	10.5	7.9	7.4±0.6
^{48}Ca	3.451	10.4	7.9	7.4±0.6
^{56}Fe	3.714	12.5	9.5	8.9±0.7
^{90}Zr	4.258	16.7	12.8	11.9±0.9
^{154}Gd	5.124	21.8	16.9	15.9±1.2
^{208}Pb	5.503	25.9	20.1	18.9±1.5

Unobserved energy (binding/removal energy)

$$\epsilon^P = S^P + \langle E_x^P \rangle + \frac{\mathbf{k}^2}{2M^*_{A-1}}$$

s^P = energy it takes to separate a proton from nucleus A to make nucleus A-1
Ex = average excitation energy of the spectator A-1 nucleus
T_{A-1}= kinetic energy of the the spectator A-1 nucleus

$$\nu + (M_P - \epsilon^P) = \sqrt{(k+q_3)^2 + M_P^2} - |U_{FSI}| + |V_{eff}^P|$$

Rearranging, we have

$$\nu + (M_{N,P} - x^{\nu,\bar{\nu}}) = \sqrt{(k+q_3)^2 + M_{P,N}^2}$$

where for neutrinos and antineutrinos we have:

$$x^{\nu}((q_3+k)^2) = \epsilon^N - |U_{FSI}| + |V_{eff}^P|$$
$$x^{\bar{\nu}}((q_3+k)^2) = \epsilon^P - |U_{FSI}|$$

if one measures both muon and proton energies in the final state one needs to add ε to get the neutrino energy

x is the interaction energy sometimes mistakinly called I binding/removal energy, but it is not correct.

x is parameter that is equal to Eb in the expression to get E_{QE} from E_{lepton} but it is not biding/removal energy.

$$\nu + (M_P - \epsilon^P) = \sqrt{(\mathbf{k} + \boldsymbol{q_3})^{\mathbf{2}} + M_P^2} - |U_{FSI}| + |V^P_{eff}|$$

$$\epsilon^P = S^P + \langle E_x^P \rangle + \frac{\langle \boldsymbol{k}^2 \rangle}{2M^*_{A-1}}$$

Kinetic energy of spectator recoil nucleus is small

${}^A_ZN$	$\langle T^{P,N} \rangle$ *average*	$T^{P,N}_{A-1}$ *average*	$\langle \epsilon^{P,N} \rangle$ Removal energy		ΔS N-P	$\langle E_x^{P,N} \rangle$ BODEK-RITCHIE	$\langle E_m^{P,N} \rangle$ *average*	ΔE_m N-P	
			$E_m + T^{P,N}_{A-1}$			$E_m^{P,N}$-$S^{P,N}$			
	nucleon $\langle KE \rangle$ T^P,T^N	A-1 nucleus $\langle KE \rangle$ P, N	use for $E_\nu^{QE-\mu}$ $Q^2_{QE-\mu}$ Q^2_{QE-P} $\langle \epsilon^P \rangle$,$\langle \epsilon^N \rangle$	S^P, S^N	$diff$	**GENIE** excitation energy $\langle E_x^P \rangle$,$\langle E_x^N \rangle$	E_m^P, E_m^N	diff E_m^N-E_m^P	**Veff**
$({}^2_1H)$	2.5, 2.5	2.5, 2.5	4.7, 4.7	2.2, 2.2	0.0	0.0, 0.0	2.2, 2.2	0.0	-
${}^6_3Li$	9.1, 9.1	1.8, 1.8	18.4, 19.7	4.4, 5.7	1.3	12.2, 12.2	16.6, 17.9	(1.3)	1.4
${}^{12}_6C$	15.5, 15.5	1.4, 1.4	27.5, 30.1	16.0, 18.7	2.7	10.1, 10.0	26.1, 28.7	2.6	3.1
${}^{16}_8O$	16.0, 16.0	1.1, 1.1	24.1, 27.0	12.1, 15.7	3.6	10.9, 10.2	23.0, 25.9	2.9	3.4
${}^{27}_{13}Al$	17.9, 18.4	0.7, 0.7	30.6, 35.4	8.3, 13.1	4.8	21.6, 21.6	29.9, 34.7	(4.8)	5.1
${}^{28}_{14}Si$	18.1, 18.4	0.7, 0.7	24.7, 30.3	11.6, 17.2	5.6	12.4, 12.4	24.0, 29.6	(5.6)	5.5
${}^{40}_{18}Ar$	19.9, 21.9	0.5, 0.6	30.9, 32.3	12.5, 9.9	-2.6	17.8, 21.8	30.2, 31.7	1.4	6.3
${}^{40}_{20}Ca$	19.9,19.9	0.5, 0.5	28.2, 35.9	8.3, 15.6	7.3	19.4, 19.8	27.7, 35.4	7.7	7.4
${}^{50}_{23}V$	20.2, 22.4	0.4, 0.5	25.6, 28.6	8.1, 11.1	3.0	17.0, 17.0	25.1, 28.1	(3.0)	8.1
${}^{56}_{26}Fe$	20.4, 22.6	0.4, 0.4	29.6, 30.6	10.2, 11.2	1.0	19.0, 19.0	29.2, 30.2	(1.0)	8.9
${}^{58.7}_{28}Ni$	20.9, 22.8	0.4, 0.4	25.4, 29.4	8.2, 12.2	4.0	16.8, 16.8	25.0, 29.0	(4.0)	9.8
${}^{88}_{40}Zr$				8.4, 12.0	3.6			1.9	11.9
${}^{197}_{79}Au$	23.9, 30.4	0.1, 0.1	25.4, 27.7	5.8, 8.1	2.3	19.5, 19.5	25.3, 27.6	(2.3)	18,5