

SuSAv2-MEC model: main features, implementation in generators and further works

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*Testing and Improving Models of Neutrino Nucleus Interactions in Generators,
Plenary sessions, ECT*, 3 June 2019*

- 1 General Introduction
 - SuperScaling Approach: SuSAv2 and RMF models
 - Comparison with CC ν_{μ} -nucleus experimental data
- 2 SuSAv2-MEC implementation in GENIE
 - Implementation of SuSAv2-MEC in GENIE
 - Validation of the implementation and Data comparison
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- 3 Further works and Next Steps
 - Low-energy effects and scaling violations
 - ED-RMF vs. SuSAv2
 - Treatment of the Δ propagator in 2p2h and Δ decay width

Challenges and open questions for neutrino interaction models

- 1 Are current theoretical models (CRPA, Valencia LFG+2p2h, Benhar's SF, SuSAv2-MEC, RGF, etc.) good enough to analyze 1p1h and 2p2h channels in CC inclusive neutrino interactions?
- 2 Can we extend these models to semi-inclusive ν reactions?
- 3 What is the physics behind these models?
- 4 Can these models also reproduce inclusive (e, e') data and semi-inclusive $(e, e'p)$ processes?
- 5 Is it possible to introduce sophisticated microscopic models in generators in a fully consistent way?

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SuperScaling Approach (SuSA)

(see [G.D. Megias' Thesis](#) for details)

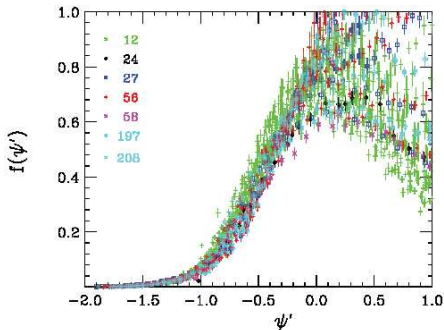
► The analysis of the large amount of existing (e, e') data at different kinematics is a solid benchmark to **test** the validity of theoretical models for neutrino reactions as well as to study the nuclear dynamics. The **SuperScaling Approach** exploits **universal features** of lepton-nucleus scattering to connect the two processes.

In inclusive QE scattering we can observe:

- ☆ Scaling of 1st kind (independence on q)
- ☆ Scaling of 2nd kind (independence on Z)



SuperScaling



$$f(\psi) \equiv f(q, \omega) \sim \frac{\sigma_{QE}(\text{nuclear effects})}{\sigma_{\text{single nucleon}}(\text{no nuclear effects})}$$

$$f(\psi') = k_F \frac{\left(\frac{d^2\sigma}{d\Omega_e d\omega} \right)_{\text{exp}}}{\sigma_{\text{Mott}}(v_L G_L^{ee'} + v_T G_T^{ee'})}$$

Good superscaling behavior at $\psi' < 0$ (below QE peak). At higher kinematics (ψ'), other contributions beyond QE and IA (2p2h, Δ , etc.) can play an important role and scaling is broken.

Separate L/T scaling functions

(see [G.D. Megias' Thesis](#) for details)

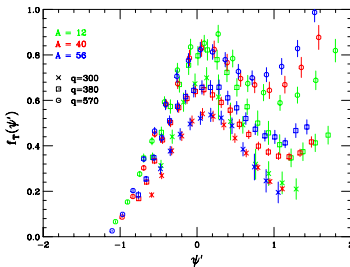
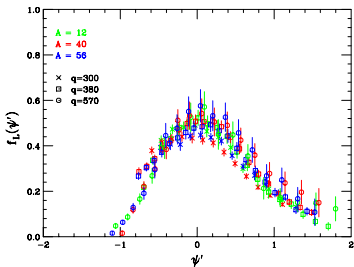
➡ The analysis of the large amount of existing (e, e') data at different kinematics is a solid benchmark to **test** the validity of theoretical models for neutrino reactions as well as to study the nuclear dynamics. The **SuperScaling Approach** exploits **universal features** of lepton-nucleus scattering to connect the two processes.

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SuperScaling

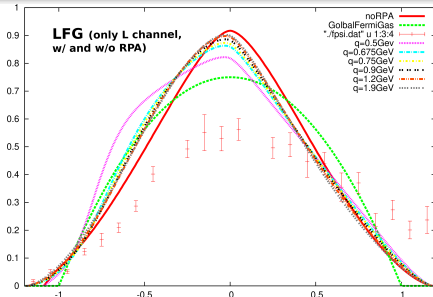
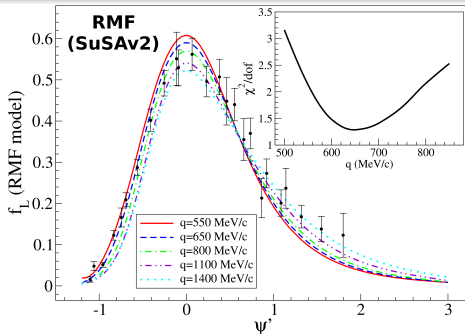


$$f_L = k_F R_L / G_L$$

$$f_T = k_F R_T / G_T$$

Scaling violations in the T channel ⇒
 2p-2h MEC, correlations, Δ -resonance
 ⇒ Mainly transverse

Testing SuperScaling for $^{12}\text{C}(e, e')$ in different nuclear models



The SuSAv2 model

PRC90, 035501 (2014)

PRD94, 013012 (2016)

★ **SuSAv2 model:** lepton-nucleus reactions addressed in the **SuperScaling Approach** and based on **Relativistic Mean Field (RMF)** theoretical scaling functions (FSI) to reproduce nuclear dynamics.

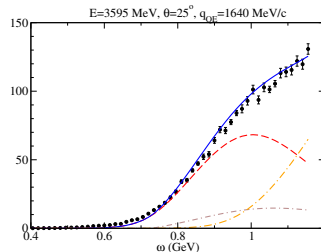
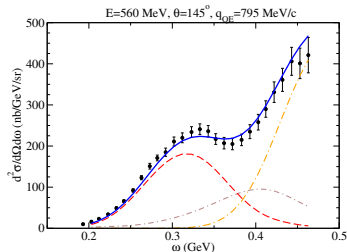
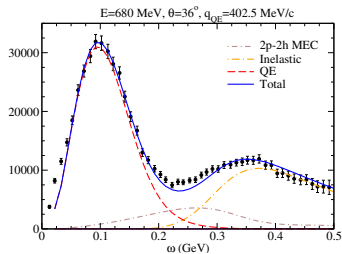
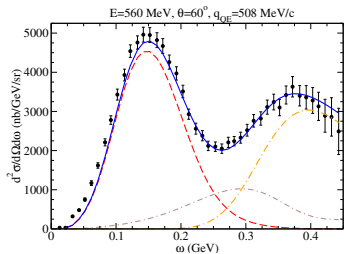
★ **RMF:** Good description of the QE (e, e') data and **superscaling properties** ($f_{L,exp}^{ee'}$).

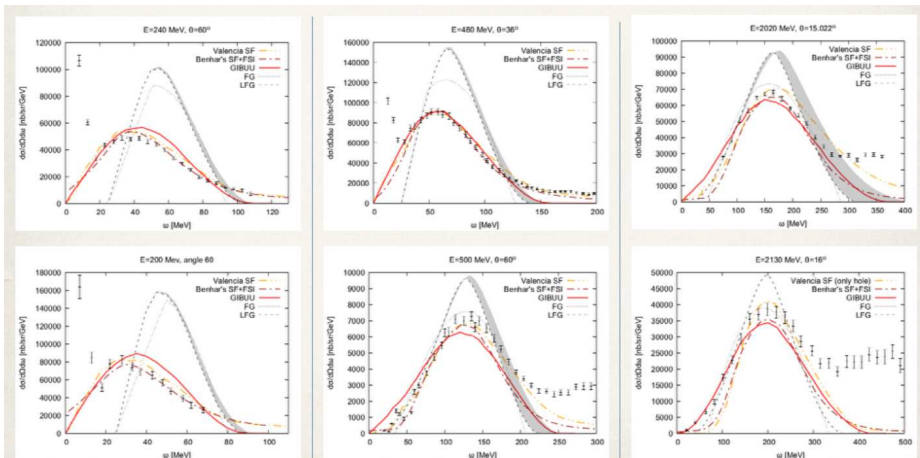
RMF predicts $f_T > f_L$ ($\sim 20\%$) as a pure relativistic effect (FSI with the residual nucleus).

Strong RMF potentials at high q_3 are corrected by RPWIA and q-dependent blending function.

Inclusive $^{12}\text{C}(e, e')$ cross sections

PRD 94, 013012 (2016)



Inclusive $^{12}\text{C}(e, e')$ cross sections with different models (J.Sobczyk's talk at NUINT18)

We plot INCLUSIVE electron scattering data and theoretical models for the QE mechanism.

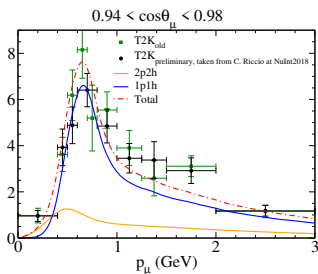
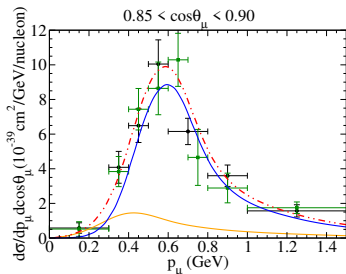
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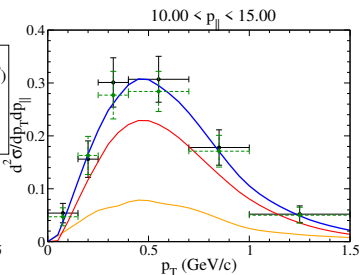
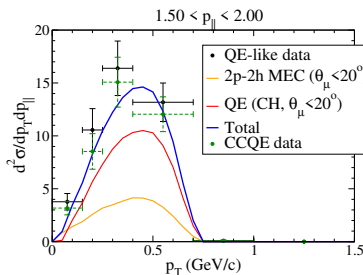
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Comparison with CC0 π data

T2K CC0 π ν_μ -C₈H₈



MINER ν A $\bar{\nu}_\mu$ -CH

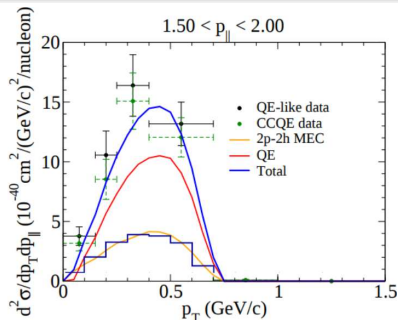
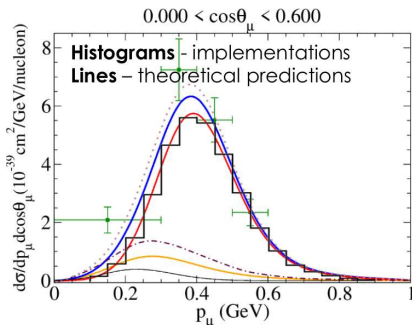


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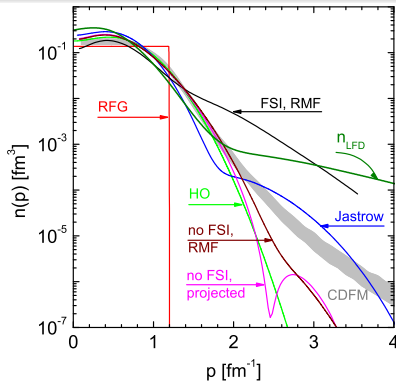
SuSAv2-MEC implementation in GENIE arXiv:1905.08556

- Implemented the SuSAv2 1p1h and 2p2h models in GENIEv3 for both (e, e') and CC ν_μ scattering. Now undergoing final validations and first physics studies before official release. **Next step: Implementation in NEUT.**
- New 1p1h and 2p2h model calculated using pre-computed hadron tensors for (e, e') and CC ν reactions. The hadron tensor elements are stored in tables which specify q_0 and q_3 in bins of 5 MeV between 0 and 2 GeV (no limits). Implementation of the hadron tensor components using the SuSA formalism (Rosenbluth-like decomposition: L and T components, V and A channels).
- Global factor / lepton tensor are easily calculated - shared by other models
- Use a GENIE's bilinear interpolation function to evaluate specific q_0, q_3 values
- Hadron tensors will be initially provided for **a few targets** (C and O so far, may add others). **Can easily scale to other nuclei.**



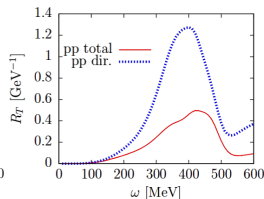
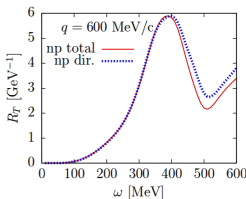
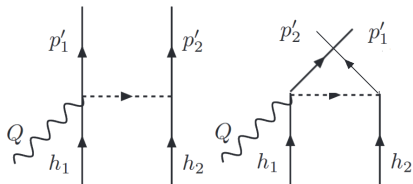
1p1h implementation: RMF and SuSAv2

- ▶ **1st step:** Implementing SuSAv2 hadron tensor $W^{\mu\nu}(q, \omega)$ + LFG on the top and comparison with original SuSAv2 model (**short term, already done**)
- ▶ **2nd step:** Adding SuSAv2 formulas, parameters and parametrization of scaling functions into GENIE to speed up simulations and to allow reweighting (**mid term**)
- ▶ **3rd step:** Introducing RMF nucleon momentum distribution in GENIE to fully test factorization approach (**mid term**)



2p-2h MEC for (e, e') and CC ν reactions PRD91, 073004 (2015)

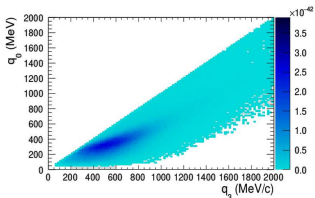
Other 2p2h models neglect direct/exchange interference terms \Rightarrow **strongly affects np/pp ratio by a factor ~ 2** (PRC94:054610,2016) \Rightarrow **Implications in nucleon multiplicity and hadron E_{reco}**



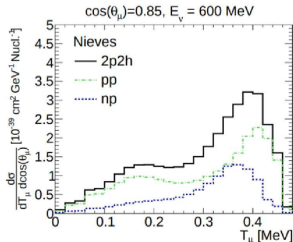
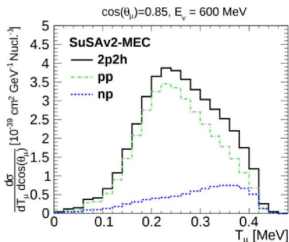
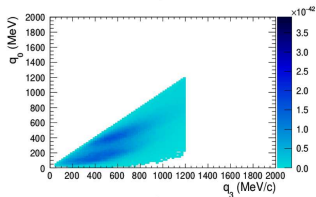
- ★ **Accurate implementation of np/pp pairs for the 2p2h channel using separate hadron tensors for np and pp pairs.**
- ★ The numerical evaluation of the hadronic tensor $W_{2p2h}^{\mu\nu}(R_K^{2p2h})$ is **performed in the RFG model** in a fully relativistic way without any approximation. It can be easily extended to all nuclei.
- ★ Separation into pp , nn and np pairs in the FS \Rightarrow also valid for $N \neq Z$ (^{40}Ar , ^{56}Fe , ^{208}Pb)
- ★ It is **computationally non-trivial** and involves 7D integrals of thousands of terms (+1 for ν -flux) \Rightarrow High increase of the computing time of $R_K^{2p2h} \Rightarrow$ **Parametrization/Implementation**

Comparison of SuSAv2-MEC^{Genie} with Nieves^{Genie} 2p2h arXiv:1905.08556

New SuSAv2 implementation



Nieves implementation



Differences in np/pp separation are mostly related to the treatment of 2p2h direct/exchange interference terms (absent in Nieves model) \rightarrow strongly affects np/pp ratio by a factor ~ 2 (PRC94:054610,2016) \Rightarrow Implications in nucleon multiplicity and hadron E_{reco}

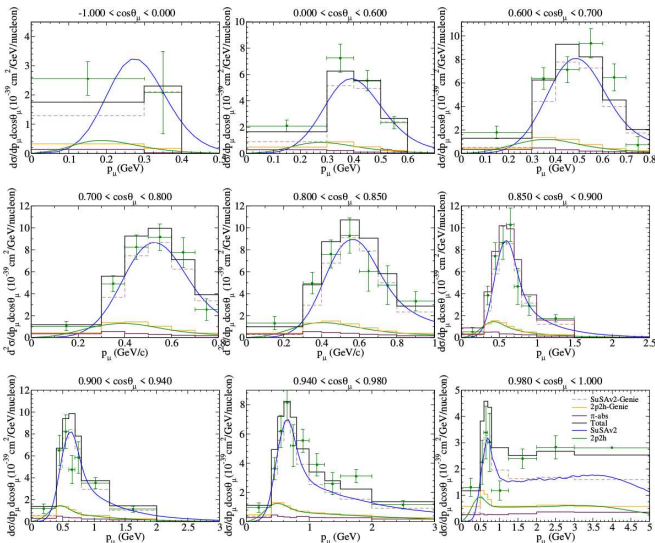
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SuSAv2-MEC implementation in GENIE: Validation plots (T2K CC0 π)

T2K CC0 π ν_μ - ^{12}C data vs. SuSAv2-MEC_{GENIE}

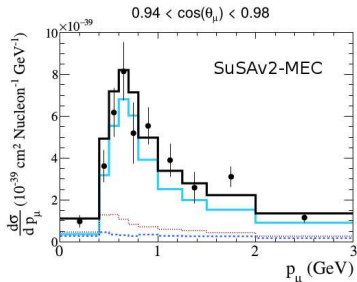
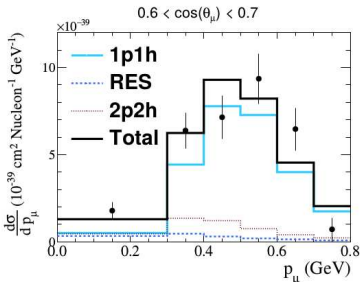
$\chi^2 = 255.8$ (67 bins)



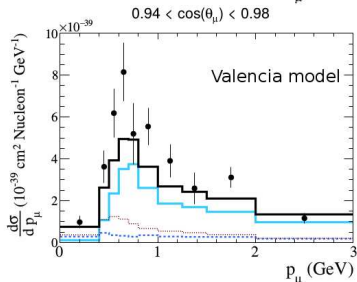
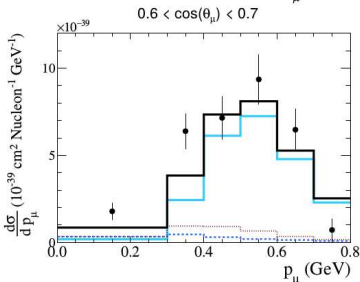
Comparison between 1p1h+2p2h models in generators *arXiv:1905.08556*

T2K CC0 π ν_μ - ^{12}C inclusive data

SuSAv2-MEC



Nieves

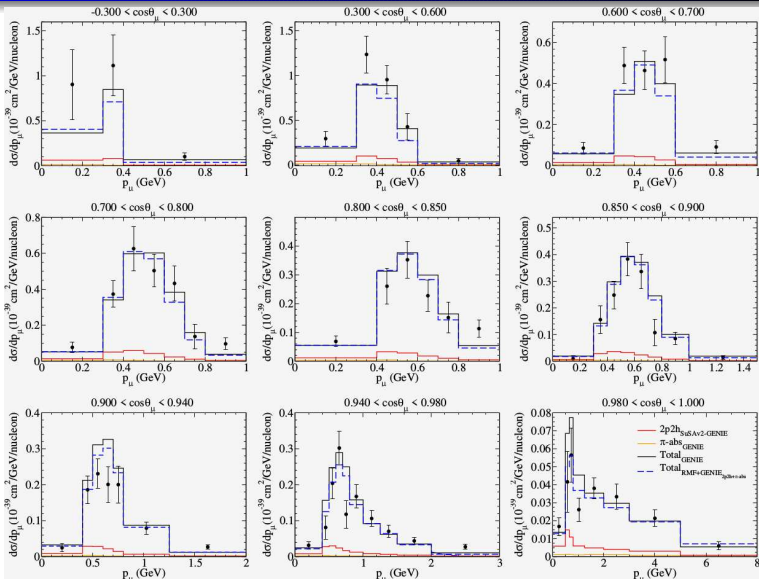


SuSAv2-MEC in GENIE: Validation plots (T2K CC0 π N p , $0p > 500$ MeV)

T2K CC0 π ν_μ - ^{12}C semi-semi-inclusive data ($0p > 500$ MeV)

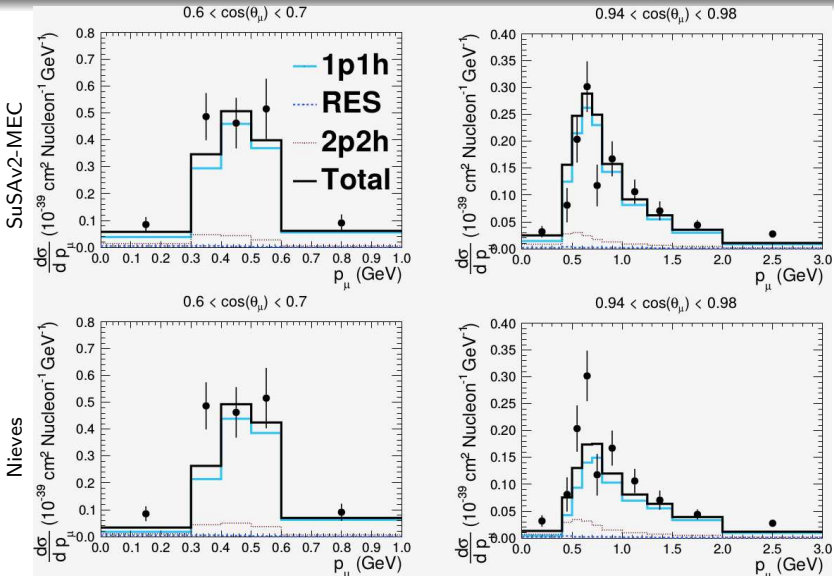
$$\chi^2_{\text{SuSAv2-MEC GENIE}} = 168.92 \text{ (60 bins)}$$

$$\chi^2_{\text{RMF+GENIE(SuSAv2-2p2h+pi-abs)}} = 171.87$$



Comparison between 1p1h+2p2h models in generators *arXiv:1905.08556*

T2K CC0 π ν_μ - ^{12}C semi-inclusive data ($0p > 500$ MeV)



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Different models can give similar inclusive CS but different semi-inclusive ones (more sensitive to nuclear-medium effects) \Rightarrow very different ν oscillation analyses (which relies on semi-inclusive predictions)

PROBLEM: Current lack of full semi-inclusive models and proper implementation in generators.

Semi-inclusive \Rightarrow Inclusive (but not viceversa) \Rightarrow Factorization approach is questionable.

- QE and 2p2h inclusive: We only need $W^{\mu\nu}(q, \omega)$ or, equivalently, $W^{\mu\nu}(p_\mu, \cos \theta_\mu)$

- QE semi-inclusive : 5D diff. CS $(\theta_\mu, p_\mu, p_N, \theta_N, \phi_N)$ - 2p2h semi-inclusive: 9D diff. CS.

Double differential **inclusive** cross section

$$\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$$

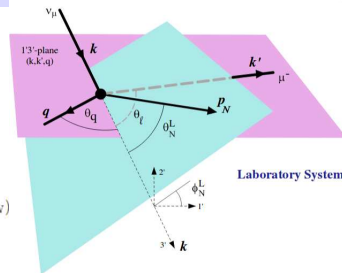
$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu} \right]_\chi = \sigma_0 \left(V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_{TT}R_{TT} + \chi \left[2V_{T'}R_{T'} \right] \right)$$

Double differential **semi-inclusive** cross section

$$\chi = +(-) \equiv \nu_\mu(\bar{\nu}_\mu)$$

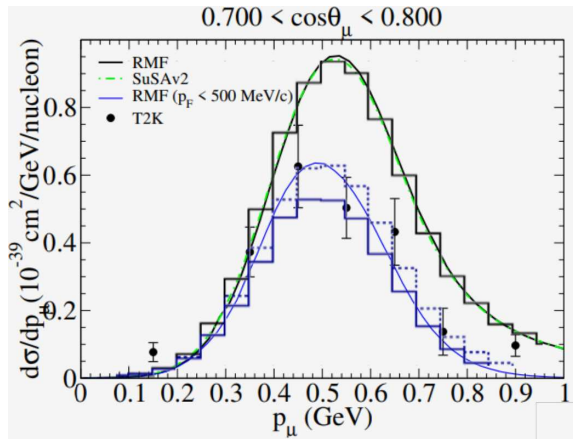
$$\frac{d\sigma}{dk'_\mu d\Omega'_\mu dp_N^2 d\Omega'_N} = \frac{G^2 \cos^2 \theta_c m_N k'^2 \varepsilon p_N^2 W_{A-1}^{v_0}}{2(2\pi)^5 k \varepsilon' E_N \sqrt{X_B^2 + m^2 a_B}} \mathcal{F}_\chi^2 \delta(k - k_0),$$

$$\begin{aligned} \mathcal{F}_\chi^2 = & \hat{V}_{CC}(w_{CC}^{VV(I)} + w_{CC}^{AA(I)}) + 2\hat{V}_{CL}(w_{CL}^{VV(I)} + w_{CL}^{AA(I)}) + \hat{V}_{LL}(w_{LL}^{VV(I)} + w_{LL}^{AA(I)}) \\ & + \hat{V}_T(w_T^{VV(I)} + w_T^{AA(I)}) + \hat{V}_{TT} \left[(w_{TT}^{VV(I)} + w_{TT}^{AA(I)}) \cos 2\phi_N + (w_{TT}^{VV(II)} + w_{TT}^{AA(II)}) \sin 2\phi_N \right] \\ & + \hat{V}_{TC} \left[(w_{TC}^{VV(I)} + w_{TC}^{AA(I)}) \cos \phi_N + (w_{TC}^{VV(II)} + w_{TC}^{AA(II)}) \sin \phi_N \right] \\ & + \hat{V}_{TL} \left[(w_{TL}^{VV(I)} + w_{TL}^{AA(I)}) \cos \phi_N + (w_{TL}^{VV(II)} + w_{TL}^{AA(II)}) \sin \phi_N \right] \\ & + \chi \left[\hat{V}_T w_T^{VA(I)} + \hat{V}_{TC} (w_{TC}^{VA(I)} \sin \phi_N + w_{TC}^{VA(II)} \cos \phi_N) + \hat{V}_{TL} (w_{TL}^{VA(I)} \sin \phi_N + w_{TL}^{VA(II)} \cos \phi_N) \right] \end{aligned}$$



Testing the factorization approach on $CC0\pi Np$ T2K data

Comparison of RMF “semi-semi-inclusive” prediction and GENIE SuSAv2 implementation to T2K data (μ kinematics with restriction of $p_{proton} < 500$ MeV/c).



Curves - theory

Histograms - GENIE

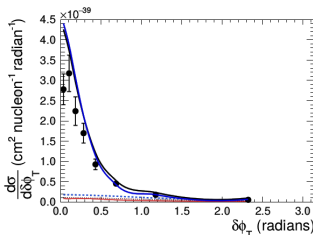
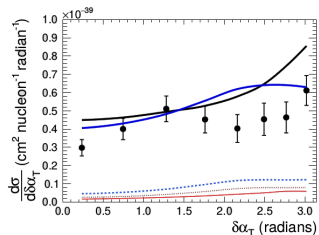
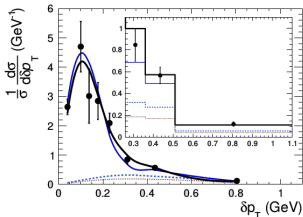
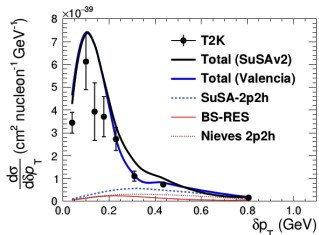
Blue: With cut in p_{proton}

Dotted line - no FSI in GENIE

Factorization approach does not seem a bad approximation for semi-semi-inclusive analysis (SuSAv2 + LFG on the top (Genie) vs. RMF code. To be done with RMF on the top).

What about more semi-inclusive measurements?

Comparison of semi-inclusive T2K STV data with SuSAv2-MEC^{Genie} + BS π abs and Valencia model + BS π abs (*arXiv:1905.08556*). Goodness of fit: For δp_T : $\chi^2_{SuSA} = 20.5$, $\chi^2_{Valencia} = 27.1$. For $\delta\alpha_T$: $\chi^2_{SuSA} = 45.3$, $\chi^2_{Valencia} = 31.4$. For $\delta\phi_T$: $\chi^2_{SuSA} = 40.1$, $\chi^2_{Valencia} = 36.8$.



Work in progress to test the factorization approach in semi-inclusive measurements when RMF momentum distribution is implemented. See S. Dolan's talk

Summary and Conclusions

What we can do:

- Valid for all nuclear targets
- No kinematical restrictions
- Works for CC neutrino and electron scattering
- Minimal calculation on the fly - fast

What we can't yet do:

- Input model is inclusive (for the moment) \Rightarrow hadron kinematic predictions are ad-hoc (like all current GENIE models)
- Implementation of RMF momentum distribution soon. Long-term: 1st implementation of a full semiinclusive formalism to produce all together lepton and hadron kinematic predictions. No factorization approach.
- Initial implementation does not allow to alter systematic parameters such as M_A^{QE} , k_F , binding energy effects, etc.

SuSAv2-MEC implementation is therefore useful as:

- Theory derived from microscopic model, predictions are significantly different from other models.
- A theory-driven mock-data studies for physics analyses.
- For model comparisons to neutrino scattering data.
- Phenomenological studies.

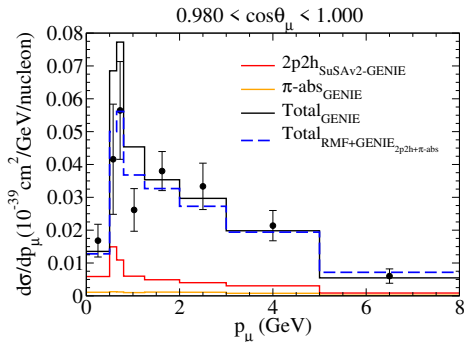
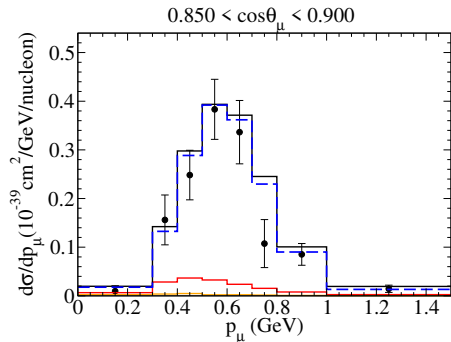
➤ Since current model is non-reweightable it cannot yet be used to calculate systematic uncertainties in physics analyses.

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Low-energy effects at T2K $CC0\pi 0p > 500$ MeV/c

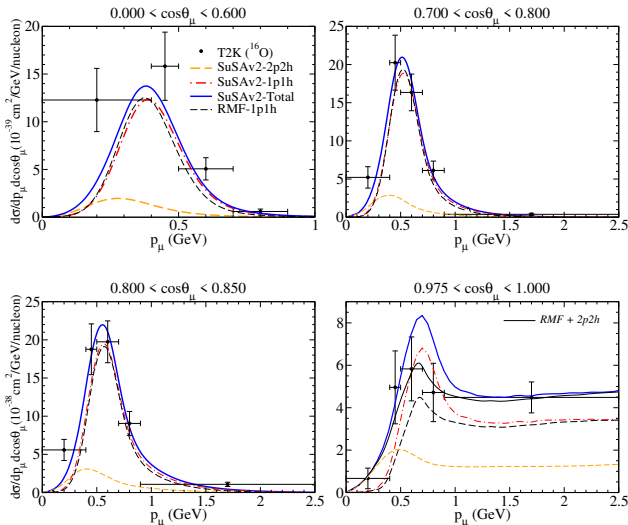
arXiv:1905.08556



Low-energy effects and scaling violations are only appreciable at very forward angles (low q_3 , q_0 values). RMF is more accurate than SuSAv2 at these kinematics.

T2K $CC0\pi \nu_\mu - H_2O$ cross sections

arXiv:1711.00771 [nucl-th] (2017)



Good comparison with T2K- ^{16}O data but some overestimations appear at very forward angles within the SuSAv2-MEC model \Rightarrow Possible RMF scaling violations at low q_0 , q_3 not completely included in the SuSAv2 formalism makes the model questionable at these kinematics.

Although RMF scaling functions are almost identical for $q_3 \gtrsim 400$ MeV/c, at very low q_3 they can differ (*scaling is broken*) \Rightarrow Solution: Determine and characterize low- q_3 RMF scaling functions to be added in the SuSAv2 formalism as well as in the implementation.

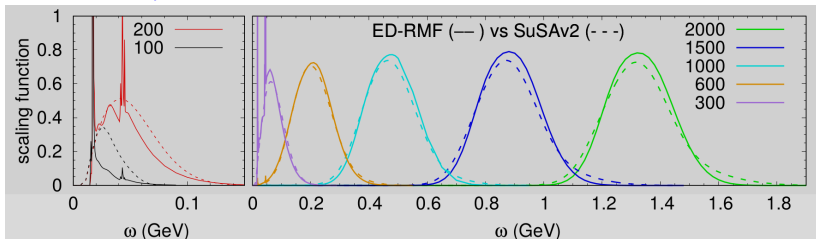
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RMF, ED-RMF and SuSAv2 models

arXiv:1904.10696

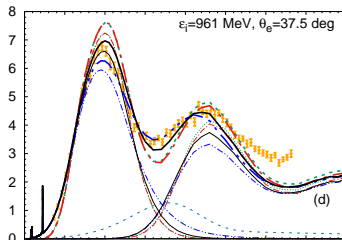
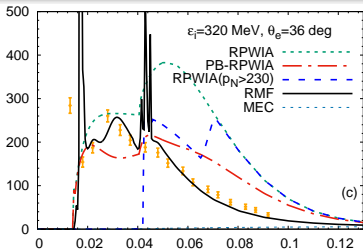
- ★ Scaling violations and low-energy effects present in RMF are not fully included in the SuSAv2-MEC model. **Solution:** **Parametrize and introduce low-q RMF effects in SuSAv2**
- ★ The issue of the strong q-dependence of RMF vector and scalar potentials at high kinematics is addressed by using a blending function to introduce RPWIA effects (no FSI) in the SuSAv2-MEC model, yielding a good representation of (e, e') and $CC0\pi$ data at intermediate and high kinematics. *To have a more consistent model and preserve orthogonality, unitarity and dispersion relations* \Rightarrow **Solution:** **ED-RMF (both inclusive and semi-inclusive for ^{12}C , ^{16}O , ^{40}Ar , etc.)**
- ★ The **ED-RMF** model introduces an Energy-Dependent potential (based on the SuSAv2 approach) to the RMF that keeps the strength for slow nucleons but makes the RMF potential softer for increasing nucleon momenta. **See A. Nikolakopoulos and R. Gonzalez talks for details**
- ★ SuSAv2 is a pure inclusive model. **Solution:** **ED-RMF (both inclusive and semi-inclusive for ^{12}C , ^{16}O , ^{40}Ar , etc.)**



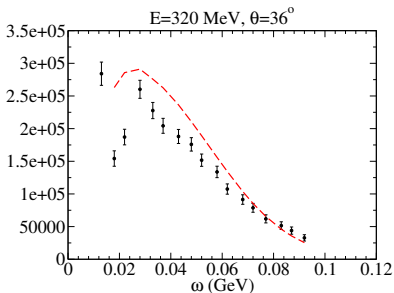
ED-RMF, RMF, SuSAv2 for $(e, e')^{12}\text{C}$

$d^2\sigma/d\Omega/d\omega$ vs. ω

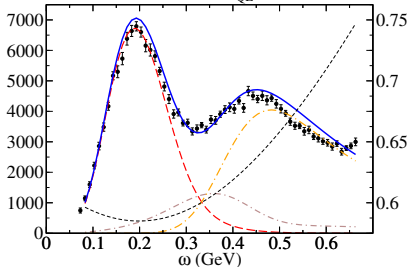
RMF, ED-RMF, others



SuSAv2



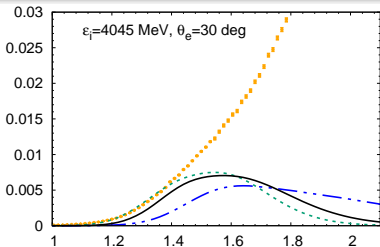
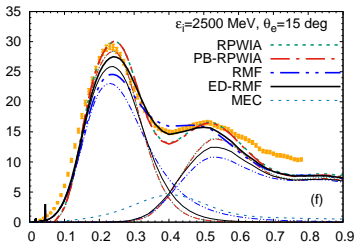
$E=961 \text{ MeV}, \theta=37.5^\circ, q_{QE}=585.8 \text{ MeV}/c$



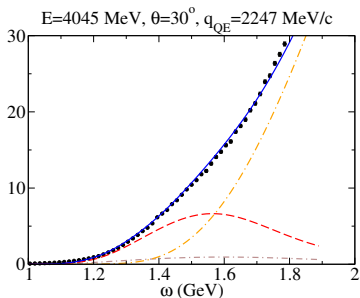
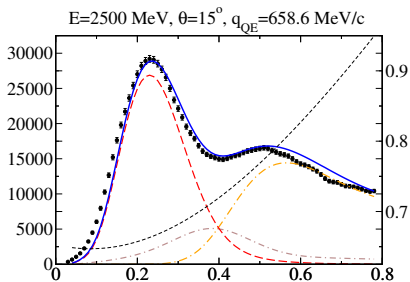
ED-RMF, RMF, SuSAv2 for $(e, e')^{12}\text{C}$

$d^2\sigma/d\Omega/d\omega$ vs. ω

RMF, ED-RMF, others



SuSAv2



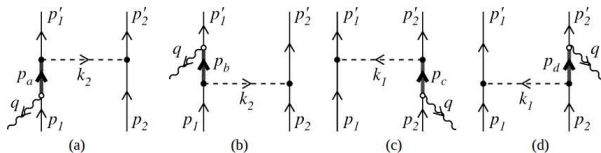
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Δ propagator in 2p2h and Δ decay width

(see also A. De Pace's talk)

- Our 2p2h-MEC model can produce semi-inclusive (e, e') and CC0 π results. Work in progress.
- An **open question** in 2p2h models (also in π production) is the **treatment of the Δ propagator** in 2-body currents and the **Δ decay width**. Taking only the real part of the propagator and assuming free Δ decay width is an approach taken by several 2p2h models and in our case it has resulted in a good empirical approach in very good agreement with (e, e') and CC0 π data.
- Next step (Solution?): Joint analysis of the 2p2h-MEC model (full propagator) and the ED-RMF (1p1h + π production)** in comparison with (e, e') data to infer a proper value of the Δ decay width with medium modifications.

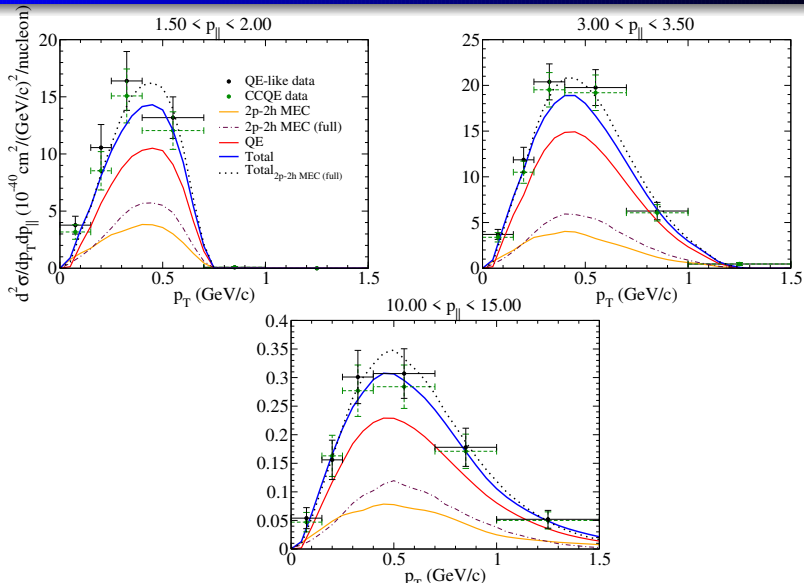


$$S_{\Delta, \alpha\beta} = \frac{-(\not{K}_{\Delta} + M_{\Delta})}{K_{\Delta}^2 - M_{\Delta}^2 + iM_{\Delta}\Gamma_{\text{width}}} (\dots)_{\alpha\beta}$$

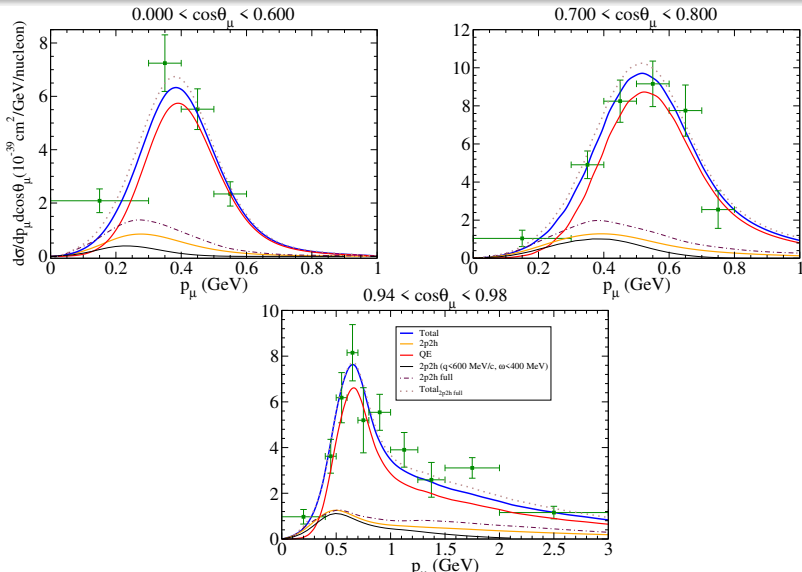
$$\Gamma_{\text{width}}^{\text{free}} \longrightarrow \Gamma_{\text{width}}^{\text{in-medium}} = \Gamma_{\text{Pauli}} - 2\Im(\Sigma_{\Delta})$$

$$M_{\Delta}^{\text{free}} \longrightarrow M_{\Delta}^{\text{in-medium}} = M_{\Delta}^{\text{free}} + \Re(\Sigma_{\Delta}) .$$

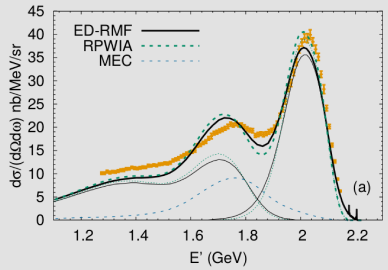
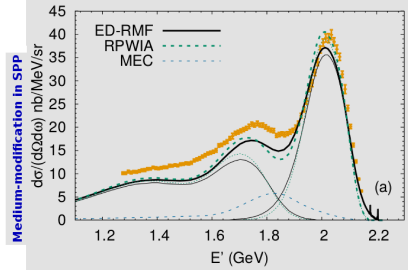
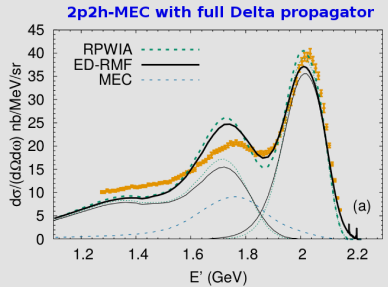
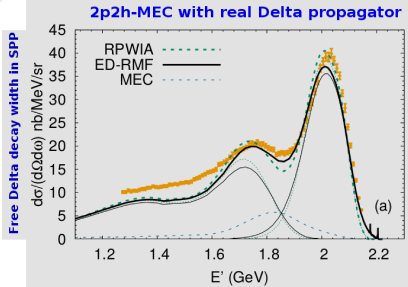
2p2h full/real Δ prop. vs MINERvA CC0 π $\bar{\nu}$ -CH data



2p2h full/real Δ prop. vs T2K CC0 π ν - ^{12}C data

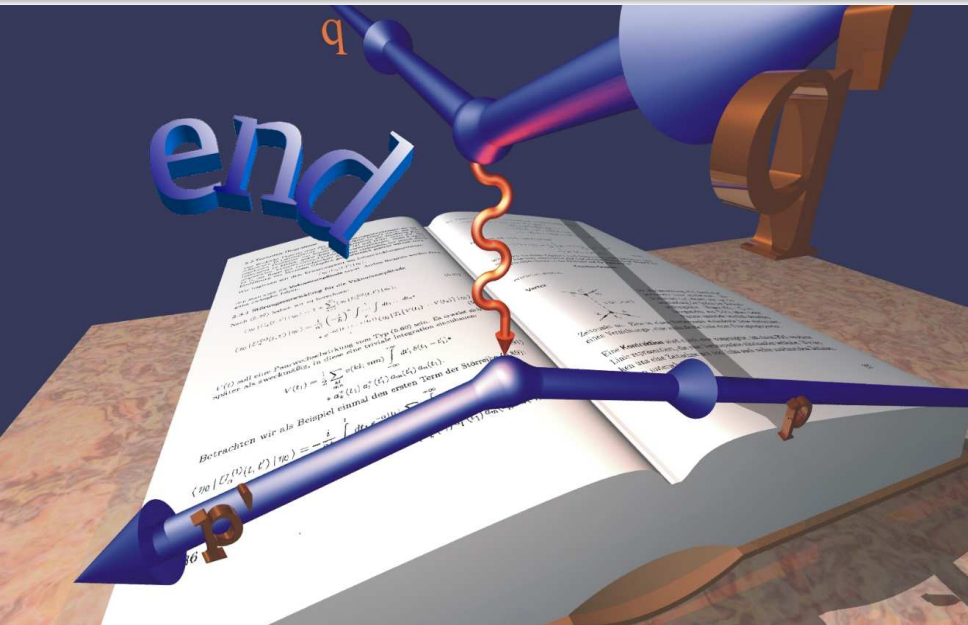


2p2h full/real Δ prop. vs JLab $(e, e')^{12}\text{C}$ data



Collaborators

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- Juan A. Caballero (University of Seville, Spain)
- Maria B. Barbaro (INFN and University of Turin, Italy)
- Raúl González-Jiménez (University Complutense of Madrid, Spain)
- J. M. Udías (University Complutense of Madrid, Spain)
- Jose E. Amaro (University of Granada, Spain)
- I. Ruiz-Simó (University of Granada, Spain)
- Martin Ivanov (Bulgarian Academy of Sciences, Bulgaria)
- Anton Antonov (Bulgarian Academy of Sciences, Bulgaria)
- W. Van Orden (Old Dominion University, JLab, USA)



$$V(t) = \frac{1}{2} \sum_{\vec{k}} \omega(\vec{k}, \text{sm}) \int d\vec{x} \psi(\vec{x} - \vec{x}_i) \cdot$$

$$+ a_2^{\vec{k}}(t) a_2^{\vec{k}}(\vec{x}_i) \psi(\vec{x}_i) \psi(\vec{x}_i)$$

Betrachten wir als Beispiel einmal den ersten Term der Störpot.

$$\langle 0_0 | U_0^{(1)}(t, t_0) | 0_0 \rangle = -\frac{i}{\hbar} \int_{t_0}^t dt \int d\vec{x} \psi(\vec{x} - \vec{x}_i) \cdot$$