Relativistic mean field approach to lepton-nucleon scattering

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Motivation

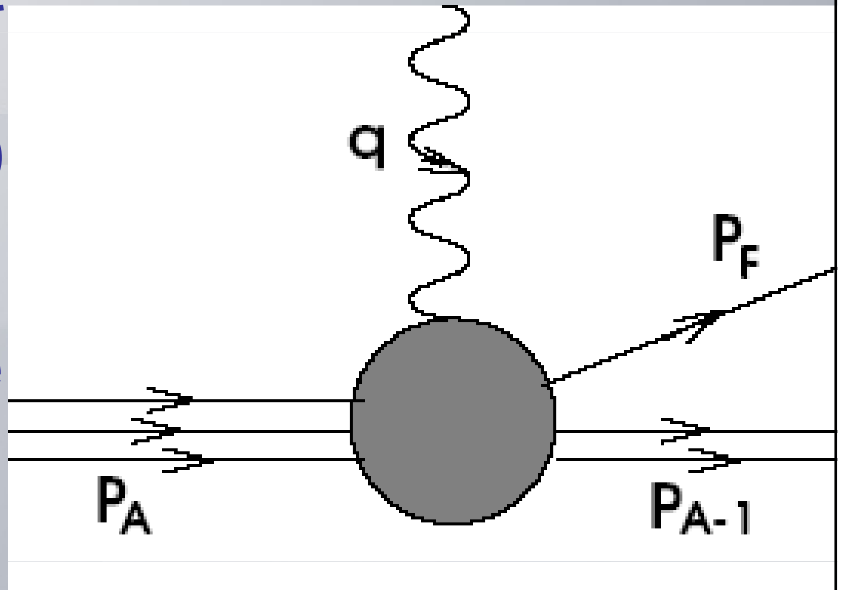
•Over the last years, the 'simple' relativistic impulse approximation has been employed successfully to describe inclusive (e,e') scattering at the quasielastic peak (K.S. Kim and L.E. Wright PRC 68 (2003) 027601, PRC 67, 054604 (2003) , Y. Jin, D.S. Onley and L.E. Wright, PRC 45 (1992) 1333, C. Maieron et al, PRC68 (2003) 048501)

•This is in spite of these relativistic models not exhibiting *explicit* correlations. All the calculations were done in a mean field level, that is, as if the nucleons were moving in an average potential, independently of one another

OVERVIEW OF THE MODEL (ingredients)

1) Weak interacting probe (e^-,ν_e ...). It allows for the simplest approach: single boson (photon, $W^\pm$,Z^0) exchange

2) Thus, the dependence on the kinematics of the exchanged boson can be extracted. For unpolarized and in plane electron scattering, this means:



$$\frac{d\sigma}{d\Omega_e d\varepsilon' d\Omega_F} = K\sigma_{Mott} f_{rec} \left[v_L R^L + v_T R^T + v_{TL} R^{TL} \cos\phi_F + v_{TT} R^{TT} \cos 2\phi_F \right]$$

$$\left(d\sigma^{Z^0/W^\pm}\right)_{\rm Free} = \delta^{(4)}(k_i^\mu - k_f^\mu + P_I^\mu - P_F^\mu)\, \sigma^{Z^0/W^\pm} \frac{1}{4\epsilon_f^2 E_I E_F}\, \omega_{\mu\nu} W^{\mu\nu}\, d^3\vec{P}_F d^3\vec{k}_f$$

The hadronic part does not need to be computed at every lepton kinematics

$$\sigma^{Z^0} = 16\, \epsilon_f^2 \cos^2(\theta/2) \left[\frac{g^2}{4\pi}\right]^2$$

$$\sigma^{W^\pm} = 16 k_f^2 \left[\frac{g^2}{4\pi}\right]^2$$

$$\begin{aligned}\omega_L W_L = \frac{1}{4\epsilon_i k_f} \Bigg\{ &\left[(\epsilon_i + \epsilon_f)^2 - |\vec{k}\,|^2 - m_l^2\right] |\rho|^2 \\ &+ \left[\frac{(\epsilon_i^2 - k_f^2)^2}{|\vec{k}\,|^2} - \omega^2 + m_l^2\right] |J_k|^2 \\ &- \left[\frac{2(\epsilon_i + \epsilon_f)(\epsilon_i^2 - k_f^2)}{|\vec{k}\,|} - 2\omega|\vec{k}\,|\right] Re\,(\rho^* J_k)\Bigg\}\end{aligned}$$

$$\begin{aligned}\omega_T W_T = \Bigg\{ &\frac{\epsilon_i k_f \sin^2\theta}{2|\vec{k}\,|^2} \cos(2\phi_F)\left(|J_{||}|^2 - |J_\perp|^2\right) \\ &+ \cancel{\left[\frac{\epsilon_i k_f \sin^2\theta}{2|\vec{k}\,|^2} - \frac{1}{2}\left(\frac{-\epsilon_f}{k_f} + \cos\theta\right)\right]} \left(|J_{||}|^2 + |J_\perp|^2\right)\Bigg\}\end{aligned}$$

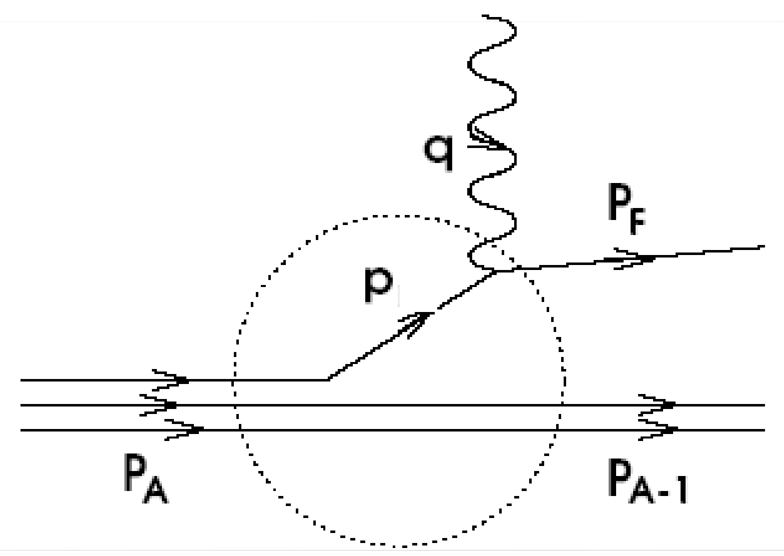
$$\omega_{TT'} W_{TT'} = -\frac{1}{|\vec{k}\,|}\left(\frac{\epsilon_i \epsilon_f}{k_f} + k_f - (\epsilon_i + \epsilon_f)\cos\theta\right) Im\,(J_{||} J_\perp^*)$$

L, T and TT' are the only responses that contribute if no nucleon is observed

OVERVIEW OF THE MODEL

A further simplification: Impulse Approximation

A weak probe will interact with similar probability with both surface nucleons or deep ones



For QE conditions and large q (a few hundreds of MeV), all nucleons contribute to the cross-section incoherently. The nuclear current is obtained as a sum over individual single-nucleon currents:

$$J^{\mu}_{N}(\omega,\vec{q}) = \int d\vec{p}\,\bar{\psi}_F(\vec{p}+\vec{q})\,\hat{J}^{\mu}_{N}(\omega,\vec{q})\,\psi_B(\vec{p})$$

one-body current

$$J_N^\mu(\omega, \mathbf{q}) = \overline{\sum_I} \sum_F \delta(E_F - E_I - \omega) \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} < \Psi_A^F | \hat{J}_N^\mu | \Psi_A^I >$$

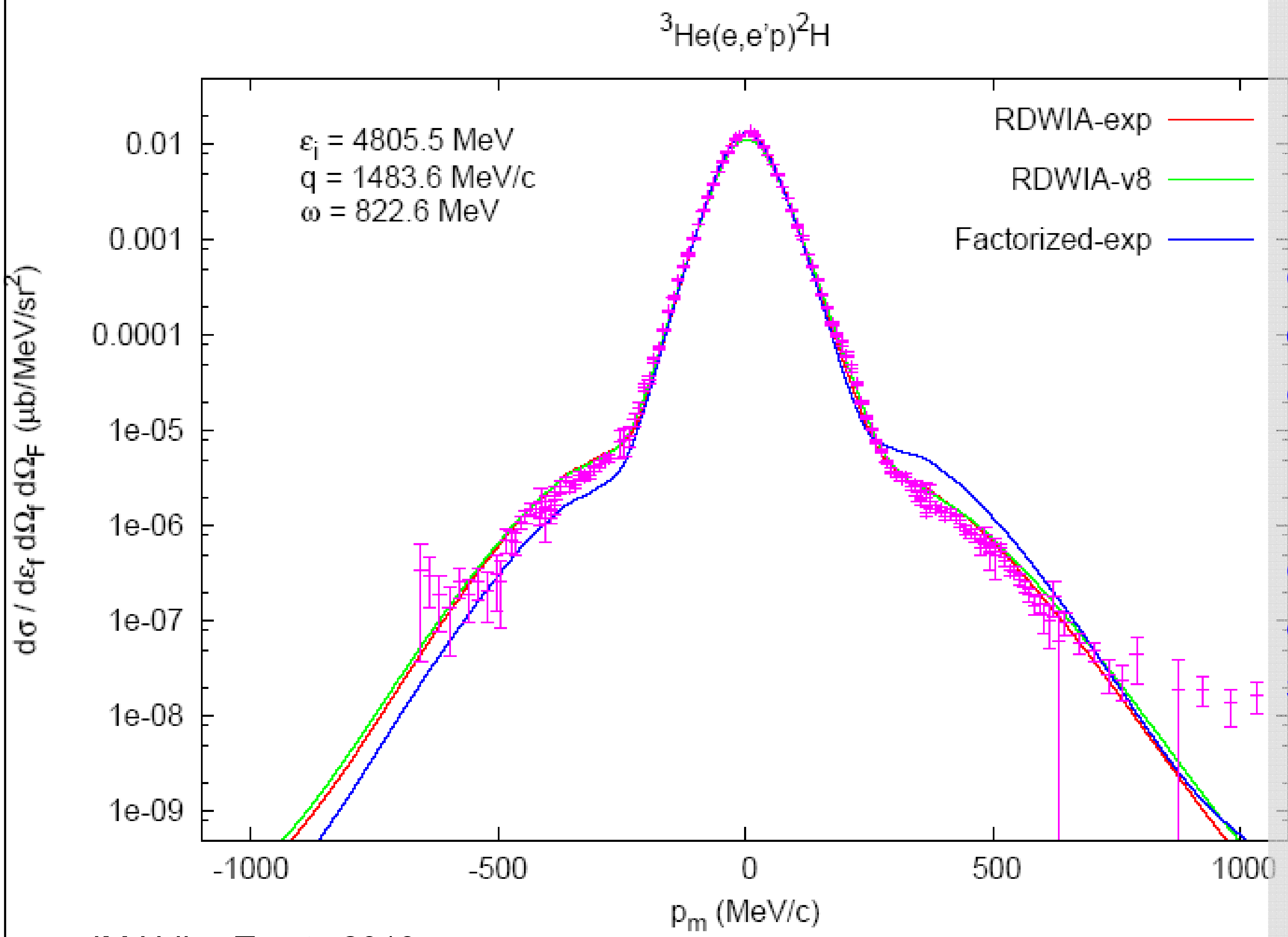
one-hole overlap function for the initial state (*quasiparticle*)

$$[S_n]^{\frac{1}{2}} \phi_{E_n}(\mathbf{p}) = < \Psi_n^{A-1}(E) | a(\mathbf{p}) | \Psi_0^A >$$

one-hole overlap function for the final state
and the nucleon in the continuum

$$\chi^{(-)}_{p_p E_n} = < \Psi_n^{A-1}(E) | a(\mathbf{p}) | \Psi_A^F >$$

For light nuclei we can do all the calculation without approximations, and essentially without free parameters, and compare to an exclusive experiment



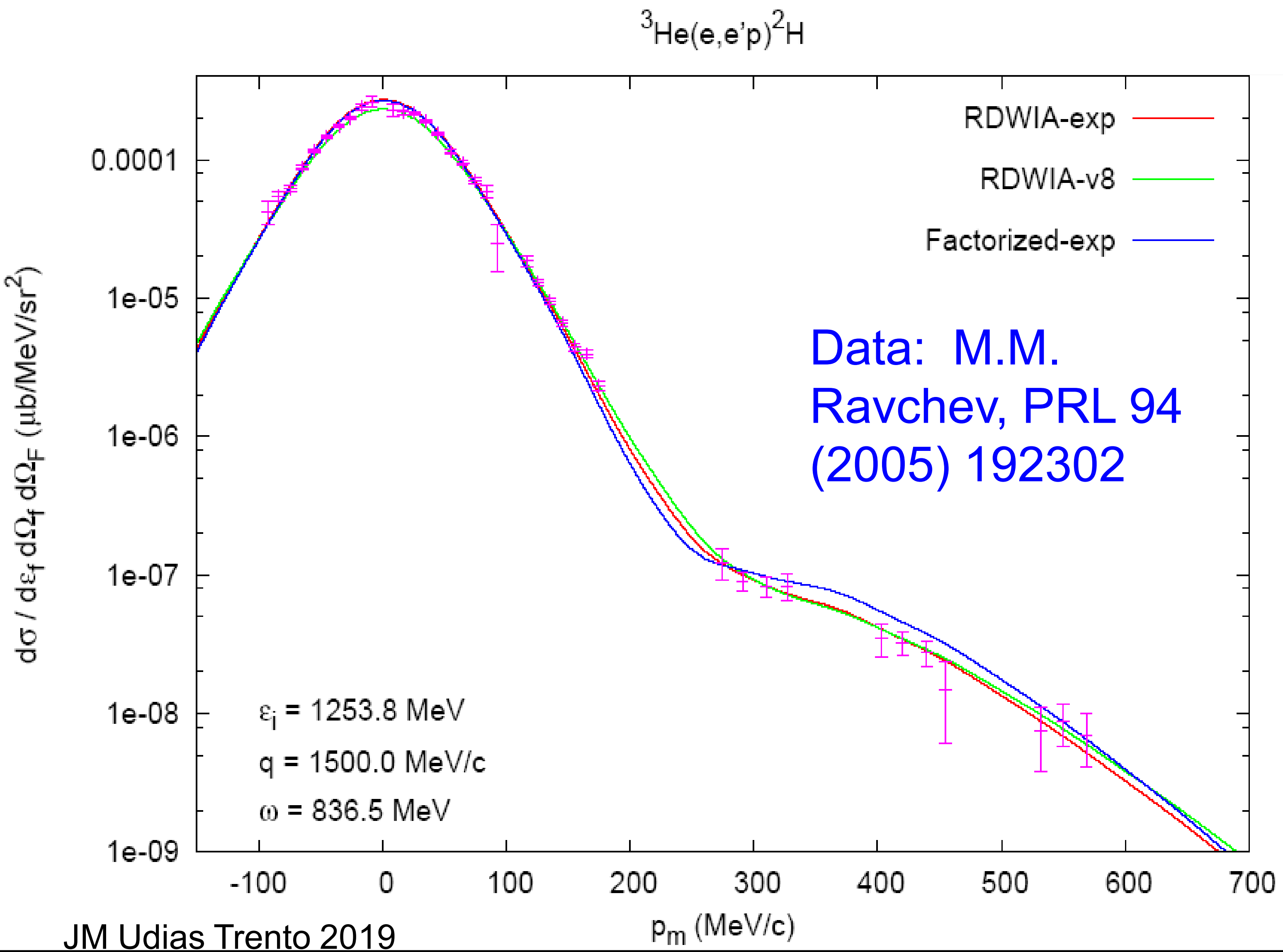
Data: M.M. Ravchev, PRL 94 (2005) 192302

Full theoretical calculation of the overlap from Faddev calculations from Pisa group. No free parameters in these calculations, not even the spectroscopic factors (of the order of 0.65)

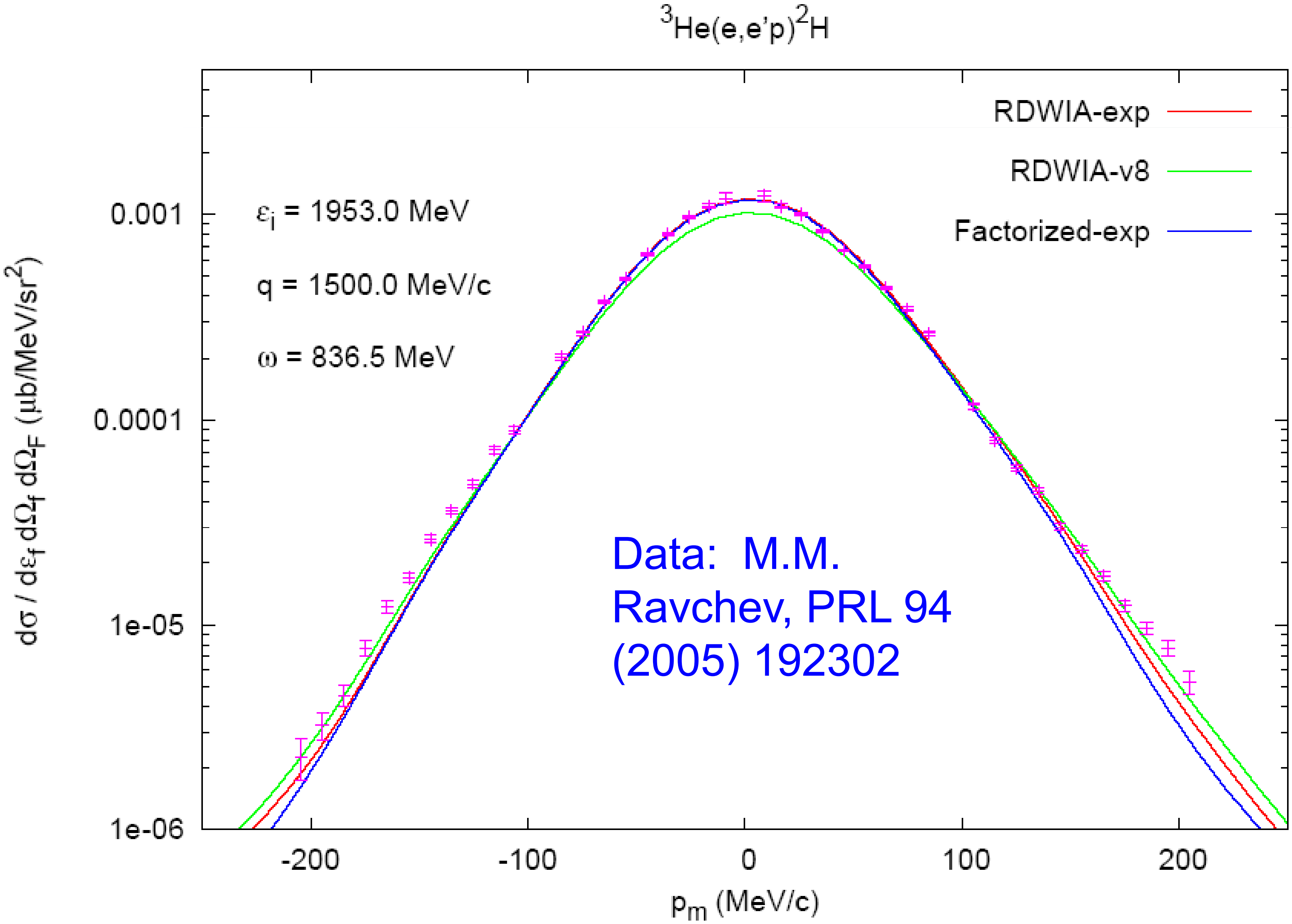
R. Álvarez et al. Few-Body Syst (2011) 50:359

Same calculation works at more backwards kinematics

FSI obtained ***folding an effective NN interaction with the deuteron (residual) density, no free parameters***



Good precision data allows for fine tuning the NN interaction



Heavier nuclei: the full calculation of the overlap with many body techniques is much more difficult, is it really worth it?

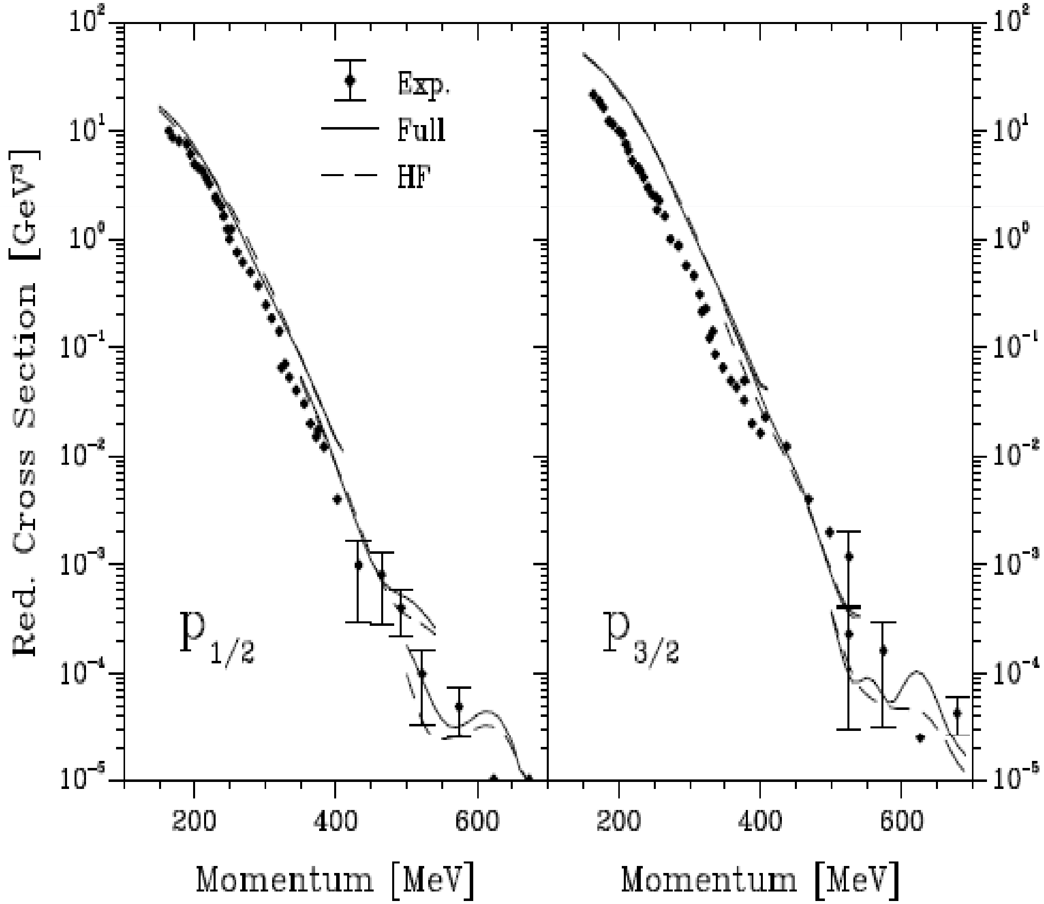
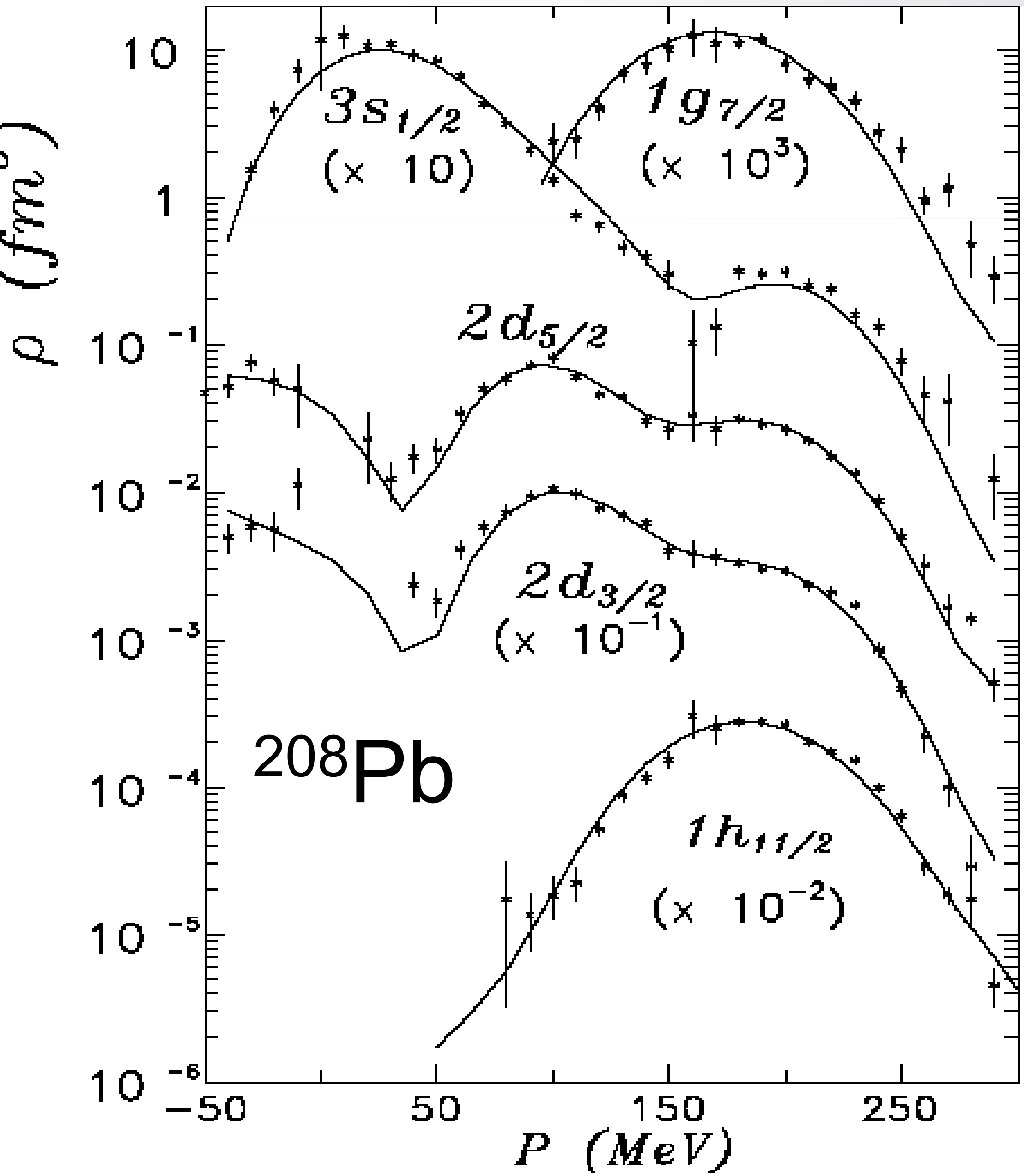


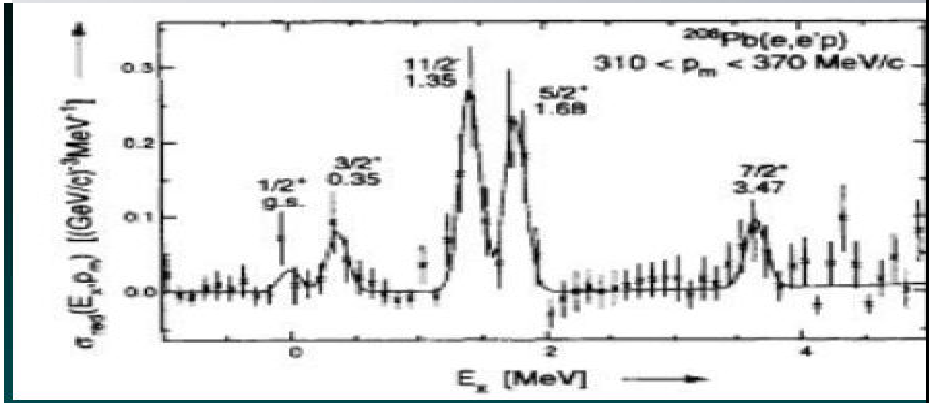
FIG. 3. Reduced cross section for the $^{16}O(e,e'p)^{15}N$ reaction leading to the ground state ($1/2^-$) and first excited state ($3/2^-$) of ^{15}N in the kinematical conditions considered in the experiment at MAMI (Mainz) [1]. Results for the mean-field description (HF) and the fully correlated spectral function (Full) are presented. The spectroscopic factors determined by a fit to low-p data from the NIKHEF experiment [29] are $S_{0p_{1/2}}$=0.60 (0.83) and $S_{0p_{3/2}}$=0.45 (0.85), where the values obtained by the theoretical approach are enclosed in parenthesis.

K. Amir et al, Nucl.Phys. A625 (1997) 633-650

The RMF yields good agreement with exclusive (e,e'p) data

JM Udias et al., PRC48, 2731 (1993), PRC51 3246 (1995)





Reasonably good agreement with data under exclusive kinematics

spectroscopic factors are now a free parameter, fitted to data. RMF tend to imply larger spectroscopic factors.

	$3s_{1/2}$	$2d_{3/2}$	$1h_{11/2}$	$2d_{5/2}$	$1g_{7/2}$
Non rel. (Ref. [41])	50 %	53 %	42 %	44 %	19 %
Non rel. (Ref. [42])	55 %	57 %	58 %	54 %	26 %
Rel. (Refs. [40, 6])	70 %	72 %	64 %	60 %	30 %

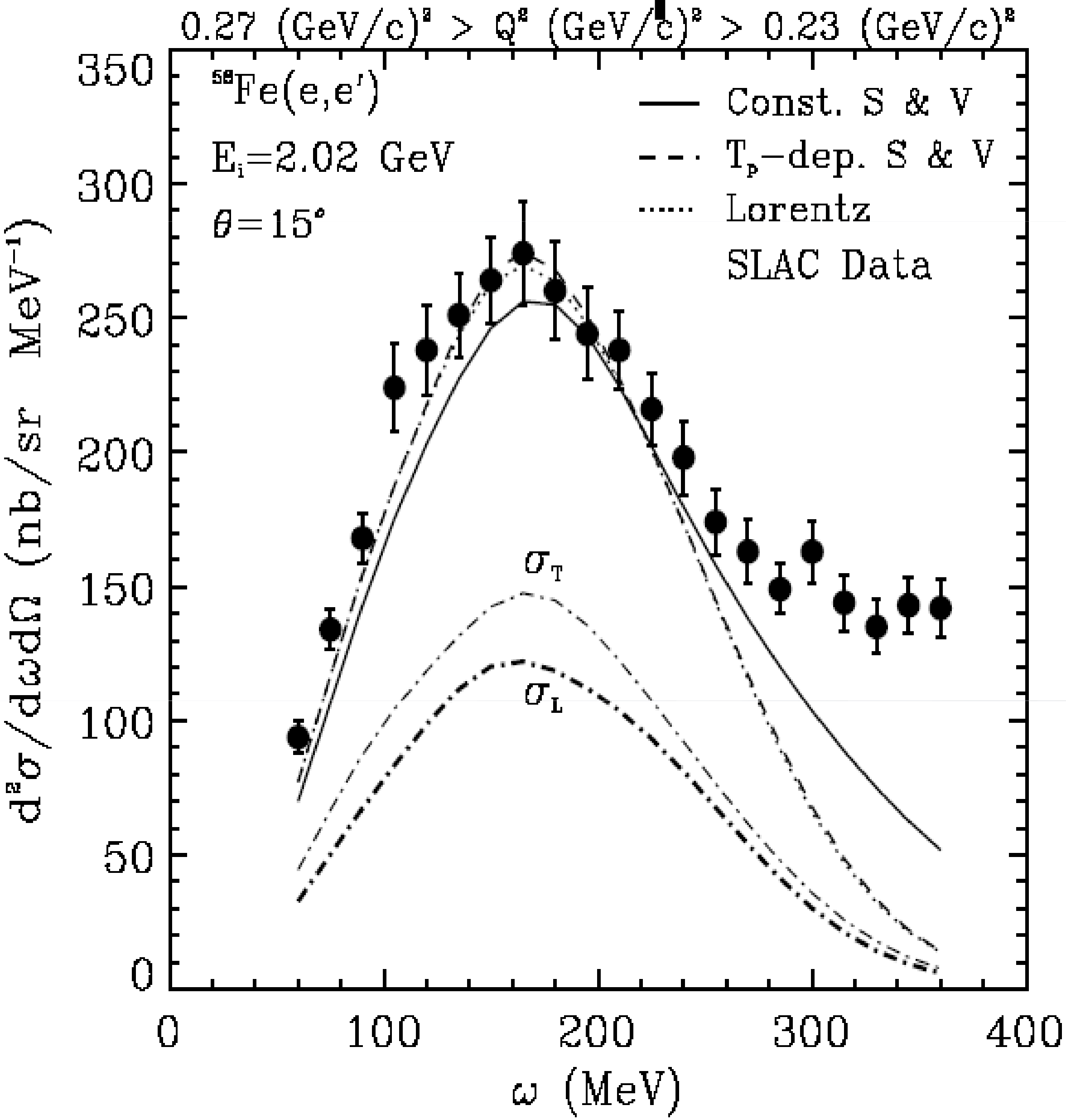
Inclusive (e,e’) reactions at the QE peak

•Spectroscopic factors should not be a problem here. We Integrate on all possible final states for the residual system. The nucleon has to go somewhere, thus once we sum on all possible final states, the whole strength should be there.

• There is a missing energy distribution (not delta-like functions), deep shells are more smeared in energy, and if the range in missing energy is not wide enough, some strength may still be missing. You can model this with a lorentzian, gaussian, etc.

•Calculations are simple and promising.

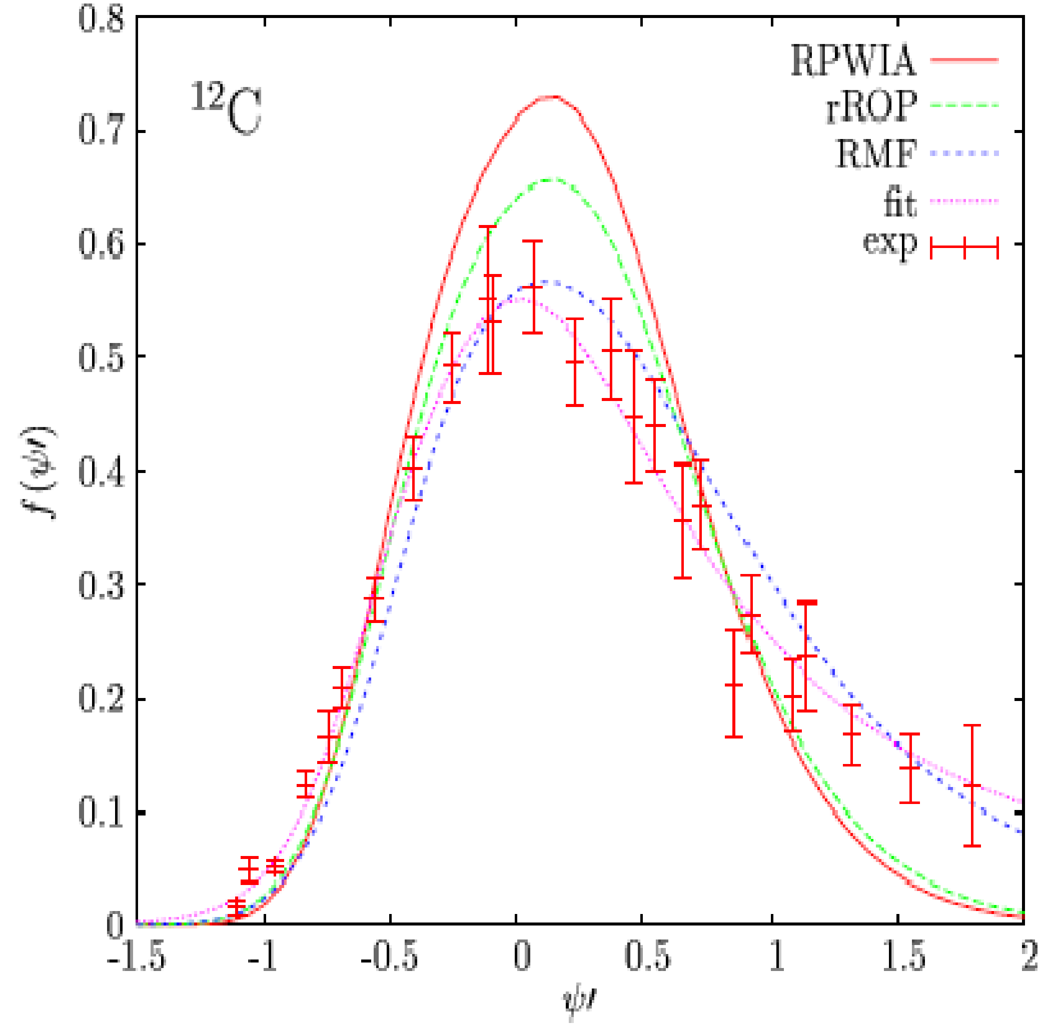
Inclusive (e,e’) reactions: comparison with the data



It was observed that constant potentials (RMF) produce an asymetric cross-section with increased strength (tail) at large ω (**K.S. Kim and L.E. Wright PRC 68 (2003) 027601)**

Comparison with data was not conclusive due to the delta peak contributing into the quasielastic region

Comparison to inclusive data: Scaling analyses (J.A. Caballero et al., PRL 95 (2005) 252502)



(When and where) to care or not to care about correlations?

Nuclei are complex interacting systems

We aim to solve the A-body problem:

$$\mathcal{H}\ \boldsymbol{\psi}(1,\ldots,A)= [\Sigma\ \mathbf{p}_i/2M + \mathcal{V}(1,\ldots.,A)]\ \boldsymbol{\psi}(1,\ldots,A)$$

This problem can be cast as a *variational* problem, looking for a $\boldsymbol{\psi}$ which minimizes

$$\frac{\langle\ \boldsymbol{\psi}(1,\ldots,A)\ |\ \mathcal{H}\ \boldsymbol{\psi}(1,\ldots,A)|\ \boldsymbol{\psi}(1,\ldots,A)\rangle}{\langle\ \boldsymbol{\psi}(1,\ldots,A)\ |\ \boldsymbol{\psi}(1,\ldots,A)\rangle}$$

Against all possible variations of $|\boldsymbol{\psi}(1,\ldots,A)\rangle$

We can minimize on a restricted subset among all possible $\boldsymbol{\psi}$, such as harmonic oscillator, single-particle Slater dertminant, etc, which simplifies the calculations.

Meaning…

Many body problem:

$$\mathcal{H}\ \boldsymbol{\psi}(1,\ldots,A) = [\Sigma\ \mathbf{p}_i/2M + \mathcal{V}(1,\ldots,A)]\ \boldsymbol{\psi}(1,\ldots,A)$$

This problem can be rewritten using the following:

$$\mathcal{V}(1,\ldots,A) = \mathcal{v}(1)+\ldots+\mathcal{v}(A)+[\mathcal{V}(1,\ldots,A)-\mathcal{v}(1)-\ldots\mathcal{v}(A)]$$

With:

$$\mathcal{H}\ \boldsymbol{\psi}(1,\ldots,A) = [\Sigma[\mathbf{p}_i/2M + \mathcal{v}(i)] + \mathcal{C}(1,\ldots,A)\ ]\boldsymbol{\psi}(1,\ldots,A)$$

We have 'split' the Hamiltonian of the system in terms of a uncorrelated part, on one side, and $\mathcal{C}(1,\ldots,A)$ with *explicit* correlation content. We aim to choose this separation in a way which maximizes the 'non correlated contribution' and minimizes the *residual correlated* interaction. That is to say, we aim to include as much as possible of the effect of correlations into the mean field potential.

Choice of non-correlated part

For the non correlated part one may pick an harmonic oscillator potential. This is easy to solve, but it may not yield the smallest *explicit* residual correlation term.

We aim try to solve the problem in perturbation theory, taking the non-correlated term as the (hopefully) 'large' term, and the explicit correlations as the perturbation. In such case, the first order solution of the problem will be given by a (possibly antisymmetrized) product of single-particle states:

$$\boldsymbol{\psi}(1,\ldots,A) = \varphi_1(1)\ldots\varphi_A(A)$$

Different separations of the central part would yield the same solutions, it sumed up on all orders of perturbation theory, if convergence is reached, which is not usually the case.

Self Consistent Mean Field

Thus we can choose to minimize only in the space of Slater determinants of single-particle wave functions:

$\boldsymbol{\psi}(1,\ldots,A)=\Pi(\text{antisym})\ \varphi_1(1)\ldots\varphi_A(A)$

One can try and solve the variational problem restricted to this space, leading to the self-consistent Hartree / Hartree-Fock equations for the single-particle functions φ, and the mean field potential.

This procedure provides a separation for which the non-correlated part is the “largest”, and the residual correlations are the “least”.

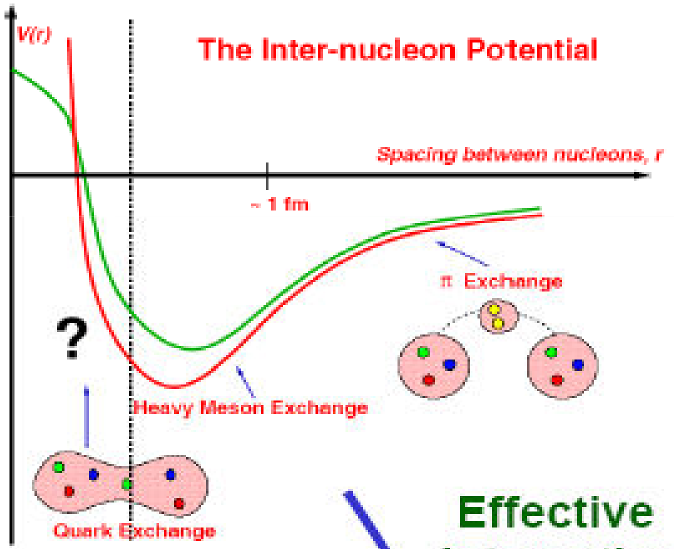
Self Consistent Mean Field+IPSM

The use of IPSM wave functions allows us to label states according to the solution of an average potential in which the particles move, quite independently. And we can use standard methods for solving single-particle equations.

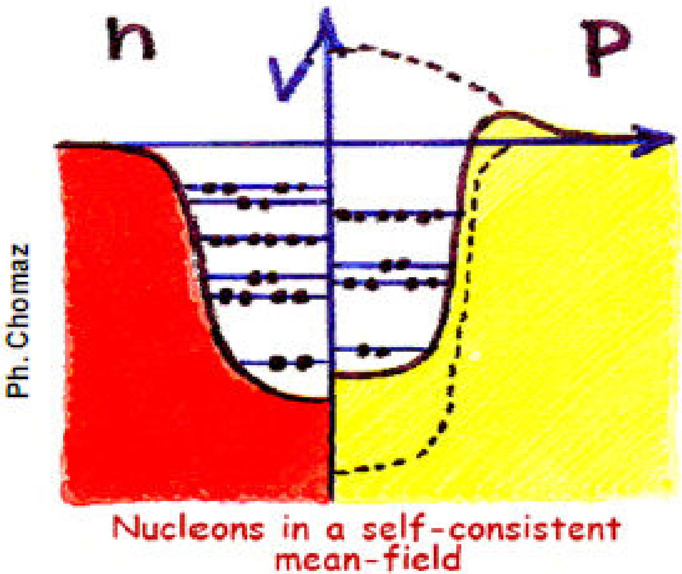
This is the foundation of the Nuclear Shell Model

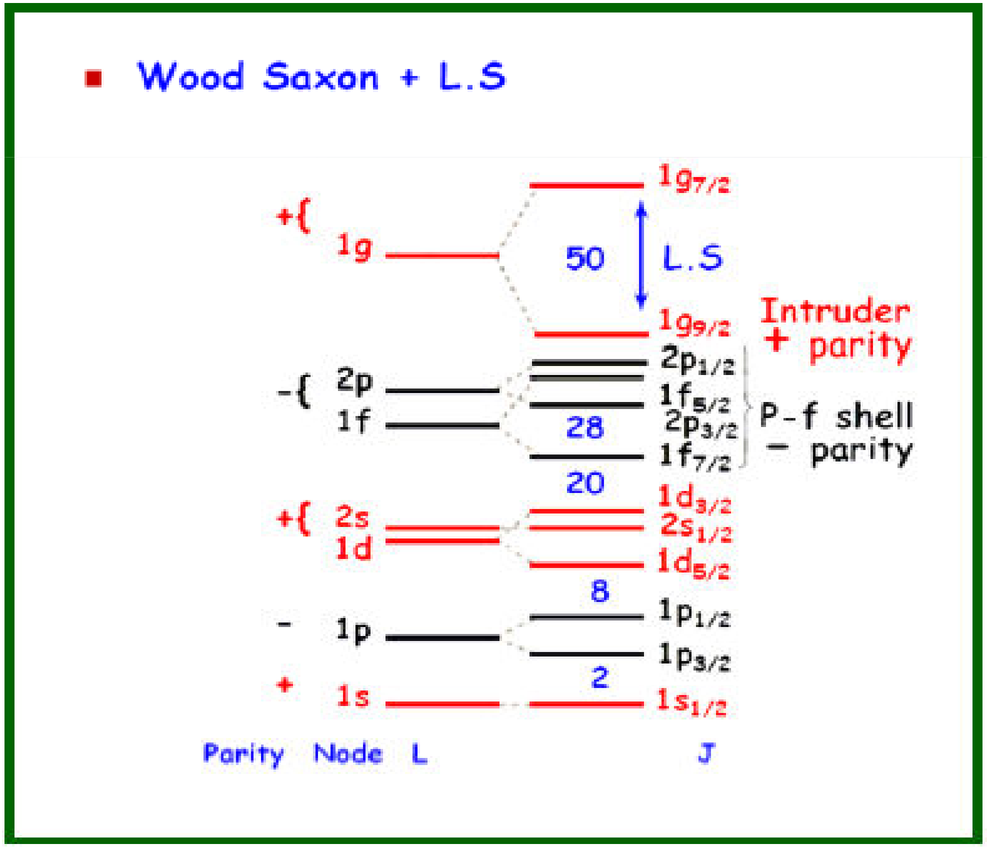
Correlations would remain *explicitly visible* only within the residual interaction. These residual correlations will change the energy of the states, introduce mixing of the different single-particle (Slater determinant) solutions

Mean Field Model of Nuclei



Effective interaction





- fermion system at low energies
- suppression of collisions by Pauli exclusion
- independent particle motion
- shell structure
- mean field approximation

Relativistic mean field (RMF)

- Dirac equation with its relativistic treatment of dynamics and kinematics as opposed to the nonrelativistic Schrödinger equation (which can also include relativistic kinematics) to describe single nucleon motion in nuclei

- The Dirac equation provides a natural description of spin-1/2 particles and, hence, provides a good framework for studying spin observables

- We could obtain the mean field in a self-consistent way from using a 'bare' **NN interaction to build the A-body hamiltonian (say derived from meson exchanges as in the Bonn family of potentials)**, this will give us an effective one-body hamiltonian and corresponding mean field potential for this problem.

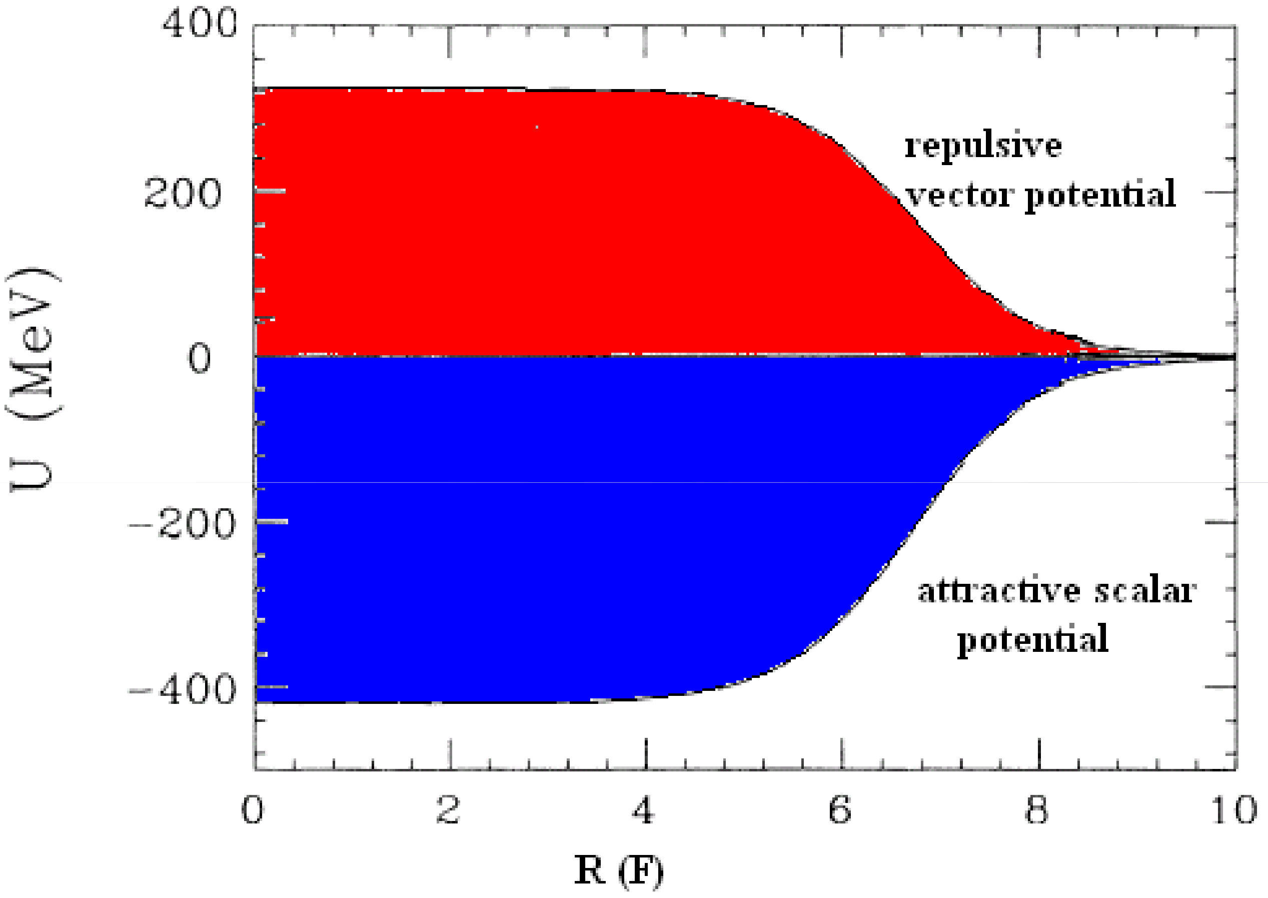
Scalar *and* Vector potentials

$$(\tilde{E}\gamma_0 - \vec{p}\cdot\vec{\gamma} - \tilde{M})\psi = 0$$

$$\tilde{E} = E - V(r)$$
$$\tilde{M} = M - S(r)$$

- Solve a Dirac-like equation
- Bound state: Phenomenological σ-ω lagrangeans (Serot and Walecka model) at mean field level

Within the (self-consistent) Relativistic Mean Field, they appear strong mean field potentials, meaning that stronger correlations can be represented



- Strong (hundreds of MeV's) repulsive vector and attractive scalar potentials are obtained with the Dirac treatment
- The small (tens of MeV) binding energy arises as a result of cancellations and is just the 'tip' of the iceberg

One can build a field theory. It can look very formal..

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}\left(\gamma_\mu\left(\mathrm{i}\partial^\mu - g_\omega\omega^\mu - g_\rho\tau\cdot\rho^\mu\right) - \left(M + g_\sigma\varphi\right)\right)\psi \\
&+ \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m_\sigma^2\varphi^2 - \frac{1}{3}g_2\varphi^3 - \frac{1}{4}g_3\varphi^4 \\
&- \frac{1}{4}\omega_{\mu\nu}\cdot\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 \\
&- \frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu
\end{aligned}$$

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial_\mu\phi}\right) - \frac{\partial\mathcal{L}}{\partial\phi} = 0.$$

$$\left(\Box + m_\omega^2\right)\omega^\mu = g_\omega\bar{\psi}\gamma^\mu\psi - c_3\left(\omega_\mu\omega^\mu\right)\omega^\mu$$

$$\left(\Box + m_\sigma^2\right)\varphi = -g_s\bar{\psi}\psi - g_2\varphi^2 - g_3\varphi^3$$

$$\gamma_\mu\left(\mathrm{i}\partial^\mu + g_\omega\omega^\mu + g_\rho\tau\cdot\rho^\mu + \left(M + g_s\varphi\right)\right)\psi = 0.$$

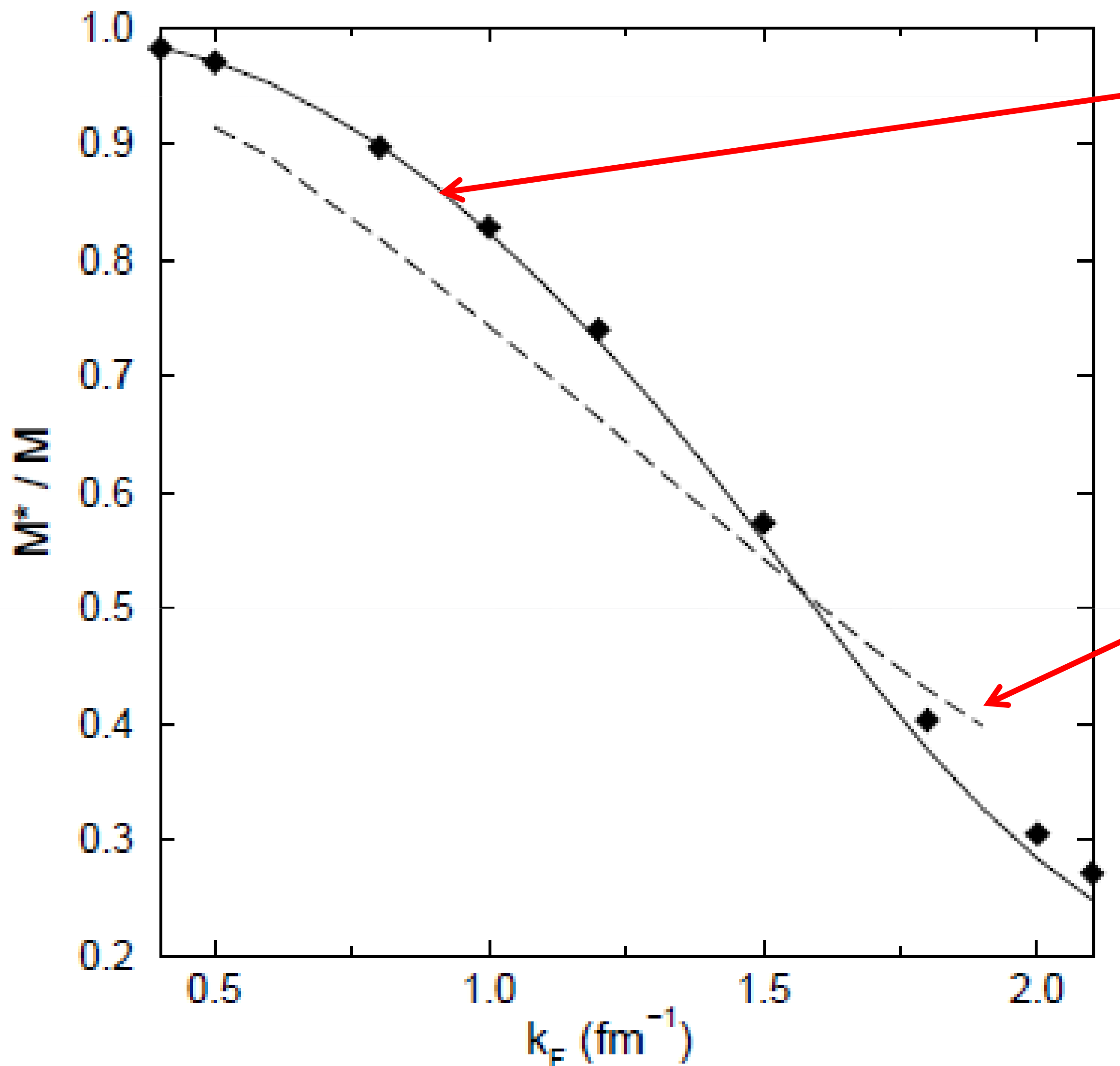
And the result is a Dirac equation for each single-nucleon state ψ_α

$$\left(-\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta\left(M + g_s\varphi_0\right) + g_\omega\omega_0 + g_\rho\tau_3\rho_0\right)\psi_\alpha = E_\alpha\psi_\alpha .$$

The relativistic variational problem can be solved without resorting to perturbating the IPSM solution.

Indeed, Dirac-Brückner techniques were developed and solutions of the more general variational many body problem have been studied

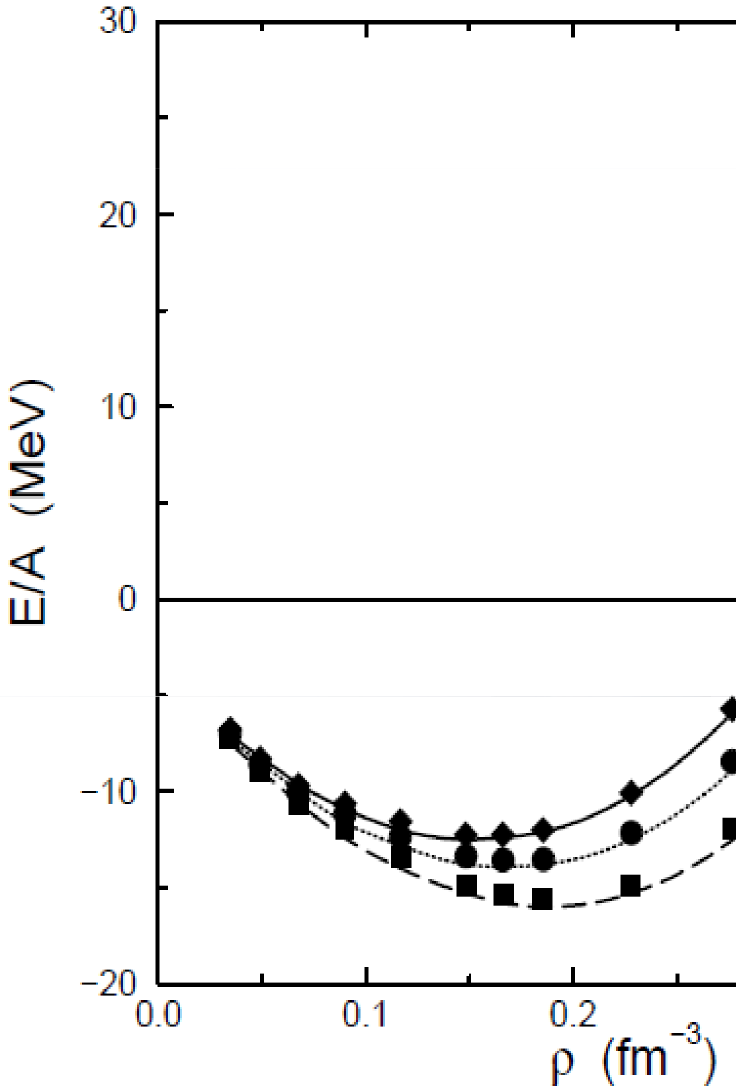
Effective mass from Relativistic Bruckner calculations



Bonn potential: solid lines and points (Sehn, Fuchs, Faessler PRC 216(1997), Haar and Malfliet Phys Rep. 149, 207(1987)

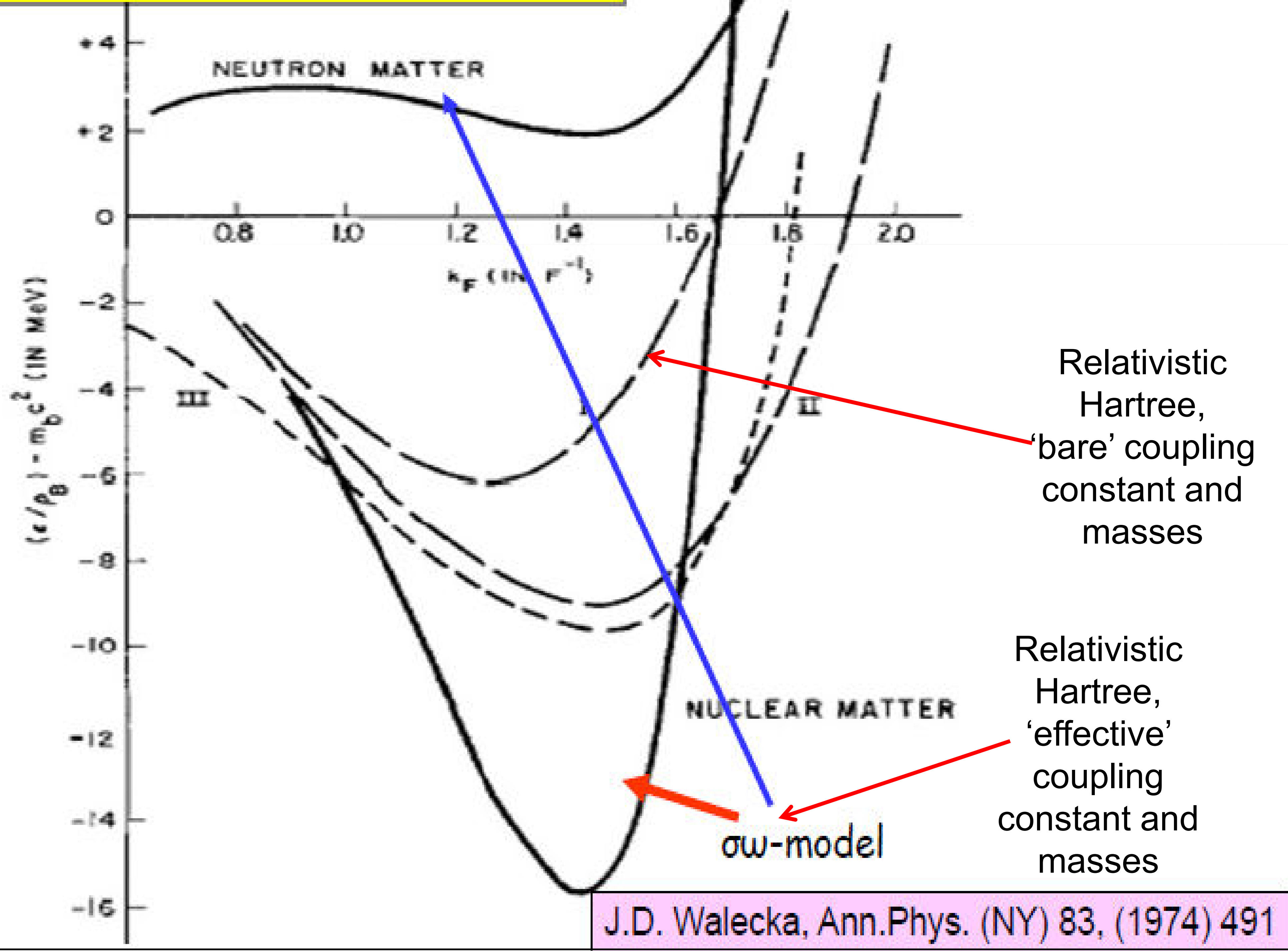
Sigma-omega model: dashed lines (Horowitz and Serot, NPA 464, 613 (1987)

Saturation curves (EoS) from relativistc Bruckner theory



Bonn potentials A-B-C (top to bottom): solid lines and points from Sehn, Fuchs, Faessler PRC 216(1997), Haar and Malfliet Phys Rep. 149, 207(1987)

Equation of state (EOS):

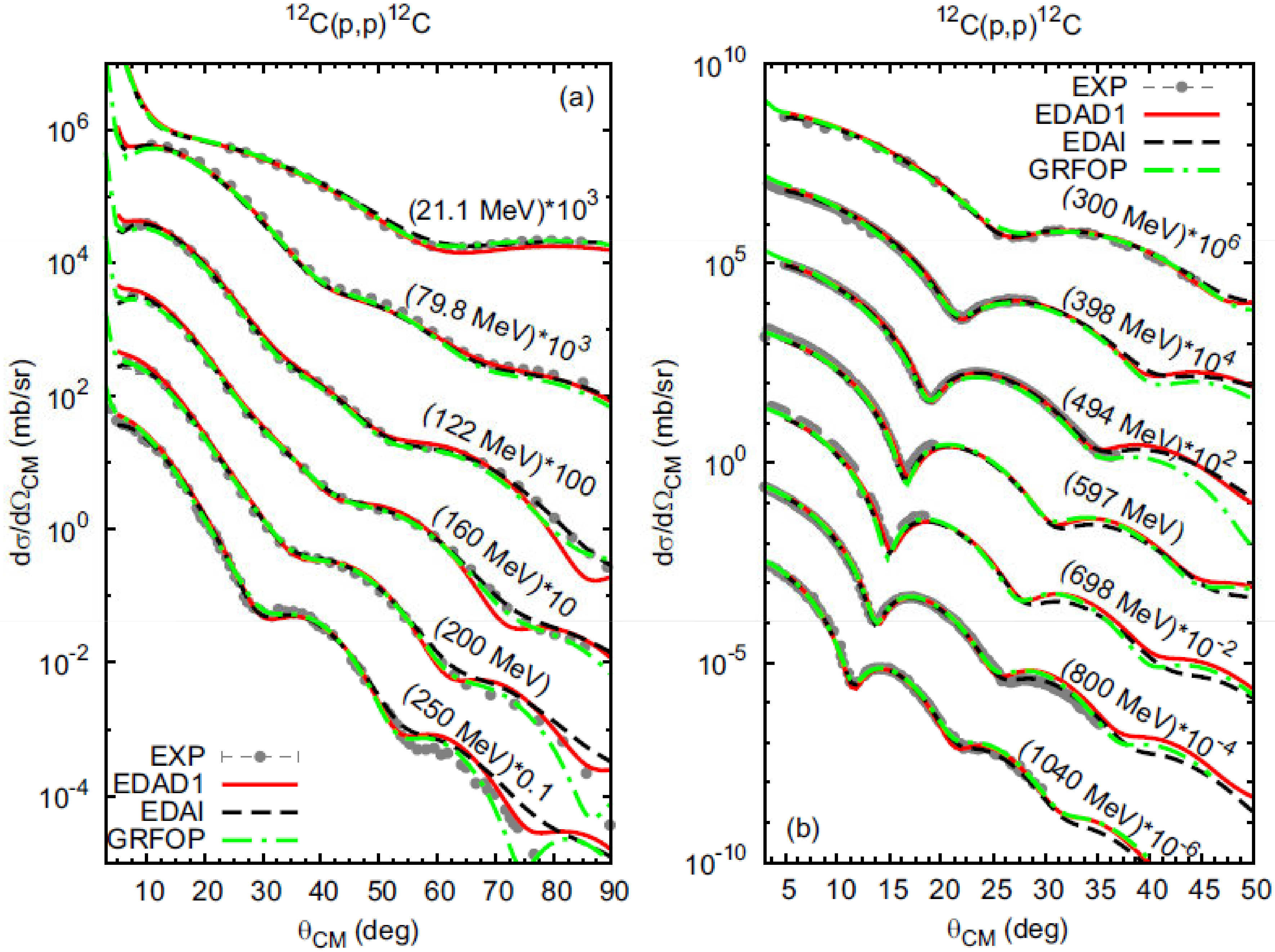


J.D. Walecka, Ann.Phys. (NY) 83, (1974) 491

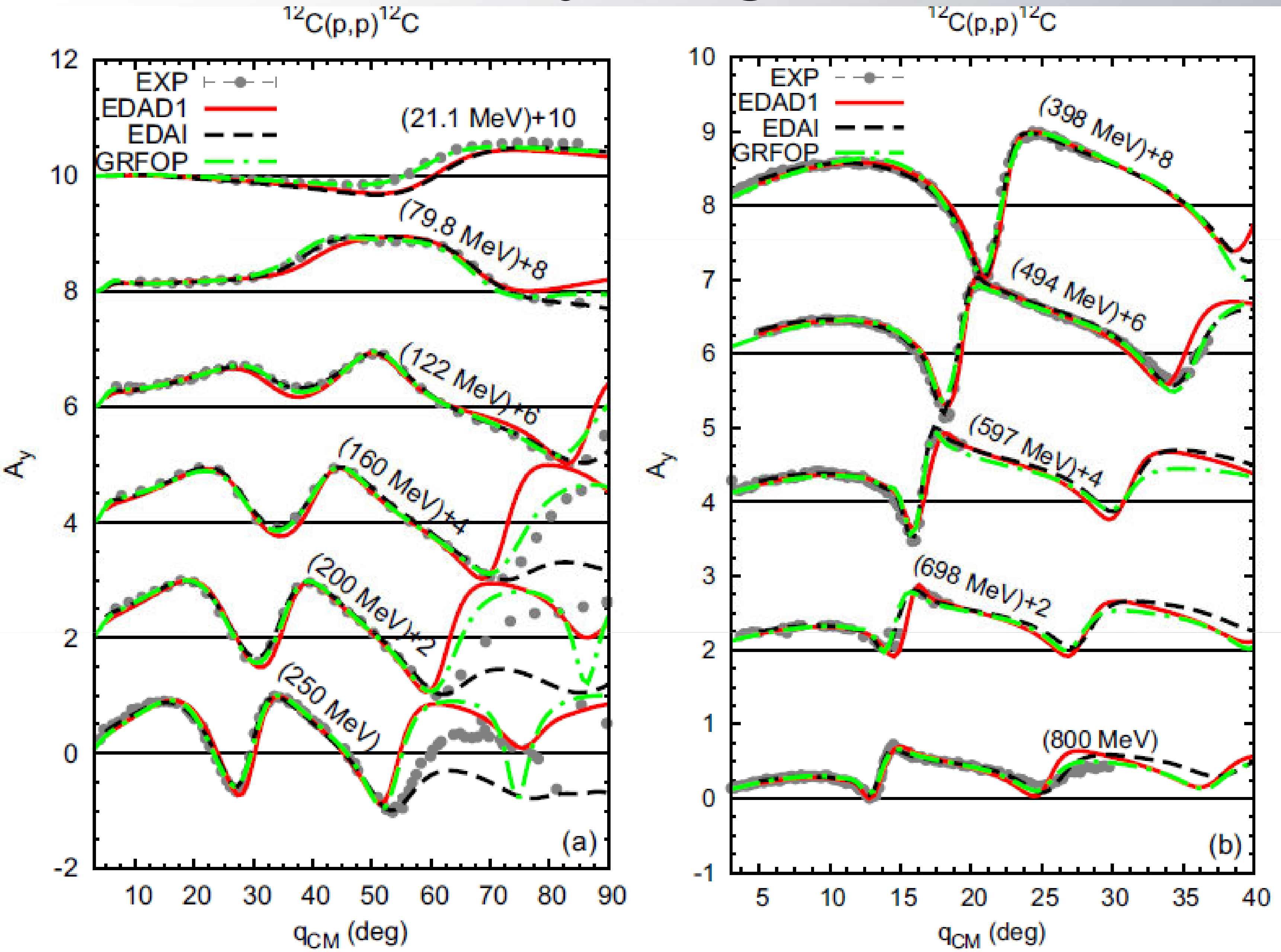
And many different versions of the lagrangians have been cooked

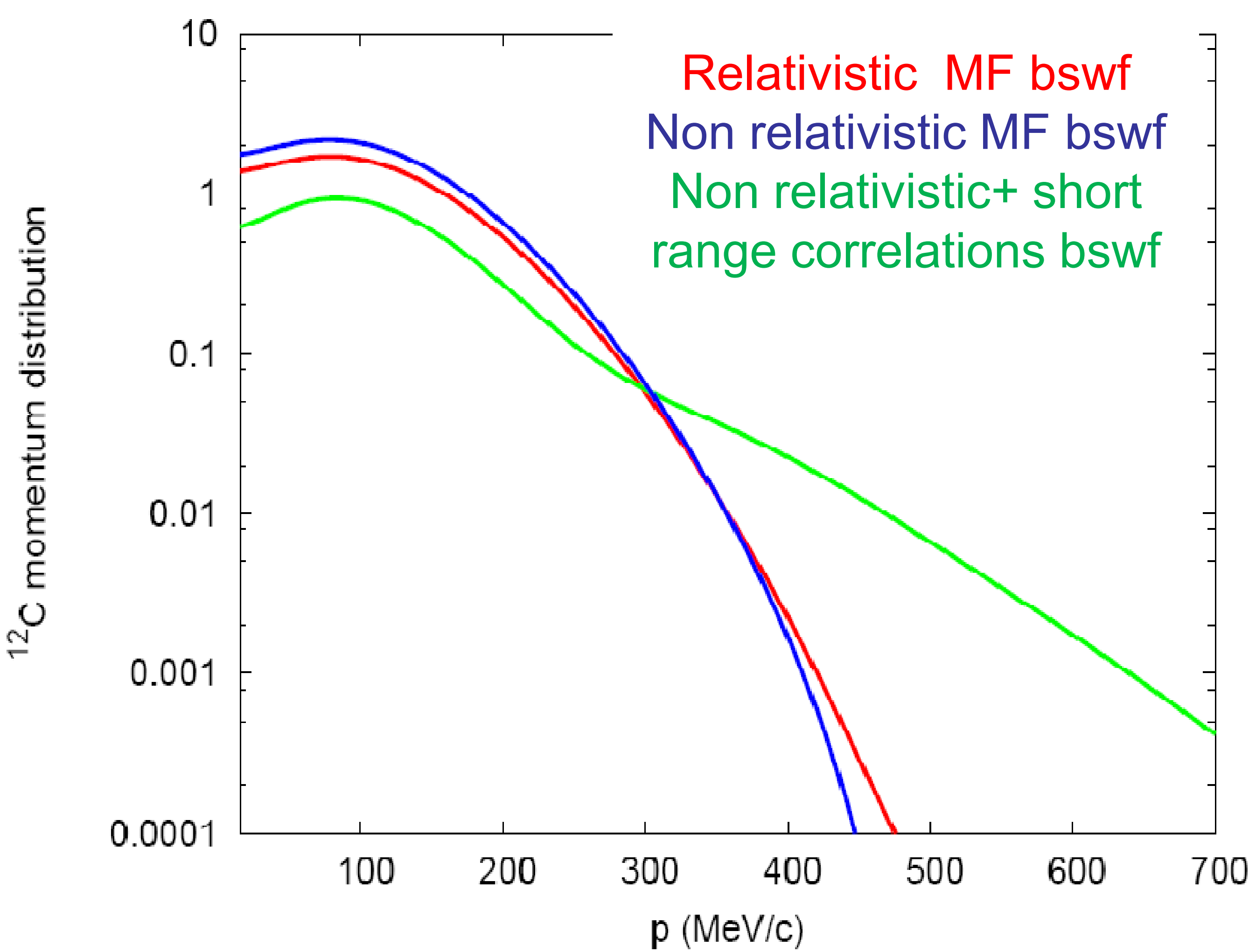
	Wa [19]	HS [8]	NL1 [15]	NL-SH [22]	TM1 [23]	TM2 [23]
M (MeV)	938.0	939.0	938.0	939.0	938.0	938.0
m_σ (MeV)	550.00	492.0	492.250	526.059	511.198	526.443
m_ω (MeV)	783.00	783.0	795.359	783.0	783.0	783.0
m_ρ (MeV)	763.00	770.0	763.0	763.0	770.0	770.0
g_s	9.58289	10.47	10.1377	10.444	10.0289	11.4694
g_ω	11.683586	13.80	13.2846	12.945	12.6139	14.6377
g_ρ	0.0	8.07	4.9757	4.383	4.6322	4.6783
g_2 (fm^{-1})	0.0	0.0	-12.1724	-6.9099	-7.2325	-4.4440
g_3	0.0	0.0	-36.2646	-15.8337	0.6183	4.6076
c_3	0.0	0.0	0.0	0.0	71.3075	84.5318
E/A (MeV)	-15.70	-15.70	-16.40	-16.32	-18.56	-14.22
ρ_B (fm^{-1})	-0.193	0.148	0.152	0.146	0.146	0.111
M^*/M	0.55	0.54	0.573	0.597	0.66	0.618

Many Optical Potentials fit of p-A cross-sections



Fit of Analyzing power

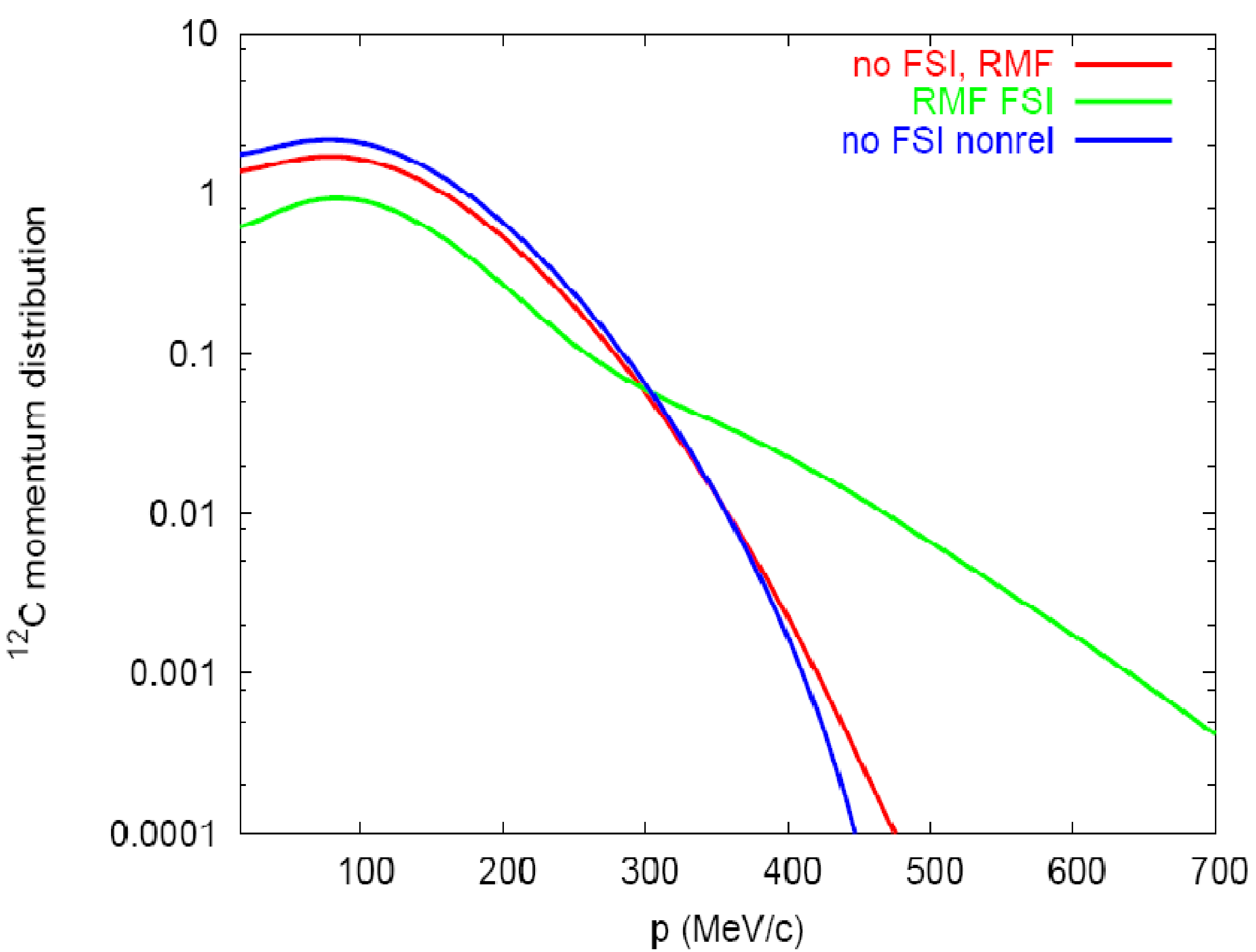




•As an example, here we show the momentum distribution for the bound nucleons

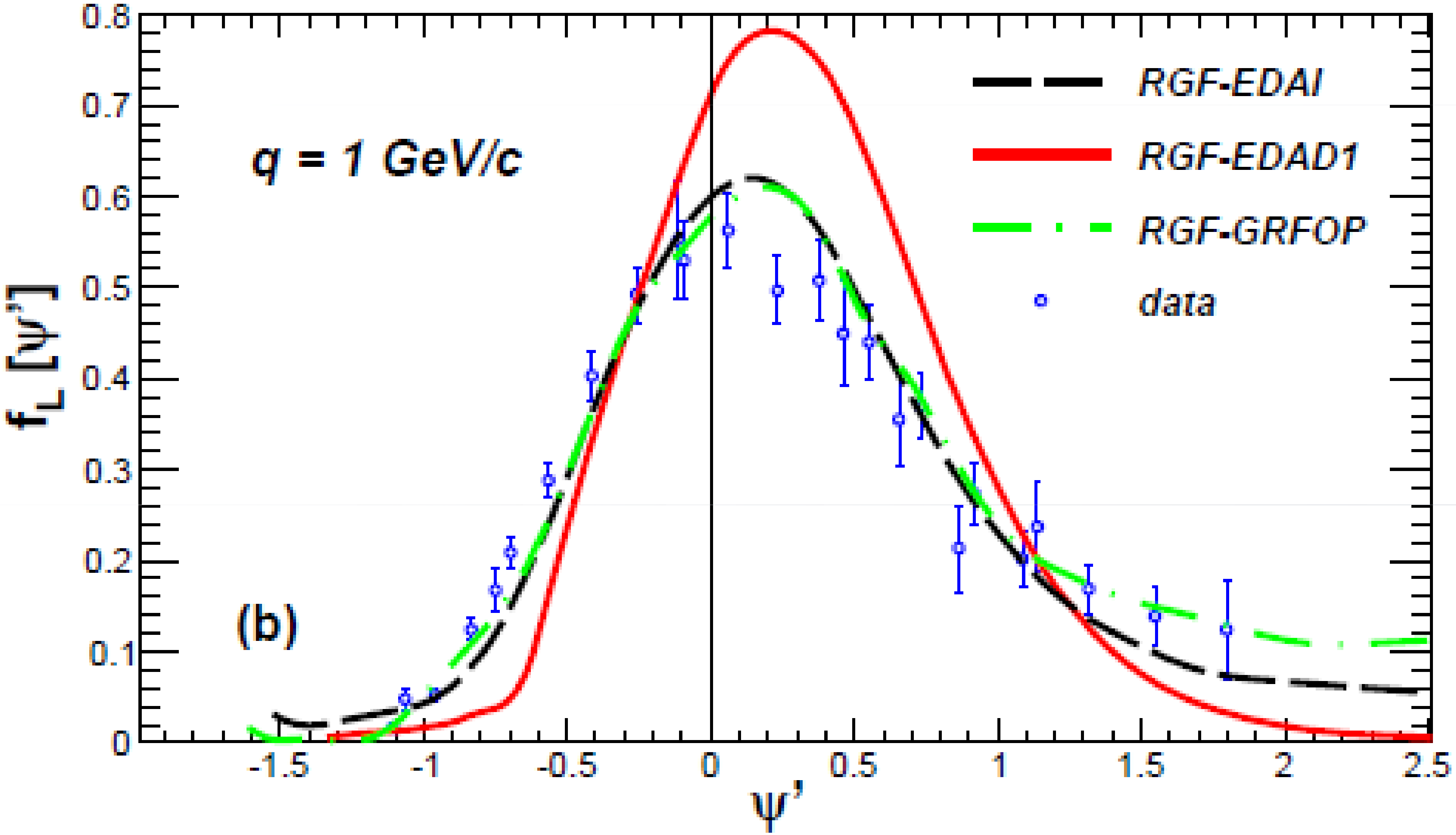
•In some representation, the high momentum tail of the momentum distributuon is due to short range correlations, it will originate in the explicit correlations term

•If we remove the pm>300 MeV/c components, the asymmetry in the quasielastic response dissappear



- As an example, here we show the distorted momentum distribution computed within RMF for the initial and final states
- The additional strength beyond coming from pm>300 MeV/c is due, in the RMF, to the strong potentials in the final state (FSI effects) which enhance the effective contribution of the nucleon at high momentum
- In this representation, the high momentum tail originates without explicit correlations

scaling function



How to shift strength to higher ω? First idea: add correlations

- Correlations introduce fragmentation of the strength in the *initial state*. Compared to mean field, the strength is distributed over a larger range of excitation energy and more uniformly. This doesn't seem to be the cause of **large** asymmetries in the cross-section

- Explicit correlations would allow for additional ***multinucleon emission*** thus giving contributions to the (e,e') cross-sections shifted by several 'separation energies' with regards to the single nucleon knockout: asymmetry

How to shift strength to higher ω? Another idea

- Strong potentials (as in the RMF) can cause large contribution to the cross-section from the high momentum components of the nucleon

- This shifts additional strength toward higher values of the energy transferred

- Recall: the exchanged boson is not interacting with free nucleons but rather with an ***effective quasiparticle object*** interacting via very strong potentials. We keep things at the mean field level, but correlations are there

Effective potential felt by the nucleon

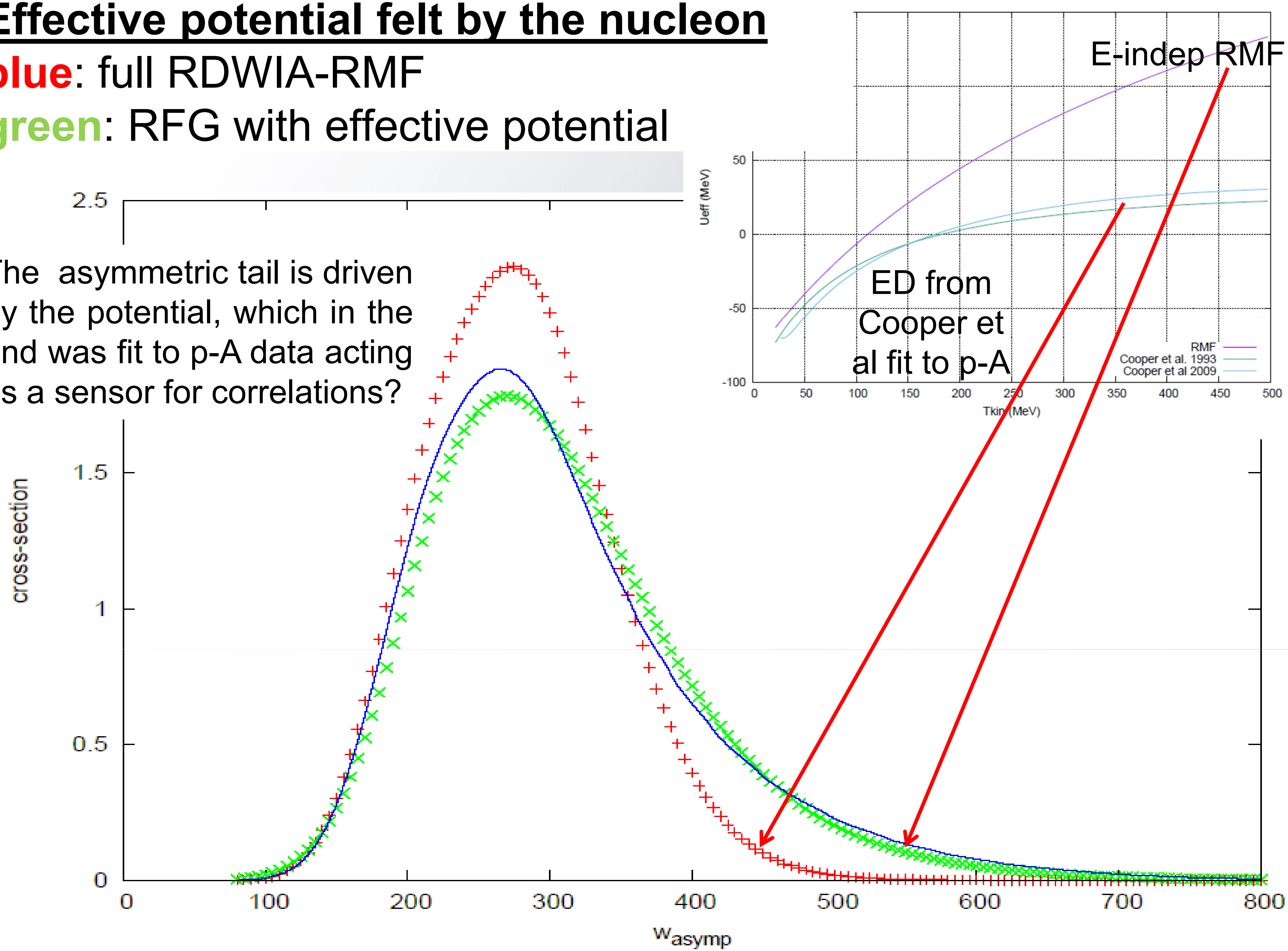
- The effect is difficult to estimate *a priori* from simple approaches, because potentials are larger in the nuclear interior but they go to zero as the nucleon approaches the surface
- The full RMF calculation provides quantitative estimates and seems to be in agreement with the data
- Within RMF, the asymmetry is due to the high momentum content of the bound nucleon contributing to the cross-section thanks to the effective strong S-V FSI

Effective potential felt by the nucleon

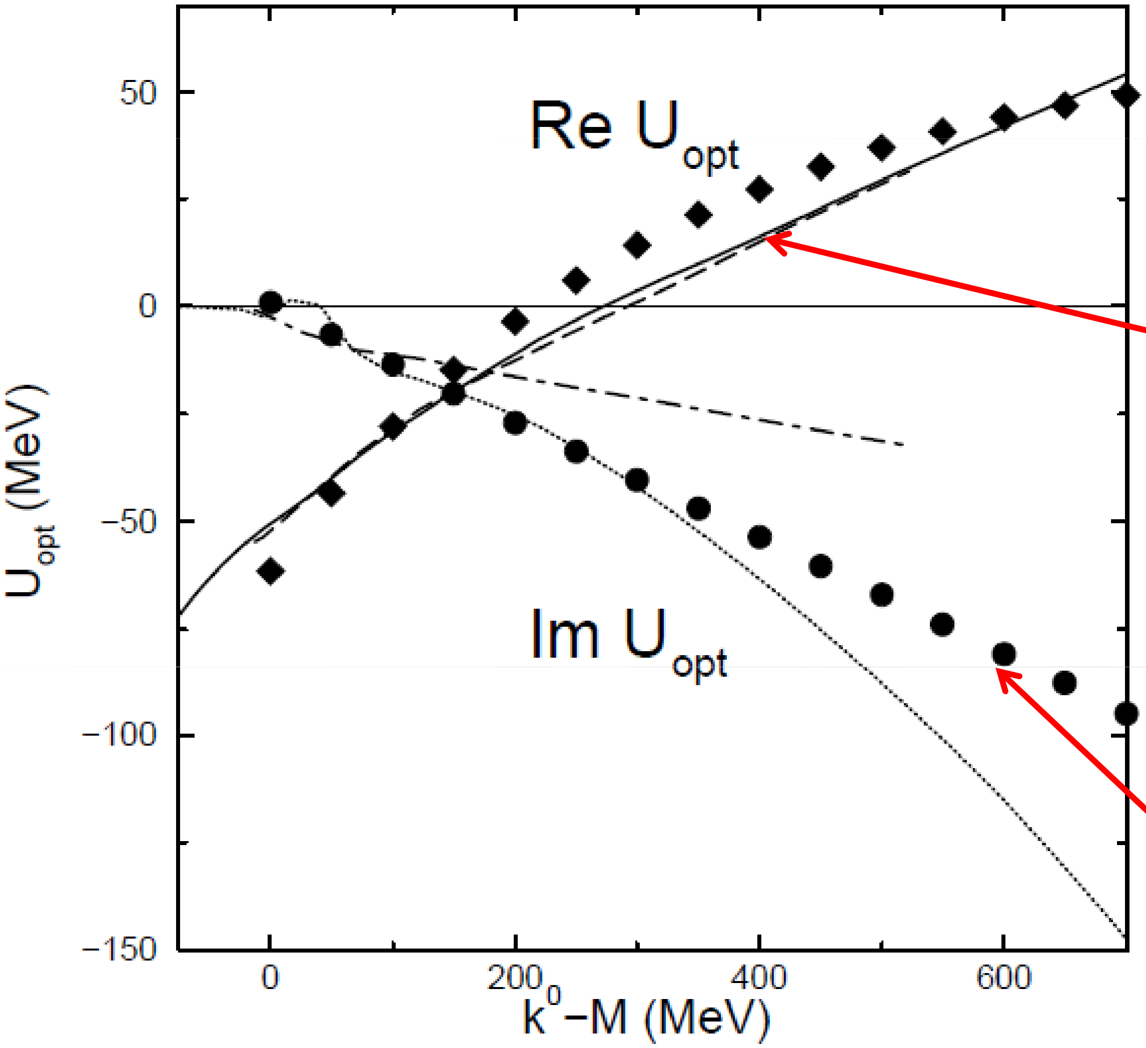
blue: full RDWIA-RMF

green: RFG with effective potential

The asymmetric tail is driven by the potential, which in the end was fit to p-A data acting as a sensor for correlations?



Effective potentials from relativistic Bruckner theory



Bonn C potential : solid and dashed lines from Sehn, Fuchs, Faessler PRC 216(1997), Haar and Malfliet Phys Rep. 149, 207(1987)

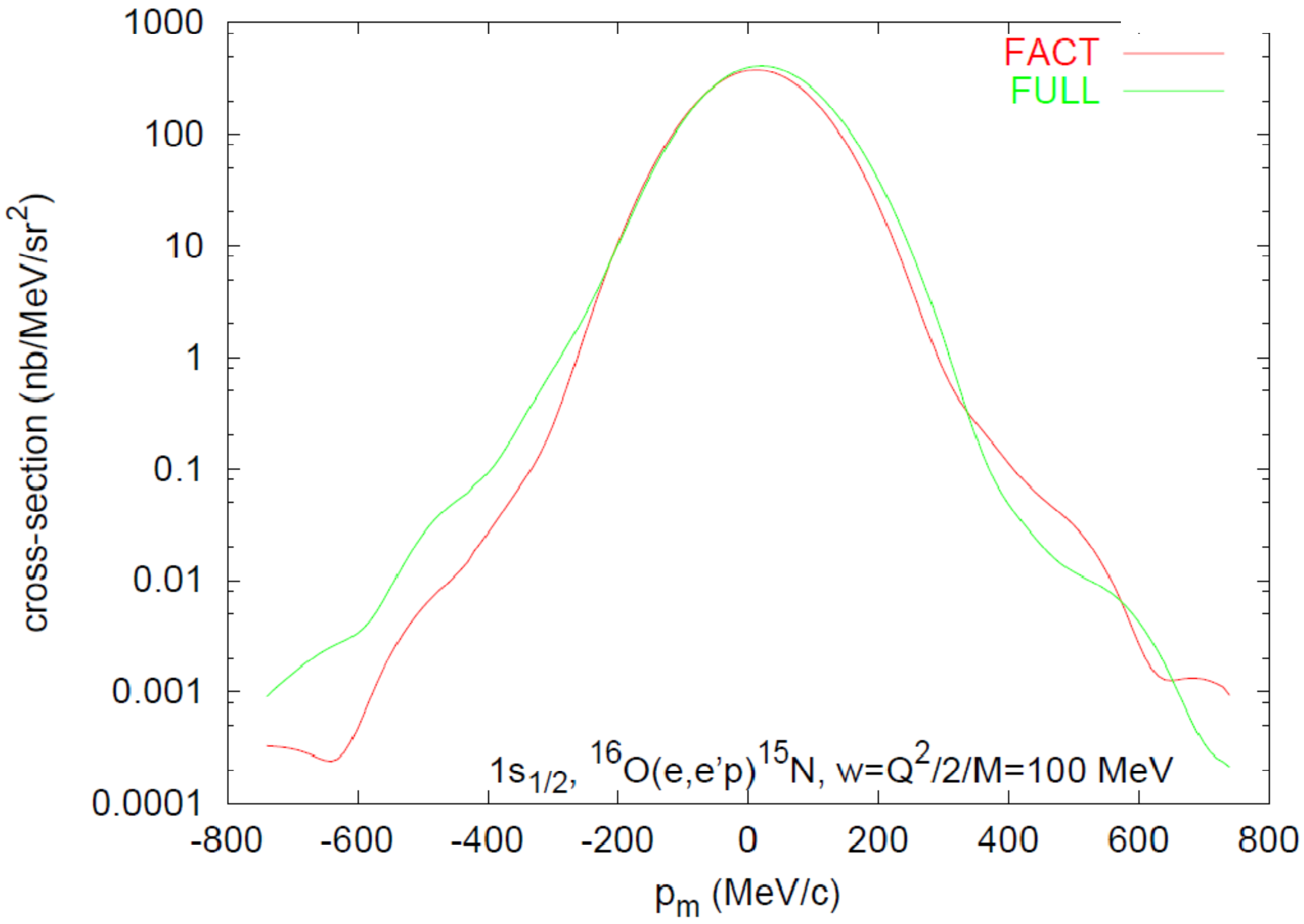
Diamonds: **phenomenological optical** potential from Cooper, Hama et al, 1993

FACTORIZATION

The one-boson exchange approxi-mation allows us to decouple the direct dependence on the *energy* and *scattering angle* of the probe *via* the Mott cross-section for electrons or the equivalent expressions for neutrinos. This is the foundation for factorization.

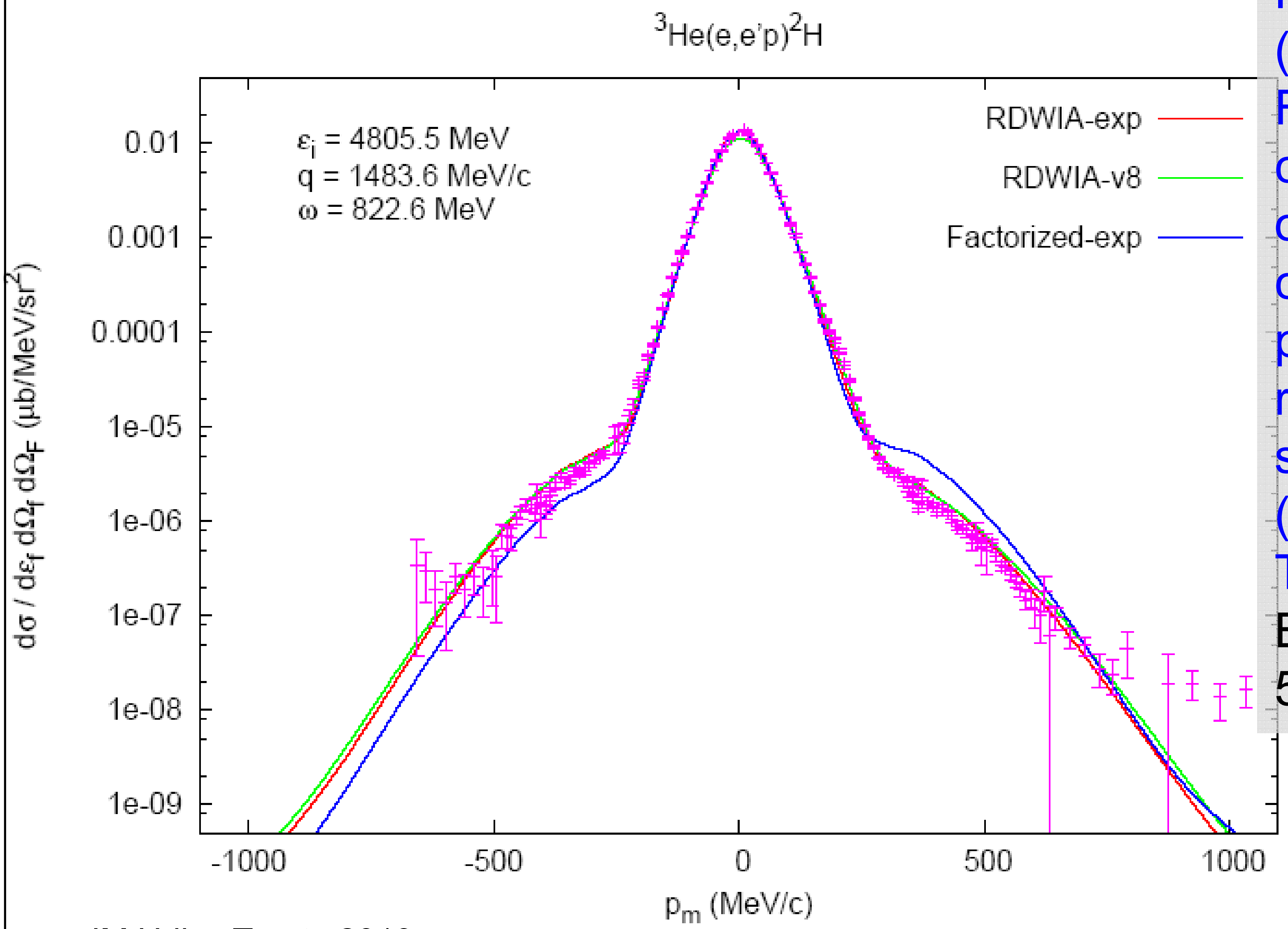
This factorization can be driven further away

$$\frac{d^5\sigma}{d\Omega_e d\varepsilon' d\Omega_F} = K\sigma_{ep} S(E_m, \vec{p}_m) \qquad \rho^{exp}(\mathbf{p}_m) = \frac{\left(\frac{d\sigma}{d\varepsilon_f d\Omega_f d\Omega_F}\right)^{exp}}{E_F p_F f_{rec}\, \sigma_{ep}}.$$



Factorization approach

Breakdown of factorization will be seen at **demanding** kinematics (q-ω constant, high momentum)



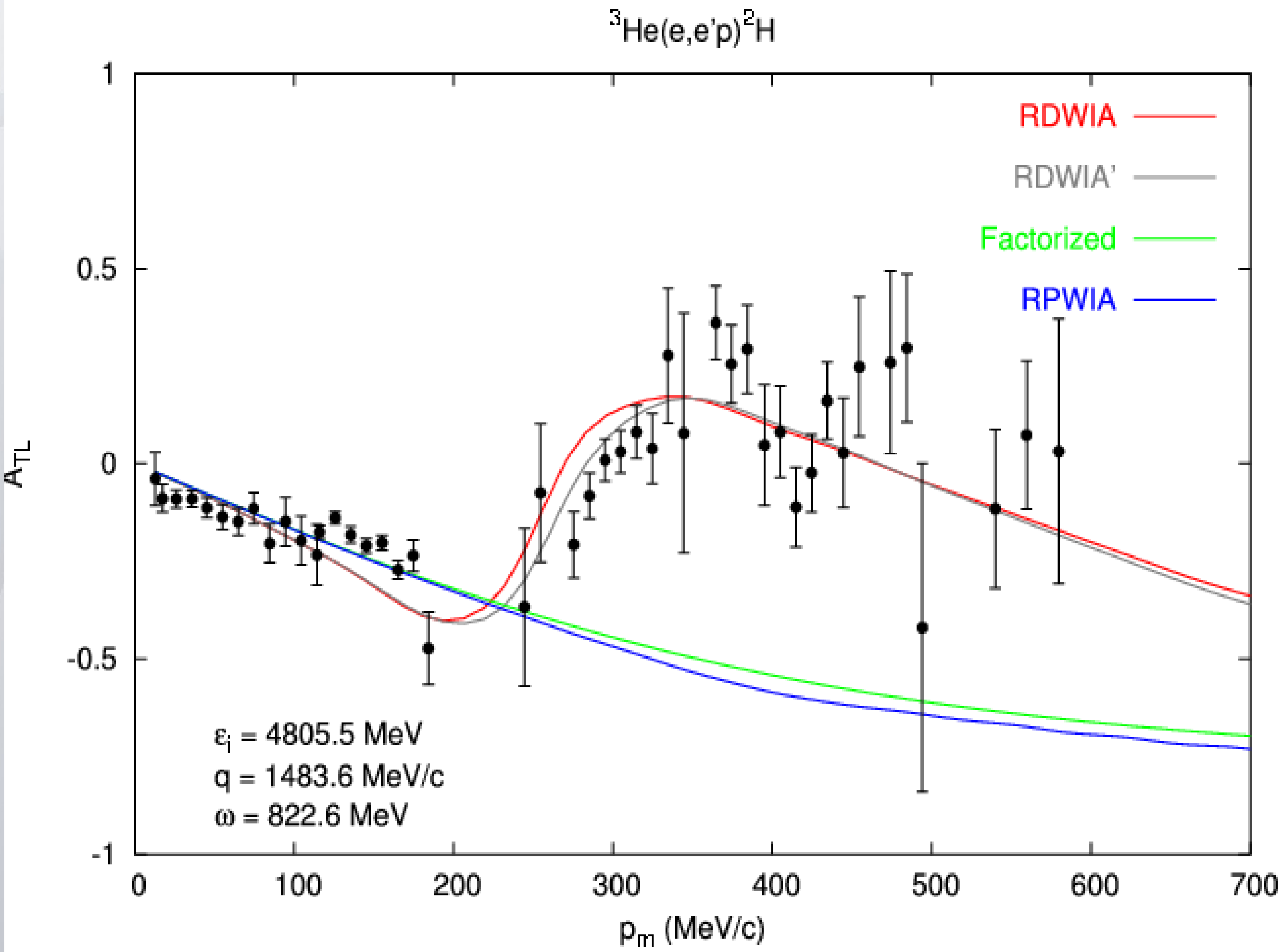
Data: M.M. Ravchev, PRL 94 (2005) 192302

Full theoretical calculation of the overlap from Faddev calculations. No free parameters in these results, not even the spectroscopic factors (of the order of 0.65)

Theory from Few-Body Syst (2011) 50:359

A_{TL} in ^{3}He, ^{4}He and ^{16}O

Asymmetry measured in (e,e'p) exclusive reactions. There are relativistic dynamical effects with a strong impact on A_{TL} which would be seen, particularly at moderate p_m.There is a noticeable difference in A_{TL} predictions for ^{3}He due to relativistic dynamics. This asymmetry is recovered with a relativistic potential in the FSI, within this approach. In other approaches it comes from MEC/beyond tree level diagrams.



M. Rvachev et al. PRL 94:12320,2005

Summary

- The RMF is successful in describing the universal inclusive scaling function representing the pure nucleonic response. But the potentials do not exhibit energy dependence in the potential, constituting a problem (too much FSI effect) for large nucleon energies.

- Optical potentials do exhibit energy dependence <u>and</u> absorption. They reproduce well exclusive data, and if used without the absroptive term (imaginary component), coupled with RMF bound state wave functions, seem to describe reasonably well the QE peak. There are more formal waves of gloing from the full optical potential to the all channels summed up (RGF) but the results are very similar to the simple prescription here

- Factorization is broken down by relativistic effects, but this should not prevent us from making successfull inclusive predictions with factorized calculations