Model implementation with hadron tensors and tests of "factorisable interfaces"

Stephen Dolan

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Overview

Based on: **arXiv:1905.08556**

Also: see Guillermo's talk before this

- Recently implemented the SuSAv2 1p1h and 2p2h models in GENIE using hadron tensors.
- Based on implementations of the Valencia 2p2h (NEUT/GENIE)

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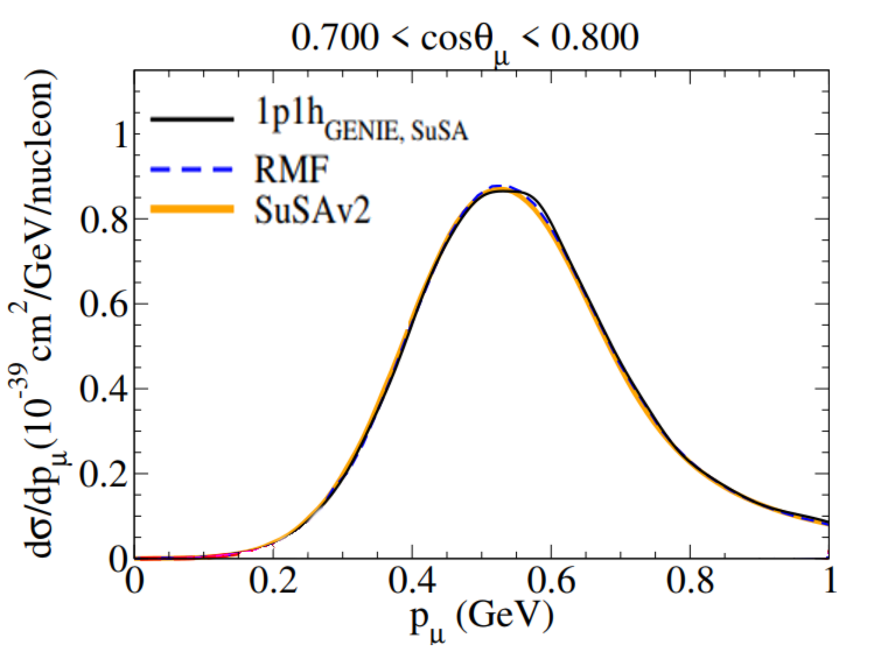

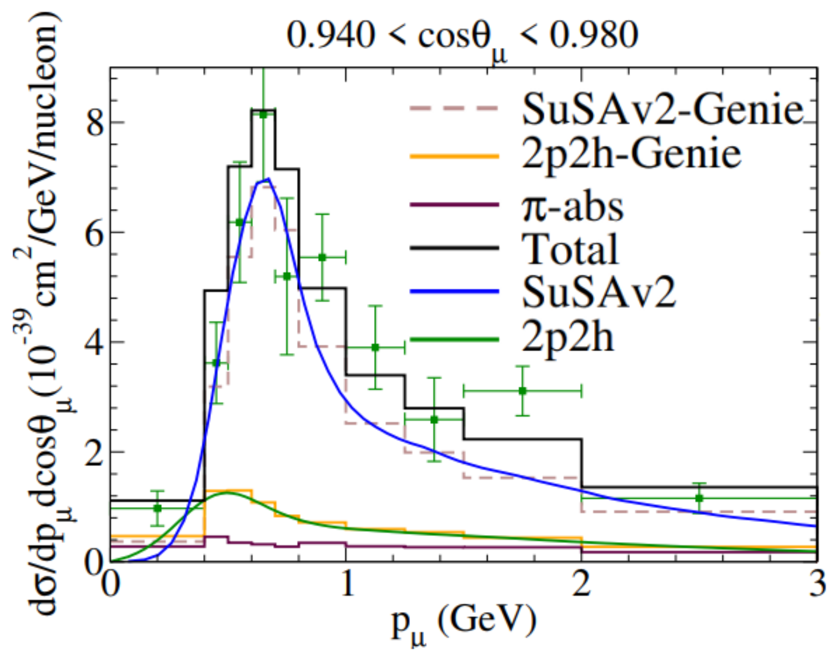

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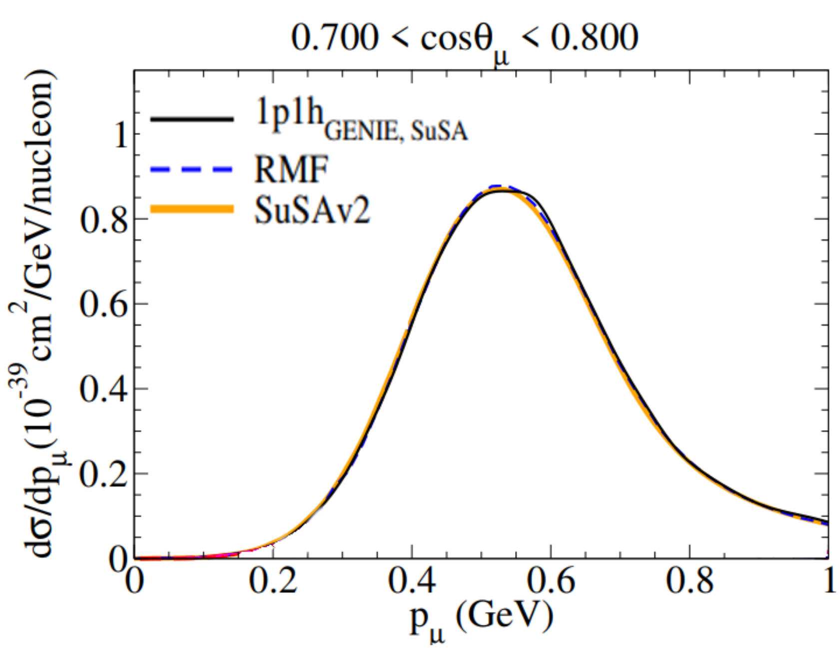
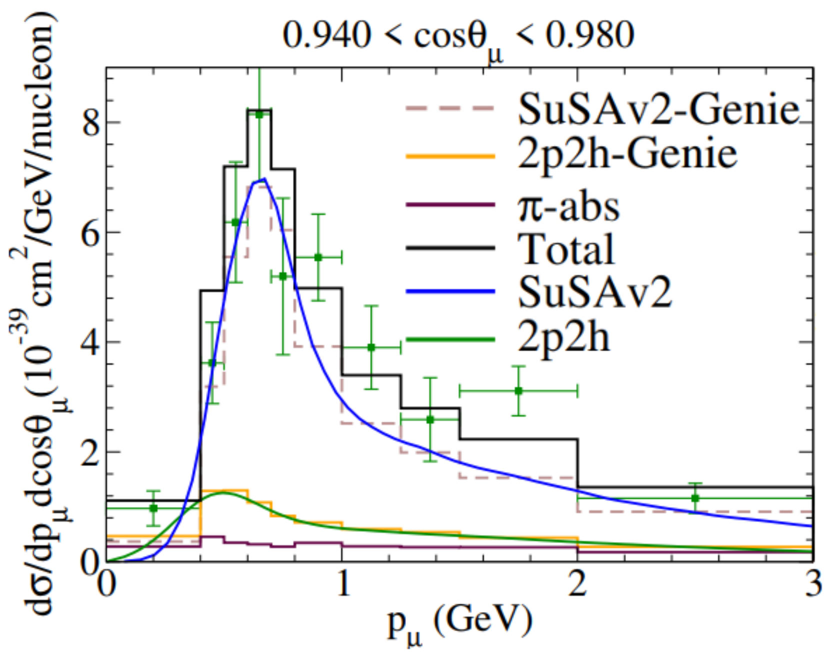
- Based on implementations of the Valencia 2p2h (NEUT/GENIE)
- Exactly reproduces the *inclusive* predictions of the models
- (Semi-)exclusive predictions are obtained using ad-hoc "*factorisation*" approximations common to most model implementations





This talk will introduce:

(More details will be in the parallel sessions tomorrow)

- Hadron tensor based model implementation
- The factorisation approximation (FA)
  - What is it?
  - Why do we care
- Testing the factorisation approximation
  - Generate hadron kinematics using FA (*SuSAv2*)
  - Do the same using a microscopic model (*RMF*)
  - Compare
- Conclusions and further work

Implementation – lepton part

Like the Nieves model, SuSAv2-MEC is able to predict only the outgoing lepton kinematics. The double differential cross section can be written:

$$\frac{d^2\sigma}{dq_0 q_3} = \sigma_0 \eta_{ij}(q_0, q_3) W_{ij}(q_0, q_3)$$

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- The hadron tensor elements are stored in tables which specify q0 and q3 in bins of 5 MeV between 0 and 2 GeV – unique SuSAv2-MEC tensors
  - Use a GENIE's bilinear interpolation function to evaluate specific q0,q3
  - Hadron tensors will be provided for a few select targets (C and O so far, may add others). Can scale to other nuclei.

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But what about the hadrons?

Semi-inclusive cross sections?

Cross sections can generally be written as a contraction of leptonic and hadronic tensors:

$$\sigma \sim \eta_{\mu\nu} W^{\mu\nu} = \eta^{s}_{\mu\nu} W_{s}^{\mu\nu} + \eta^{a}_{\mu\nu} W_{a}^{\mu\nu}$$

s and *a* indicate a splitting of the tensors into symmetric and asymmetric components – not so important here.

Adapted from: https://indico.ectstar.eu/event/19/contributions/221

Semi-inclusive cross sections?

Expanding this in the most general way, we have 10 response functions (+6 without leptonic factors)

$$\eta^s_{\mu\nu} W^{\mu\nu}_s \sim \widehat{V}_{CC} W^{CC}_{semi} + \widehat{V}_{CL} W^{CL}_{semi} + \widehat{V}_{LL} W^{LL}_{semi} + \widehat{V}_{T} W^{T}_{semi} + \widehat{V}_{TT} W^{TT}_{semi} + \widehat{V}_{TC} W^{TC}_{semi} + \widehat{V}_{TL} W^{TL}_{semi}$$

$$\eta^a_{\mu\nu} W^{\mu\nu}_a \sim \widehat{V}_{T'} W^{T'}_{semi} + \widehat{V}_{TC'} W^{TC'}_{semi} + \widehat{V}_{TL'} W^{TL'}_{semi}$$

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Semi-inclusive cross sections?

But if we integrate over hadronic kinematics it turns out most of these cancel:

(For CCQE interactions, this comes from an integral over outgoing nucleon angle)

$$\eta^{s}_{\mu\nu} W^{\mu\nu}_{s} \sim \widehat{V}_{CC} W^{CC}_{semi} + \widehat{V}_{CL} W^{CL}_{semi} + \widehat{V}_{LL} W^{LL}_{semi} + \widehat{V}_{T} W^{T}_{semi} + \cancel{\widehat{V}_{TT} W^{TT}_{semi}} + \cancel{\widehat{V}_{TC} W^{TC}_{semi}} + \cancel{\widehat{V}_{TL} W^{TL}_{semi}}$$

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This is what is done in most models (and in most of what is implemented in the generators)

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Semi-inclusive cross sections?

Calculating **this** allows us to predict muon and nucleon kinematics (and correlations):

$$\frac{d\sigma_{CCQE}}{dp_\mu d\theta_\mu dp_p d\theta_p d\theta_{\mu p}}$$

But this is a large calculation

Calculating **this** allows us to predict muon kinematics …

$$\frac{d\sigma_{CCQE}}{dp_\mu dcos\theta_\mu}$$

Most of the terms cancel – this is easy*!

…. but doesn't necessarily allow us to say **anything** about nucleon kinematics

* Read as less difficult

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In the generators we (mostly) do this

(And so do most microscopic models)

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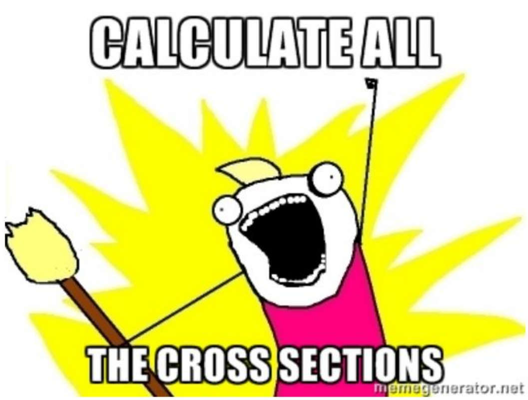
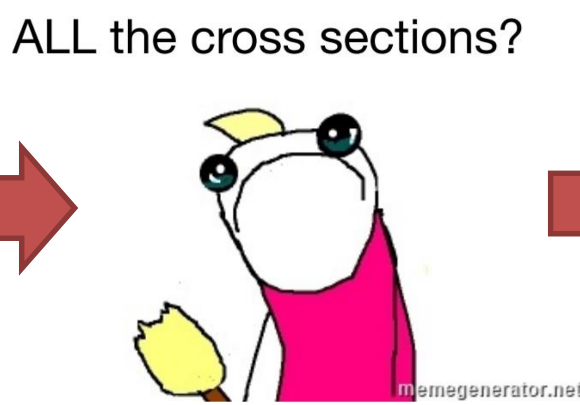
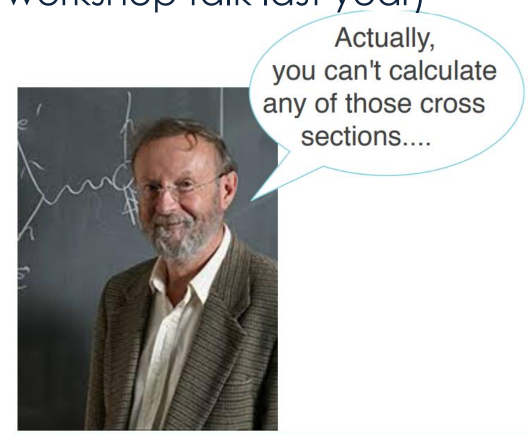
Progression of last year's workshop (from G. Perdue's GENIE ECT* workshop talk last year)







How do generators predict hadron kinematics? (Much more detail tomorrow)

- Start with the **inclusive** prediction
- Pick random initial-state nucleon momentum and binding energy based on some spectral function
- Conserve energy / momentum at the vertex to predict hadron kinematics (under impulse approximation)
- Add an FSI cascade to deal with all the stuff that we missed out

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Another way of stating the problem

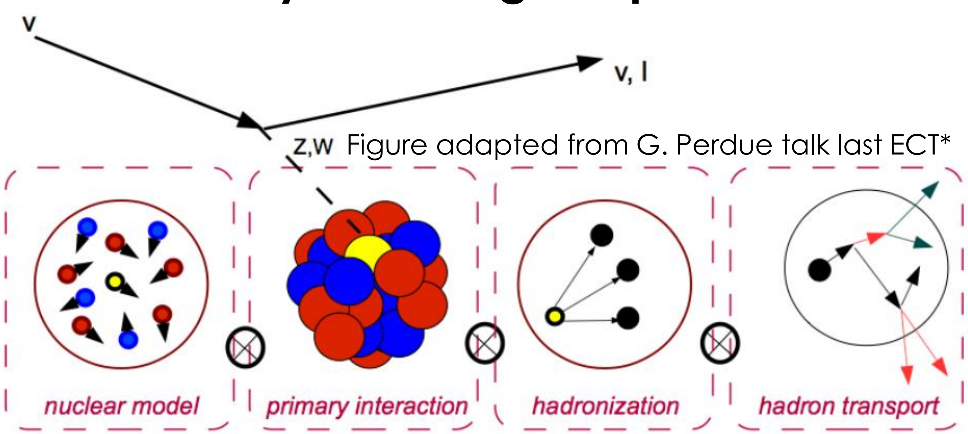

Figure adapted from G. Perdue talk last ECT*

We use an inclusive model for this bit

- We ***factorise*** aspects of neutrino interactions
- But in reality these components really aren't separable
- E.g. the hadron tensors in the *primary interaction* should encapsulate the *nuclear model*.

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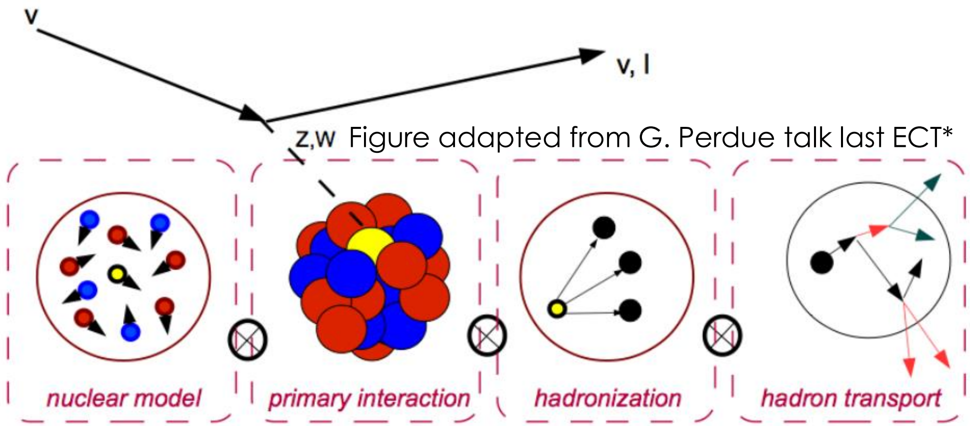

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How different are they?

i.e. how good is this "***factorisation approach***" (FA)?

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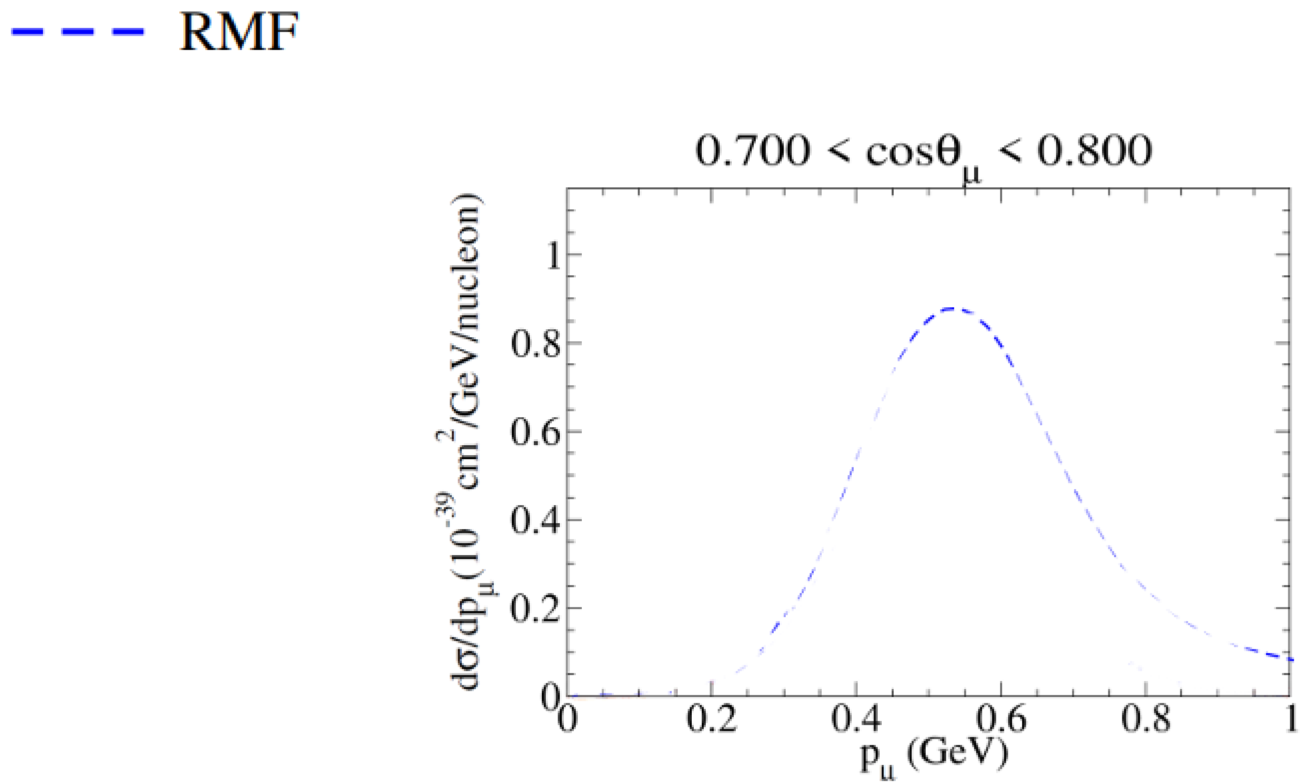
Caveats:

- Even for the inclusive case, SuSAv2 and RMF are not quite identical at very high and low kinematics – will stick to a good kinematic region
- For the FA, will use LFG rather than the real RMF spectral function (work in progress)

A first test of the FA

arXiv:1905.08556

These lines show the ***inclusive*** 1p1h prediction (no proton constraints)



- RMF detailed microscopic model calculation of inclusive 1p1h for T2K flux

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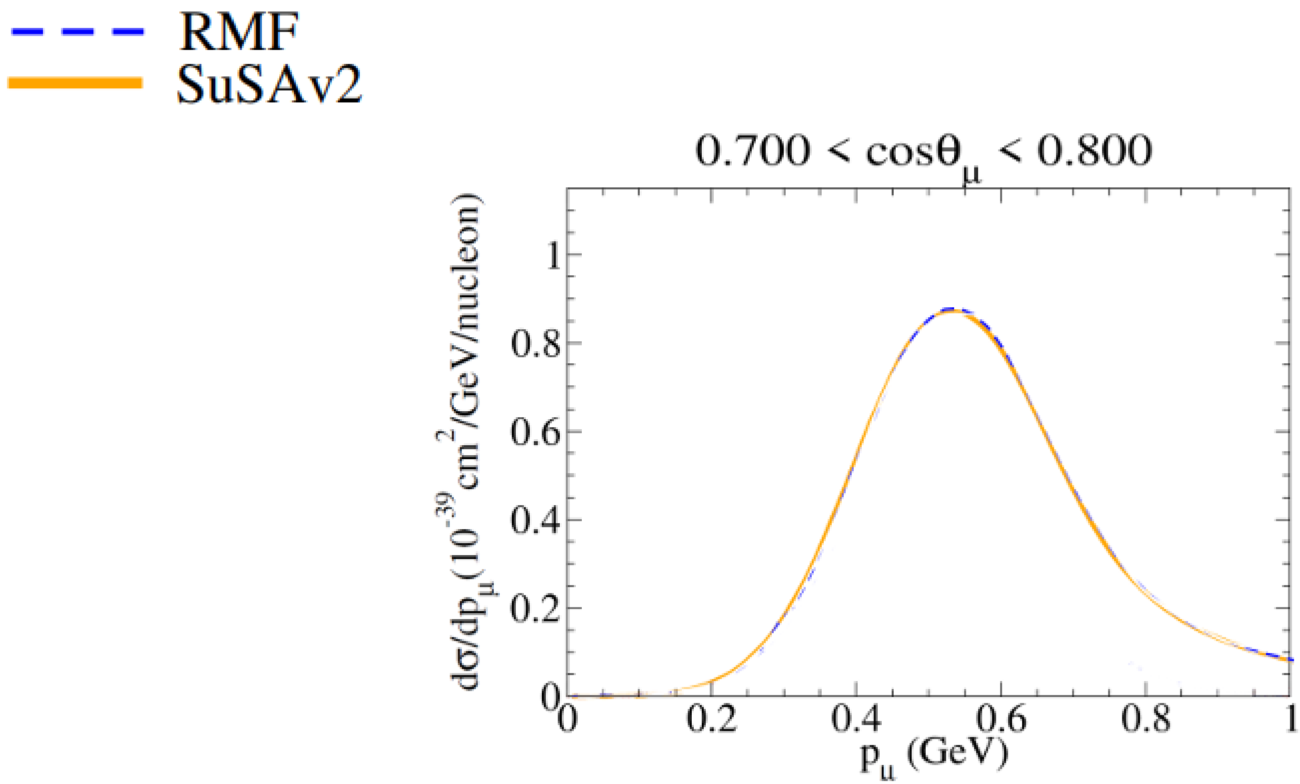

- RMF detailed microscopic model calculation of inclusive 1p1h for T2K flux
- SuSA is identical in this kinematic region (not true if we move to very small or steep angles)

[arXiv:1905.08556](https://arxiv.org/abs/1905.08556)

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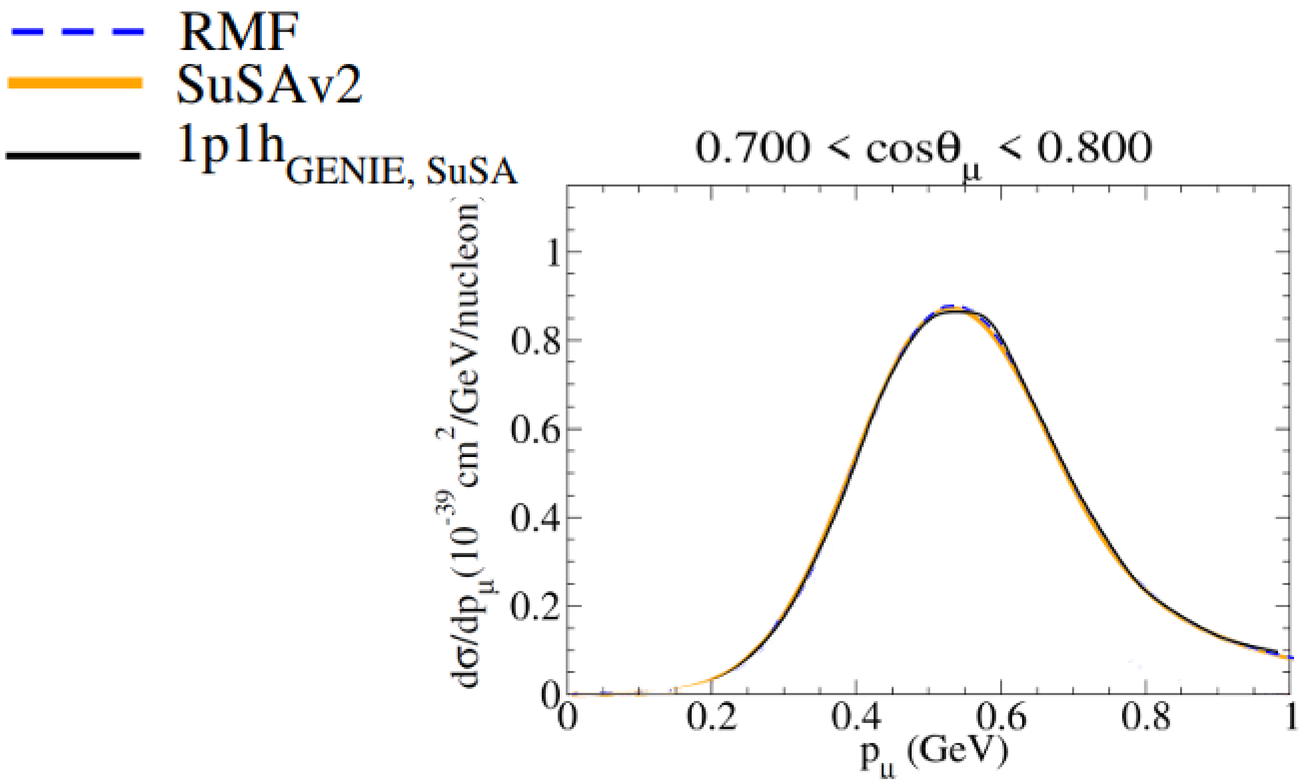

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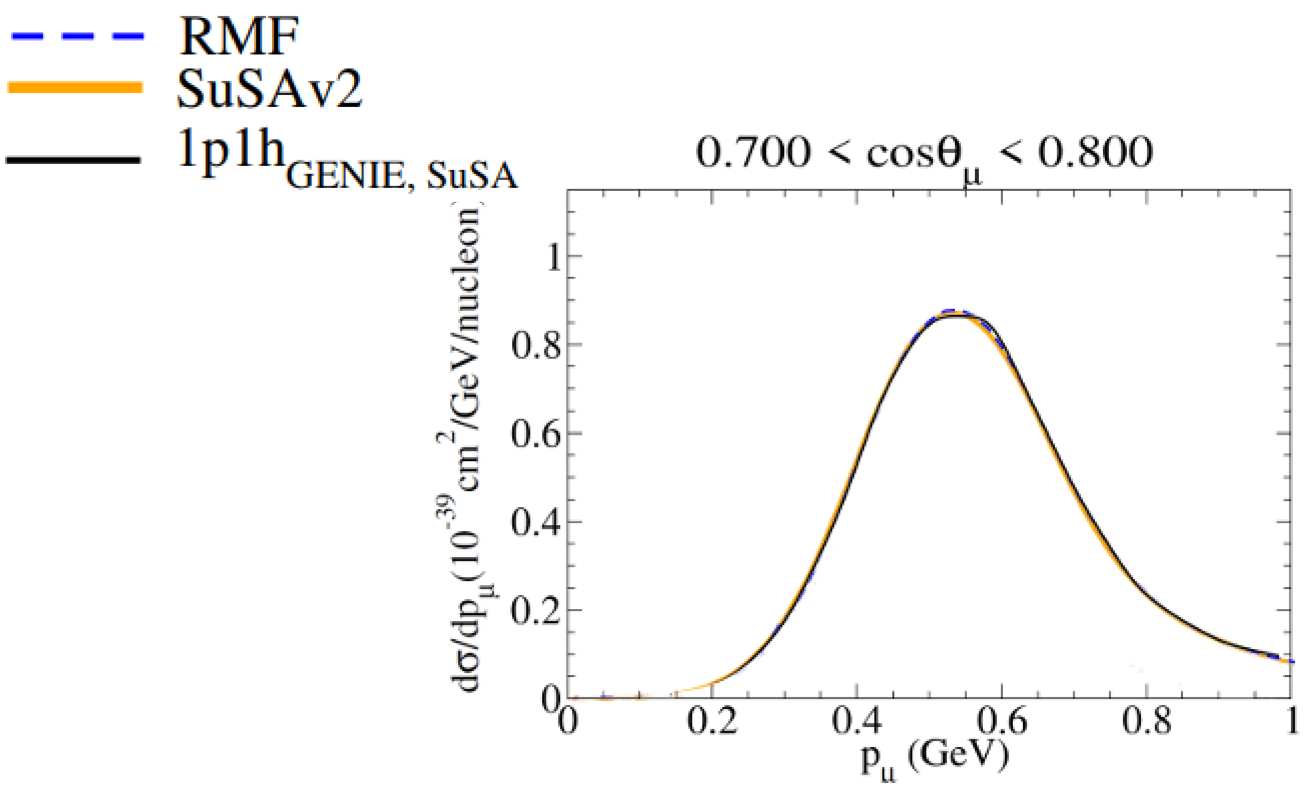

- The GENIE implementation works.
- Great, for inclusive calculations the microscopic base model (RMF), the inclusive theory (SuSAv2) and the implementation (in GENIE) all agree.

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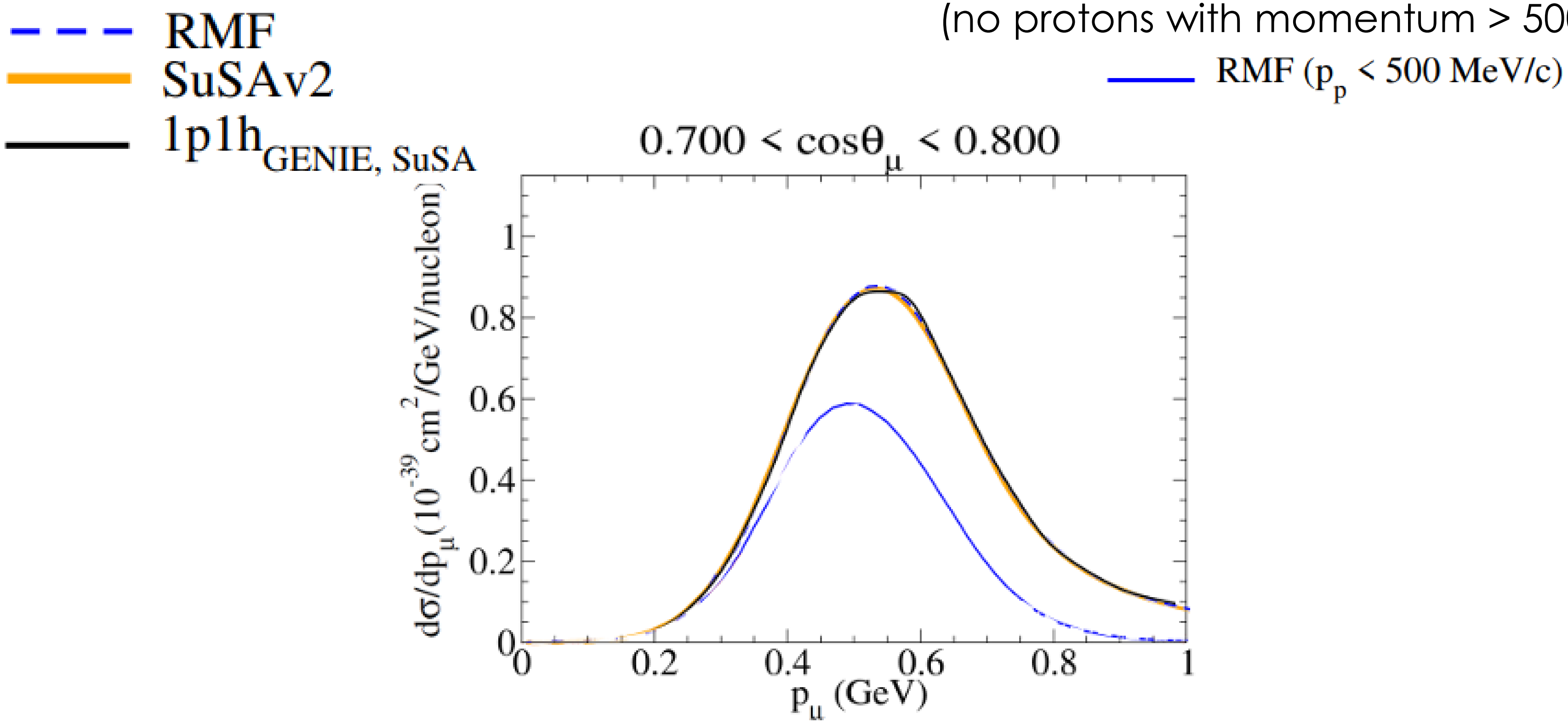

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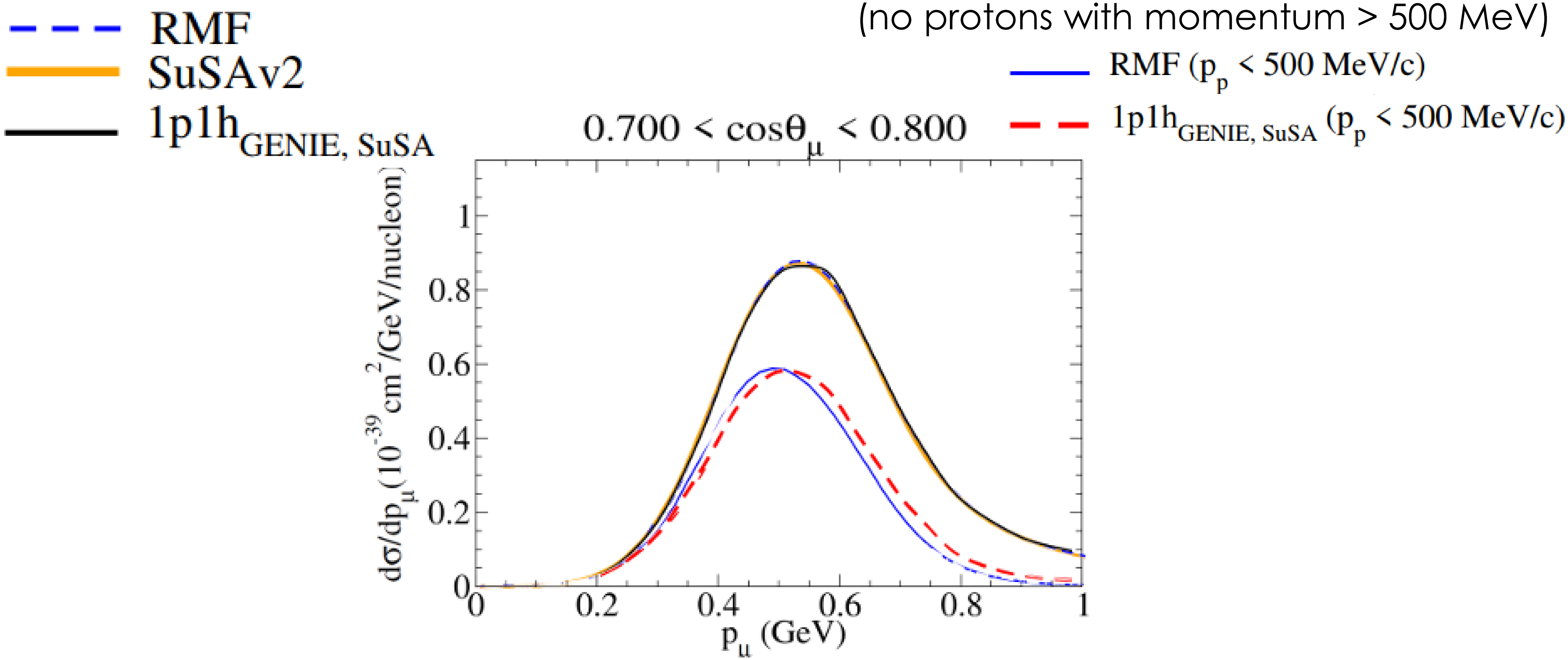


- FA implementation in this simple situation is surprisingly good!
- Still not perfect – exclusive kinematics are not quite right

A first test of the FA

arXiv:1905.08556

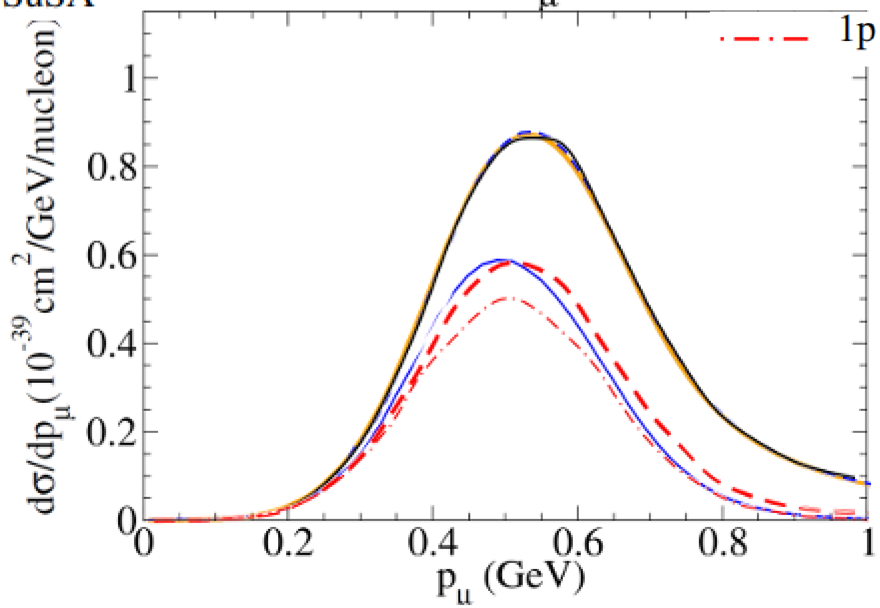
These lines show the ***inclusive*** 1p1h prediction (no proton constraints)

- - - RMF
— SuSAv2
— $1p1h_{GENIE,\ SuSA}$

These lines show the ***exclusive*** 1p1h prediction
(no protons with momentum > 500 MeV)

— RMF (p_p < 500 MeV/c)
– – $1p1h_{GENIE,\ SuSA}$ (p_p < 500 MeV/c)
-·- $1p1h_{GENIE\ no\ FSI,\ SuSA}$ (p_p < 500 MeV/c)

$0.700 < \cos\theta_\mu < 0.800$



- No FSI cascade in GENIE → less slow protons → smaller cross section
- FSI is (unsurprisingly) important to get the FA to work at all
- FSI maybe too strong at larger kinematics (shared in other angular bins)

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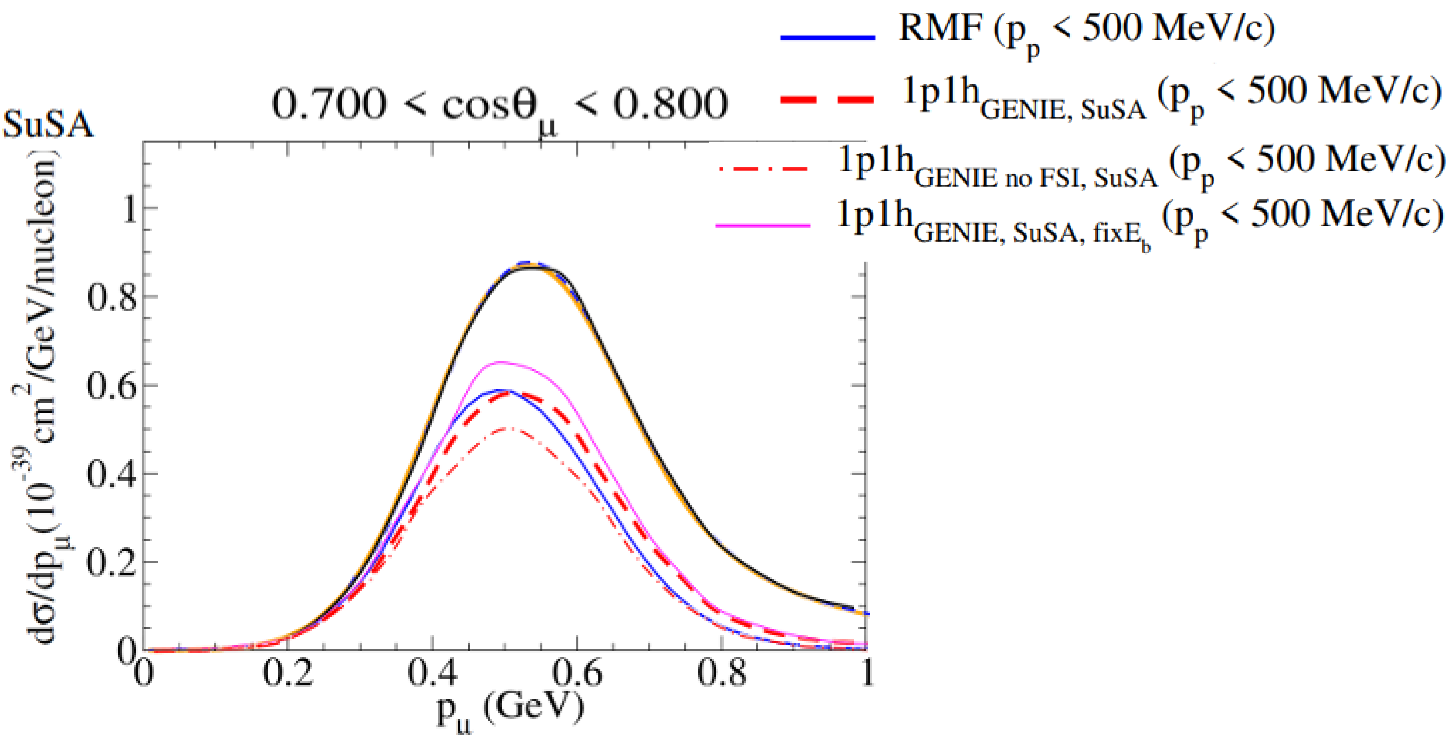


- Our SuSAv2 implementation uses a q_3 dependent removal energy
  - One step away from full factorisation
- If we use a fixed binding energy of ~25 MeV (common) then things don't look so good in the peak region

Summary

- Implemented the inclusive SuSAv2 1p1h and 2p2h models in GENIEv3 using hadron tensors.
- To get hadron kinematics, rely on the "factorization approximation" (FA) – as most models do
- FA has the potential to impact oscillation analyses. We have a poor idea of how valid it is.

This talk: **a first test of the factorisation approximation in GENIEv3**

- In a very simple test case it does surprisingly well!
- Still not perfect: how can we do better?

Further work

- Implement RMF initial nucleon momentum distribution into GENIEv3 and re-test
- Investigate how improved semi-classical approaches to FSI might help mitigate help the FA do better:
  - E.g. as in: arXiv:1902.05618
- Mitigate factorization:
  - Kinematic dependent Fermi motion / removal energy
  - One hadron tensor + spectral function per nuclear shell?
  - Better methods of calculating hadron kinematics?
- Avoid factorization:
  - Implement fully exclusive models?
  - Spectral function implementations sort of already do this

Discussion topics

Hadron tensor implementations

- What does the calculation of an xsec using a hadron tensor look like?
- How should this be implemented in the generators?
- Is this the same for 1p1h, 2p2h and pion production?
- What choices do have for making semi-inclusive predictions in the generators? How do we currently make these choices?

Factorization approximations

- Can we quantify the impact? Develop uncertainties to cover the difference?
- What are the possible biases from this for neutrino oscillation analyses?
- What can we learn about its validity from electron scattering data? (E.g. to what extent does the missing energy and momentum depend on the kinematics?)
- What can we measure in neutrino scattering to test this (transverse imbalance as a function of lepton kinematics?)

Factorization mitigation

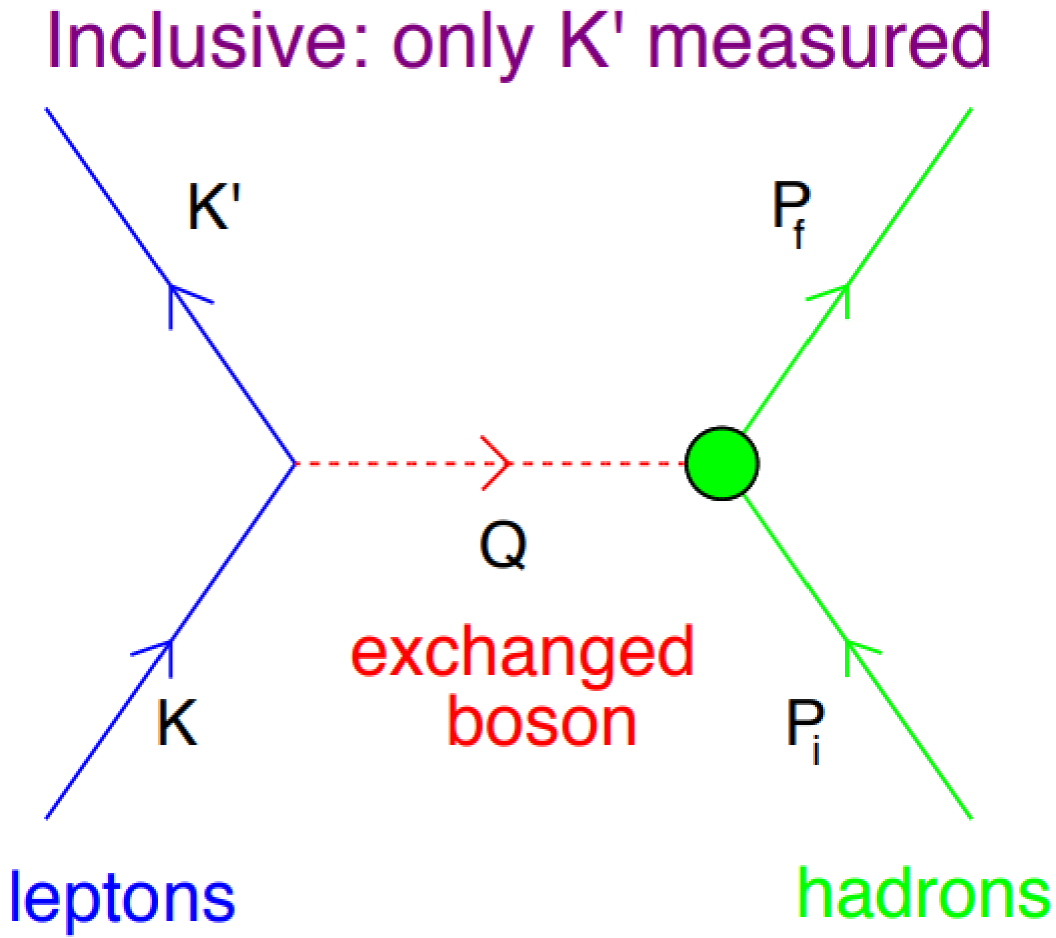
- Can we simply implement full semi-inclusive calculations directly?
  - Would probably require a new paradigm for event generation
  - 15 vs 5 nuclear responses – is this too hard or too slow?
  - Did we already do this for electron scattering? Were models for e,e'p fully exclusive?
- Even if we do this, how should we treat FSI?
- SF models are a bit different – are they immune to factorisation issues?
- Can we use some information from semi-inclusive predictions to make better choices in the factorisation scheme?
- Can we implement separate hadron tensors and spectral functions for each shell?

Bonus topic: What can we learn from LHC experiences? Can they tell us how far we can go in complexity in our MC generators and what tricks that we can use to do so?

Inclusive

E.g. CCInclusive

Measure only muons, don't care what happened on the hadronic side

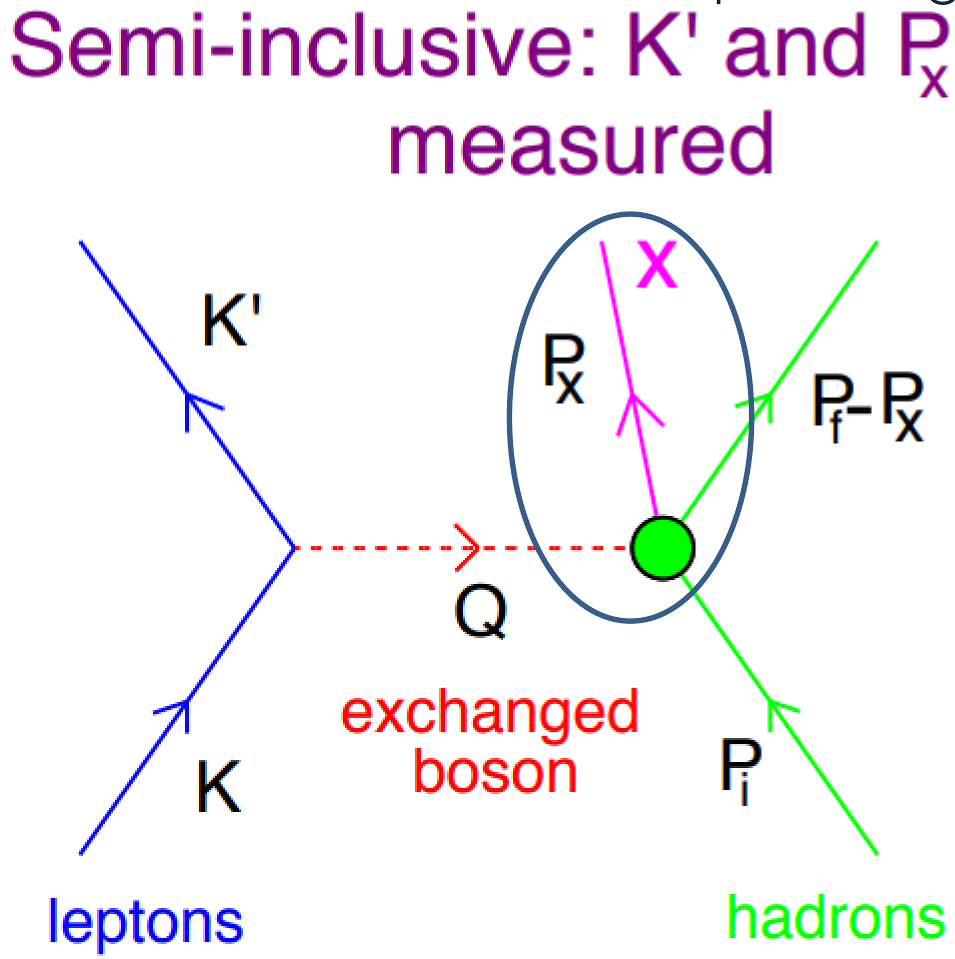


https://indico.ectstar.eu/event/19/contributions/221

Semi-inclusive

E.g. CC0π+Np

Measure muons and highest momentum proton, ignore the rest



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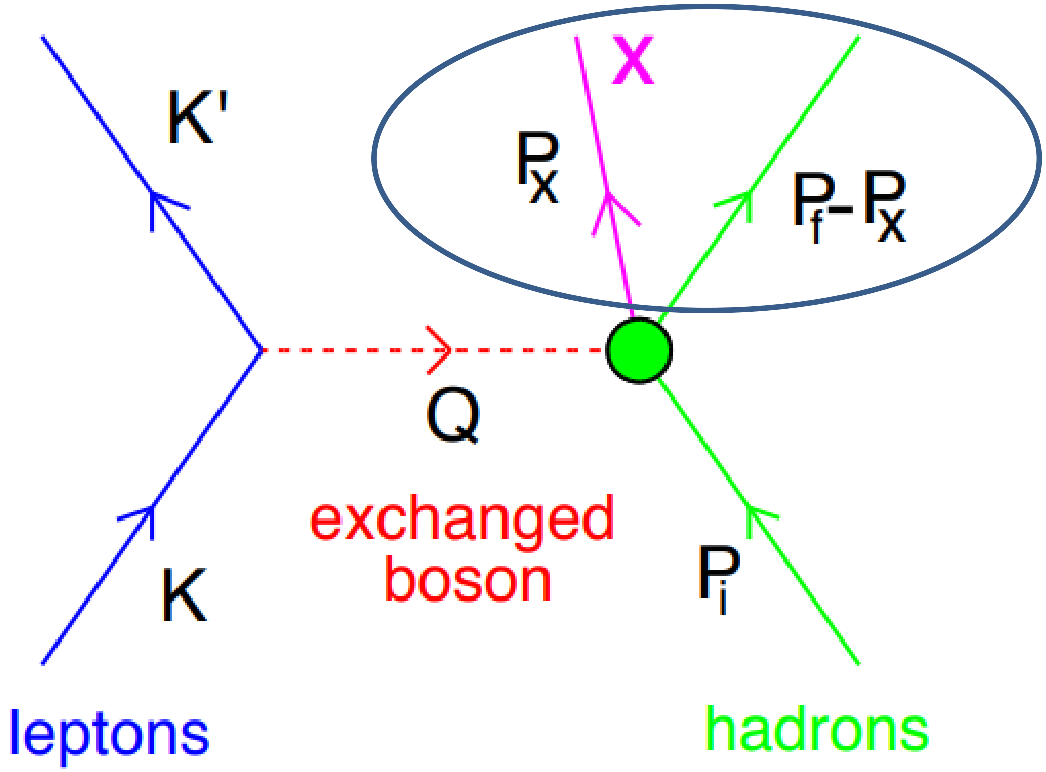
Exclusive

E.g. CC0π+1p

Measure muons only for interactios with exactly one final state proton

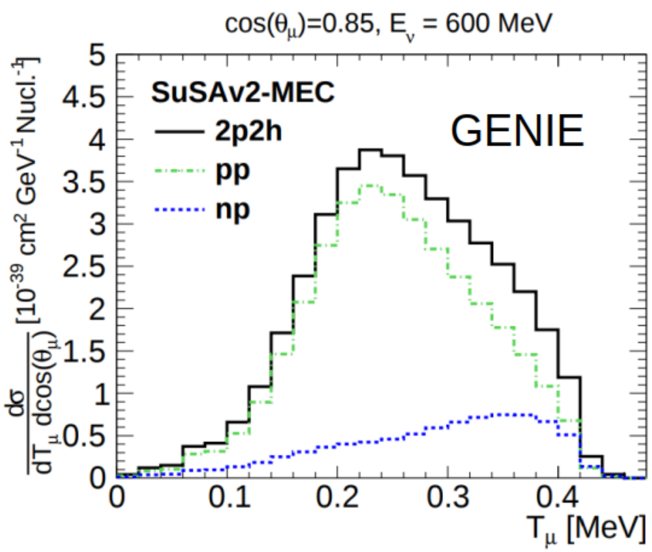
Measure everything in one particular final state

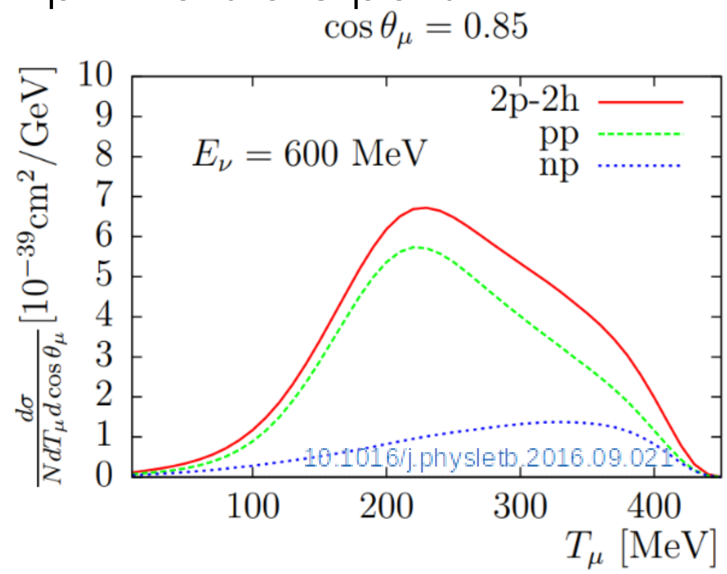


Only possible to measure with a perfect detector (no phase-space restrictions)

Implementation – hadron pairs

- Use a separate hadron tensor for only np initial state pairs





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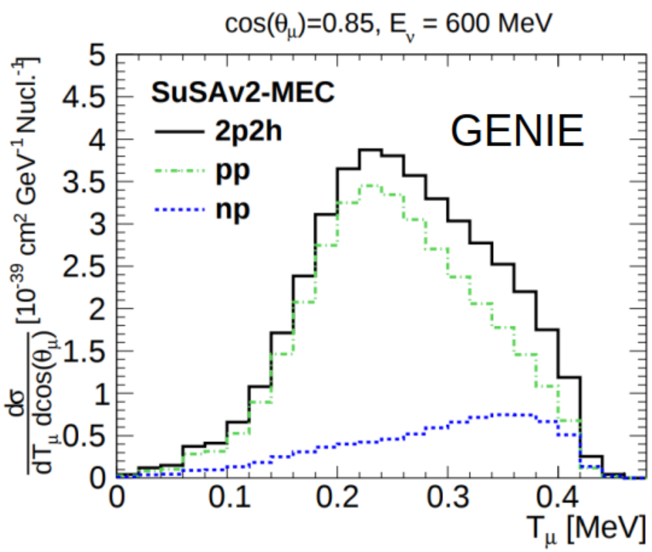

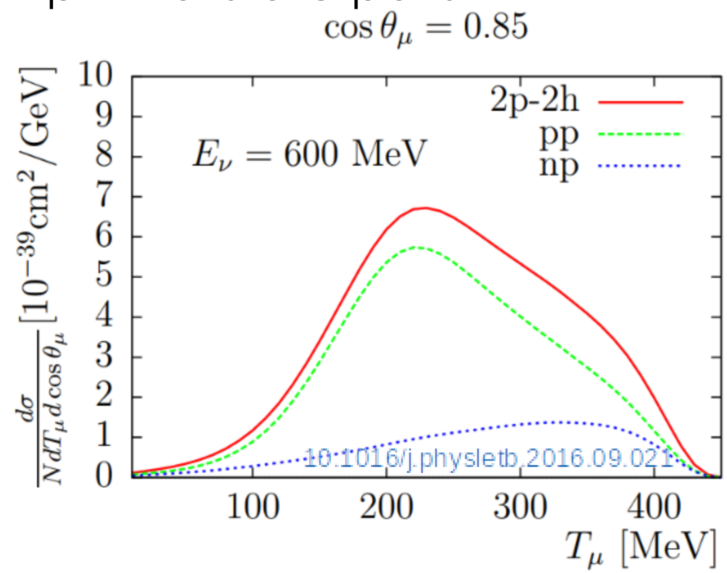

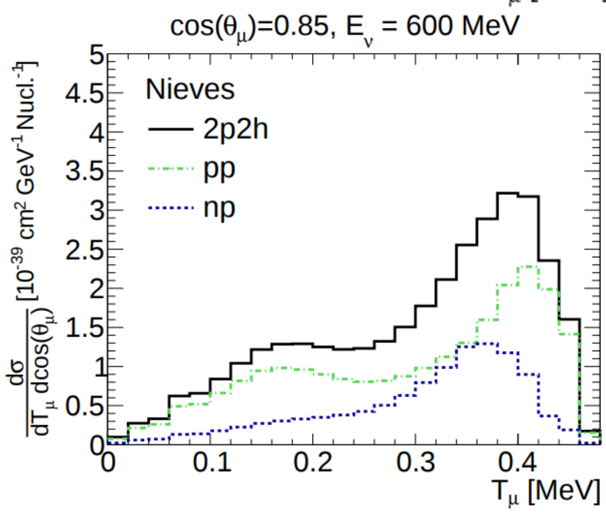

- Very different final state nucleon pair distribution than in Nieves model
- Possibly quite significant effect for calorimetric energy reconstruction

FA in SuSAv2 implementations

Xsec calculation:

- Perform inclusive calculation (q_0, q_3) using SuSAv2 hadron tensor

Hadronic side: more complicated …
Bold/*italics* **bits represent very**/*slightly* **questionable physics** in the FA, but this is no worse than in most other model implementations.

- Draw target nucleon from chosen nuclear model **irrespective of q_0, q_3**
- Get removal energy from RMF-like treatment, *re-throw from nuclear model if nucleon is Pauli blocked*
- Transfer all of ω, q to nucleon, **none to remnant**
- Subtract removal energy, **put proton on-shell with adjustment of p** (only needed for 1p1h) **then conserve momentum by adjusting remnant kinematics**
- *Do FSI cascade and rest of interaction using standard GENIE methods*

Factorisation Approximation (FA)

Assumptions to get hadron kinematics from inclusive models:

- *Inclusive interaction kinematics are independent of the initial-state nucleon kinematics and binding energy*
- *Semi-classical FSI-cascade, unrelated to the inclusive model's nuclear dynamics*
- *Simplistic treatment of energy(/momentum) transfer to the nuclear remnant (treatment depends on generator)*

More general than just the impulse approximation

But without semi-inclusive model predictions, we can't do much better…

Why do I care (oscillations)

Inclusive interaction kinematics are independent of the initial-state nucleon kinematics and binding energy

Simple example of what I mean:

- Consider a model where deeper nucleons have a larger maximum momentum p_{max} and larger E_b (LFG or a shell model)
- In reality we might expect higher q_0 would allow interactions with deeper nucleons. So the p_{max} and E_b will depend on q_0.
- But our choice of p in generators is entirely factorized from q_0

- If the Fermi motion and binding energy sampled depends on the initial-state nucleon kinematics then reconstruction of the neutrino energy will be affected → impact on neutrino oscillation analyses

Why do I care (oscillations)

Inclusive interaction kinematics are independent of the initial-state nucleon kinematics and binding energy

For T2K/HK we can mock this effect up with a simple toy:

- Let's say (arbitrarily) the real Fermi motion/E_b behaves more like an SF for low q_0 and like an LFG at higher q_0
- We can compare the $E_\nu^{CCQE} - E_\nu^{true}$ for this mixed model with what we get for a pure LFG