Coulomb and Optical Potentials Working Group

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Reference
Removal Energies and Final State Interaction in Lepton Nucleus Scattering
Arie Bodek, Tejin Cai 10.1140/epjc/s10052-019-6750-3
https://arxiv.org/abs/1801.07975 published in Eur. Phys. J. C. (2019) 79: 293

Talks in parallel session so far

1. Artur Ankowski, "Final state interactions in the spectral function approach
Summary of Artur M. Ankowski,[1,*] Omar Benhar,[2] and Makoto Sakuda[1]
PHYSICAL REVIEW D 91, 033005 (2015)

2. Jose Manuel Udias “QE response in RDWIA with the different potentials”

If anybody wants to talk in the parallel session, please let me know

The following parameters can make impulse approximation calculations for 1p1h process in neutrino MC closer to more complete QM calculations

1. **Removal energy in the initial state** (or a 2-D spectral function) . We should use the correct parmeters for the Fermi gas implemented in the current Monte Carlo generators
2. **Effects of Coulomb potential on initial and final state leptons and hadrons.** In general, this has not been implemented yet.
3. **Effects of nuclear effective Optical Potential on final state hadrons.** This potential is a function of final state kinetic energy. In general, this has not been implemented yet.

The effects of **removal energy** and **optical potential** are of similar magnitude (but in opposite direction at low Q, and same direction at high Q). Both must be accounted for.

Effect of Coulomb and Nuclear Fields

- **Quantum vs. Classical**: In general, the scattering from the nucleon is treated as **one boson exchange** (**quantum mechanically).** The effects of the Coulomb and Nuclear mean fields are treated **classically** as scattering in a potential.

- **Coulomb field** is treated using the **Effective Momentum Approximation** (EBA), which has been confirmed in comparisons of quasielastic scattering cross sections of incident electrons and positrons.

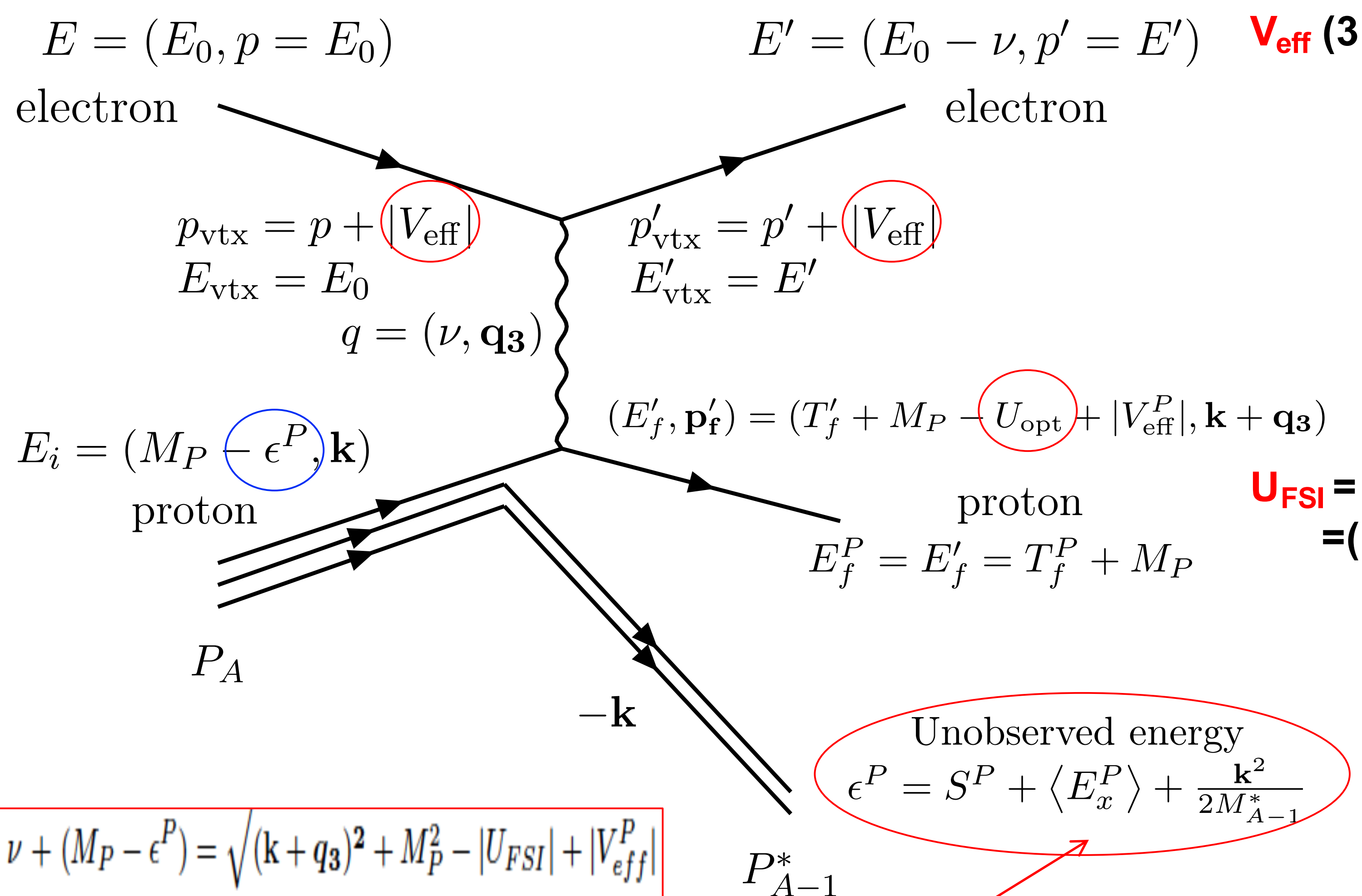
- **Nuclear Mean field can be** treated as an effective **optical potential**.
  a) **The real part of the potential affects the energy of the final state nucleon.**
  b) The **imaginary part accounts for elastic and inelastic interactions** of a final state nucleon with other nucleons in the the nucleus. Theorists call both the real and imaginary components as final state interaction **(FSI).**

- **However, Experimentalists have been using the term FSI to account only for elastic and inelastic interactions** of a final state nucleon with other nucleons in the the nucleus Consequently, the real part is has not been included.

Electron QE Scattering in a Coulomb and Nuclear potential

Energy (sun of kinetic and potential energy) is conserved at every step. Momentum changes and momentum conservation is taken care of by the spectator nucleus (with negligible energy)

Electron scattering on proton



$V_{\rm eff}$ **(3 to 7 MeV)**

U_{FSI} **=** $-U_{opt}$
=(-20 to -30 MeV) low Q2

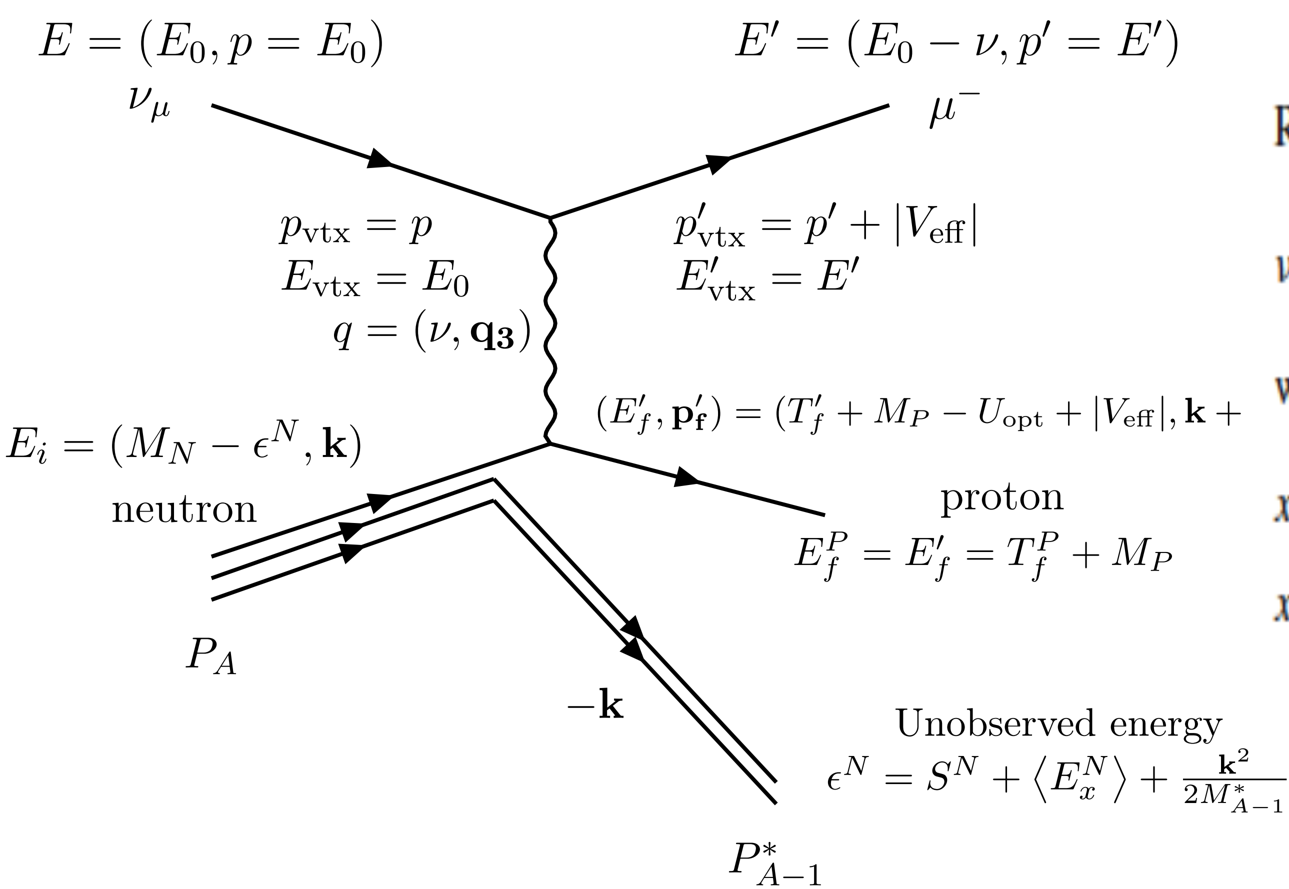
$$\nu + (M_P - \epsilon^P) = \sqrt{(\mathbf{k} + \mathbf{q_3})^2 + M_P^2} - |U_{FSI}| + |V^P_{eff}|$$

Use average Ex If a spectral function is not used

Neutrino QE Scattering in a Coulomb and Nuclear potential

Energy (sun of kinetic and potential energy) is conserved at every step. Momentum changes by momentum conservation is taken care of by the spectator nucleus (with negligible energy)

Neutrino scattering on neutron



$$\nu + (M_P - \epsilon^P) = \sqrt{(k + q_3)^2 + M_P^2} - |U_{FSI}| + |V^P_{eff}|$$

Rearranging, we have

$$\nu + (M_{N,P} - x^{\nu,\bar{\nu}}) = \sqrt{(k + q_3)^2 + M^2_{P,N}}$$

where for neutrinos and antineutrinos we have:

$$x^{\nu}((q_3 + k)^2) = \epsilon^N - |U_{FSI}| + |V^P_{eff}|$$

$$x^{\bar{\nu}}((q_3 + k)^2) = \epsilon^P - |U_{FSI}|$$

Bodek-Ritchie (off shell) – corresponds to spectral function notation

$$\nu + (M_P - \epsilon^P) = \sqrt{(\mathrm{k} + q_3)^2 + M_P^2} - |U_{FSI}| + |V_{eff}^P|$$

vs Smith Moniz (on Shell)

The Smith-Moniz [20] formalism uses on-shell description of the initial state. In the on-shell formalism, the energy conserving expression is

$$\nu + M - \epsilon = E_f$$

is replaced with

$$\nu + \sqrt{k^2 + M^2} - \epsilon_{SM}^{\prime P,N} = E_f.$$

Therefore,

$$\epsilon'_{SM} = \epsilon + \langle T^{P,N} \rangle,$$

At Q^2=0.2 GeV^2 (T=0.1 GeV^2) optical potential is negative

$$\nu + (M_{N,P} - x^{\nu,\bar{\nu}}) = \sqrt{(k+q_3)^2 + M_{P,N}^2}$$

where for neutrinos and antineutrinos we have:

$$x^{\nu}((q_3+k)^2) = \epsilon^N - |U_{FSI}| + |V_{eff}^P|$$
$$x^{\bar{\nu}}((q_3+k)^2) = \epsilon^P - |U_{FSI}|$$

Neutrino QE scattering on neutron

$$\nu + (M_P - \epsilon^P) = \sqrt{(k+q_3)^2 + M_P^2} - |U_{FSI}| + |V_{eff}^P|$$

$$\epsilon'_{SM} = \epsilon + \langle T^{P,N} \rangle,$$

	Carbon MeV	Argon MeV
$\lvert V_{eff}^P \rvert$	3.1	6.3
ε^N	30.1	32.1
x	**33.2** -20 =**13.2**	**38.3** -30=**8.3**

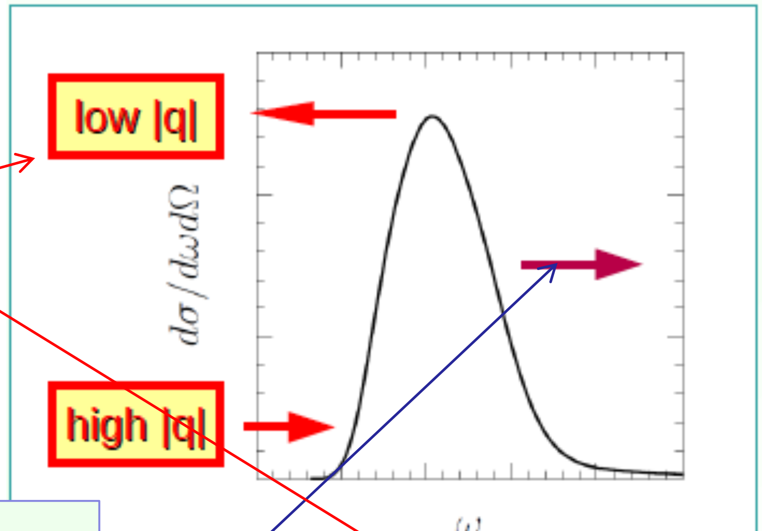
T^P	15.5	21.9
ε'^N_{SM}	**48.7**-20=**28.7**	**60.3**-30=30.3

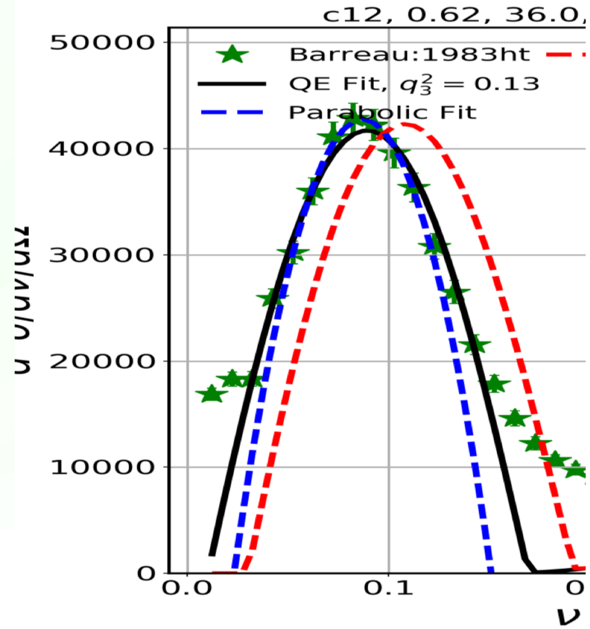
|Ufsi|	**-20.0**	**-30.0**
T=0.1 GeV^2		
	Carbon	Argon

Shift in QE peak position from two sources

Real part of the OP

- acts in the final state
- shifts the QE peak to low ω at low $|\mathbf{q}|$ (to high ω at high $|\mathbf{q}|$)



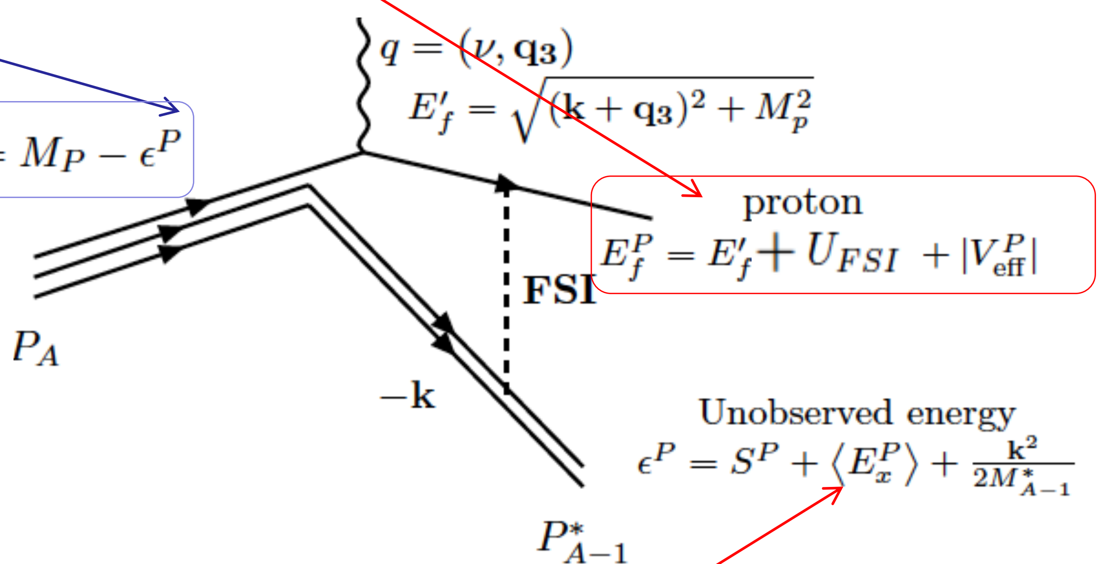


Binding energy in RFG

acts in the initial state

shifts the QE peak to high ω

$\mathbf{k}, E_i = M_P - \epsilon^P$



$$\nu + (M_P - \epsilon^P) = \sqrt{(k+q_3)^2 + M_P^2} - |U_{FSI}| + |V^P_{eff}|$$

$$\nu + (M_{N,P} - x^{\nu,\bar{\nu}}) = \sqrt{(k+q_3)^2 + M^2_{P,N}}$$

where for neutrinos and antineutrinos we have:

$$x^{\nu}((q_3+k)^2) = \epsilon^N - |U_{FSI}| + |V^P_{eff}|$$

$$x^{\bar{\nu}}((q_3+k)^2) = \epsilon^P - |U_{FSI}|$$

Use average Ex If a spectral function is not used

$$\nu + (M_P - \epsilon^P) = \sqrt{(\mathbf{k} + \mathbf{q_3})^2 + M_P^2} + U_{FSI} + |V_{eff}^P|$$

$$\epsilon^P = S^P + \langle E_x^P \rangle + \frac{\langle k^2 \rangle}{2M^*_{A-1}} \qquad U_{FSI} = U_{FSI}((q_3 + k)^2)$$

Removal energy has 3 components

Separation Energies S (P,N)

are tabulated in nuclear mass tables

This is the energy to Separate a nucleon from Nucleus A,

For the case where the nucleus A-1 is Left in the ground state.

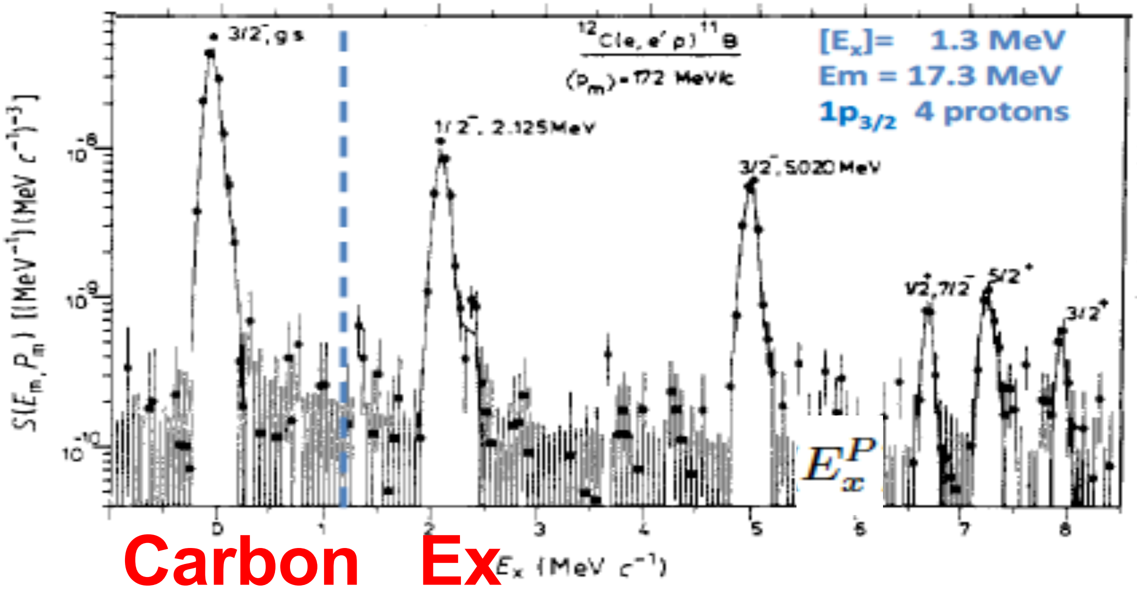
It is 16 MeV for Carbon.

$^A_Z Nucl$	remove proton Spectator	S^P	remove neutron Spectator	S^N	S^{N+P}
$^2_1$H	N	2.2	P	2.2	2.2
$^6_3$Li 1+	$^5_2$He $\frac{3}{2}$-	4.4	$^5_3$Li $\frac{3}{2}$-	5.7	4.0
$^{12}_6$C 0+	$^{11}_5$B $\frac{3}{2}$-	16.0	$^{11}_6$C $\frac{3}{2}$-	18.7	27.4
$^{16}_8$O 0+	$^{15}_7$N $\frac{1}{2}$-	12.1	$^{15}_8$O $\frac{1}{2}$-	15.7	23.0
$^{24}_{12}$Mg 0+	$^{23}_{11}Na$ $\frac{3}{2}$+	11.7	$^{23}_{12}$ Mg $\frac{3}{2}$+	16.5	24.1
$^{27}_{13}$Al$\frac{5}{2}$+	$^{26}_{12}Mg$ 0+	8.3	$^{23}_{12}$ Al 5+	13.1	19.4
$^{28}_{14}$Si 0+	$^{27}_{13}$Al$\frac{5}{2}$+	11.6	$^{27}_{14}$Si $\frac{5}{2}$+	17.2	24.7
$^{40}_{18}$Ar$\frac{3}{2}$+	$^{39}_{17}$CL $\frac{3}{2}$+	12.5	$^{39}_{18}$Ar $\frac{7}{2}$-	9.9	20.6
$^{40}_{20}$Ca 0+	$^{39}_{19}$K $\frac{3}{2}$ +	8.3	$^{39}_{20}$Ca $\frac{3}{2}$+	15.6	21.4
$^{51}_{23}$V $\frac{7}{2}$-	$^{50}_{22}$Ti 0+	8.1	$^{50}_{23}$V 6+	11.1	19.0
$^{56}_{26}$Fe 0+	$^{55}_{25}$Mn $\frac{5}{2}$ -	10.2	$^{55}_{26}$Fe $\frac{3}{2}$-	11.2	20.4
$^{58}_{28}$Ni $\frac{3}{2}$ -	$^{58}_{27}$Co 2+	8.2	$^{58}_{87}$Ni 0+	12.2	19.5
$^{89}_{39}$Y $\frac{1}{2}$ -	$^{88}_{38}$Sr $\frac{1}{2}$-	7.1	$^{88}_{39}$Y 4-	11.5	18.2
$^{90}_{40}$Zr 0+	$^{89}_{39}$Y $\frac{1}{2}$ -	8.4	$^{88}_{40}$Zr$\frac{9}{2}$+	12.0	17.8
$^{120}_{50}$Sn 0+	$^{119}_{49}$In $\frac{9}{2}$ +	10.1	$^{119}_{50}$Sn $\frac{1}{2}$+	8.5	17.3
$^{181}_{73}$Ta$\frac{7}{2}$ -	$^{180}_{72}$Hf 0+	5.9	$^{180}_{73}$Ta 1+	7.6	13.5
$^{197}_{79}$Au$\frac{3}{2}$+	$^{196}_{78}$Pt 0+	5.8	$^{196}_{79}$Au 2-	8.1	13.7
$^{208}_{82}$Pb 0+	$^{207}_{81}$TI $\frac{1}{2}$+	8.0	$^{207}_{82}$Pb $\frac{1}{2}$-	7.4	14.9

$$\nu + (M_P - \epsilon^P) = \sqrt{(\mathbf{k} + q_3)^2 + M_P^2} + U_{FSI} + |V_{eff}^P|$$

$$\epsilon^P = S^P + \langle E_x^P \rangle + \frac{\langle k^2 \rangle}{2M_{A-1}^*} \qquad U_{FSI} = U_{FSI}((q_3+k)^2)$$

<Ex> Mean excitation energy of (A-1)*

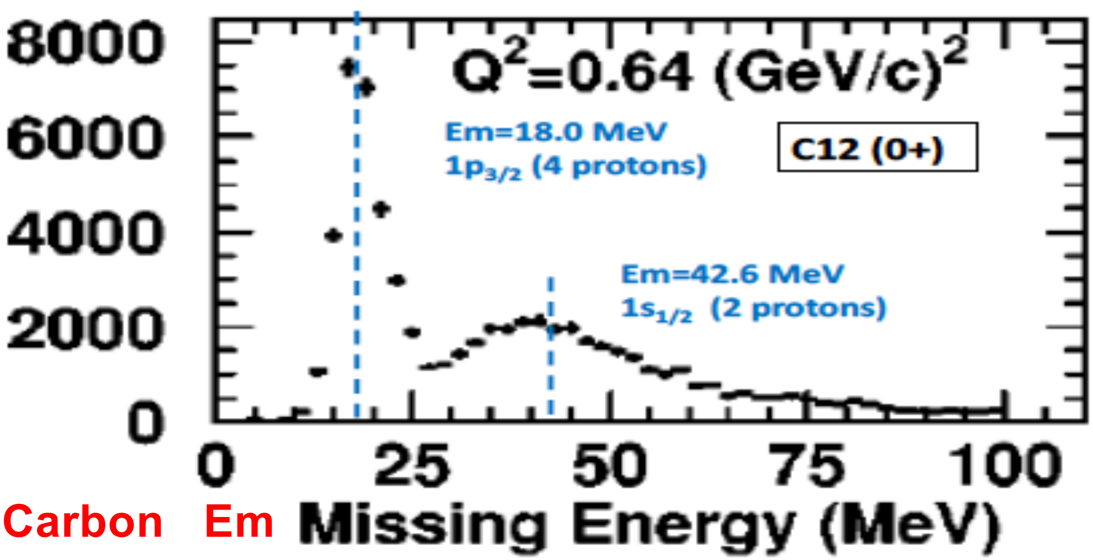


Carbon Ex

We obtain <Ex> from spectral functions measured in exclusive ee'P

ee'P high resolution

Here Ex is measured



Carbon Em

Carbon
Sp=16 MeV <Ex> = 10 MeV

ee'P high resolution 1p1h
here Em is measured

$$\langle E_m^P \rangle = S^P + \langle E_x^P \rangle.$$

$\langle E_m^P$ **2p2h Em>80 GeV)**

5.1 Direct measurements of $\langle E_m^P \rangle^{SF}$ and $\langle T \rangle^{SF}$

<Ex> continued

Method 1: excitation energy
From tests of Koltun sum rule

Spectral functions are used to test Koltun Sum rule

These two quantities are directly extracted from spectral function measurements in analyses that test the Koltun sum rule [12]. The Koltun's sum rule states that

$$\frac{E_0}{A} = \frac{1}{2} [\, \langle T \rangle^{SF} \frac{A-2}{A-1} - \langle E_m^P \rangle^{SF}], \qquad (34)$$

Carbon
Sp=16 MeV
<Ex> = 10 MeV

where E_0/A is the nuclear binding energy per particle obtained from nuclear masses and includes a (small) correction for the Coulomb energy,

Ave. < KE>

Exactly what we need For momentum distribution K_F

$$\langle T \rangle^{SF} = \int d^3k \; dE_m \; \frac{k^2}{2M} P_{SF}(k, E_m) \; , \qquad (35)$$

and

Ave. $\langle E_m \rangle$

Get: Average excitation $\langle Ex \rangle$ from $\langle E_m \rangle$

$$\langle E_m \rangle^{SF} = \int d^3k \; dE_m \; E_m \; P_{SF}(k, E_m) \; . \qquad (36)$$

$$\langle E_m^P \rangle = S^P + \langle E_x^P \rangle .$$

For precise tests of the Koltun sum rule a small contribution from three-nucleon processes should taken into account.

Source Source	$^{A}_{Z}Nucl$ Nucl.	k_F^P, k_F^N Moniz ±5 updated MeV/c	$\lvert V_{eff}\rvert$ Gueye ref.[15] MeV		ϵ^P (MeV)
	$^{2}_{1}H$	88,88			***4.7±1**
ee′p Tokyo[29–31]	$^{6}_{3}Li$	169,169	1.4	ϵ^{levels}	***18.4±3**
ee′p Tokyo[29–31]	$^{12}_{6}C$	221,221	3.1±0.25	ϵ^{levels}	24.0±3
ee′p NIKHEF[33]	$^{12}_{6}C$			ϵ^{levels}	27.1±3
ee′p Saclay[28]	$^{12}_{6}C$			ϵ^{levels} $\langle\epsilon\rangle^{SF}$	25.8±3 24.8±3.0
Shell Model binding E	$^{12}_{6}C$			$\epsilon^{levels}_{shell-model}$	24.9 ±5
ee′p Jlab Hall C [27]	$^{12}_{6}C$			$\langle\epsilon\rangle^{SF}$	***27.5±3**
ee′p Jlab Hall A [34]	$^{16}_{8}O$	225,225	3.4	ϵ^{levels}	***24.1±3**
Shell Model binding E	$^{16}_{8}O$			$\epsilon^{levels}_{shell-model}$	23.5±5
ee′p Tokyo[29–31]	$^{27}_{13}Al$	238,241	5.1	ϵ^{levels}	**30.6 ±3**
ee′p Saclay[28]	$^{28}_{14}Si$	239,241	5.5	ϵ^{levels} $\langle\epsilon\rangle^{SF}$	28.3±2 ***24.7±3**
$^{40}_{20}Ca \rightarrow ^{40}_{18}Ar$ Shell Model	$^{40}_{18}Ar$	251,263	6.3	ϵ^{levels}	***30.9±4**
ee′p Tokyo[29–31]	$^{40}_{20}Ca$	251,251	7.4±0.6	Δ^{levels}	26.3±3
ee′p Saclay[28]	$^{40}_{20}Ca$			ϵ^{levels} $\langle\epsilon\rangle^{SF}$	27.0±3 ***28.2±3**
Shell-model binding E	$^{40}_{20}Ca$			$\epsilon^{levels}_{shell-model}$	23.6±5
ee′p Tokyo[29–31]	$^{50}_{23}V$	253,266	8.1	ϵ^{levels}	***25.6±3**
ee′p Jlab hall C [27]	$^{56}_{26}Fe$	254,268	8.9±0.7	$\langle\epsilon\rangle^{SF}$	***29.6±3**
ee′p Saclay[28]	$^{58.7}_{28}Ni$	257,269	9.8	ϵ^{levels} $\langle\epsilon\rangle^{SF}$	25.7±3 ***25.4±3**
Shell-model binding E	$^{88}_{40}Zr$		11.9±0.9	$\epsilon^{levels}_{shell-model}$	**25.1±5**
ee′p Jlab Hall C [27]	$^{197}_{79}Au$	275,311	18.5	$\langle\epsilon\rangle^{SF}$	***25.4±3**

$$\nu + (M_P - \epsilon^P) = \sqrt{(\mathbf{k} + \boldsymbol{q_3})^2 + M_P^2} - |U_{FSI}| + |V_{eff}^P|$$

$$\epsilon^P = S^P + \langle E_x^P \rangle + \frac{\langle \boldsymbol{k}^2 \rangle}{2M^*_{A-1}}$$

Kinetic energy of spectator recoil nucleus is small

${}^A_Z N$	$\langle T^{P,N} \rangle$ *average*	$T^{P,N}_{A-1}$ *average*	$\langle \epsilon^{P,N} \rangle$ Removal energy		ΔS N-P	$\langle E_x^{P,N} \rangle$ BODEK-RITCHIE	$\langle E_m^{P,N} \rangle$ *average*	ΔE_m N-P	
			$E_m + T^{P,N}_{A-1}$			$E_m^{P,N}$-$S^{P,N}$			
	nucleon $\langle KE \rangle$ T^P,T^N	A-1 nucleus $\langle KE \rangle$ P, N	use for $E_\nu^{QE-\mu}$ $Q^2_{QE-\mu}$ Q^2_{QE-P} $\langle \epsilon^P \rangle$,$\langle \epsilon^N \rangle$	S^P, S^N	$diff$	**GENIE** excitation energy $\langle E_x^P \rangle$,$\langle E_x^N \rangle$	E_m^P, E_m^N	diff E_m^N-E_m^P	**Veff**
$({}^2_1H)$	2.5, 2.5	2.5, 2.5	4.7, 4.7	2.2, 2.2	0.0	0.0, 0.0	2.2, 2.2	0.0	-
${}^6_3Li$	9.1, 9.1	1.8, 1.8	18.4, 19.7	4.4, 5.7	1.3	12.2, 12.2	16.6, 17.9	(1.3)	1.4
${}^{12}_6C$	15.5, 15.5	1.4, 1.4	27.5, 30.1	16.0, 18.7	2.7	10.1, 10.0	26.1, 28.7	2.6	3.1
${}^{16}_8O$	16.0, 16.0	1.1, 1.1	24.1, 27.0	12.1, 15.7	3.6	10.9, 10.2	23.0, 25.9	2.9	3.4
${}^{27}_{13}Al$	17.9, 18.4	0.7, 0.7	30.6, 35.4	8.3, 13.1	4.8	21.6, 21.6	29.9, 34.7	(4.8)	5.1
${}^{28}_{14}Si$	18.1, 18.4	0.7, 0.7	24.7, 30.3	11.6, 17.2	5.6	12.4, 12.4	24.0, 29.6	(5.6)	5.5
${}^{40}_{18}Ar$	19.9, 21.9	0.5, 0.6	30.9, 32.3	12.5, 9.9	-2.6	17.8, 21.8	30.2, 31.7	1.4	6.3
${}^{40}_{20}Ca$	19.9,19.9	0.5, 0.5	28.2, 35.9	8.3, 15.6	7.3	19.4, 19.8	27.7, 35.4	7.7	7.4
${}^{50}_{23}V$	20.2, 22.4	0.4, 0.5	25.6, 28.6	8.1, 11.1	3.0	17.0, 17.0	25.1, 28.1	(3.0)	8.1
${}^{56}_{26}Fe$	20.4, 22.6	0.4, 0.4	29.6, 30.6	10.2, 11.2	1.0	19.0, 19.0	29.2, 30.2	(1.0)	8.9
${}^{58.7}_{28}Ni$	20.9, 22.8	0.4, 0.4	25.4, 29.4	8.2, 12.2	4.0	16.8, 16.8	25.0, 29.0	(4.0)	9.8
${}^{88}_{40}Zr$				8.4, 12.0	3.6			1.9	11.9
${}^{197}_{79}Au$	23.9, 30.4	0.1, 0.1	25.4, 27.7	5.8, 8.1	2.3	19.5, 19.5	25.3, 27.6	(2.3)	18,5

Coulomb corrections

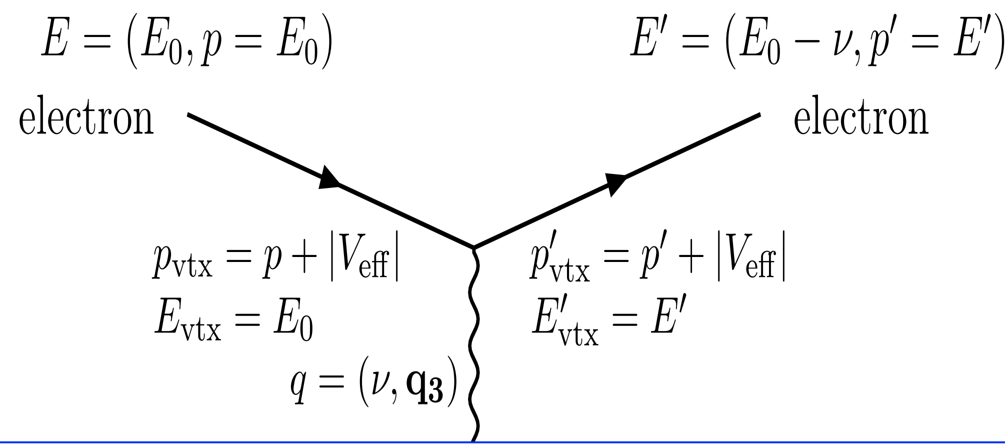
Needed for both initial state electrons
And final state protons

$$V(r) = \frac{3\alpha(Z)}{2R} + \frac{r\alpha(Z)}{2R^2}$$

$$R = 1.1A^{1/3} + 0.775A^{-1/3}$$

$$V_{eff} = -0.8V(r=0) = -0.8\frac{3\alpha(Z)}{2R}.$$

Electron scattering on proton



$$\mathbf{q_3^2} = Q^2 + \nu^2$$

$$Q^2 = 4(E_0 + |V_{eff}|)(E_0 - \nu + |V_{eff}|)\sin^2\frac{\theta}{2}$$

$$\nu + (M_p - \epsilon^P) = \sqrt{(\mathbf{k} + \mathbf{q_3})^2 + M_p^2} + U_{FSI} + V_{eff}^P$$

$$\epsilon^P = S^P + \langle E_x \rangle + \frac{k^2}{2M_{A-1}^*}$$

$$U_{FSI} = U_{FSI}((q_3 + k)^2)$$

$$\nu + (M_P - \epsilon^P) = \sqrt{(\mathbf{k} + \mathbf{q_3})^2 + M_P^2} + U_{FSI} + |V_{eff}^P|$$

$$\epsilon^P = S^P + \langle E_x^P \rangle + \frac{\langle k^2 \rangle}{2M_{A-1}^*}$$

Veff (Coulomb) From comparison of inclusive e+ A and e- A

PHYSICAL REVIEW C, VOLUME 60, 044308

Coulomb distortion measurements by comparing electron and positron quasielastic scattering off ^{12}C and ^{208}Pb

P. Guèye,* M. Bernheim, J. F. Danel, J. E. Ducret, L. Lakéhal-Ayat, J. M. Le Goff, A. Magnon, C. Marchand, J. Morgenstern, J. Marroncle, P. Vernin, and A. Zghiche-Lakéhal-Ayat†
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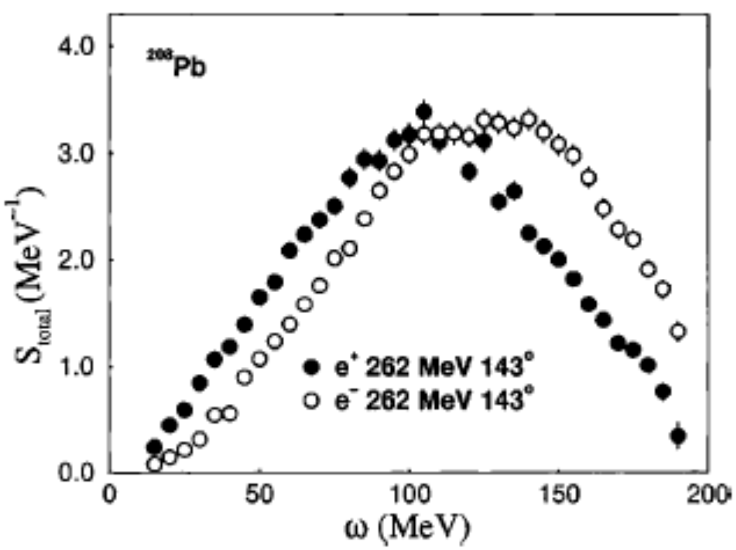


FIG. 6. Positron and electron response functions for the kinematics ^{208}Pb 262 MeV-143°.

TABLE II. Coulomb potential energies of several nuclei evaluated using the experimental charge densities of Ref. [26]. Both the Coulomb potential at the origin $|V_C(0)|$ and its averaged value $|V_C|$ from Eq. (10) are shown. The values of the fit of Eq. (9) are also shown together with the experimental charge mean-square radii.

Nucleus	$\langle r^2 \rangle_{exp}^{1/2}$ (fm)	$\|V_C(0)\|$ (MeV)	$\|V_C\|$ (MeV)	$\|V_C\|_{fit}$ (MeV)
^{12}C	2.464	4.6	3.3	3.1±0.25
^{40}Ca	3.450	10.5	7.9	7.4±0.6
^{48}Ca	3.451	10.4	7.9	7.4±0.6
^{56}Fe	3.714	12.5	9.5	8.9±0.7
^{90}Zr	4.258	16.7	12.8	11.9±0.9
^{154}Gd	5.124	21.8	16.9	15.9±1.2
^{208}Pb	5.503	25.9	20.1	18.9±1.5

$$\nu + (M_p - \epsilon^P) = \sqrt{(\mathbf{k} + \mathbf{q_3})^2 + M_p^2} + U_{FSI} + V_{eff}^P$$

$$\epsilon^P = S^P + \langle E_x \rangle + \frac{k^2}{2M^*_{A-1}}$$

$S^{P,N}$ is tabulated in nuclear mass tables

Veff (Coulomb) From comparison of inclusive e+ A and e- A

	S^P, S^N	$\langle E_x^P \rangle$, $\langle E_x^N \rangle$	$\lvert V_{eff} \rvert$
(^2_1H)	2.2, 2.2	0.0, 0.0	-
^6_3Li	4.4, 5.7	12.2, 12.2	1.4
$^{12}_6C$	16.0, 18.7	10.1, 10.0	3.1
$^{16}_8O$	12.1, 15.7	10.9, 10.2	3.4
$^{27}_{13}Al$	8.3, 13.1	21.6, 21.6	5.1
$^{28}_{14}Si$	11.6, 17.2	12.4, 12.4	5.5
$^{40}_{18}Ar$	12.5, 9.9	17.8, 22.1	6.3
$^{40}_{20}Ca$	8.3, 15.6	19.4, 19.8	7.4
$^{50}_{23}V$	8.1, 11.1	17.0, 17.0	8.1
$^{56}_{26}Fe$	10.2, 11.2	19.0, 19.0	8.9
$^{58.7}_{28}Ni$	8.2, 12.2	16.8, 16.8	9.8
$^{88}_{40}Zr$	8.4, 12.0		11.9
$^{197}_{79}Au$	5.8, 8.1	19.5, 19.5	18,5
$^{208}_{82}Pb$	8.0, 7.4	Assume same as Au	18.9

All in MeV

<Ex>
from exclusive e-e'P spectral functions
(previous slide)

Ufsi
From Inclusive e-A (next slide)

Electron scattering on pro (QE, Resonance production, W (inelastic)

$$\nu^{QE} + (M_P - \varepsilon) = \sqrt{(\vec{k} + \vec{q}_3)^2 + M_P^2} + U_{FSI}^{QE} + |V_{eff}^P|$$

$$\nu^{\Delta} + (M_P - \varepsilon) = \sqrt{(\vec{k} + \vec{q}_3)^2 + M_\Delta^2} + U_{FSI}^{\Delta} + |V_{eff}^P|$$

$$\nu^{W} + (M_P - \varepsilon) = \sqrt{(\vec{k} + \vec{q}_3)^2 + M_W^2} + U_{FSI}^{W} + |V_{eff}^P|$$

$$\vec{q}_3^{\,2} = Q^2 + \nu^2 = 4(E_0 + |V_{eff})(E_0 - \nu + |V_{eff}|) \sin^2\frac{\theta}{2} + \nu^2$$

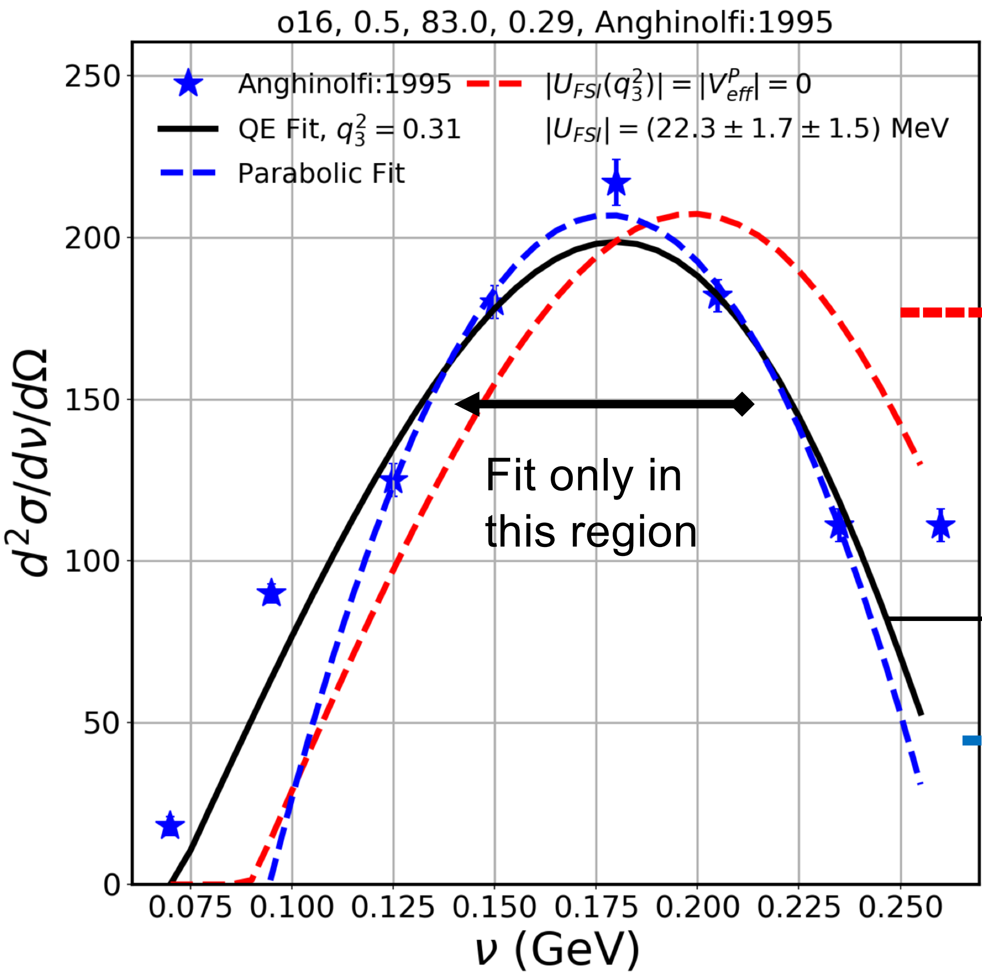
$$\nu^{QE} + (M_P - \varepsilon) = \sqrt{\vec{k}^2(k_z)\rangle + 2k_z\vec{q}_3 + \vec{q}_3^{\,2} + M_P^2} + U_{FSI}^{QE} + |V_{eff}^P$$

$$\nu + (M_p - \epsilon^P) = \sqrt{(\mathbf{k}+\mathbf{q_3})^2 + M_p^2} + U_{FSI} + V_{eff}^P$$

$$\epsilon^P = S^P + \langle E_x \rangle + \frac{k^2}{2M_{A-1}^*}$$

Fit to Inclusive e-A for Ufsi using RFG model

Spectra: 4 Li6, 35 C12 + 8 O16, 8 Al27, 29 Ca40+2 Ar40, 30 Fe56, 22 Pb208+1 Gold



Dashed red line: RFG no Ufsi

Solid Black line: RFG with Ufsi

Dashed blue line: Simple parabola

Update Trento: change Pb208 excitation energy to be the same as Au197.

Models of Ufsi extracted from proton scattering data on nuclei

E. D. Cooper, S. Hama, and B. C. Clark
Phys. Rev. C **80**, 034605 – Published 8 September 2009

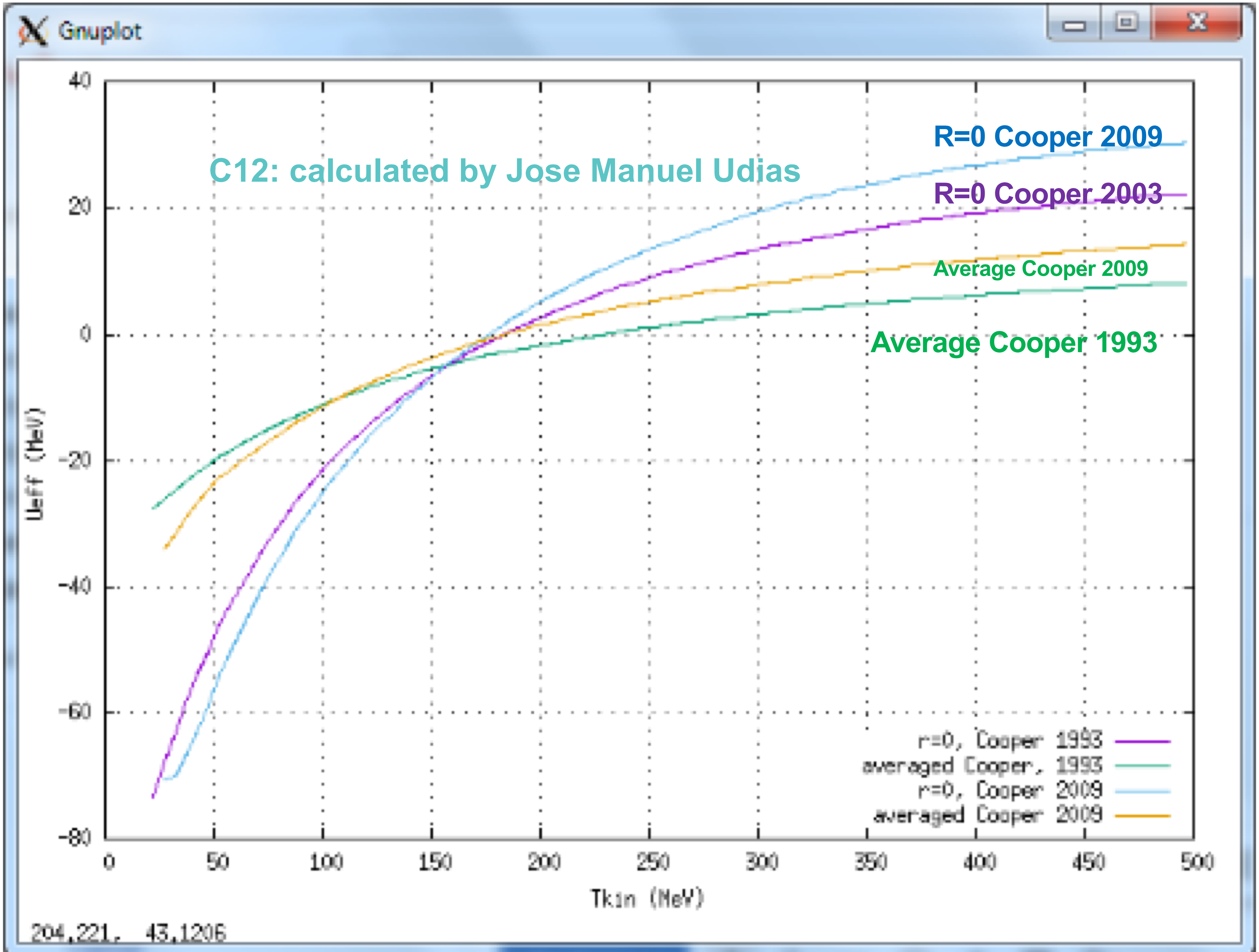
Artur M. Ankowski,[1,*] Omar Benhar,[2] and Makoto Sakuda[1]
PHYSICAL REVIEW D 91, 033005 (2015)

E. D. Cooper, S. Hama, B. C. Clark, and R. L. Mercer, Phys. Rev. C 47, 297 (1993).

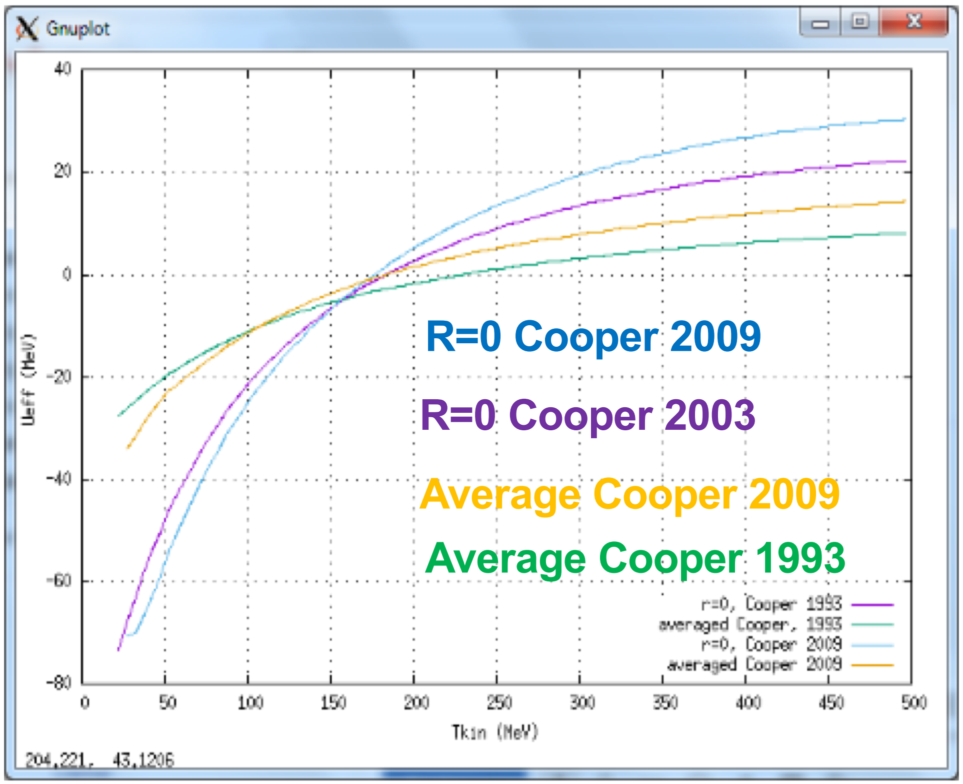
- - - $r=0$ U_{opt}
—— Average U_{opt}
calculated by Jose Manuel Udias using Cooper 2009

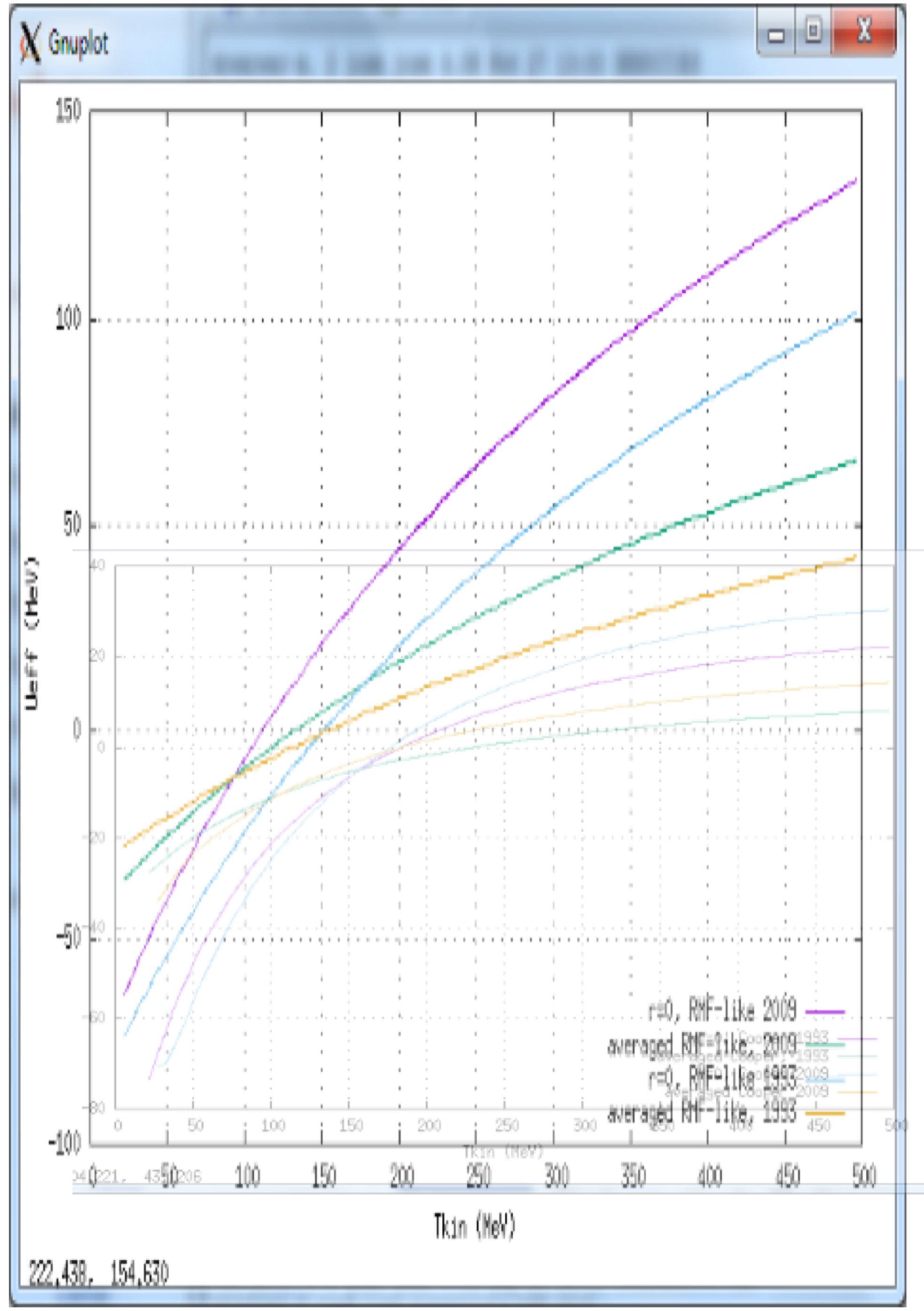
—— Average U_{opt}
calculated by Artur Ankowski using Cooper et al. 2009

—— Average U_{opt}
calculated by Artur Ankowski Using Cooper 1993


Gnuplot
C12: calculated by Jose Manuel Udias
R=0 Cooper 2009
R=0 Cooper 2003
Average Cooper 2009
Average Cooper 1993
r=0, Cooper 1993
averaged Cooper, 1993
r=0, Cooper 2009
averaged Cooper 2009
Ueff (MeV)
Tkin (MeV)
40
20
0
-20
-40
-60
-80
0
50
100
150
200
250
300
350
400
450
500
204.221, 43.1206

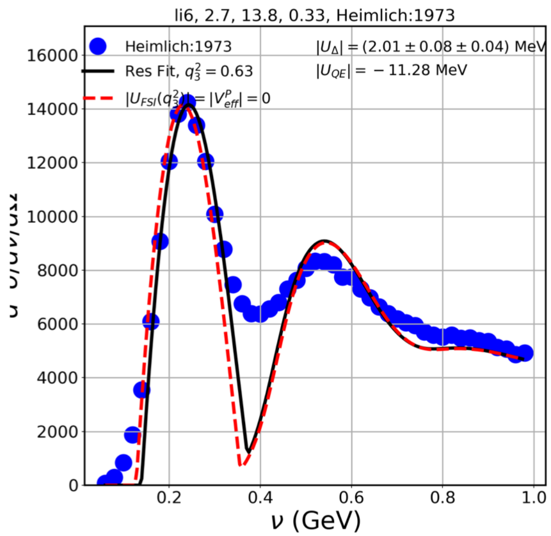
C12: calculated by Jose Manuel Udias

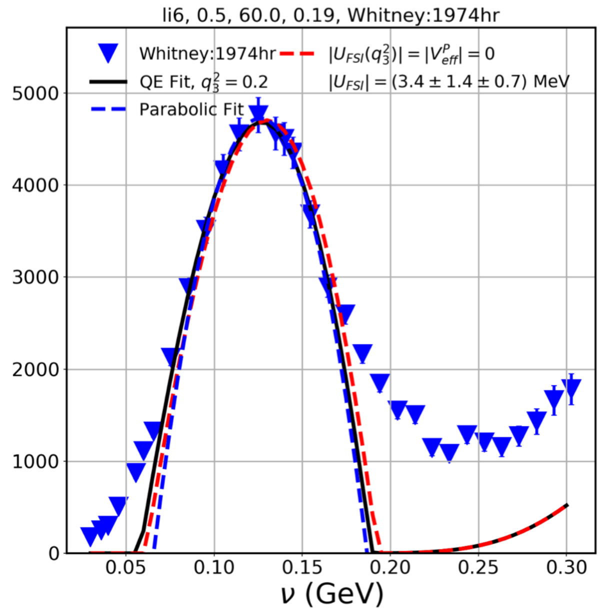




May 2019 Trento

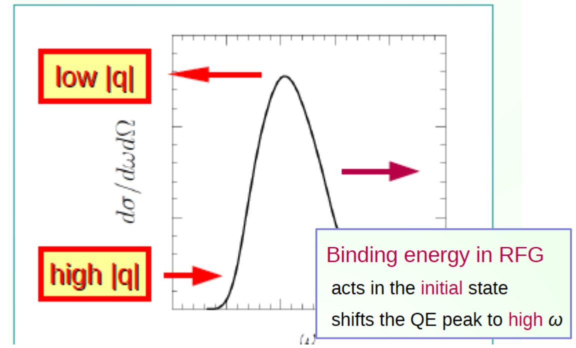
4 Li6 spectra

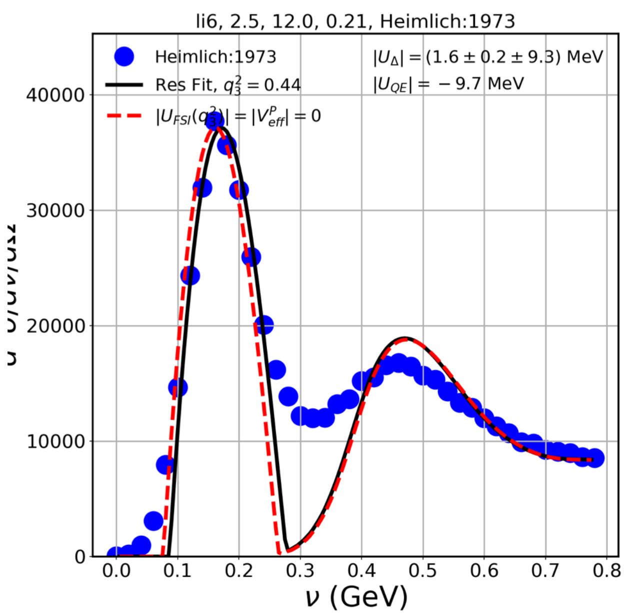


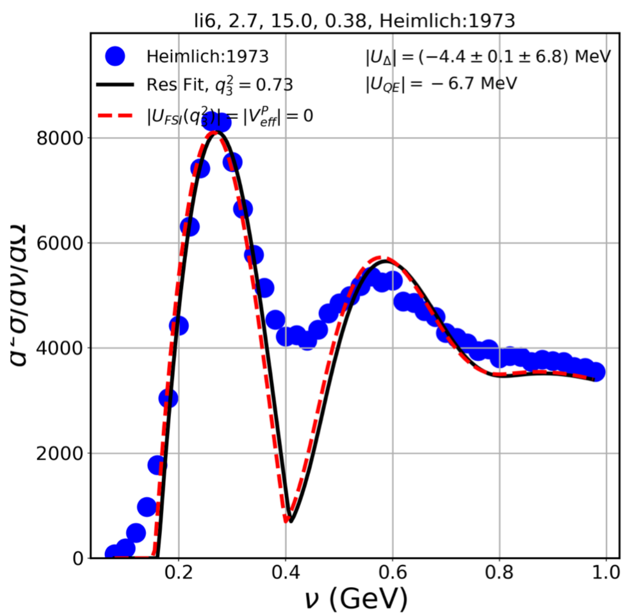


Real part of the OP

- acts in the final state
- shifts the QE peak to low ω at low $|\mathbf{q}|$ (to high ω at high $|\mathbf{q}|$)







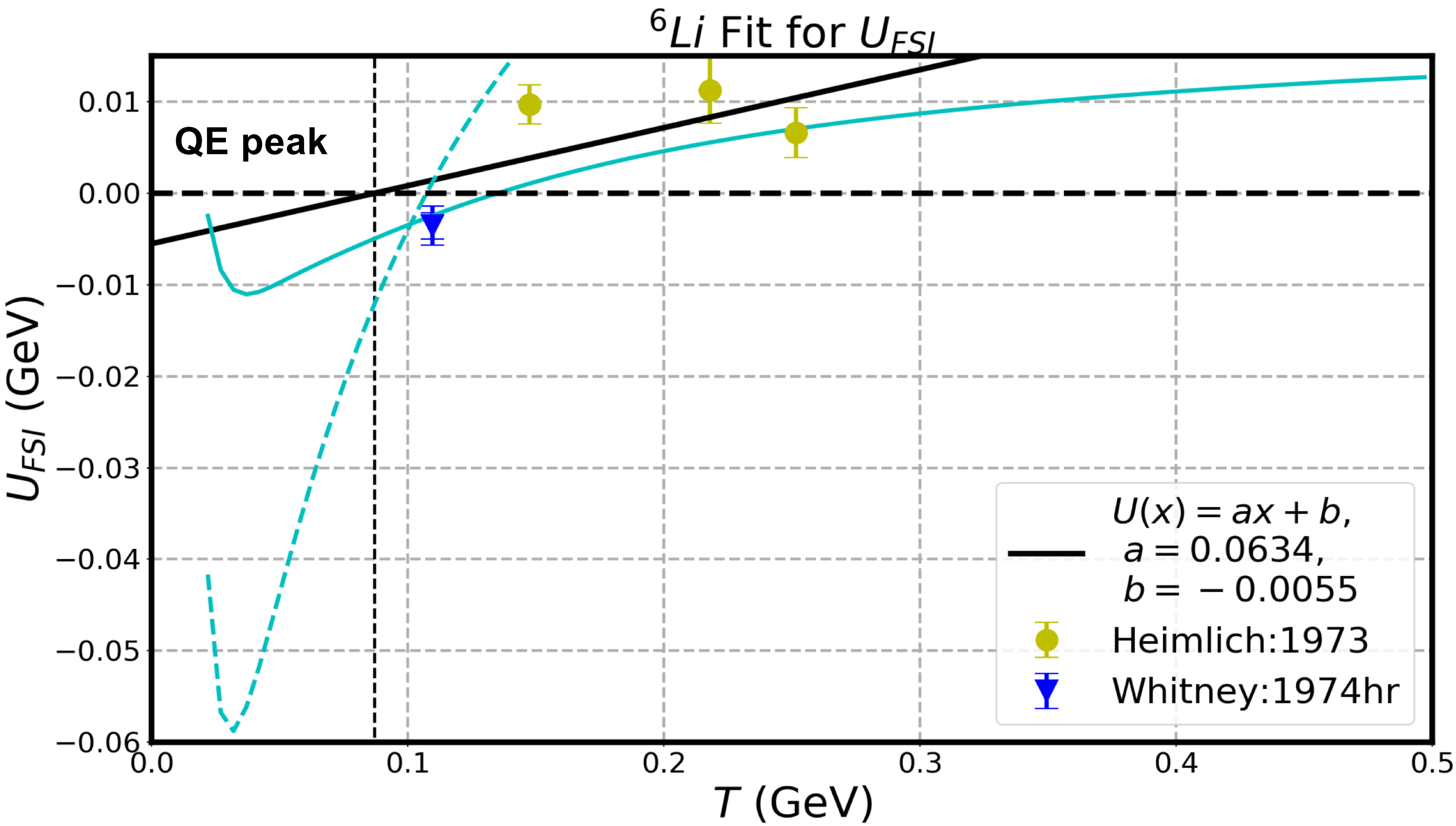
We have not included 2p2h.

Therefore, we only fit the data in the top 1/3 of the QE peak.

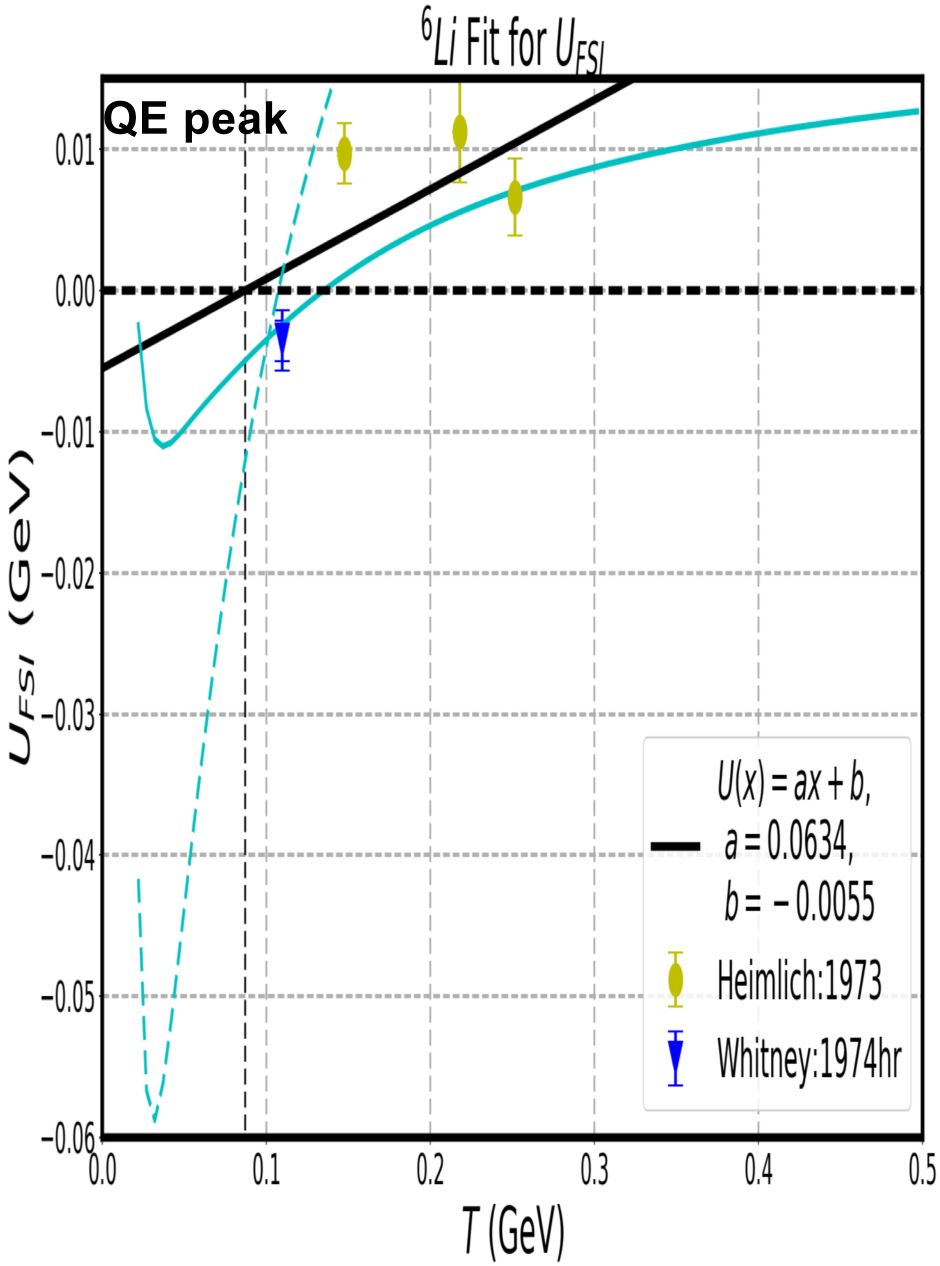
4 Li6 spectra - Trento

Data in agreement with Cooper 2009 within 5 MeV

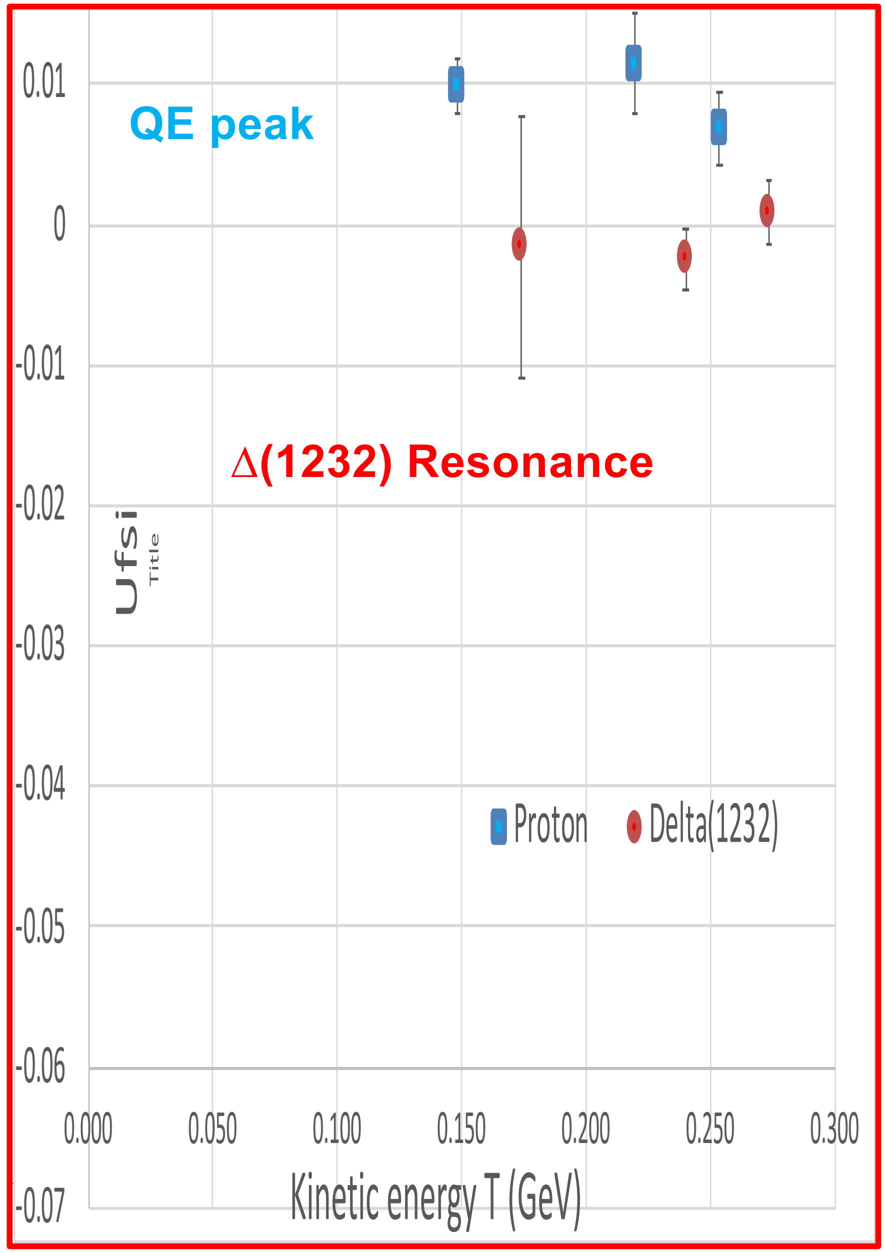
$r=0$ U_{opt}
Average U_{opt}
calculated by Jose Manuel Udias using model of Cooper et al. PRC 80, 034605 (2009)

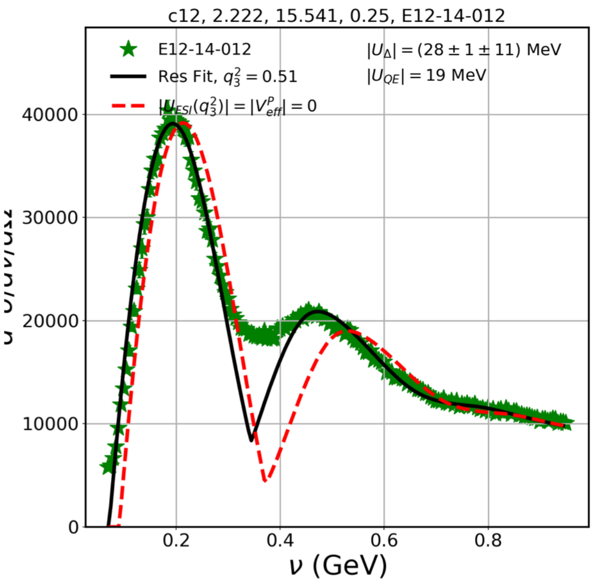


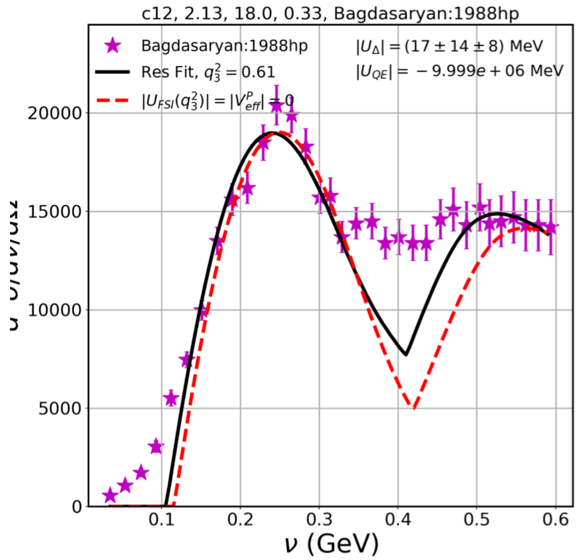
Li 6 Ufsi for QE peak positive

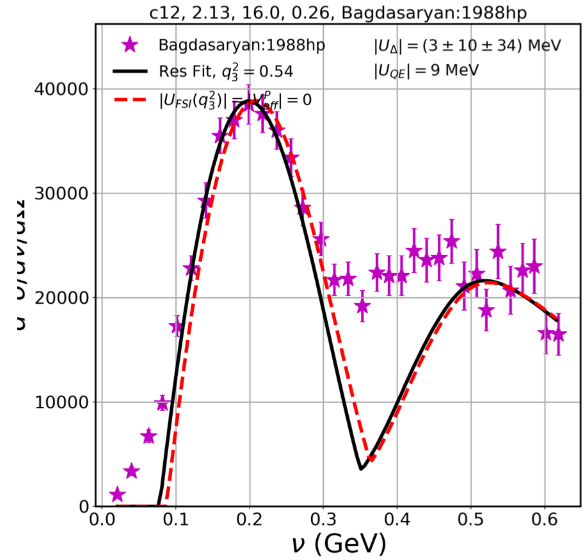


Li 6 Ufsi for Δ(1232) Resonance zero.
Smaller by 5 to 10 MeV than for QE

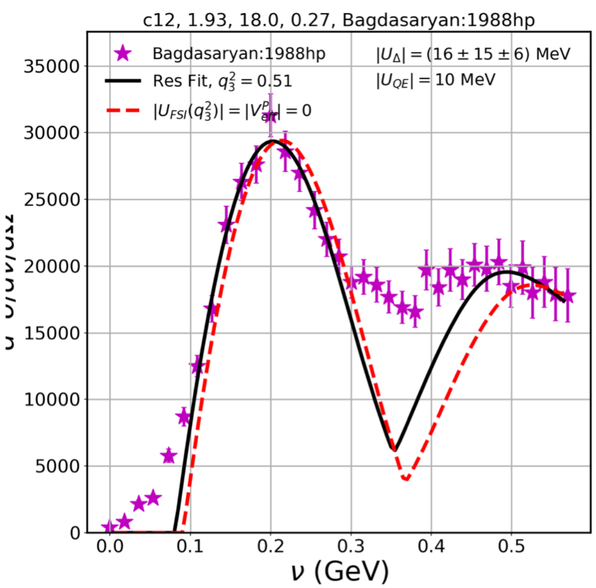


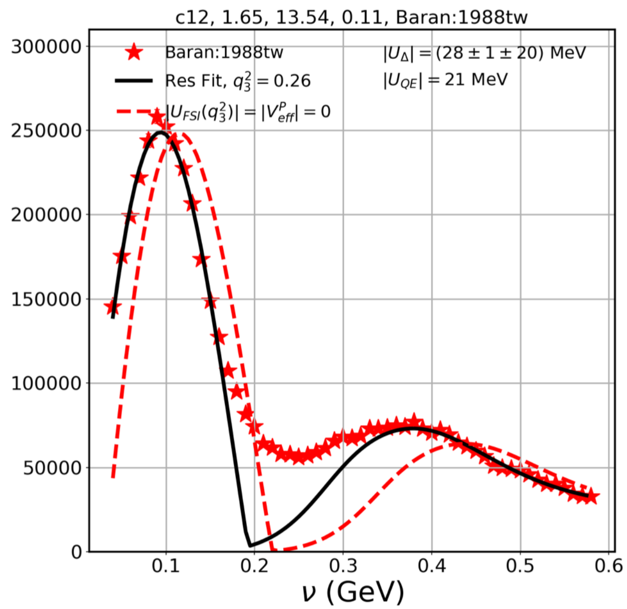


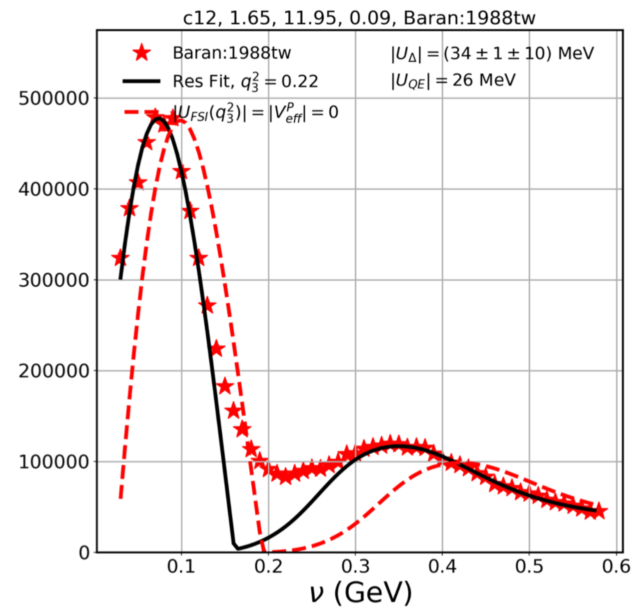


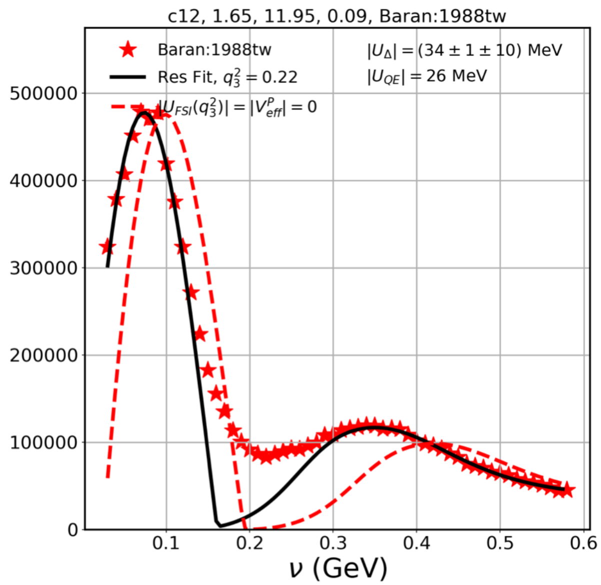


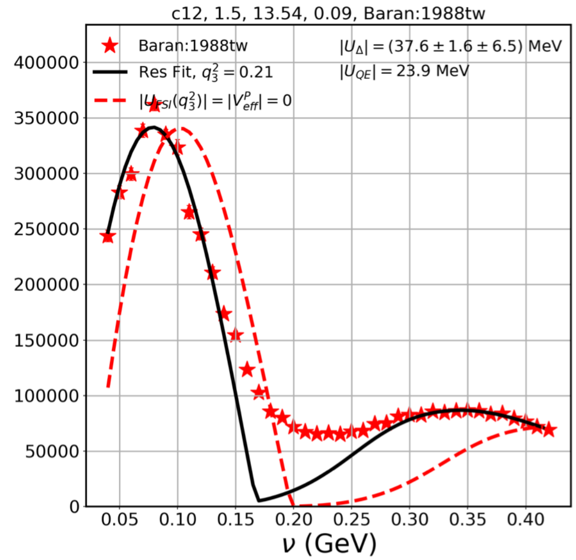
Carbon 12 page 1

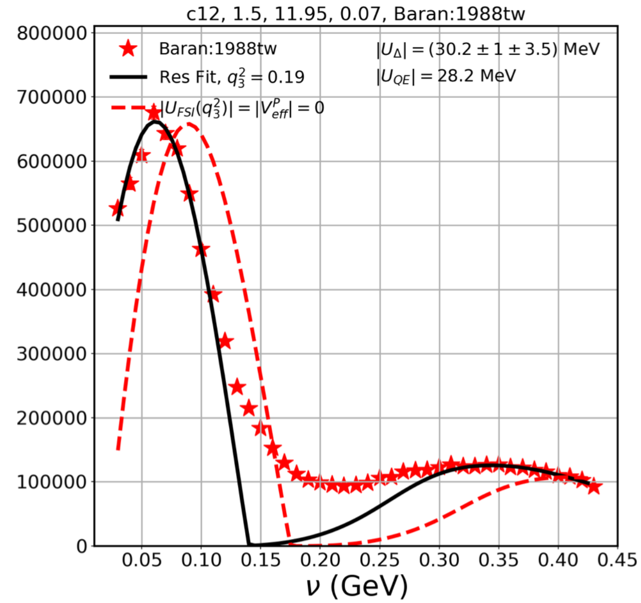




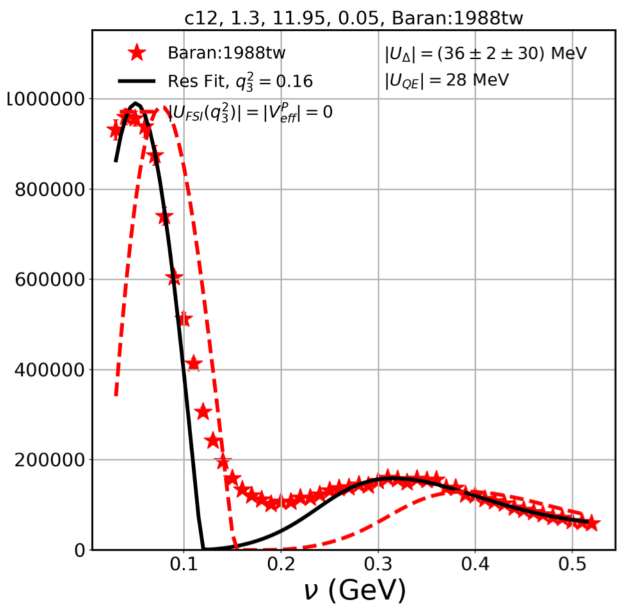


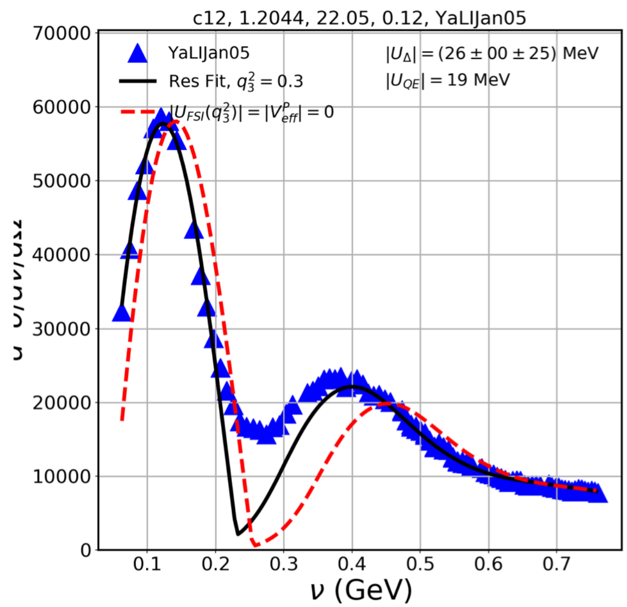


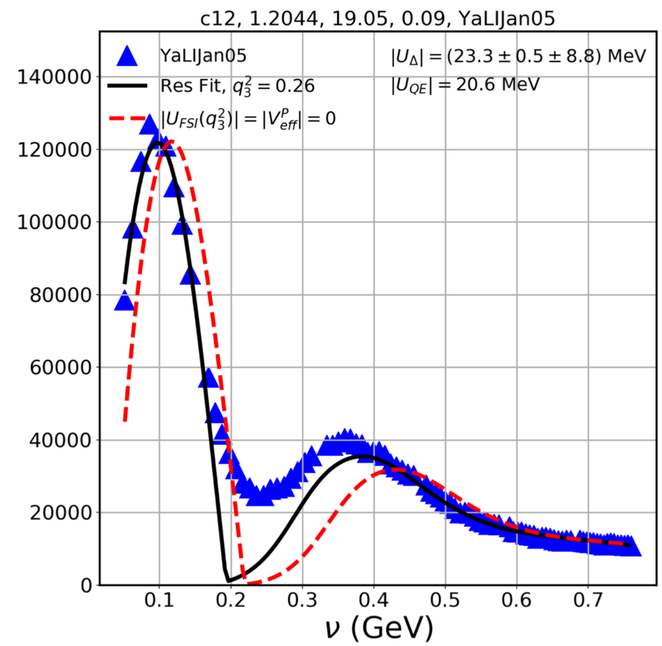


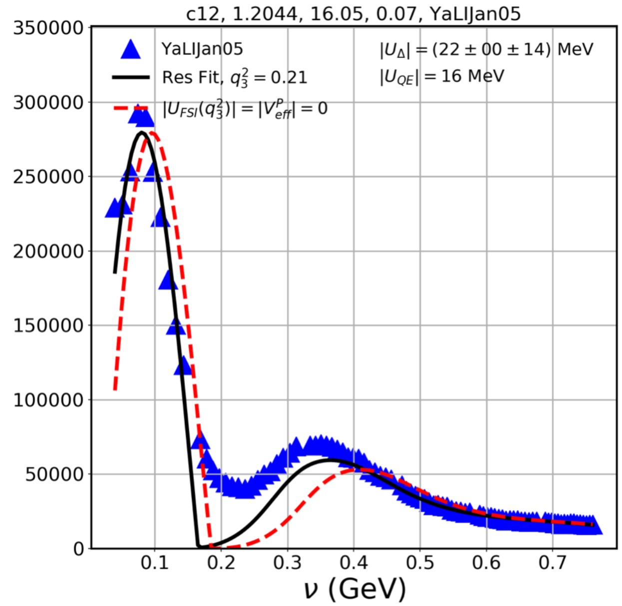


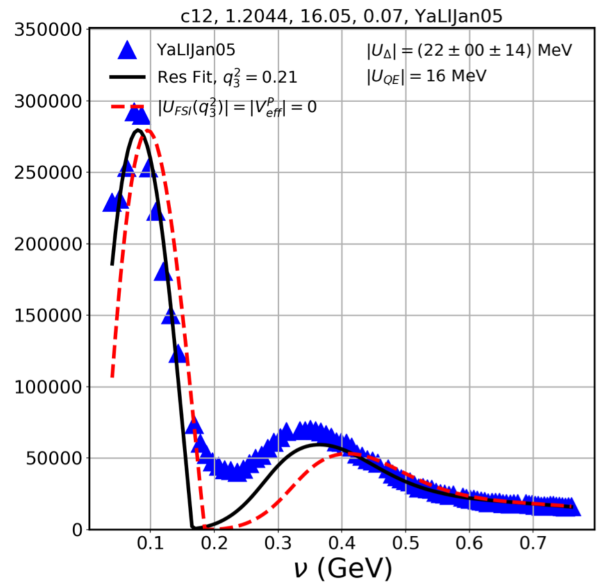
Carbon 12 page 2

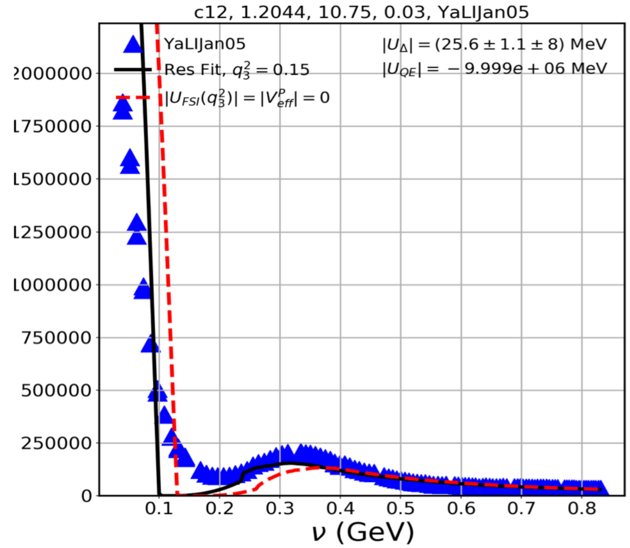




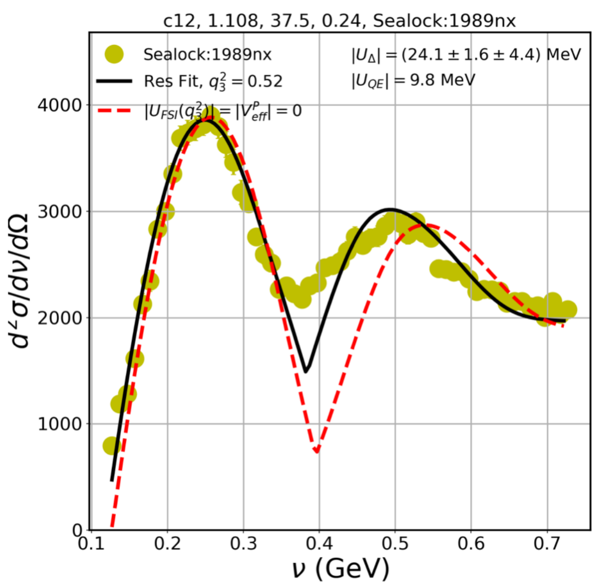


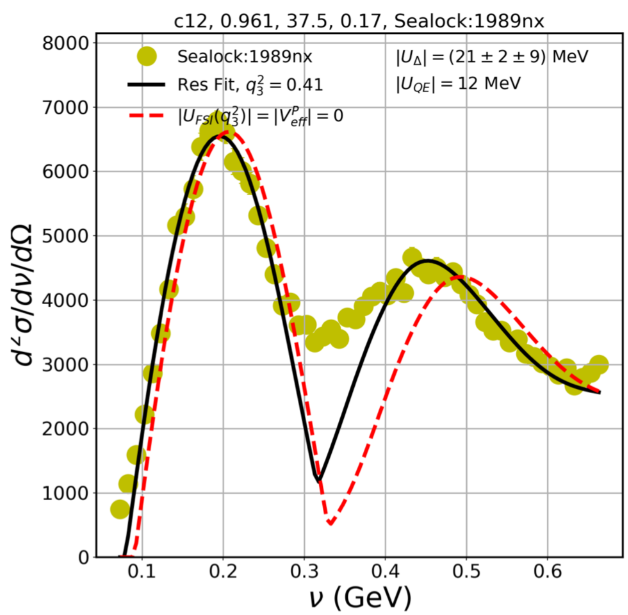


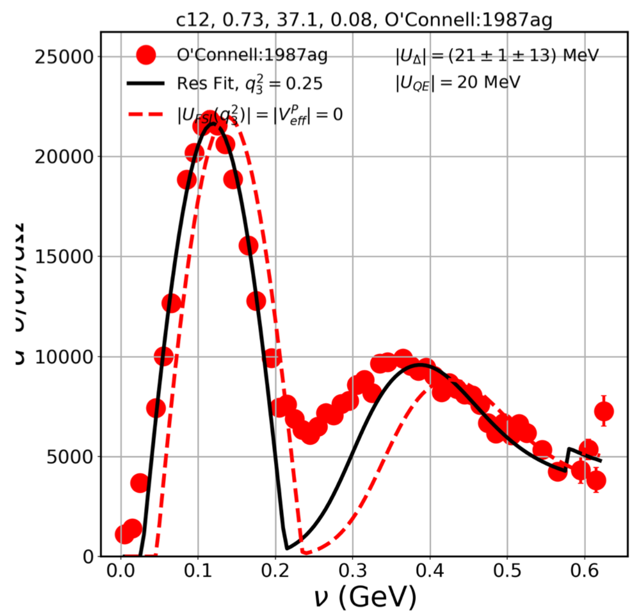


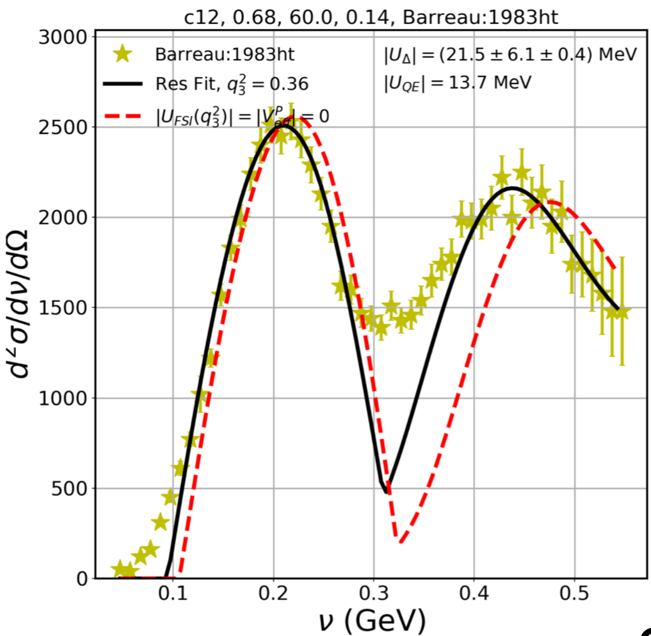


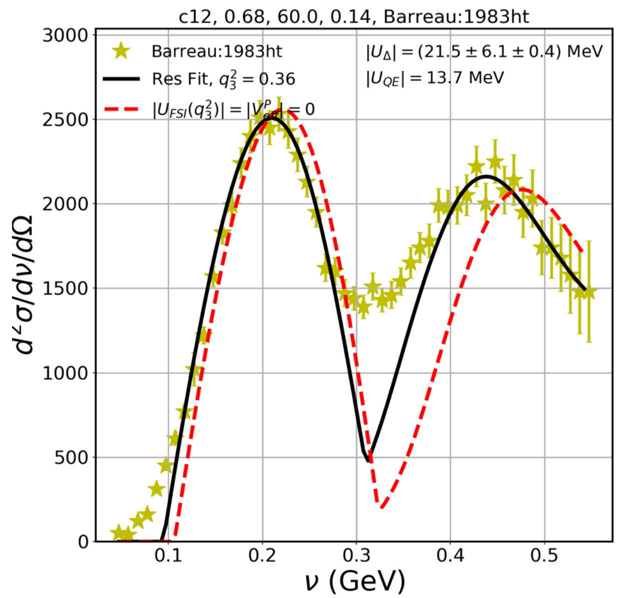
Carbon 12 page 3

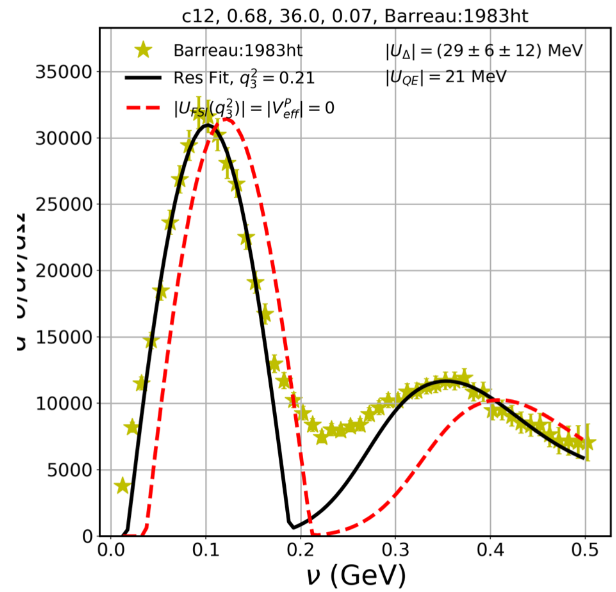




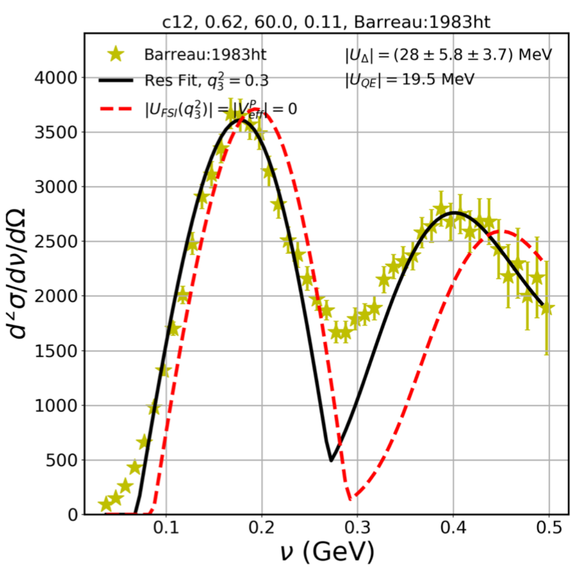


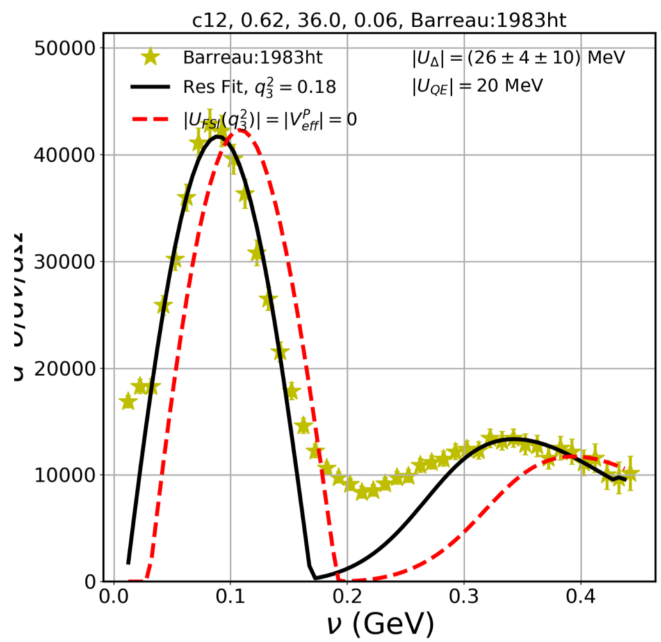


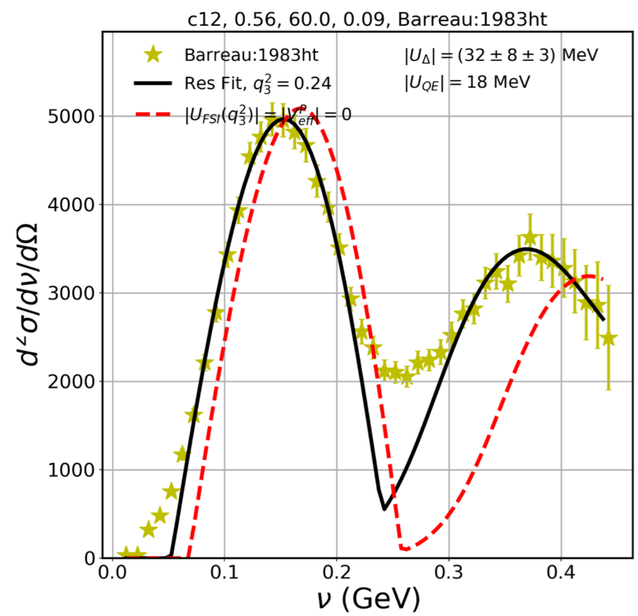


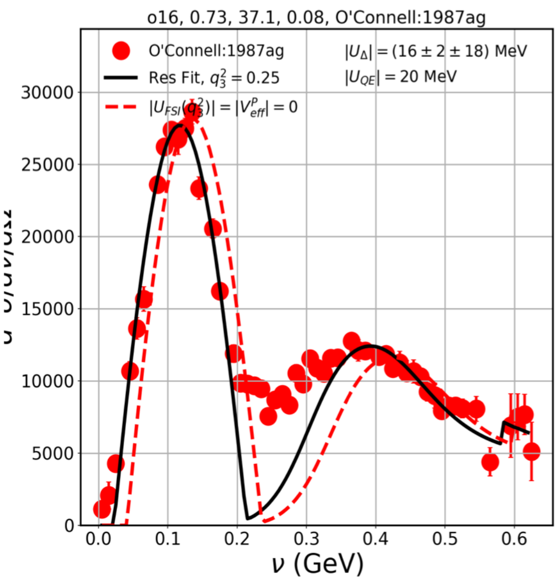


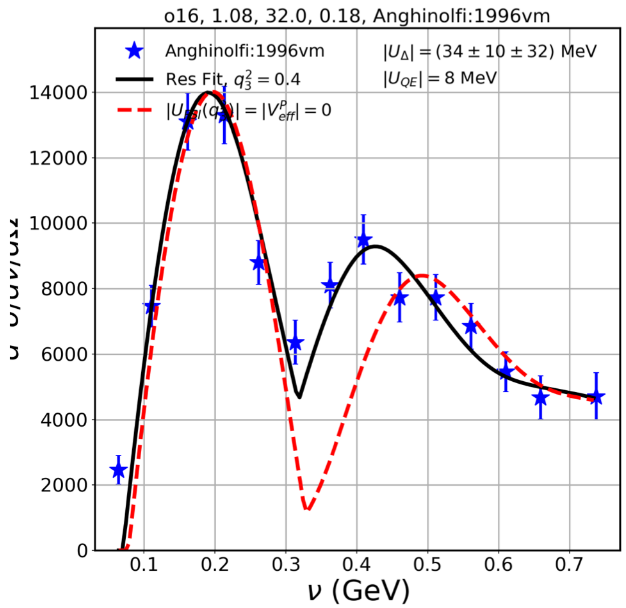
Carbon 12 page 4

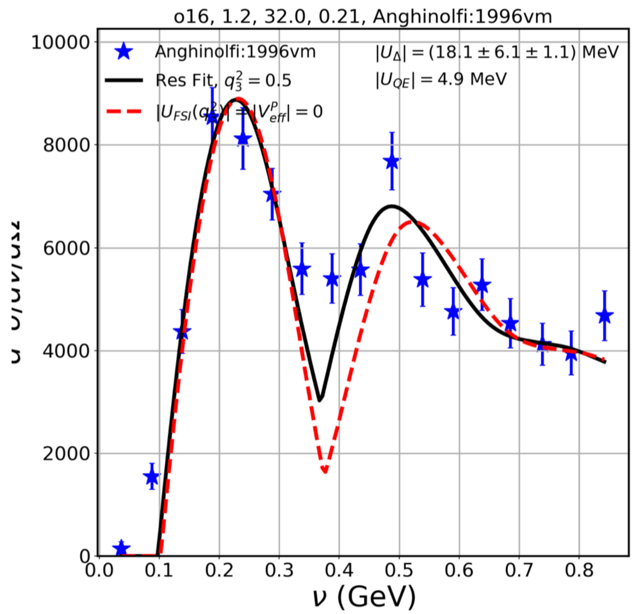










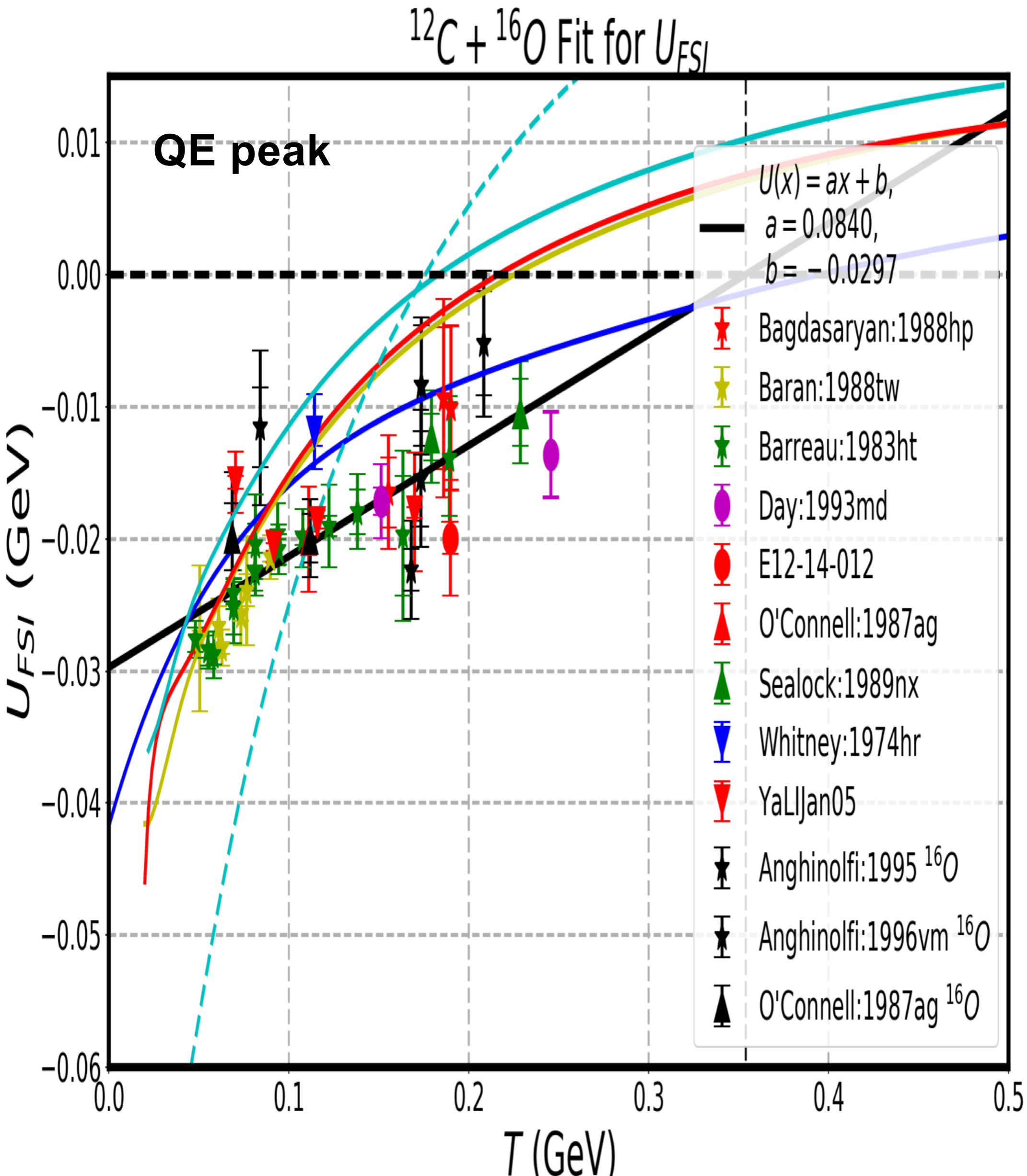


Oxygen 12 page 5

These are 35 Carbon (C12) spectra (12 include Delta_

There are 8 Oxygen spectra (3 include Delta)

35 C12 and 8 O16 spectra Trento



r =0 U_{opt}

Average U_{opt}

Cooper 2009 calculated by Jose Manuel Udias

PRC 80, 034605 (2009) Cooper et al

Average U_{opt}

Cooper 2009 calculated by Artur Ankowski

Average U_{opt}

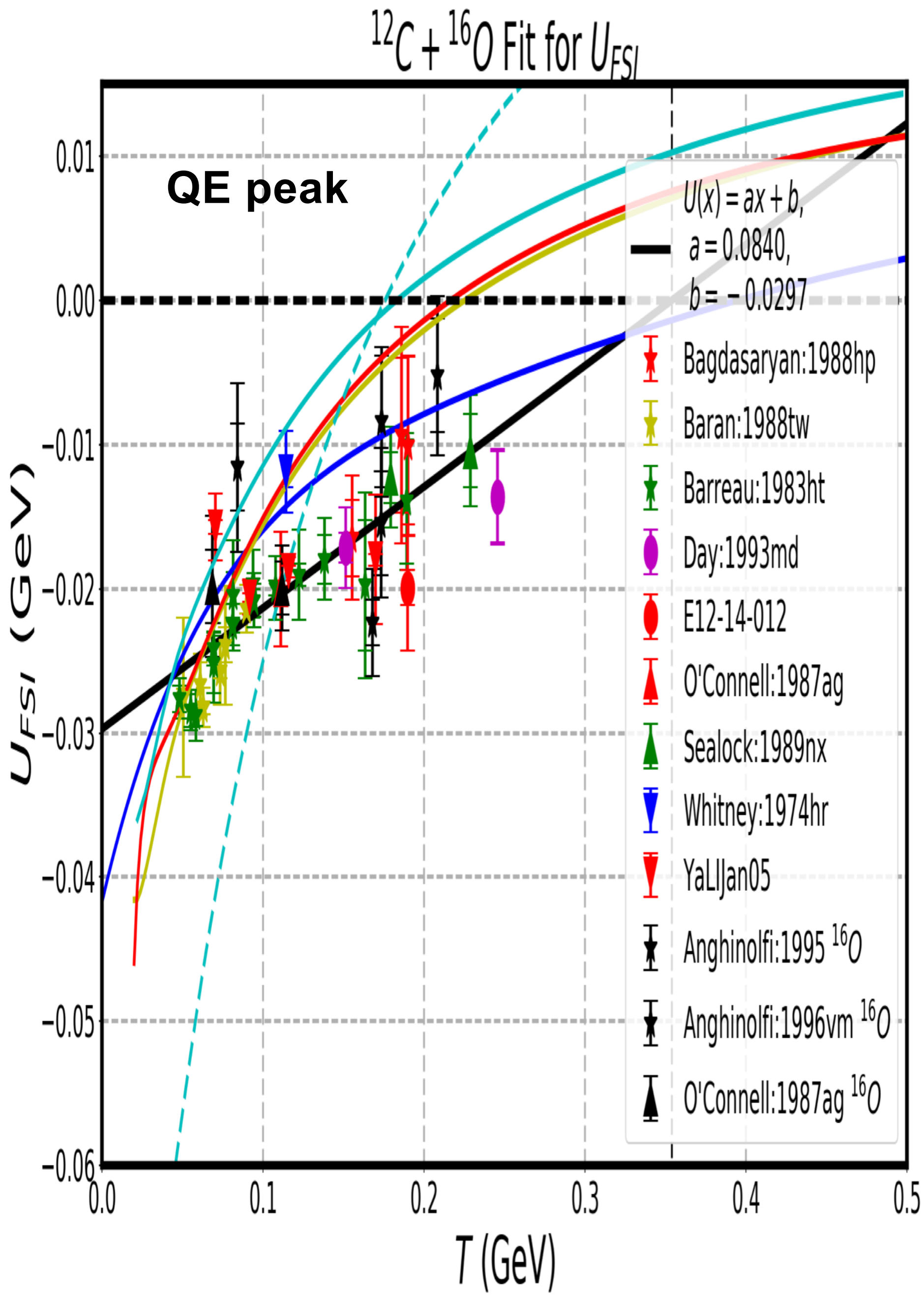
Cooper 1993 calculated by Artur Ankowski

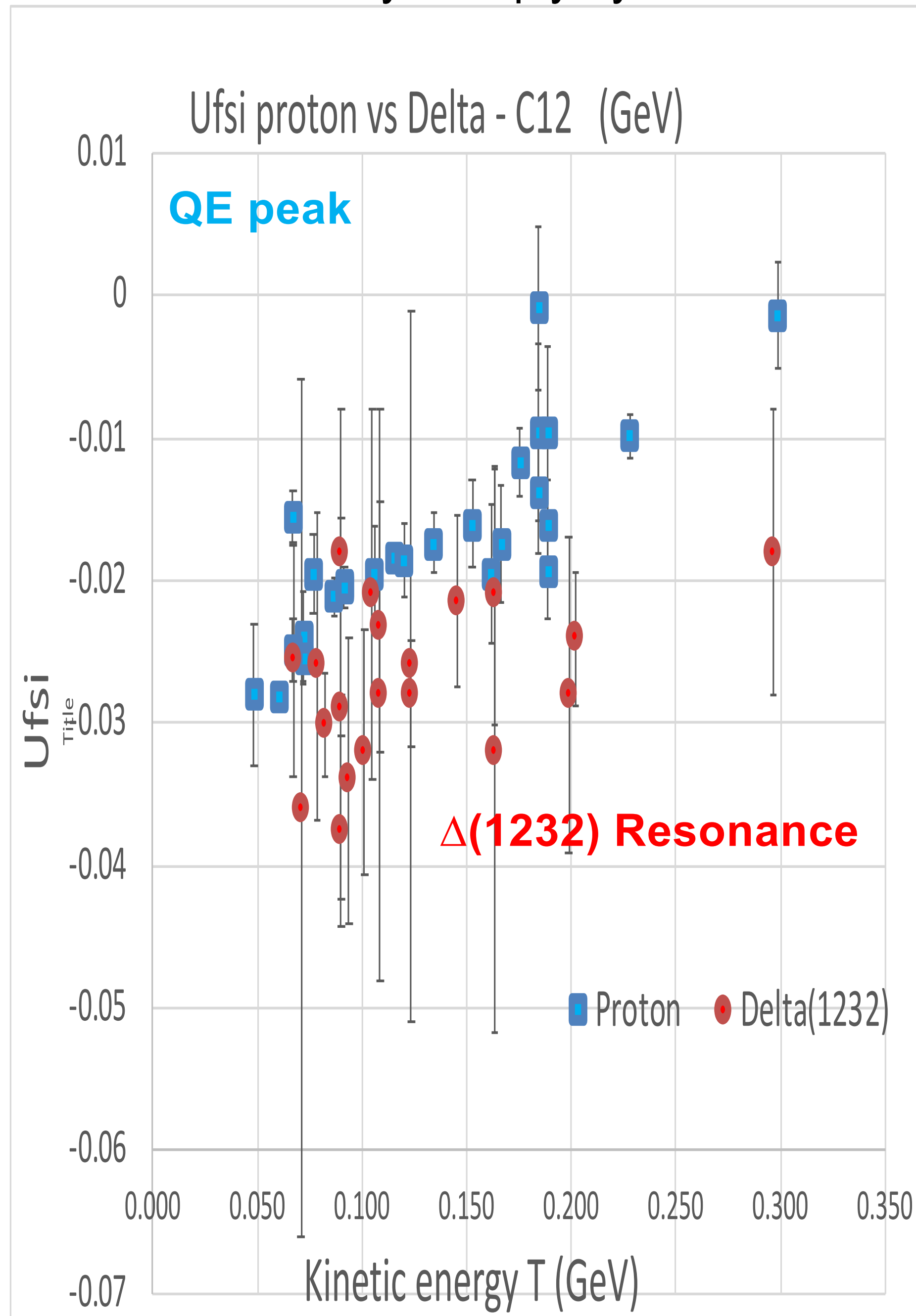
Cooper-et al PRC 47, 297(1993)

Data in better agreement with Cooper 2003 within 5 MeV.

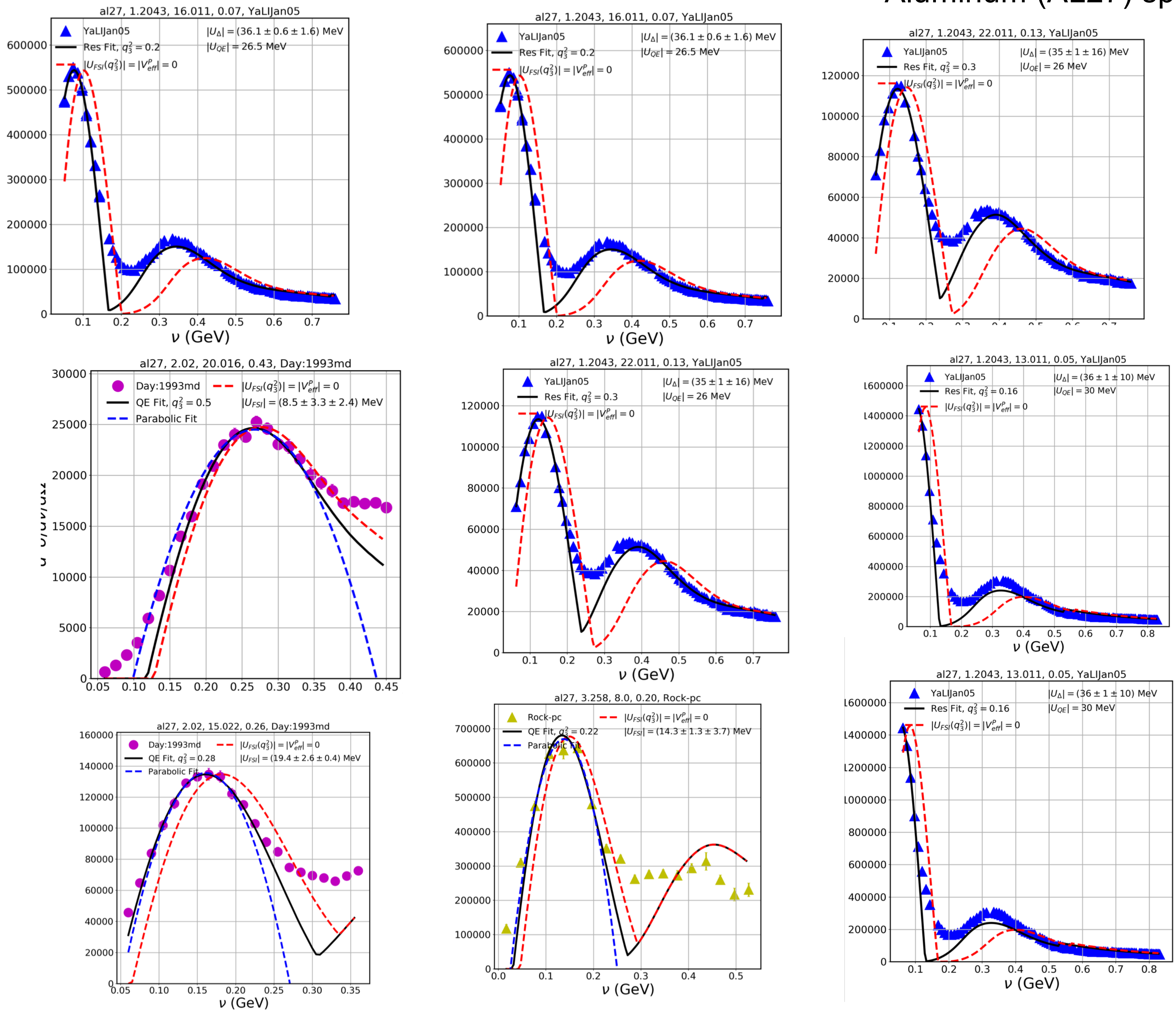
Data about 10 MeV lower than Cooper 2009

Gibuu uses same Ufsi for everything except Delta for which they multiply by 3/2.

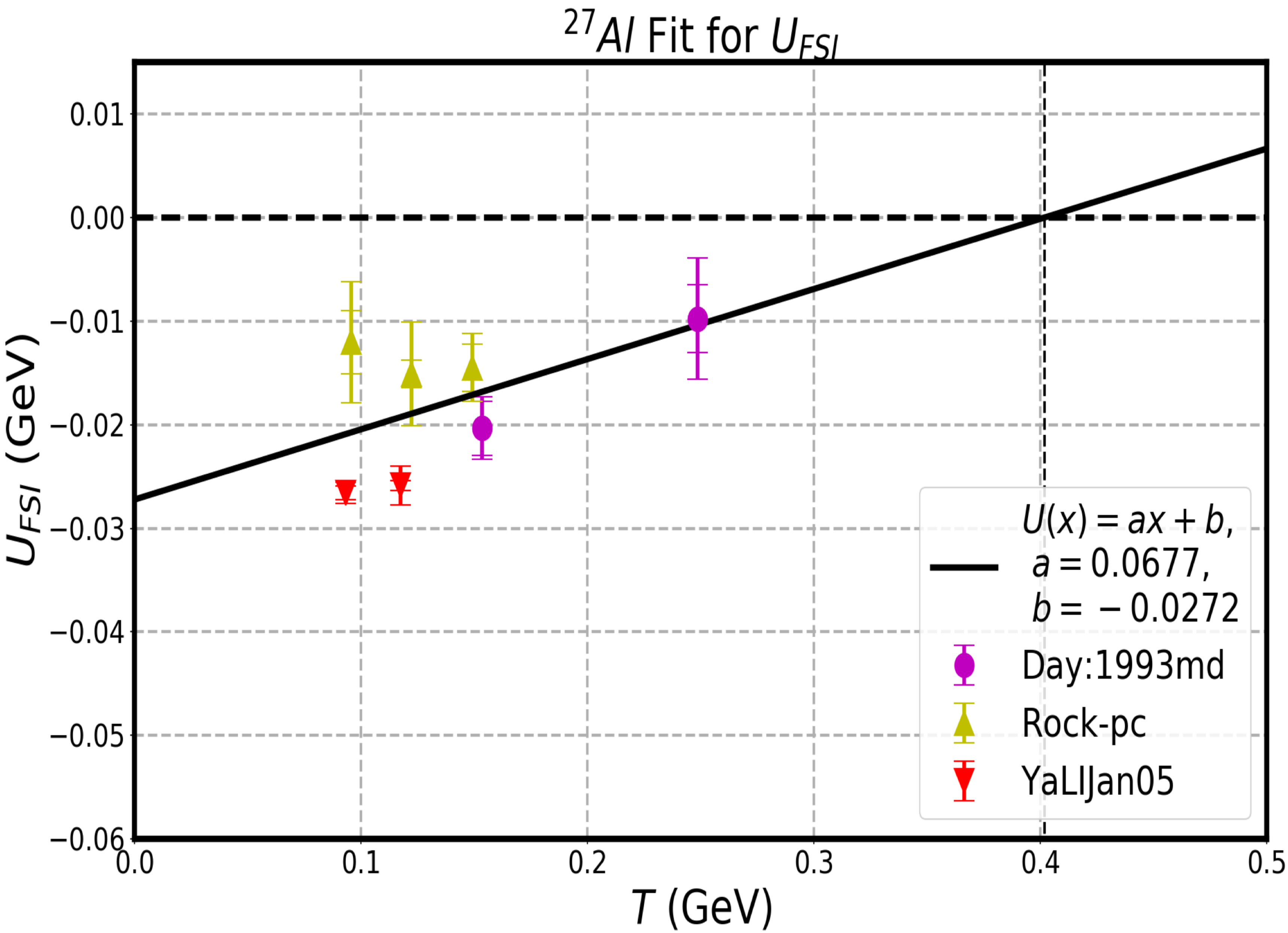


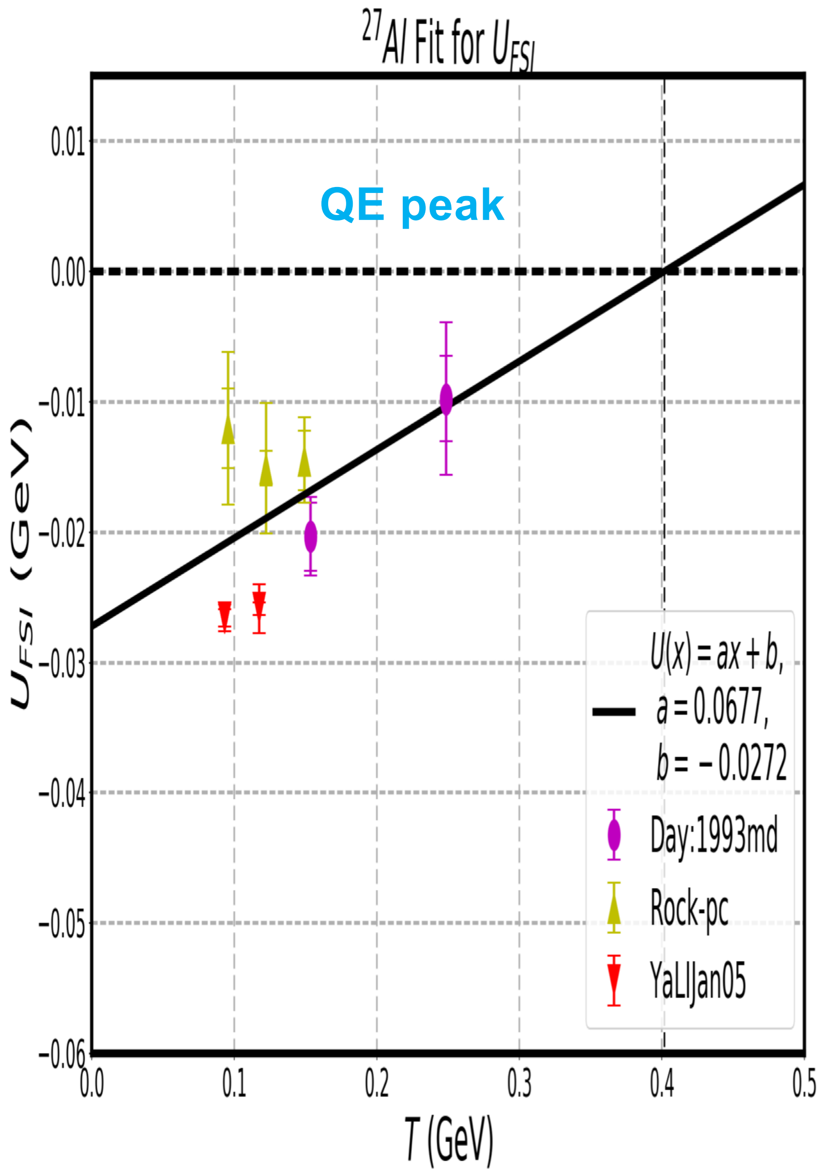


Aluminum (AL27) spectra

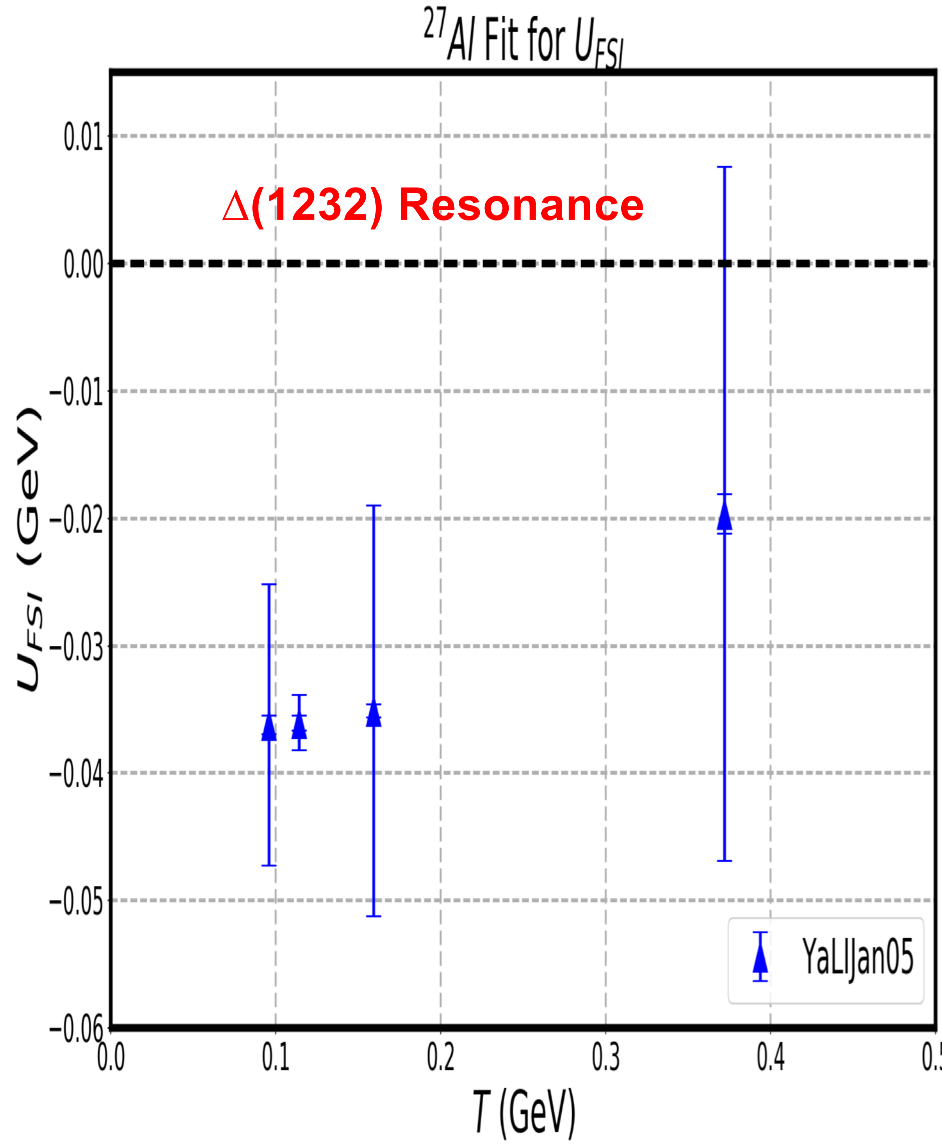


8 Aluminum (AL27) spectra Trento





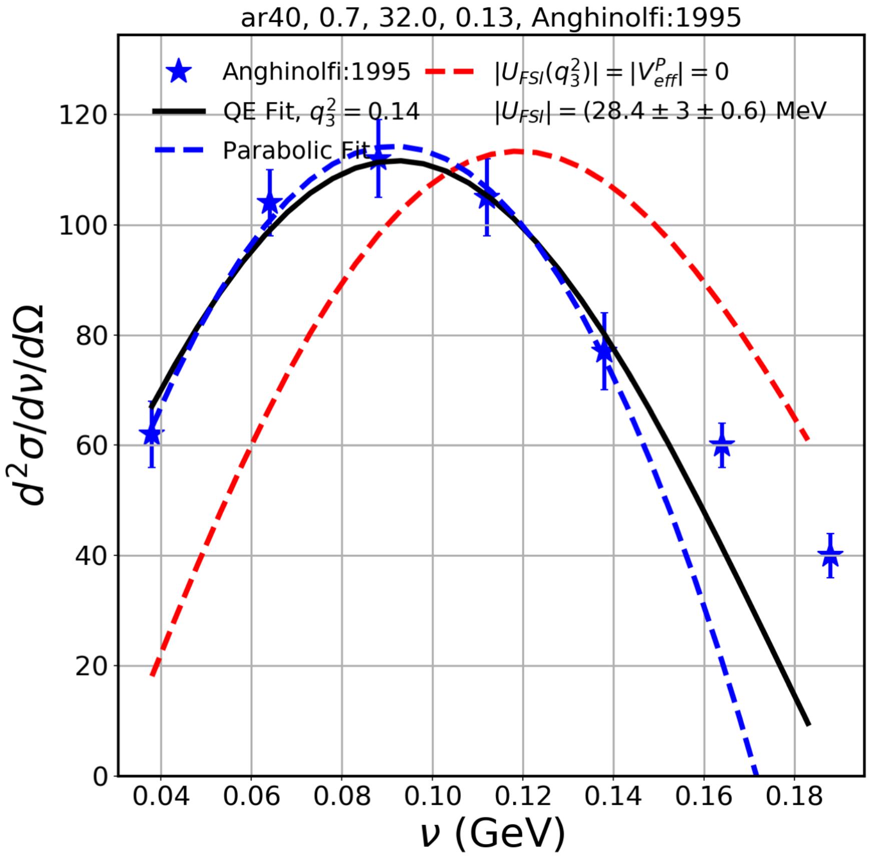
^{27}Al Fit for U_{FSI}
QE peak
$U(x) = ax + b$,
$a = 0.0677$,
$b = -0.0272$
Day:1993md
Rock-pc
YaLIJan05
U_{FSI} (GeV)
T (GeV)

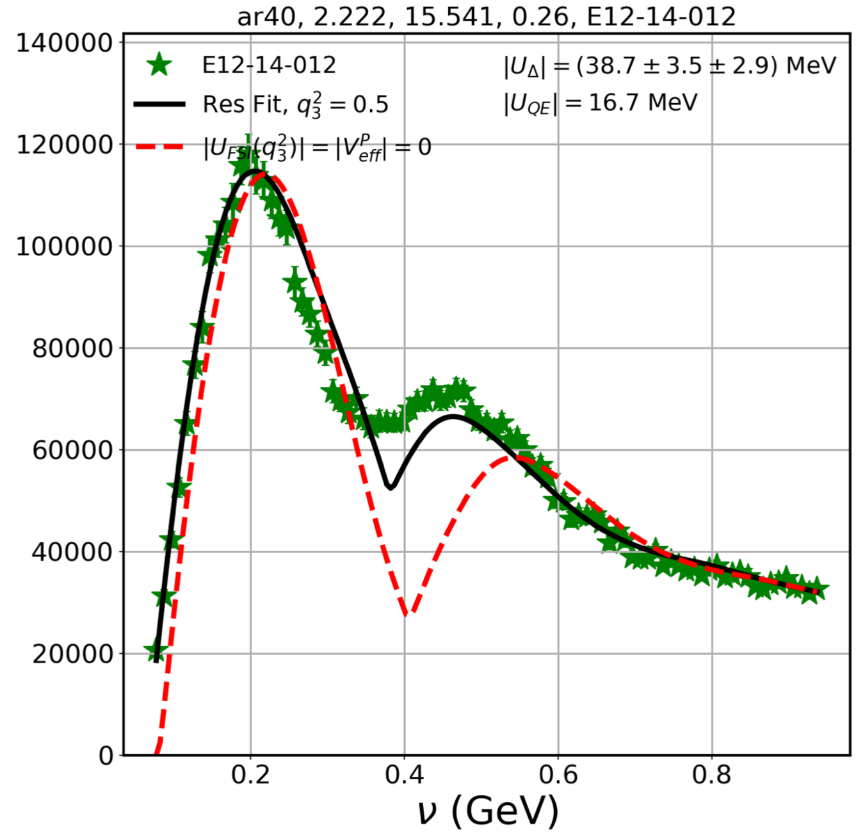


^{27}Al Fit for U_{FSI}
Δ(1232) Resonance
YaLIJan05
U_{FSI} (GeV)
T (GeV)

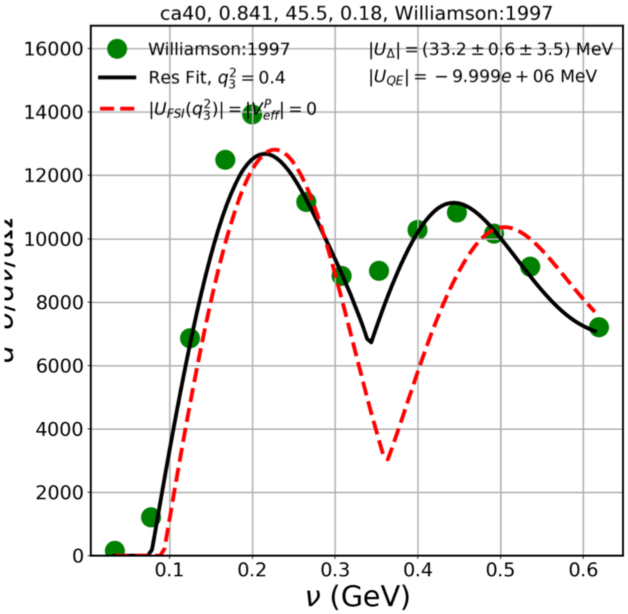
$(q_3)^2$=0.14 GeV^2

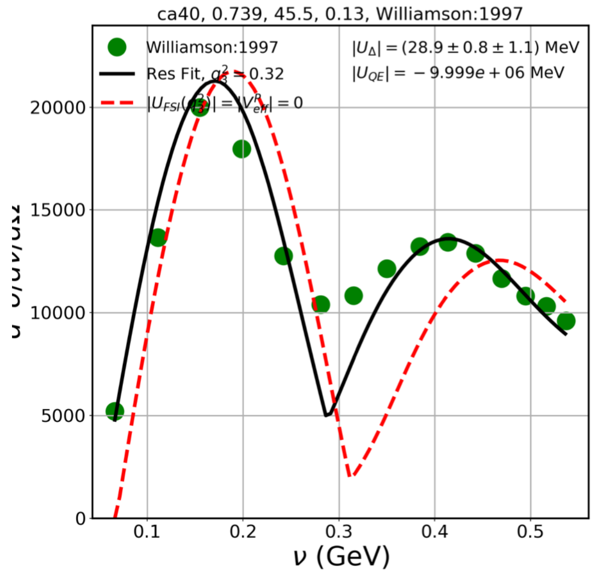
$(q_3)^2$=0.37 GeV^2

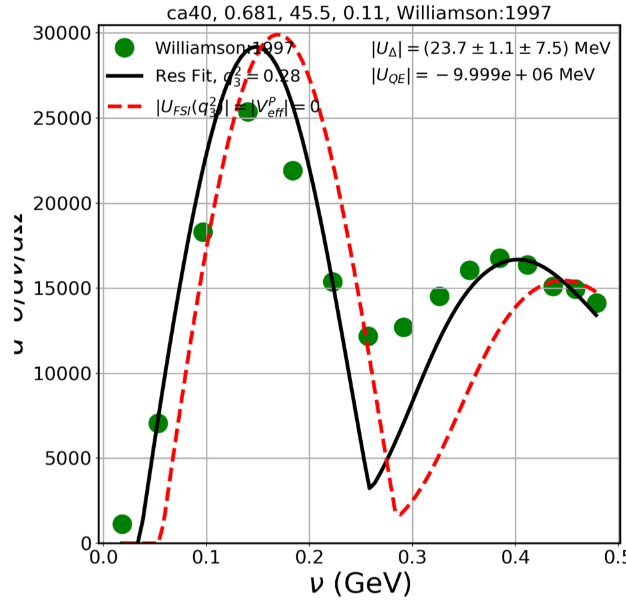


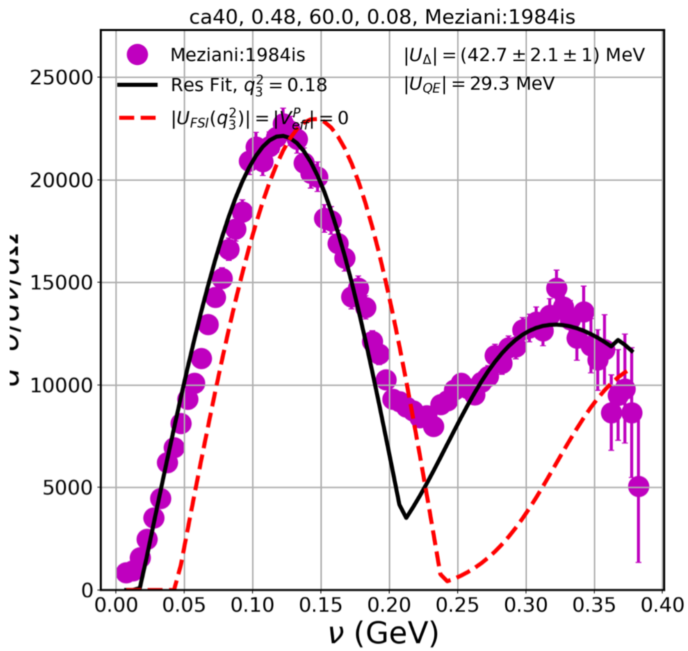


Two Argon40 Spectra one with Delts

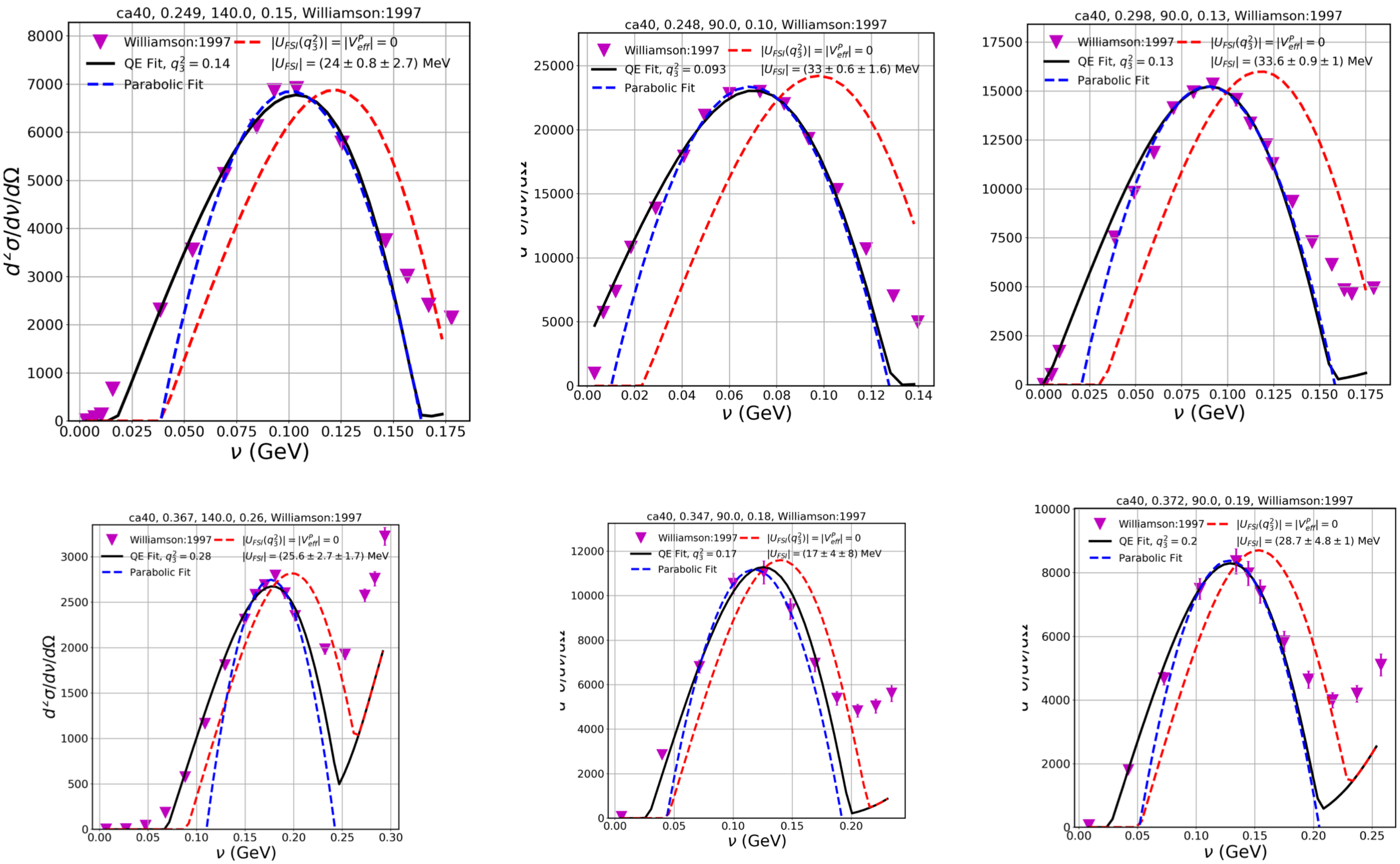


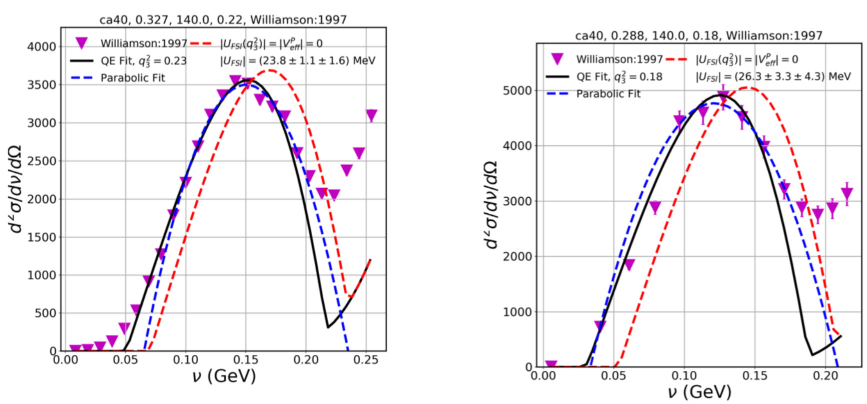






4 out of 29 Calcium 40 spectra with Delta





8 out of 29
Calcium 40
spectra with
no Delta

29 CA 40 spectra and 2 Ar40 spectra -Trento

r =0 U_{opt}

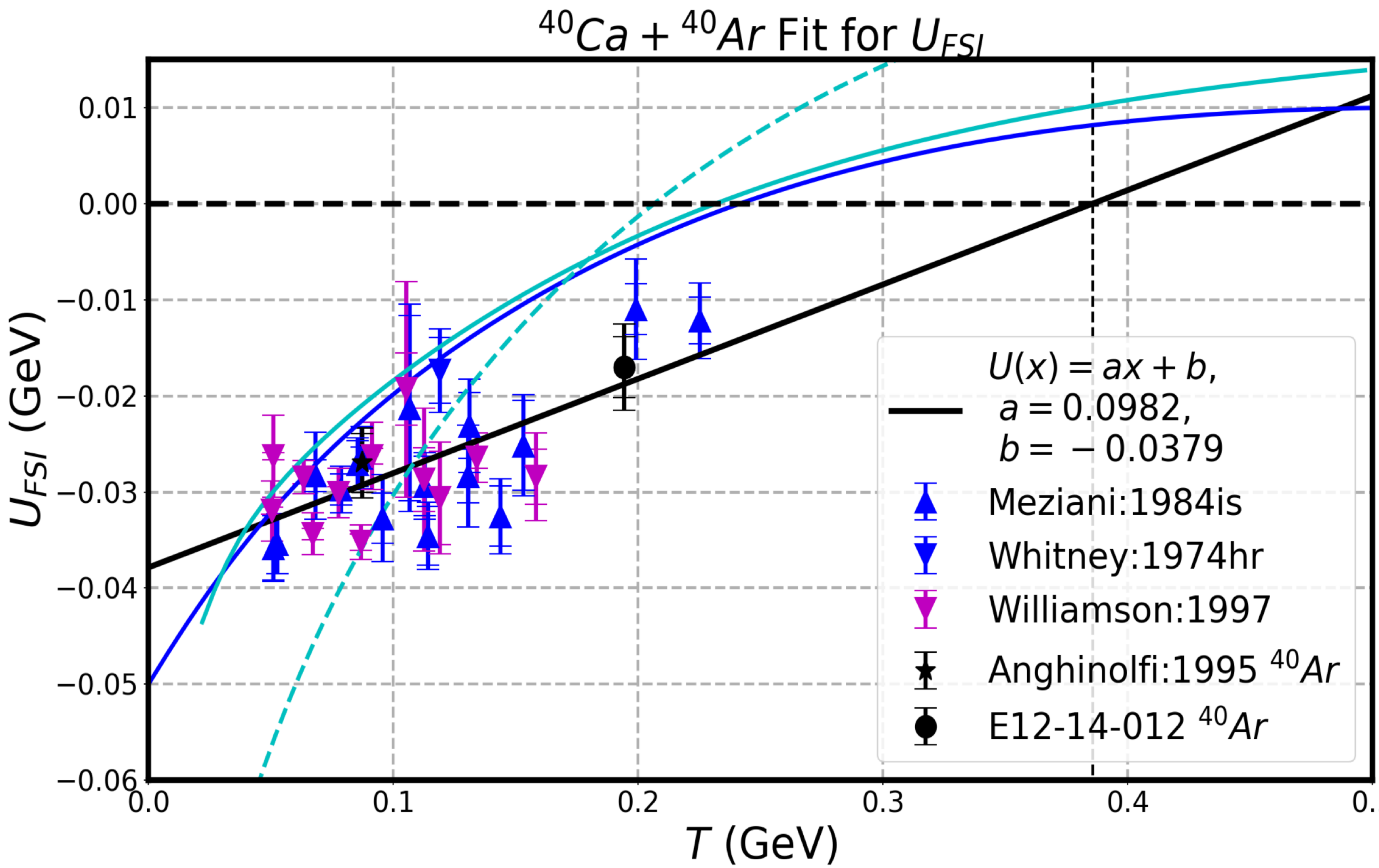
Average U_{opt}

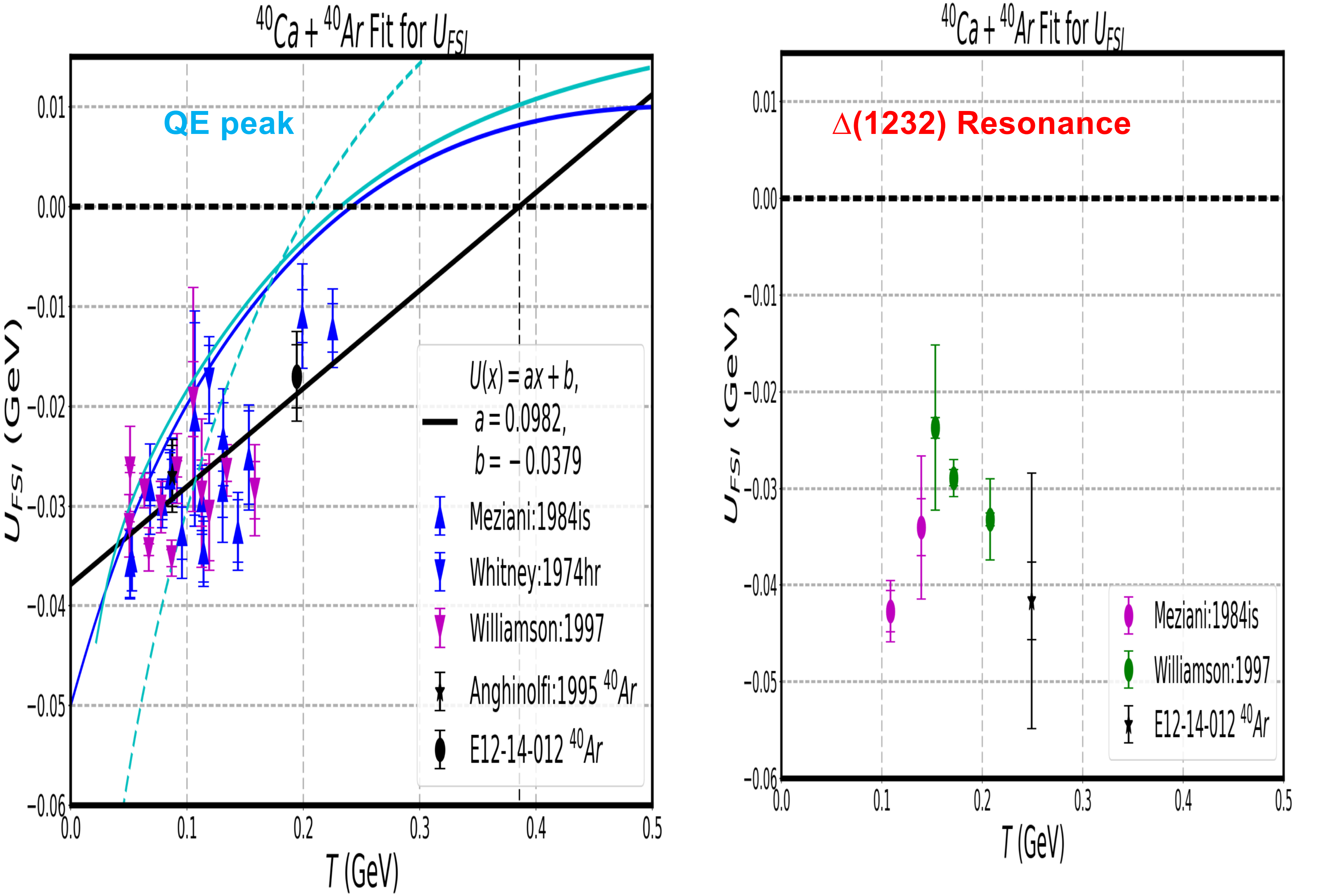
Cooper 2009 calculated by Jose Manuel Udias

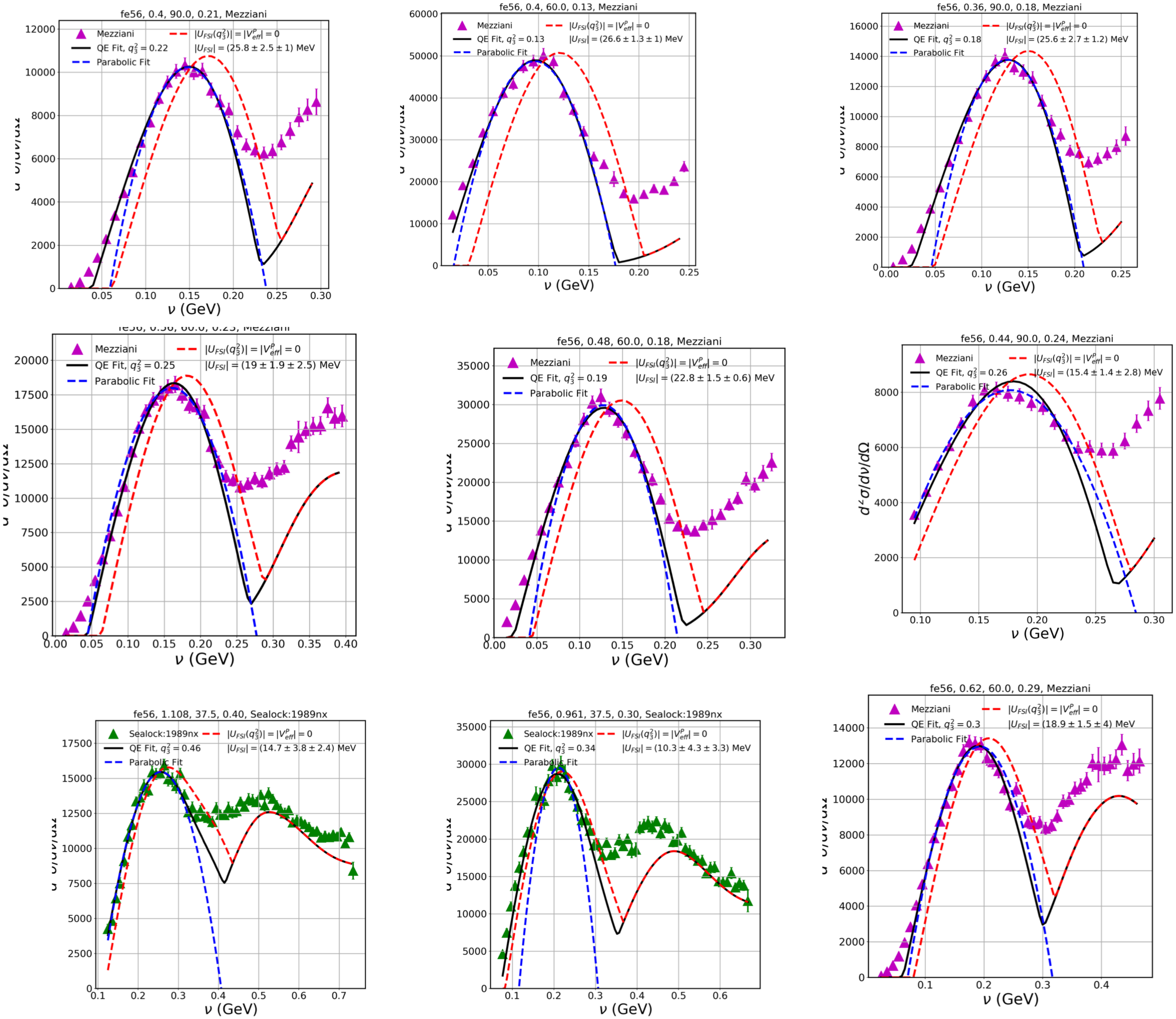
Average U_{opt}

Cooper 2009 calculated by Artur Ankowski

Data about 10 MeV lower than Cooper 2009



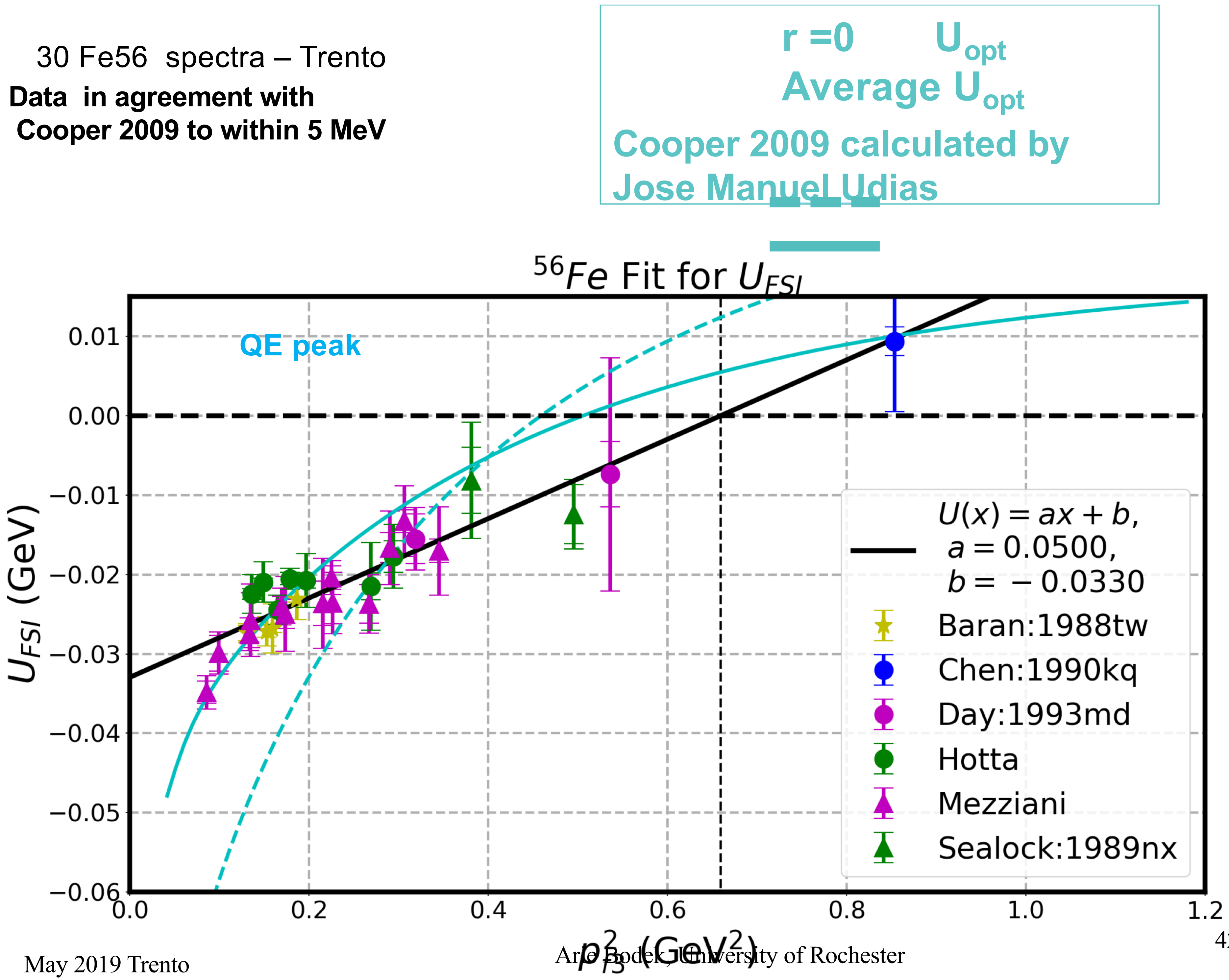

$^{40}Ca + ^{40}Ar$ Fit for U_{FSI}
QE peak
$U(x) = ax + b,$
$a = 0.0982,$
$b = -0.0379$
Meziani:1984is
Whitney:1974hr
Williamson:1997
Anghinolfi:1995 ^{40}Ar
E12-14-012 ^{40}Ar
U_{FSI} (GeV)
T (GeV)
$^{40}Ca + ^{40}Ar$ Fit for U_{FSI}
Δ(1232) Resonance
Meziani:1984is
Williamson:1997
E12-14-012 ^{40}Ar
U_{FSI} (GeV)
T (GeV)

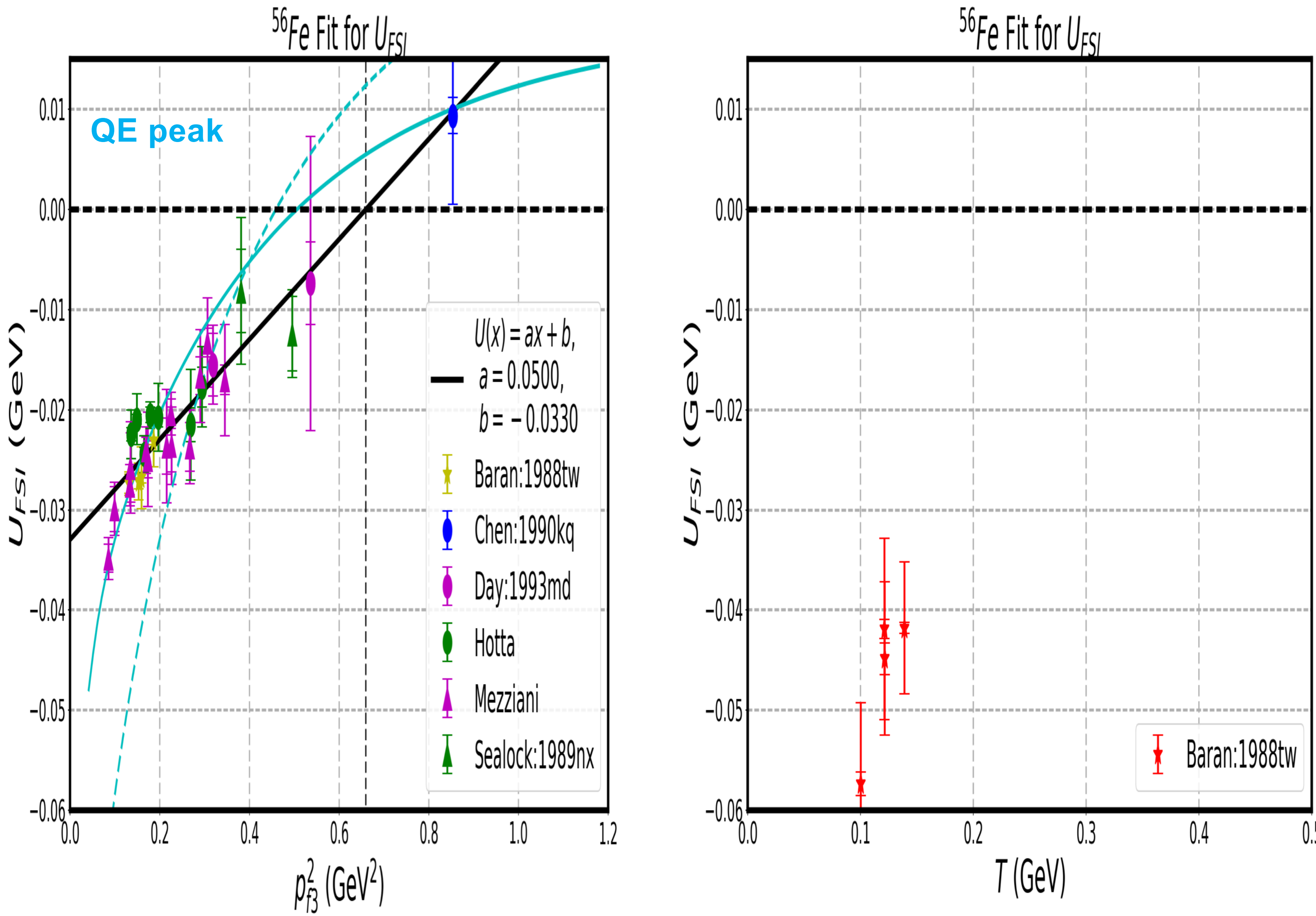


9 Fe56 Spectra out of 30 spectra

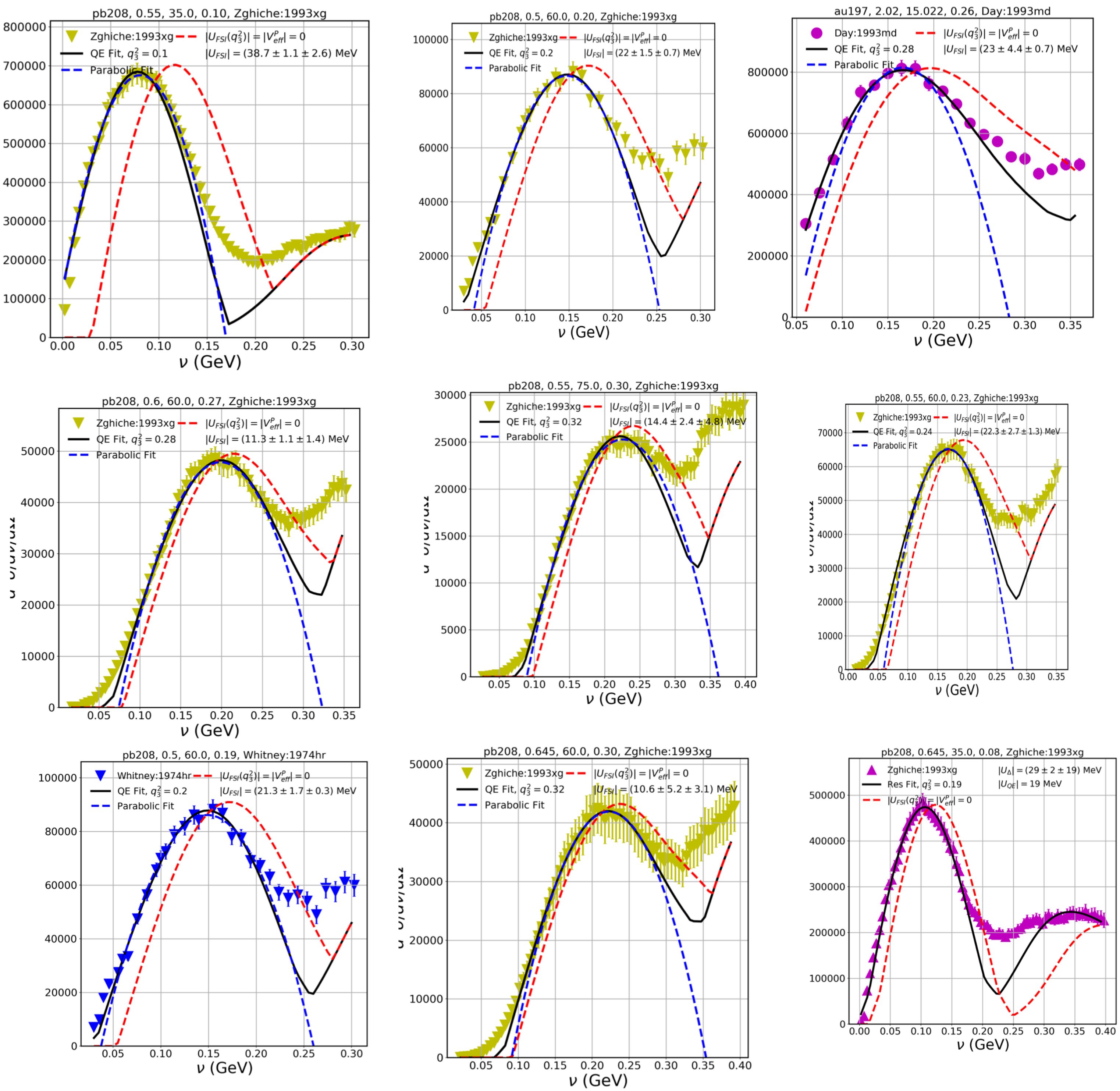
30 Fe56 spectra – Trento

Data in agreement with Cooper 2009 to within 5 MeV





^{56}Fe Fit for U_{FSI}
QE peak
$U(x) = ax + b$,
$a = 0.0500$,
$b = -0.0330$
Baran:1988tw
Chen:1990kq
Day:1993md
Hotta
Mezziani
Sealock:1989nx
U_{FSI} (GeV)
p_{f3}^2 (GeV^2)
0.01
0.00
−0.01
−0.02
−0.03
−0.04
−0.05
−0.06
0.0
0.2
0.4
0.6
0.8
1.0
1.2
^{56}Fe Fit for U_{FSI}
Baran:1988tw
U_{FSI} (GeV)
T (GeV)
0.1
0.3
0.5



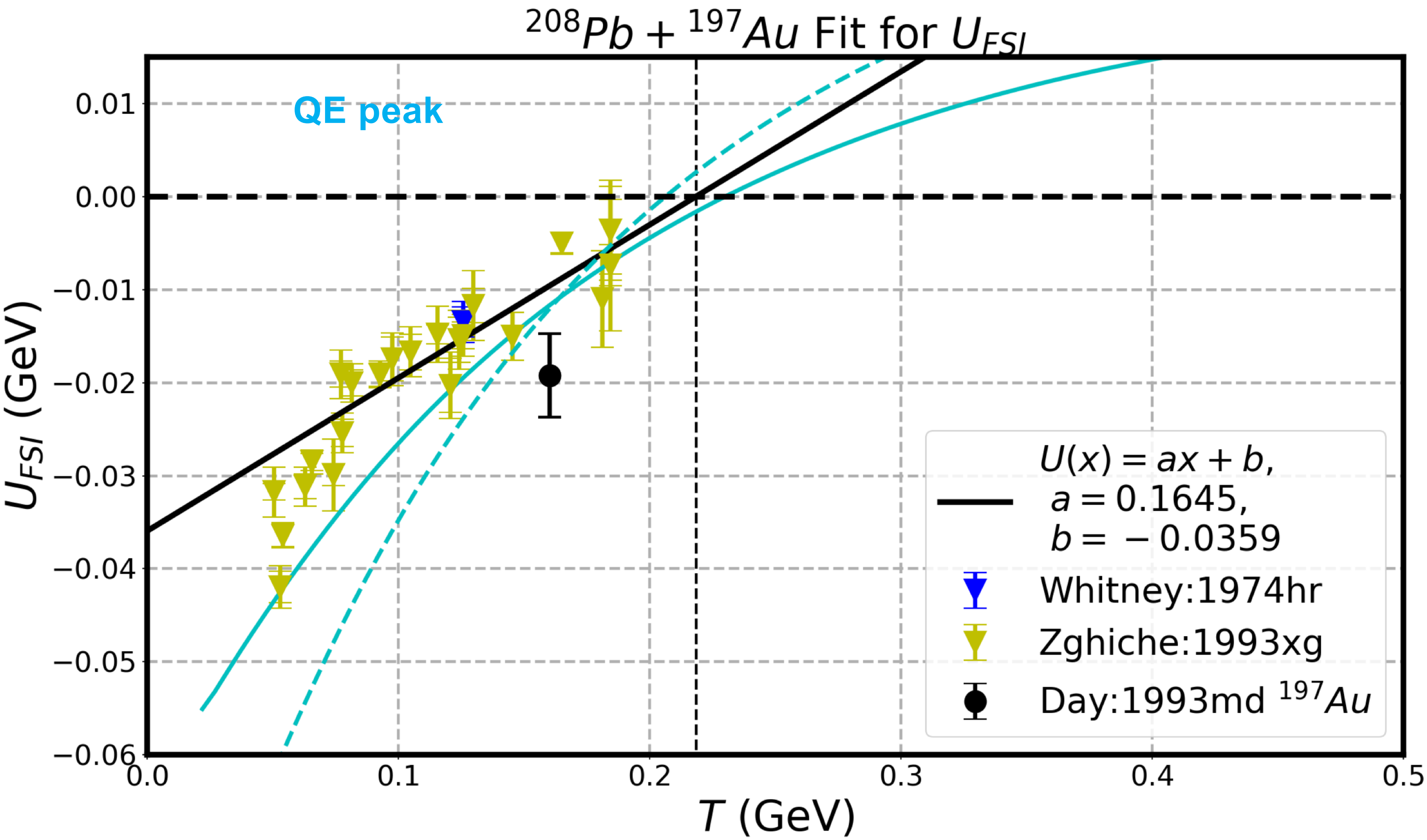
1 Au197 and 8 out of 22 Pb208 spectra.

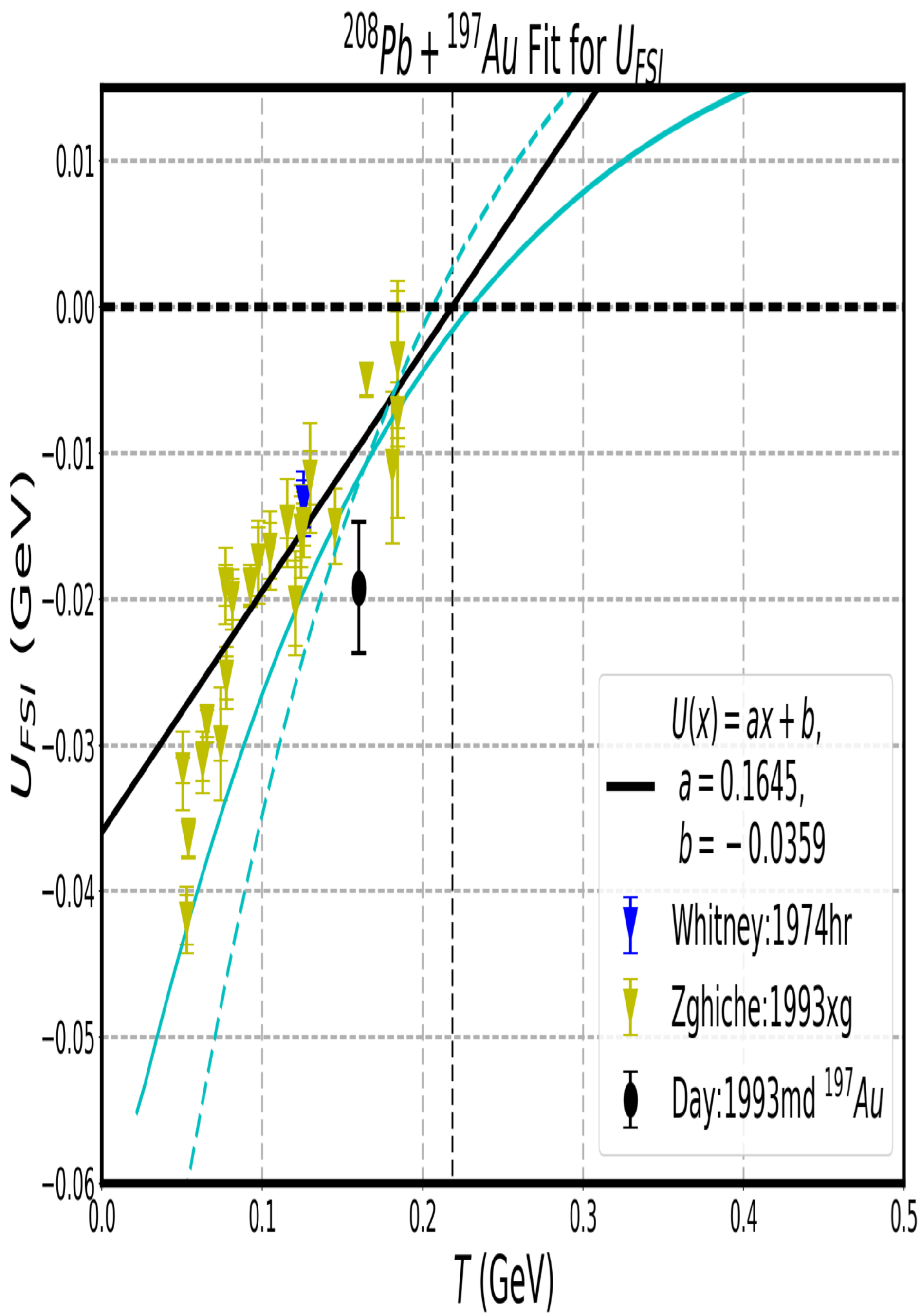
Only one has Delta

22 Pb208 spectra and 1 Au197 spectra
Au<Ex> =Pb<Ex>

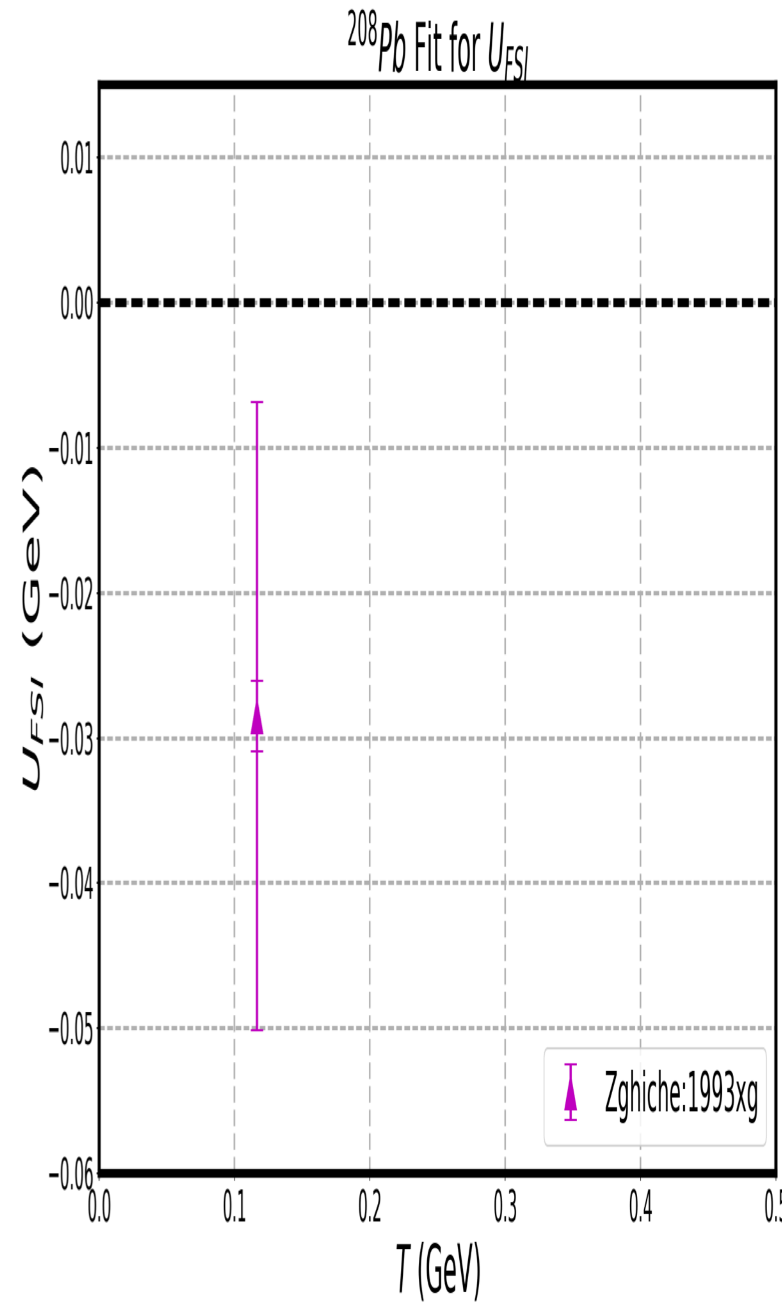
Data in agreement with
Cooper 2009 to within 10 MeV

r =0 U_{opt}
Average U_{opt}
Cooper 2003 calculated by
Jose Manuel Udias





$^{208}Pb + ^{197}Au$ Fit for U_{FSI}
U_{FSI} (GeV)
T (GeV)
$U(x) = ax + b$,
$a = 0.1645$,
$b = -0.0359$
Whitney:1974hr
Zghiche:1993xg
Day:1993md ^{197}Au



^{208}Pb Fit for U_{FSI}
U_{FSI} (GeV)
T (GeV)
Zghiche:1993xg

Comments

1. We plan to repeat the studies with effective spectral function for QE (this mimics super-scaling results).

2. It would be nice to have some theoretical input on the difference in Ufsi for longitudinal and transverse virtual bosons,

3. Similarly for the W dependence.

4. It would be interesting if experts to run GENIE (and other MC) for electron neutrinos. The neutrino energies should be the same for the ~100 electron scattering spectra (plus Veff) and the scattering angle should also be the same. This allows for a direct comparison with the location of QE peak and Delta resonance and extraction of the Ufsi from the electron scattering data. Studies can done with various options (Fermi gas, local Fermi gas, spectral function etc).

Appendix

Approximate extraction of Ufsi from the peak position of the QE peak

$$\nu^{QE} + (M_P - \varepsilon) = \sqrt{\vec{k}^2(k_z)\rangle + 2k_z\vec{q}_3 + \vec{q}_3^{\,2} + M_P^2} + U_{FSI}^{QE} + |V_{eff}^P$$

In the peak region of the QE distribution $k_z \approx 0$. Therefore, from the location of the peak in ν we extract $U_{FSI}(\mathbf{p_f'^2})_{peak}$ for

$$\mathbf{p_f'^2}_{peak} = (\vec{q}_3 + \vec{k})^2_{peak} \approx \langle \vec{k}^2(k_z = 0)\rangle + \vec{q}_3^{\,2} = \frac{1}{2}k_F^2 + \vec{q}_3^{\,2} \approx \mathbf{0.02\ GeV^2 + q_3^2} \quad (for\ K_F = 0.2)$$

$$\nu_{peak}^{QE} + M_P - \varepsilon - U_{FSI}^{QE} - |V_{eff}^P| = \sqrt{\frac{1}{2}k_F^2 + (\vec{q}_3^{\,2})_{peak}^{QE} + M_P^2}$$

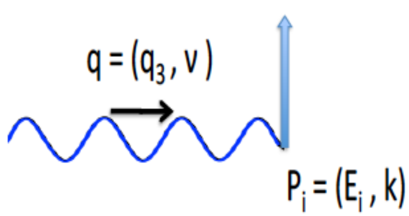


Fig. 3. Scattering from an off-shell bound nucleon of momentum k which is perpendicular to the direction of the virtual photon. This is the configuration at the *peak* of the Fermi motion smearing. At the *peak* of the distribution the z component of the nucleon momentum (k_z) is zero.

Because form factors vary with Q^2, the QE peak position is not exactly at Kz=0 so the extraction of Ufsi from the peak position is approximate. In addition the Coulomb effects are different for neutrons and protons

A better extraction compares the QE distribution to a model and changes Ufsi within the model to fit the data