Hadron tensors implementations and factorisation* summary

Stephen Dolan

Stephen.Dolan@llr.in2p3.fr

* Possible meanings: "decoupling the momentum distribution", "separation into spectral function and nucleon cross section, "random guesses about the final state"

Hadron tensors implementations

$$\frac{d^2\sigma}{dq_0 q_3} = \sigma_0 \eta_{ij}(q_0, q_3) W_{ij}(q_0, q_3)$$

- Lepton tensor and global factors are common between models
- Hadron tensor encodes nuclear dynamics, one for each model
- Hadron tensor tables are tabulated:
  - 2 variables (q_0, q_3 ; $T_\ell, \cos(\theta_\ell)$ etc)
  - 5 elements ($W_{00}, W_{01}, W_{11}, W_{12}, W_{33}$)
  - Fine bins (5 – 20 MeV), interpolation methods to extrapolate between
- Lepton tensors are simple: calculated on the fly
- Used for: Valencia 2p2h, SuSAv2 1p1h+2p2h
- WIP: SF-based 1p1h and 2p2h, others?

Hadron tensors implementations

- Hadron tensors are unique to the specific model configuration used – bake in a nuclear model with specific parameters (E_b, k_f etc)

- Current implementations have tensors for a few select targets and then scaling to others
  - 1p1h and 2p2h scale differently
  - Account for altered removal energy
    - Currently shift the value of q_0 evaluated from the tensor

- For 2p2h we have separate tensors for different initial state nucleon pairs
  - Can predict pre-FSI final state directly from the model

- Where available, electron scattering hadron tensors are calculated separately, but from the same model
  - Suggestion: split relevant tensor elements into vector and axial parts – no need for separate tensors

Hadron tensors implementations

Advantages over coding in the model directly:

- Fast
- Easy to have a unified framework for many models
- Hard to get wrong – easier to guarantee reproduction of the theory

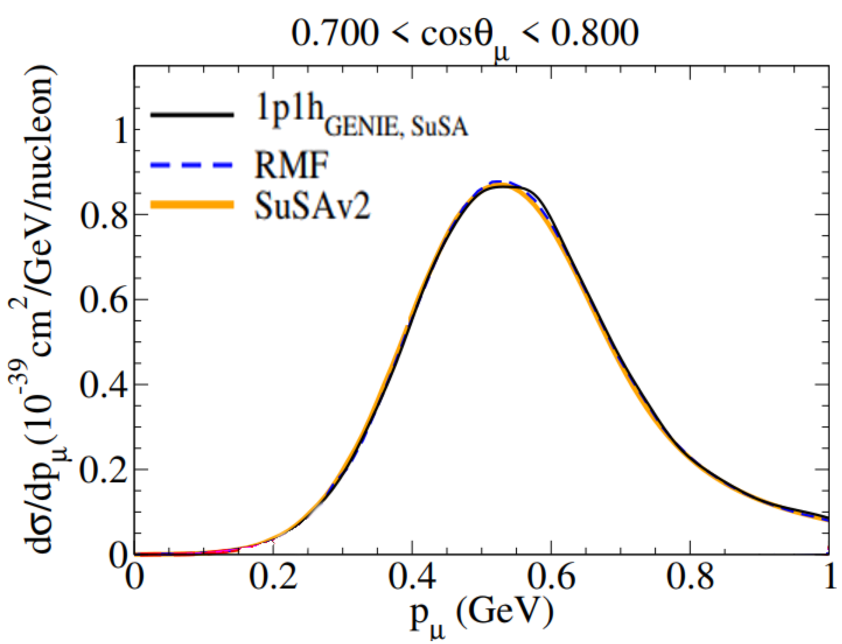
Disadvantages:

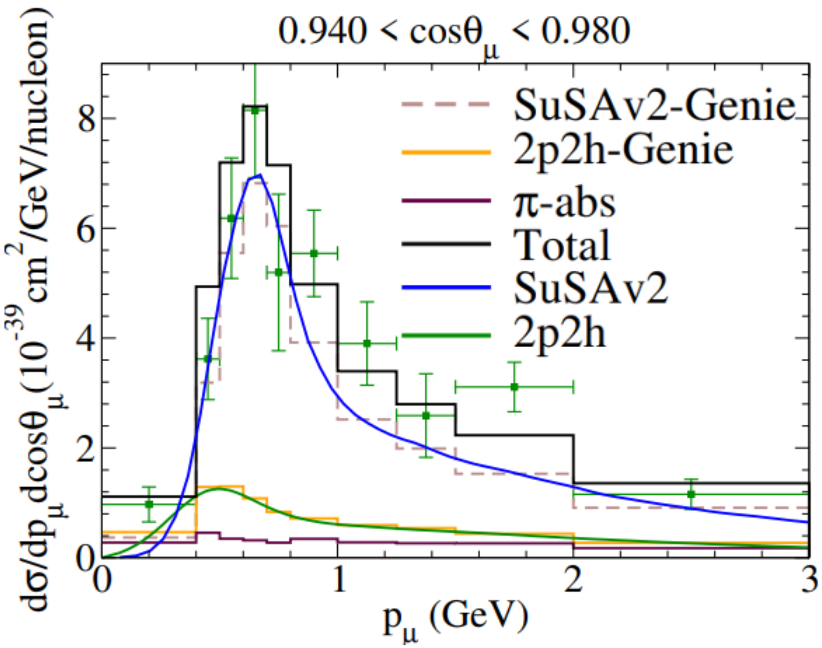
- Not very flexible
- Reweighting model parameters will be difficult
- More difficult to maintain consistency in model implementation: Can be unclear what actually went into the hadron tensor (what form factors, binding energy etc)

Hadron tensors implementations

Proper use of this prescription guarantees reproduction of *inclusive* cross-section predictions (remember we implemented only this):

$$\frac{d^2\sigma}{dq_0 q_3} = \sigma_0 \eta_{ij}(q_0, q_3) W_{ij}(q_0, q_3)$$





What about the hadrons?

Different models can give similar inclusive CS but different semi-inclusive ones (more sensitive to nuclear-medium effects) $\Rightarrow$ **very different ν oscillation analyses (which relies on semi-inclusive predictions)**

PROBLEM: Current lack of full semi-inclusive models and proper implementation in generators.

Semi-inclusive $\Rightarrow$ Inclusive (but not viceversa) $\Rightarrow$ Factorization approach is questionable.

- QE and 2p2h inclusive: We only need $W^{\mu\nu}(q,\omega)$ or, equivalently, $W^{\mu\nu}(p_\mu, \cos\theta_\mu)$
- QE semi-inclusive : 5D diff. CS $(\theta_\mu, p_\mu, p_N, \theta_N, \phi_N)$ - 2p2h semi-inclusive: 9D diff. CS.

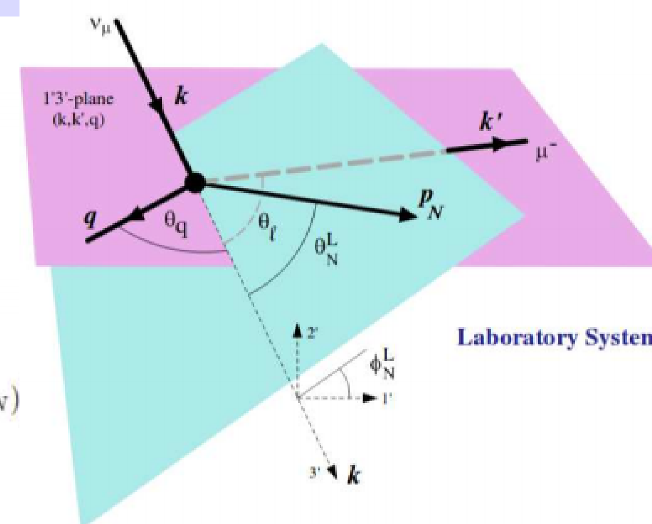
Double differential inclusive cross section $\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$

$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu}\right]_\chi = \sigma_0 \left(V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_T R_T + \chi\left[2V_{T'}R_{T'}\right]\right)$$

Double differential semi-inclusive cross section $\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$

$$\frac{d\sigma}{dk' d\Omega_{k'} dp_N^2 d\Omega_N^L} = \frac{G^2\cos^2\theta_c m_N k'^2 \varepsilon p_N^2 W_{A-1} v_0}{2(2\pi)^5 k\varepsilon' E_N \sqrt{X_B^2 + m^2 a_B}} \mathcal{F}_\chi^2 \delta(k-k_0),$$

$$\begin{aligned}
\mathcal{F}_\chi^2 =& \hat{V}_{CC}(w_{CC}^{VV(I)} + w_{CC}^{AA(I)}) + 2\hat{V}_{CL}(w_{CL}^{VV(I)} + w_{CL}^{AA(I)}) + \hat{V}_{LL}(w_{LL}^{VV(I)} + w_{LL}^{AA(I)}) \\
&+ \hat{V}_T(w_T^{VV(I)} + w_T^{AA(I)}) + \hat{V}_{TT}\left[(w_{TT}^{VV(I)} + w_{TT}^{AA(I)})\cos 2\phi_N + (w_{TT}^{VV(II)} + w_{TT}^{AA(II)})\sin 2\phi_N\right] \\
&+ \hat{V}_{TC}\left[(w_{TC}^{VV(I)} + w_{TC}^{AA(I)})\cos\phi_N + (w_{TC}^{VV(II)} + w_{TC}^{AA(II)})\sin\phi_N\right] \\
&+ \hat{V}_{TL}\left[(w_{TL}^{VV(I)} + w_{TL}^{AA(I)})\cos\phi_N + (w_{TL}^{VV(II)} + w_{TL}^{AA(II)})\sin\phi_N\right] \\
&+ \chi\left[\hat{V}_{T'}w_{T'}^{VA(I)} + \hat{V}_{TC'}(w_{TC'}^{VA(I)}\sin\phi_N + w_{TC'}^{VA(II)}\cos\phi_N) + \hat{V}_{TL'}(w_{TL'}^{VA(I)}\sin\phi_N + w_{TL'}^{VA(II)}\cos\phi_N)\right.
\end{aligned}$$



What about the hadrons?

Different models can give similar inclusive CS but different semi-inclusive ones (more sensitive to nuclear-medium effects) $\Rightarrow$ **very different ν oscillation analyses (which relies on semi-inclusive predictions)**

PROBLEM: Current lack of full semi-inclusive models and proper implementation in generators.

We're clearly missing the ingredients needed for a full semi-inclusive cross section, but how much does this matter ... ?

Double differential inclusive cross section $\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$

$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu}\right]_\chi = \sigma_0 \left(V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_T R_T + \chi\left[2V_{T'}R_{T'}\right]\right)$$

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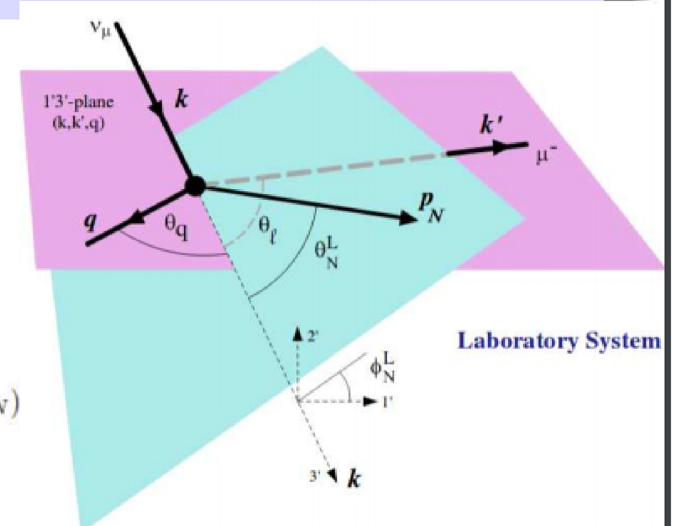
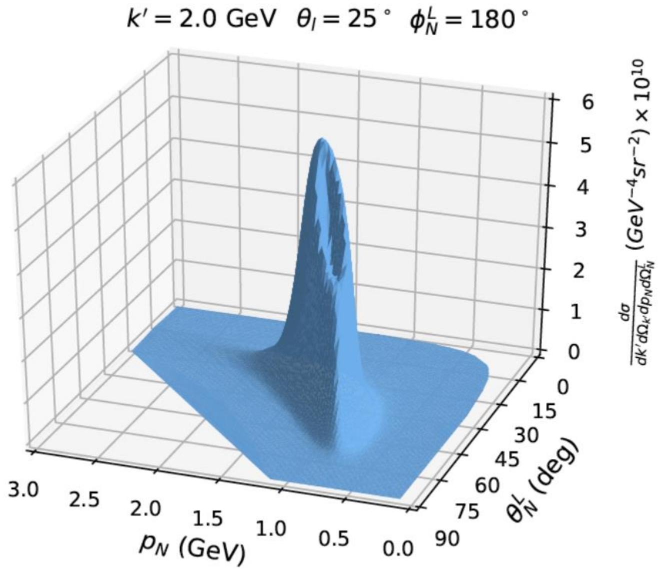
\end{aligned}$$



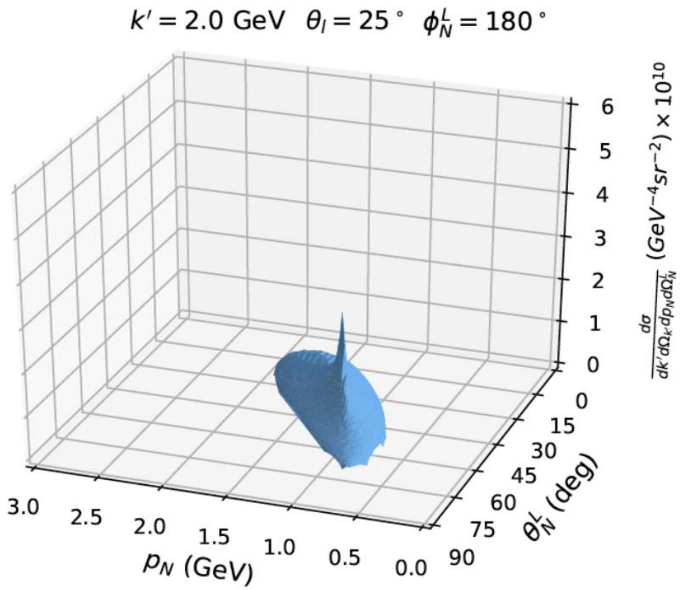
The worst case

Last year:
https://indico.ectstar.eu/event/19/contributions/221/

CC neutrino semi-inclusive cross section based on the spectral function from Omar



CC neutrino semi-inclusive cross section based on the LDA (local Fermi gas) approximation



6. Discussion of implications

- For inclusive scattering integrals must be done over the "landscape" whose boundaries are determined by the lepton kinematics
- Most modeling is done for inclusive reactions and there one finds that, as long as the models used are relativistic, the results are not dramatically different (see our recent analysis of T2K oxygen results)
- However, inevitably experimental studies must rely on semi-inclusive simulations, or, indeed, may want the extra hadronic information as in measurements using TPCs
- Despite the fact that inclusive relativistic modeling is reasonably robust, semi-inclusive modeling using naïve models which have been designed for studies of inclusive scattering are at best suspect

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What we do for SuSA implentations

Implementation of SuSAv2-MEC hadronic part (FA):

- Draw target nucleon from chosen nuclear model **irrespective of q_0, q_3**
- Get removal energy from RMF-like treatment, *re-throw from nuclear model if nucleon is Pauli blocked*
- Transfer all of ω, q to nucleon, **none to remnant**
- Subtract removal energy, **put proton on-shell with adjustment of p** (only needed for 1p1h) **then conserve momentum by adjusting remnant kinematics**
- *Do FSI cascade and rest of interaction using standard GENIE methods*

Lots of scope for improvement!

Avoid by sampling full exclusive xsec?

Mitigate by making nuclear model q_0, q_3 dependant?

Different model for each shell?

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Removal energy should also depend on chosen initial nucleon momentum which should depend on inclusive kinematics …

The remnant should take *some* momentum … how much?

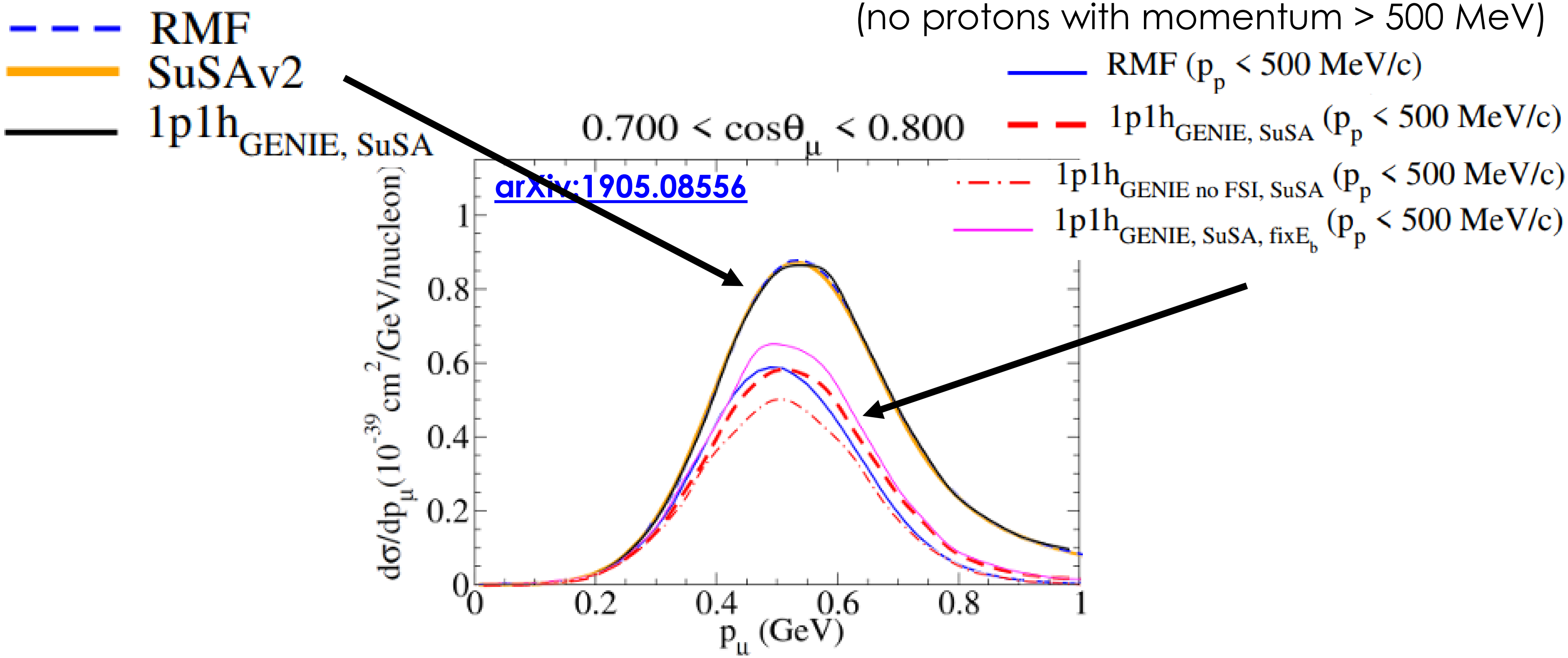
This is just bad, we're working on this. Have some ideas.

How much can we improve FSI? Better motivated cascades? GiBUU-like hadron transport?

How bad is it?

These lines show the ***inclusive*** 1p1h prediction (no proton constraints)

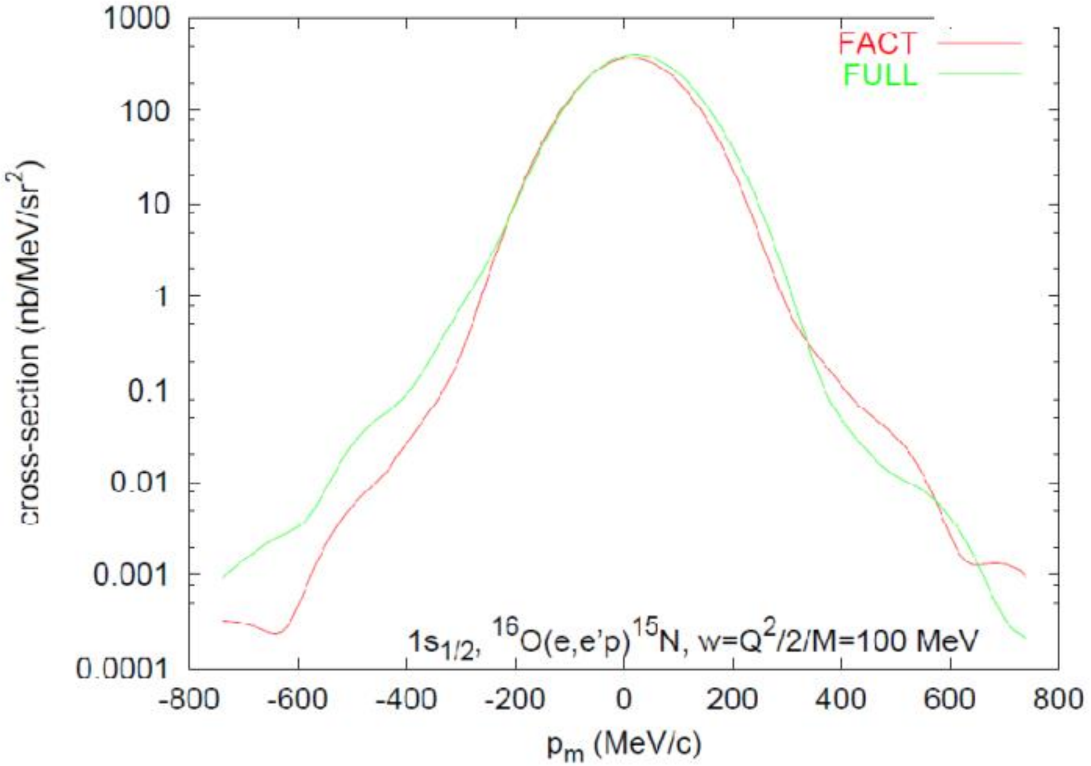
These lines show the ***exclusive*** 1p1h prediction
(no protons with momentum > 500 MeV)



- For *inclusive* calculations the microscopic base model (RMF), the inclusive theory (SuSAv2) and the implementation (in GENIE) all agree.
- *Exclusive* GENIE calculations do not match RMF. Varying the ingredients to the FA leads to quite different predictions.

Factorization in RMF

$$\frac{d^5\sigma}{d\Omega_e d\varepsilon' d\Omega_F} = K\sigma_{ep} S(E_m, \vec{p}_m) \qquad \rho^{exp}(\mathbf{p}_m) = \frac{\left(\frac{d\sigma}{d\varepsilon_f d\Omega_f d\Omega_F}\right)^{exp}}{E_F p_F f_{rec}\sigma_{ep}}.$$



Factorization approach

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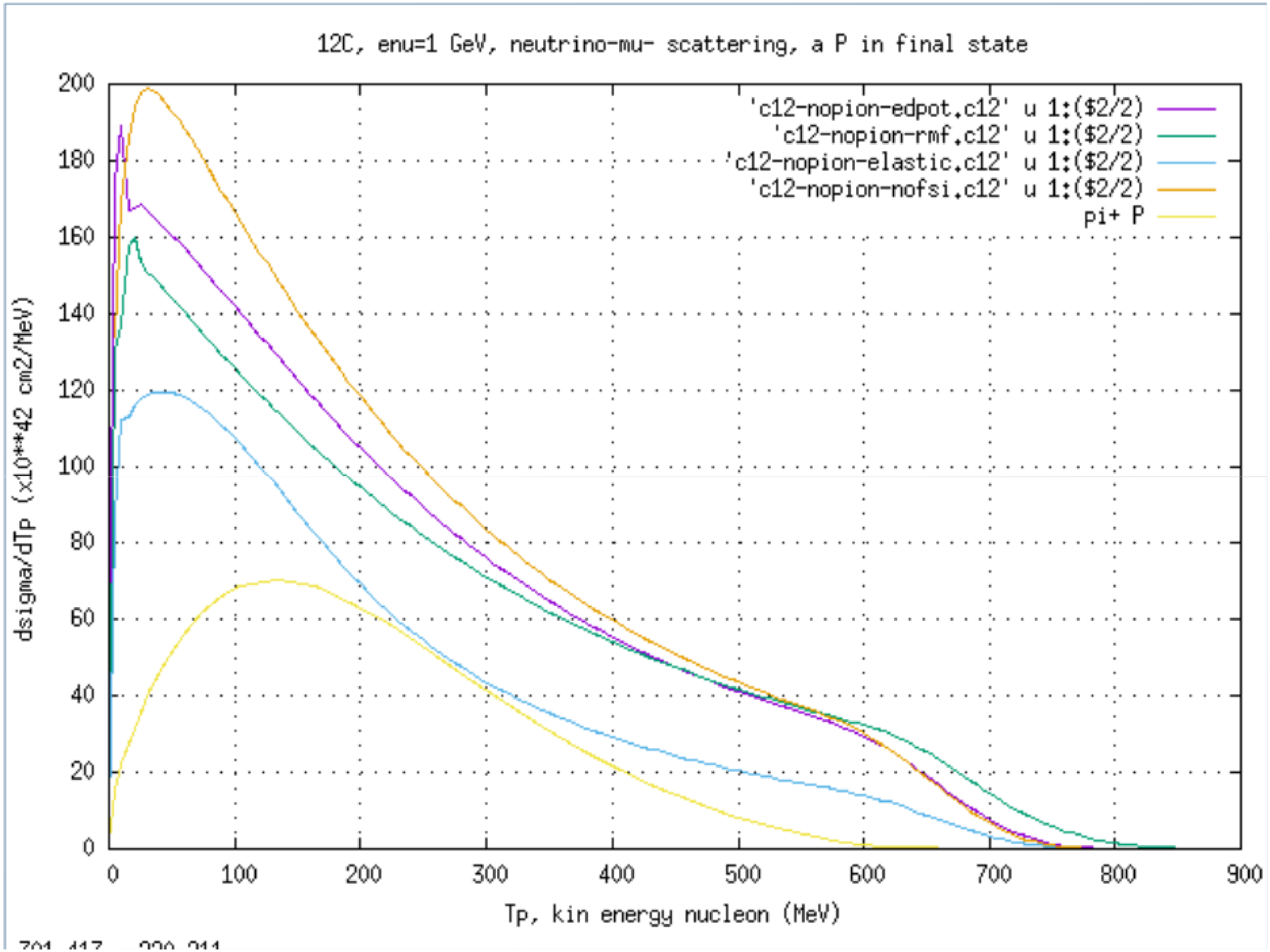
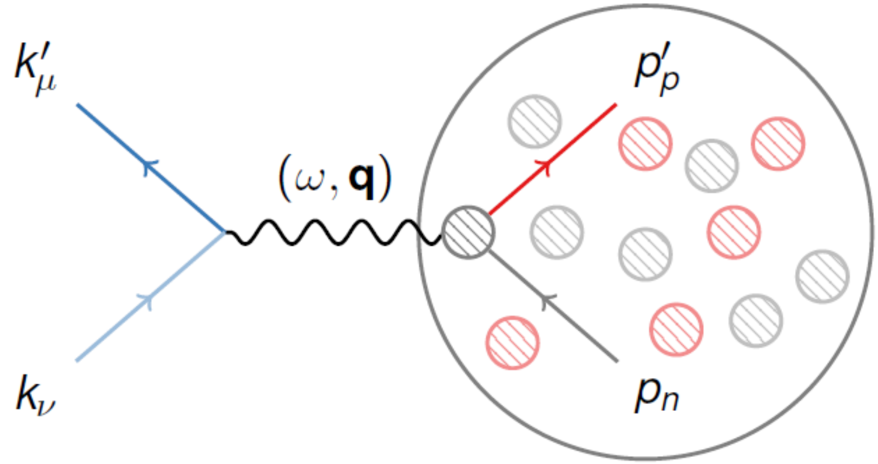
JM Udias Trento 2019

RMF exclusive



Other approaches: SF in NuWro

Plane-wave impulse approximation



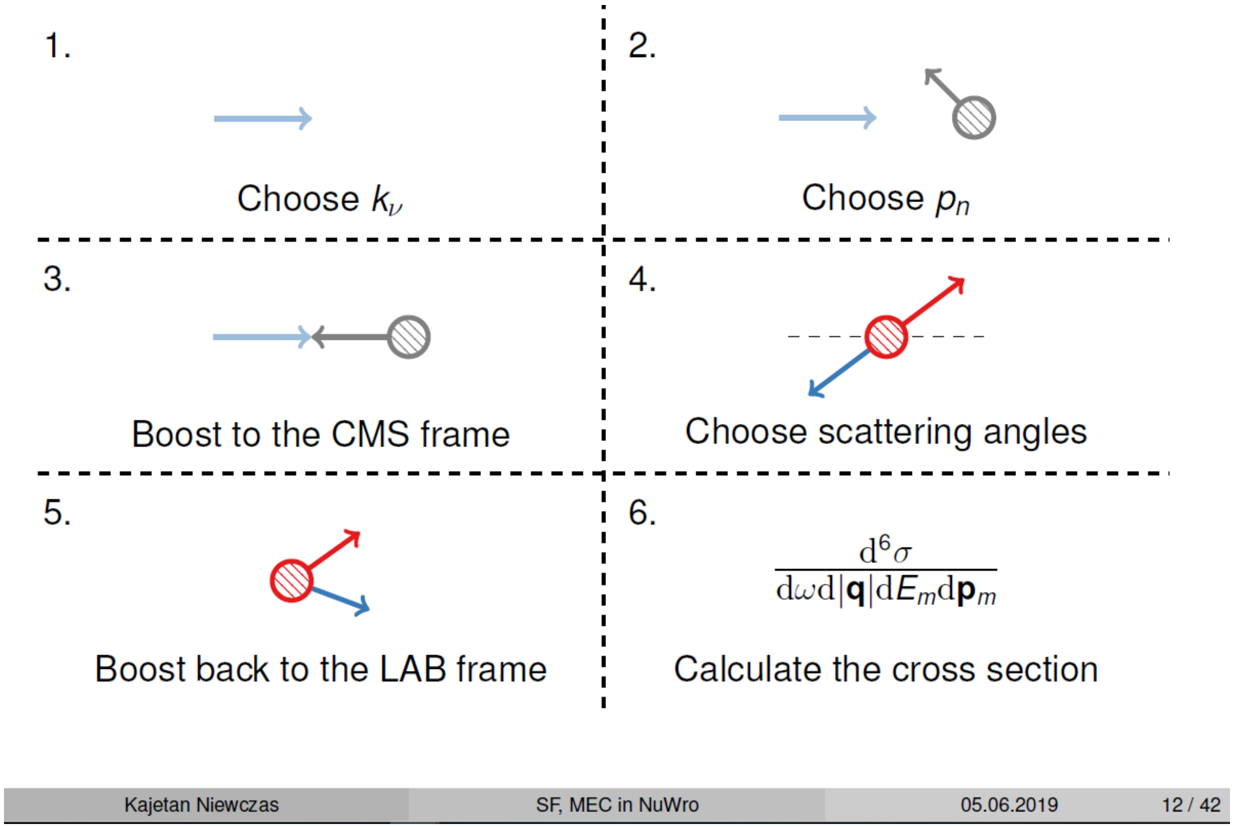
Factorization of the cross section in the **absence of FSI**:

$$\frac{\mathrm{d}^6\sigma^{\mathrm{PWIA}}}{\mathrm{d}\omega \mathrm{d}|\mathbf{q}| \mathrm{d}E_m \mathrm{d}\mathbf{p}_m} = \frac{G_F^2 \cos^2\theta_C |\mathbf{q}|}{4\pi E_\mathbf{k}^2 E_\mathbf{p} E_{\mathbf{p}'}} P_{(n)}(E_m, \mathbf{p}_m) L_{\mu\nu} \widetilde{H}^{\mu\nu} \delta(\omega + M - E_m - E_{\mathbf{p}'})$$

$P_{(n)}(E_m, \mathbf{p}_m)$ - probability density of initial nucleons

$L_{\mu\nu} \widetilde{H}^{\mu\nu} \delta(\omega + M - E_m - E_{\mathbf{p}'})$ - interaction dynamics for a given nucleon

Other approaches: SF in NuWro



Therefore, **NuWro calculates**

$$\sigma^{\mathrm{PWIA}} = \int_V \frac{d^6\sigma^{\mathrm{PWIA}}}{d\omega d|\mathbf{q}| dE_m d\mathbf{p}_m} \frac{1}{S(E_m, |\mathbf{p}_m|)} \left[d\Omega^*_\mu S(E_m, |\mathbf{p}_m|) dE_m d\mathbf{p}_m \right]$$

Other approaches: SF in NuWro

FSI for SF

The procedure after **O. Benhar et al., Phys.Rev. C44 (1991) 2328**

The cross section is folded as

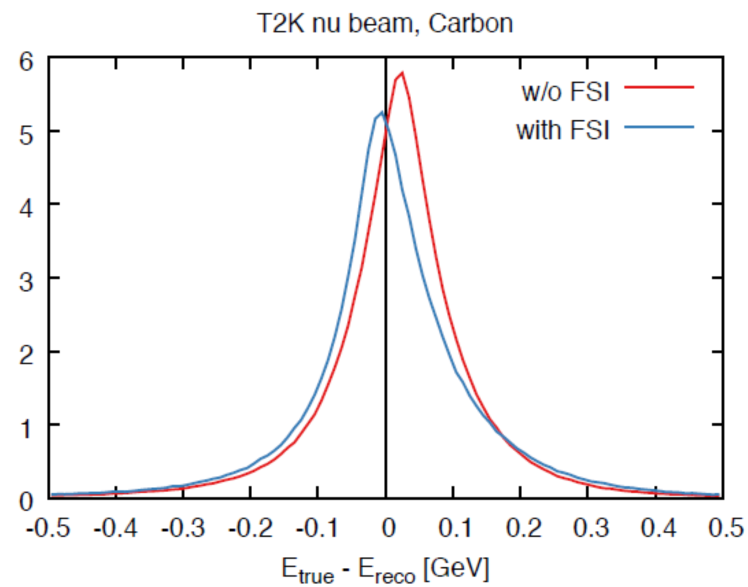
$$\frac{\mathrm{d}\sigma^{\mathrm{FSI}}}{\mathrm{d}\omega\mathrm{d}\Omega} = \int \mathrm{d}\omega' f_{\mathbf{q}}(\omega-\omega')\frac{\mathrm{d}\sigma^{\mathrm{IA}}}{\mathrm{d}\omega\mathrm{d}\Omega}$$

where the folding function is

$$f_{\mathbf{q}}(\omega) = \delta(\omega)\sqrt{T_A} + (1-\sqrt{T_A})F_{\mathbf{q}}(\omega)$$

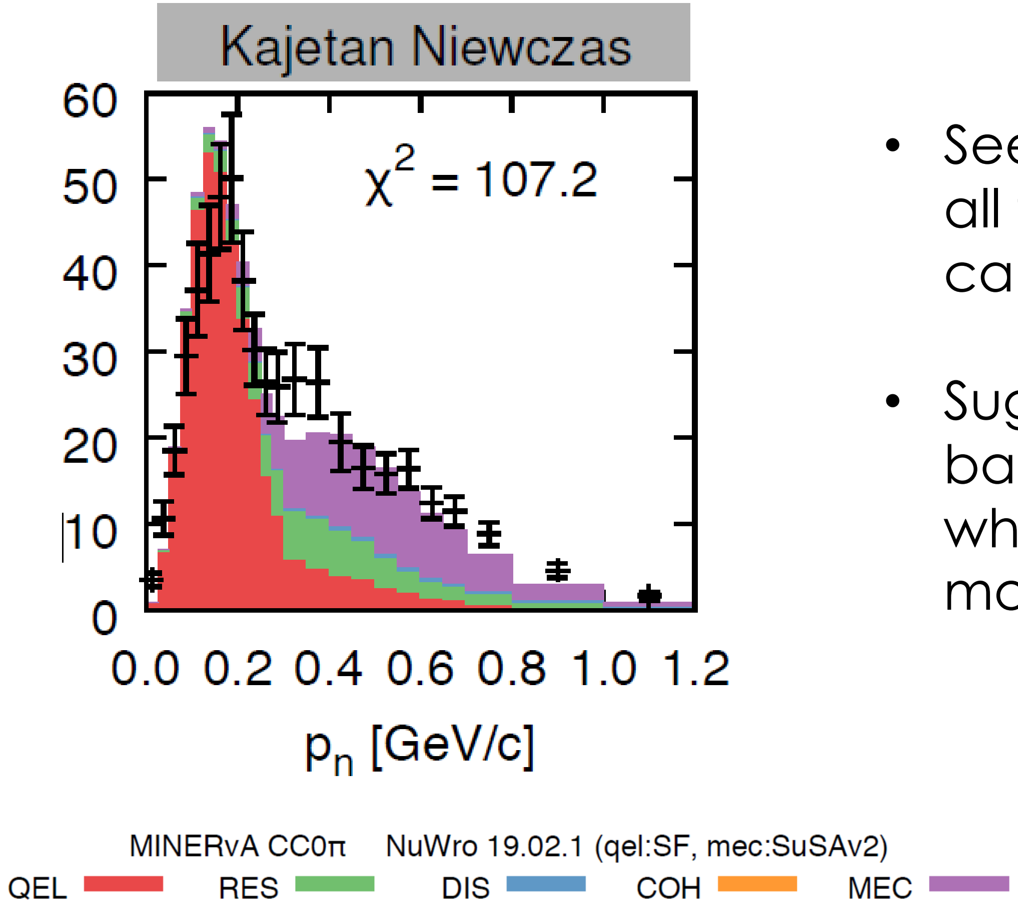
and $F_{\mathbf{q}}(\omega)$ smears the energy transfer according to the **NN cross section** weighted with nuclear **transparency** T_A

interaction between the knocked-out **nucleon** and the spectator system $\rightarrow$ affects the **conservation of energy** and therefore the **kinematics** of the final **lepton**



Caveat: not entirely clear whether this FSI is double counting with what's in NuWro's cascade

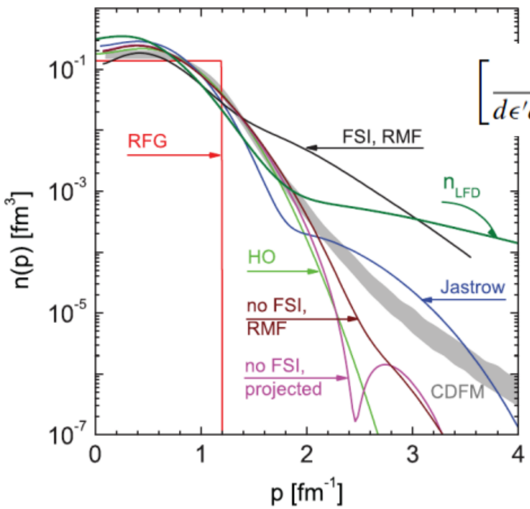
Other approaches: SF in NuWro



- Seems to work fairly well – includes all the ingredients for semi-inclusive calculations. Not many "tricks".
- Suggestion: could we "piggy back" off the hadron part of SF when implementing other inclusive models?

Future implementations of RMF

Decoupling momentum distributions



In the RPWIA there is exact factorization:

$$\left[\frac{d\sigma}{d\epsilon' d\Omega' dp_N d\Omega_N}\right]^{\mathrm{PWIA}}_{(e,e'N)} = K\sigma^{eN}(q,\omega;p,\mathcal{E},\phi_N)S(p,\mathcal{E}),$$

This is not true in a model with distorted waves

Compare RMF with and without FSI.

We can not decouple the momentum distribution and the inclusive cross section

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4 June 2019, ECT* A. Nikolakopoulos

$$\rho(p) = n^{\mathrm{dist}}(p) = \frac{\left[\frac{d\sigma}{d\Omega' d\epsilon' d\Omega_N}\right]_{\mathrm{FSI}}}{K\sigma^{eN}}.$$

One can not directly measure a momentum distribution.

One can only measure the cross section

- To capture the full model, really would need to implement: $\dfrac{d^5\sigma}{dE_f d\Omega_f d\Omega_N}$
    - Could do this will large hadron tensor tables
    - May be some tricks to reduce dimensionality
- But then we fix the final nucleon momentum from the model leaving no room for a FSI cascade (no nuclear emission) …

Future implementations of RMF

(e,e'p) is a different game

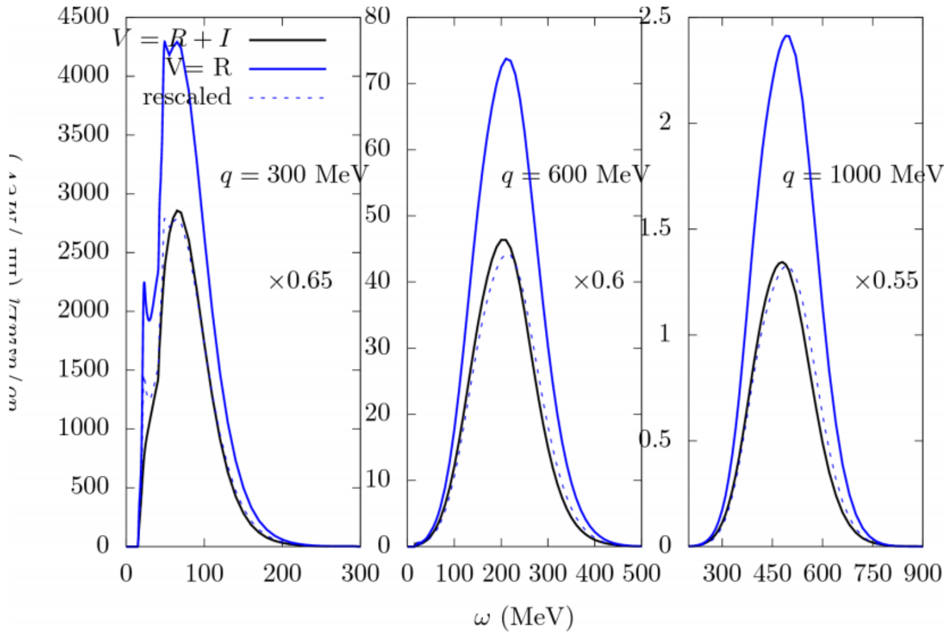
All kinematic variables are determined and selected to probe a specific missing energy region.

One needs to introduce an imaginary potential and/or spectroscopic factors to account for **reduction** and correlations

Observation/Assumption:

The effect of the optical potential accounts almost only for 'hard' rescattering events.

- Can implement the model in order to leave room for a cascade
- Promising direction, stay tuned



Observation/Assumption:

The effect of the optical potential accounts almost only for 'hard' rescattering events.

So the MC can take care of this but the model should take into account the real part of the potential to give A good inclusive cross section

Summary

- We now have a well established framework to implement new models in GENIE (and beyond) using hadron tensors
- Exactly reproduces *inclusive* input model predictions
- Hadron kinematic predictions are made using "factorisation" approximations (FA) – ad-hoc and possibly unreliable
- Showed some very simple tests of FA – need more detailed analysis to better assess validity
- Semi-inclusive SF approaches avoid some of the issues but need FSI added on top of the base model (which alters the lepton kinematics)
- More exclusive inputs from theory will help us improve our implementations

Backups

Discussion topics

Hadron tensor implementations
- What does the calculation of an xsec using a hadron tensor look like?
- How should this be implemented in the generators?
- Is this the same for 1p1h, 2p2h and pion production?
- What choices do have for making semi-inclusive predictions in the generators? How do we currently make these choices?

Factorization approximations
- Can we quantify the impact? Develop uncertainties to cover the difference?
- What are the possible biases from this for neutrino oscillation analyses?
- What can we learn about its validity from electron scattering data? (E.g. to what extent does the missing energy and momentum depend on the kinematics?)
- What can we measure in neutrino scattering to test this (transverse imbalance as a function of lepton kinematics?)

Factorization mitigation
- Can we simply implement full semi-inclusive calculations directly?
  - Would probably require a new paradigm for event generation
  - 15 vs 5 nuclear responses – is this too hard or too slow?
  - Did we already do this for electron scattering? Were models for e,e'p fully exclusive?
- Even if we do this, how should we treat FSI?
- SF models are a bit different – are they immune to factorisation issues?
- Can we use some information from semi-inclusive predictions to make better choices in the factorisation scheme?
- Can we implement separate hadron tensors and spectral functions for each shell?

Bonus topic: What can we learn from LHC experiences? Can they tell us how far we can go in complexity in our MC generators and what tricks that we can use to do so?