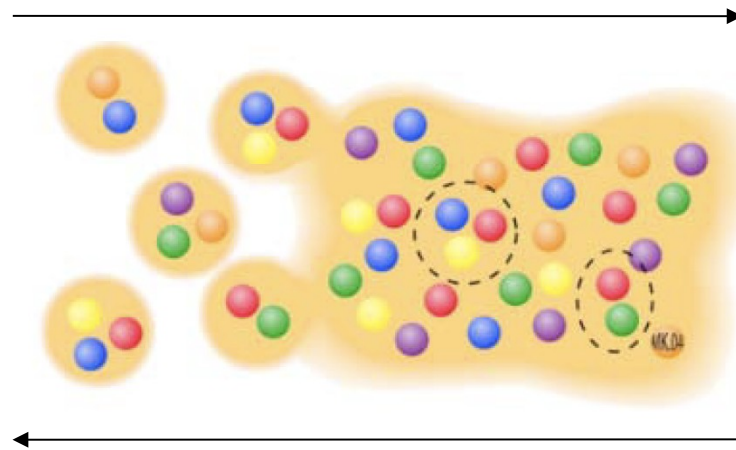


# Production of hadrons and nuclei as quark clusters at chemical freezeout

**David Blaschke**

University of Wroclaw, Poland & JINR Dubna & MEPhI Moscow, Russia

Mott delocalization = dissociation, deconfinement



Hadrons

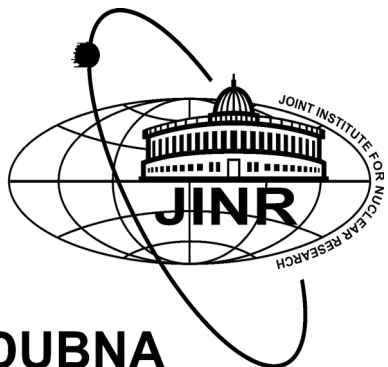
Nuclei

Quark-  
Gluon

Plasma

Mott localization = hadronization, confinement

**Light Clusters in Nuclei and Nuclear Matter, ECT\* Trento, 05.09.2019**



DUBNA



Uniwersytet  
Wrocławski



Grant No. 17-12-01427

# Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

The basic idea: Localization of (certain) multiquark states (“cluster”) = hadronization;  
Reverse process = delocalization by quark exchange between hadrons

Freeze-out criterion:

$$H_{\text{exp}}(\tau) = \frac{\dot{R}(\tau)}{R(\tau)} = \tau_{\text{coll},i}^{-1}(T, \mu),$$

$$\tau_{\text{coll},i}^{-1}(T, \mu) = \sum_j \sigma_{ij} v n_j(T, \mu)$$

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

$$r_{\pi}^2(T, \mu) = \frac{3}{4\pi^2} f_{\pi}^{-2}(T, \mu)$$

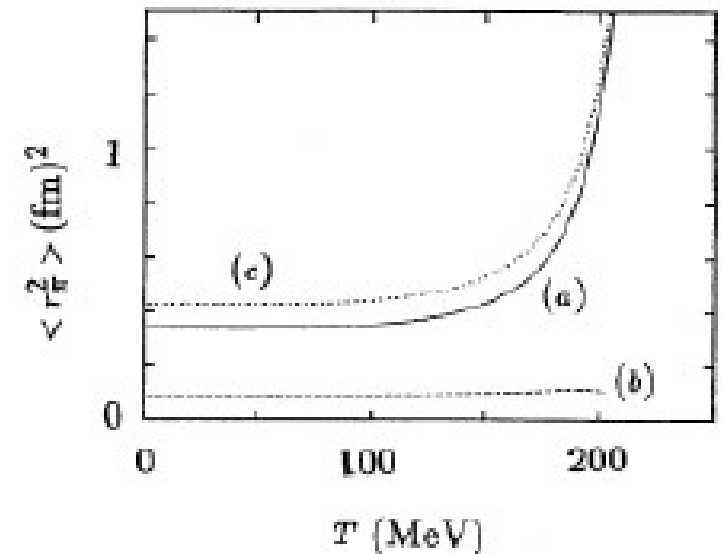
$$f_{\pi}^2(T, \mu) = -m_0 \langle \bar{q}q \rangle_{T, \mu} / M_{\pi}^2$$

$$r_{\pi}^2(T, \mu) = \frac{3 M_{\pi}^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1}$$

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\text{MF}} \left[ 1 - \frac{T^2}{8f_{\pi}^2(T, \mu)} - \frac{\sigma_N n_{s,N}(T, \mu)}{M_{\pi}^2 f_{\pi}^2(T, \mu)} \right]$$



Hippe & Klevansky, PRC 52 (1995) 2172



# Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

Povh-Huefner law behaviour for quark exchange between hadrons

PHYSICAL REVIEW C

VOLUME 51, NUMBER 5

MAY 1995

## Quark exchange model for charmonium dissociation in hot hadronic matter

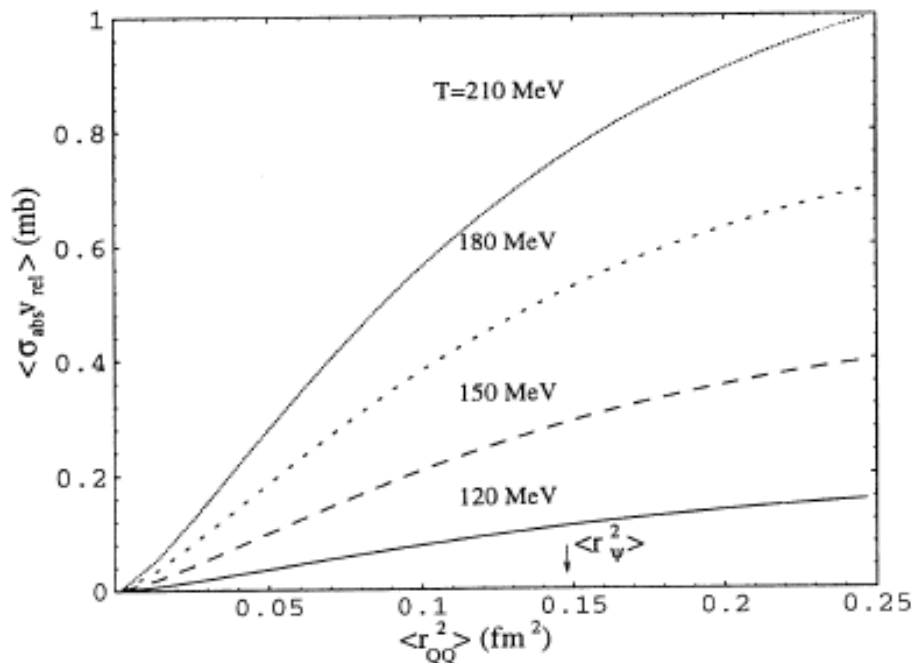
K. Martins\* and D. Blaschke†

*Max-Planck-Gesellschaft AG "Theoretische Vielteilchenphysik," Universität Rostock, D-18051 Rostock, Germany*

E. Quack‡

*Gesellschaft für Schwerionenforschung mbH, Postfach 11 05 52, D-64220 Darmstadt, Germany*

(Received 15 November 1994)



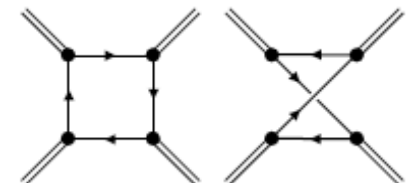
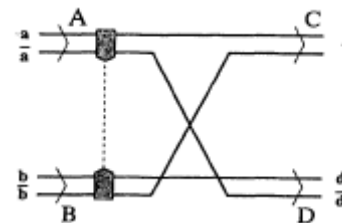
$$\langle \sigma_{abs} v_{rel} \rangle \propto \langle r^2 \rangle_{Q\bar{Q}} \langle r^2 \rangle_{q\bar{q}}$$

### Flavor exchange processes



Nonrelativistic  $\rightarrow$  rel. quark loop integrals

$M_{fi} =$



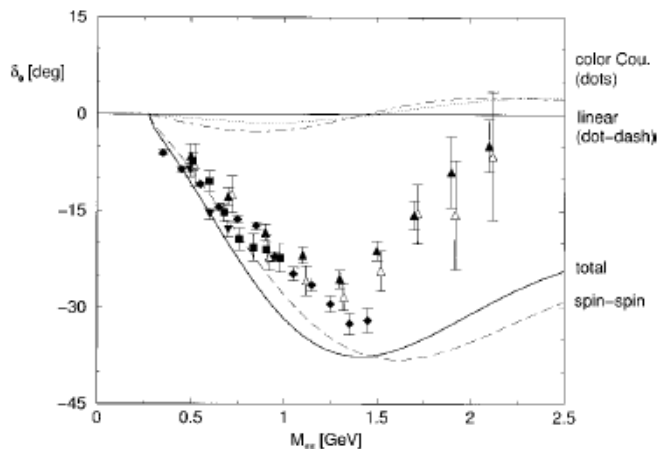
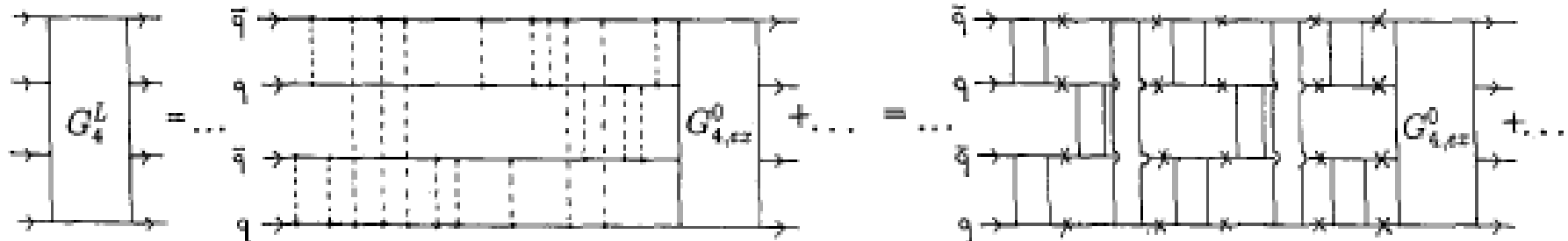
# Quark exchange in meson-meson scattering

DB, G. Roepke, Phys. Lett. B 299 (1993) 332; T. Barnes et al., PRC 63 (2001) 025204

Povh-Huefner law behaviour for quark exchange between hadrons ?

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle \quad r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1}$$

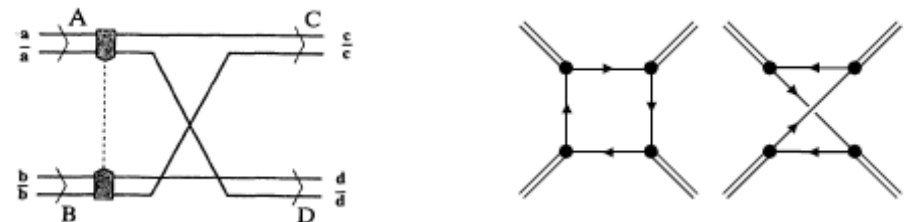
$$\mathcal{M}^{ss}(12, 1'2') = \frac{16}{3\sqrt{3}} C_{\text{SFC}}(12, 1'2') \frac{(2\pi)^3}{\Omega_0} \frac{\alpha_s}{3\pi^2 m_q^2} \exp\left(-\frac{1}{4b^2} (k'^2 + \frac{1}{3}k^2)\right) \delta_{K,K'}$$



Quark exchange process in M-M scattering

Nonrelativistic  $\rightarrow$  rel. quark loop integrals

$$M_{t1} =$$



# Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

Model results:

$$\tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu)$$

Collision time strongly T, mu dependent !

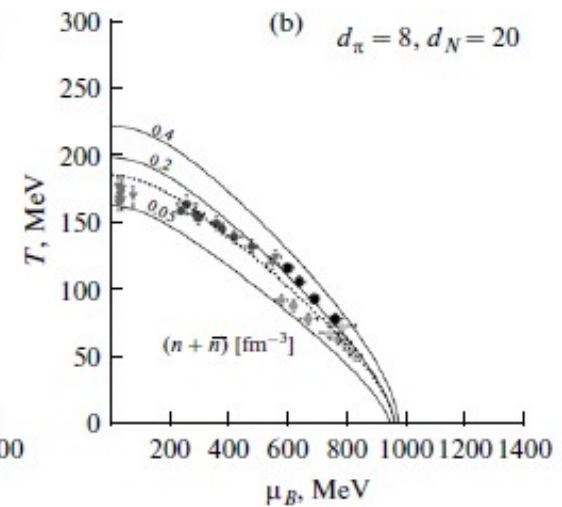
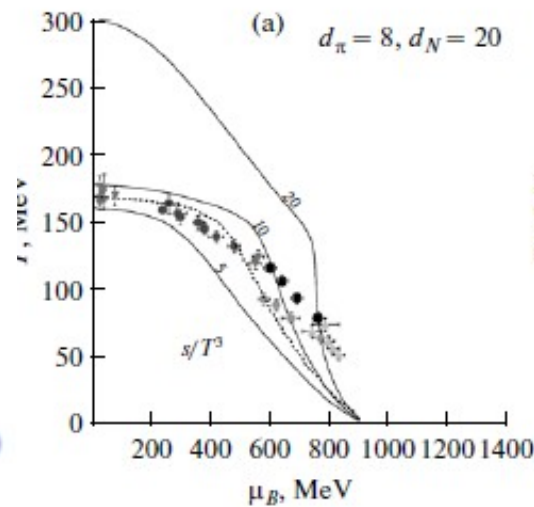
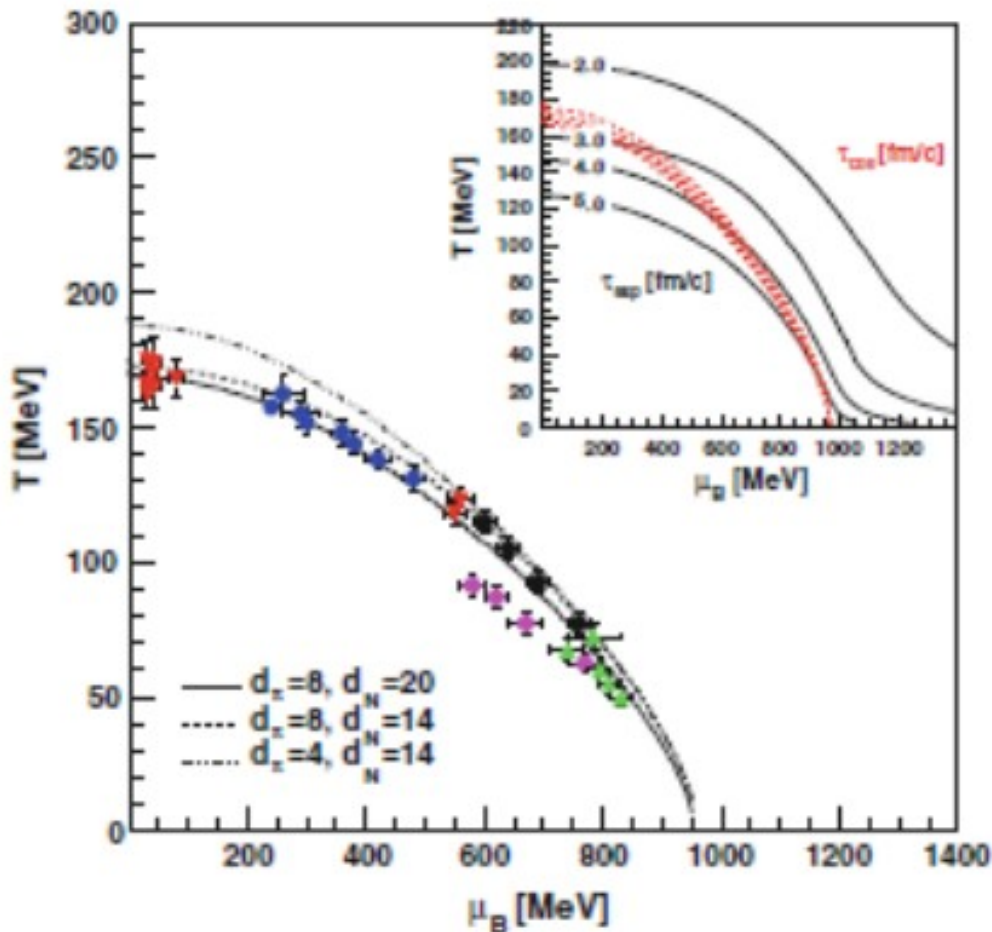
Schematic resonance gas:  $d\pi$  pions,  $dN$  nucleons

Expansion time scale from entropy conservation:

$$s(T, \mu) V(\tau_{\text{exp}}) = \text{const}$$

$$\tau_{\text{exp}}(T, \mu) = a s^{-1/3}(T, \mu),$$

Thermodynamics consistent with phenomenological Freeze-out rules:



# Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, *Few Body Syst.* 53 (2012) 99

## Model results:

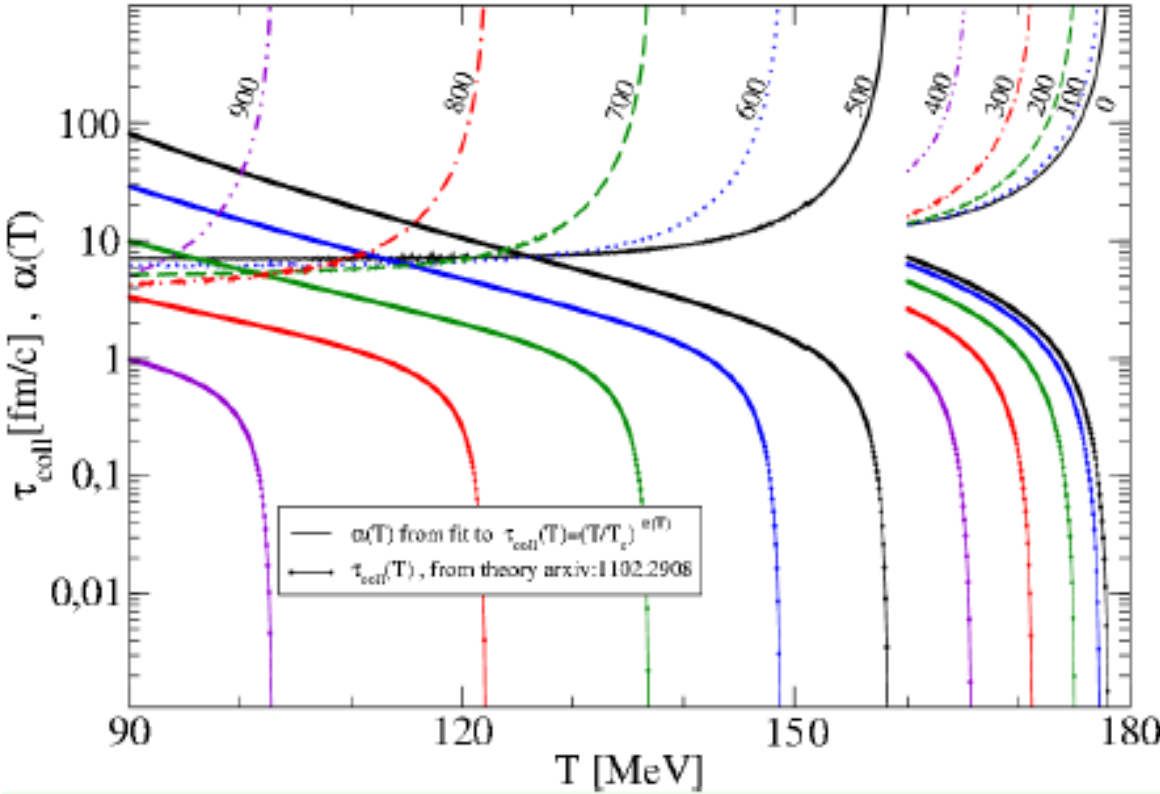
### Full hadron resonance gas model

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle ;$$

$$r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1}$$

$$r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu)$$

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{\text{vac}}} = 1 - \frac{m_0}{F_\pi^2 m_\pi^2} \left[ 4N_c \int \frac{dp p^2}{2\pi^2} \frac{m}{\varepsilon_p} [f_\Phi^+ + f_\Phi^-] \right. \\ \left. + \sum_{M=f_0, \omega, \dots} d_M (2 - N_s) \int \frac{dp p^2}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p)) \right. \\ \left. + \sum_{B=N, \Lambda, \dots} d_B (3 - N_s) \int \frac{dp p^2}{2\pi^2} \frac{m_B}{E_B(p)} [f_B^+(E_B(p)) + f_B^-(E_B(p))] \right] \\ - \sum_{G=\pi, K, \eta, \eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp \frac{p^2}{E_G(p)} f_G(E_G(p)).$$



Collision time follows a power law

$$\tau_{\text{coll}} \sim (T/T_c)^\alpha$$

with a large exponent  $\alpha \sim 20$

See also: P. Braun-Munzinger, J. Stachel, C. Wetterich, *PLB* (2004)

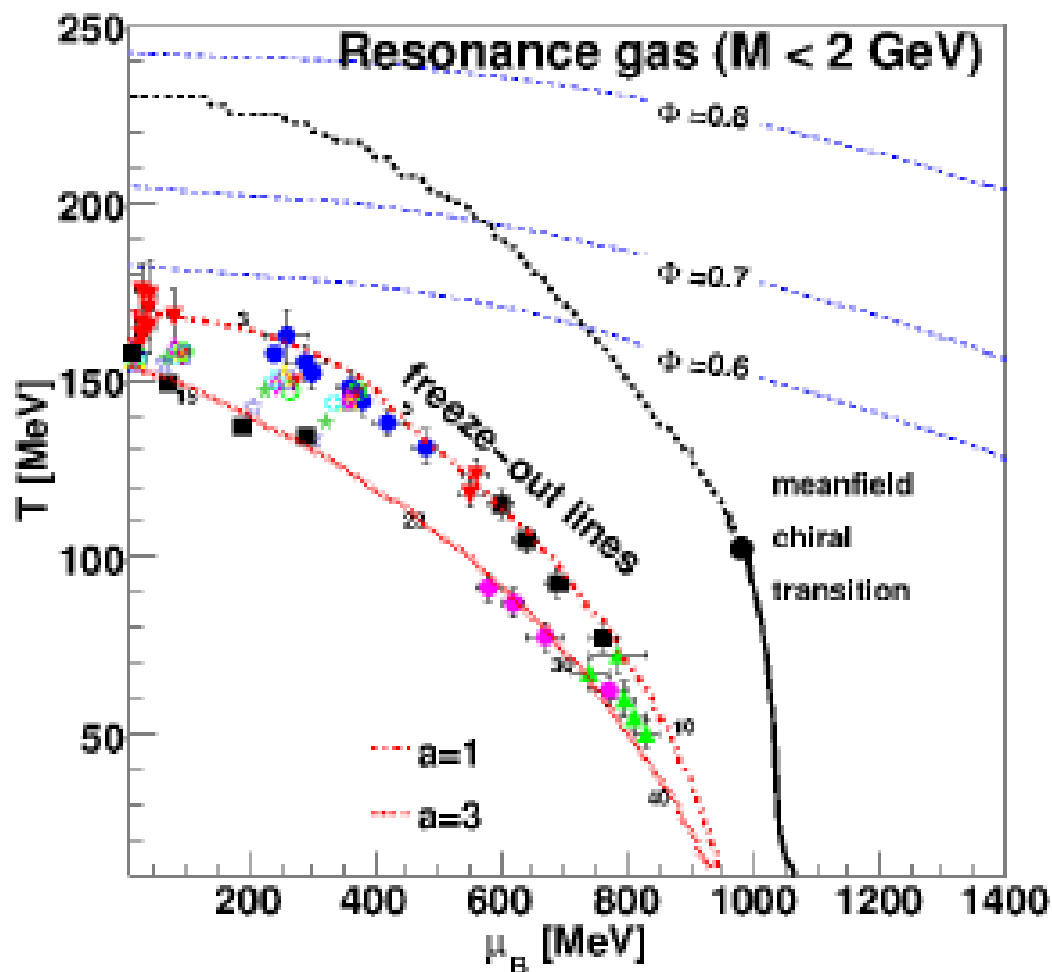
# 1. Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, *Few Body Syst.* 53 (2012) 99

## Model results:

Full hadron resonance gas model

See also: S. Leupold, *J. Phys. G* (2006)



$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{\text{vac}}} = 1 - \frac{m_0}{F_\pi^2 m_\pi^2} \left[ 4N_c \int \frac{dp p^2}{2\pi^2} \frac{m}{\varepsilon_p} [f_\Phi^+ + f_\Phi^-] \right. \\ \left. + \sum_{M=f_0, \omega, \dots} d_M (2 - N_s) \int \frac{dp p^2}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p)) \right. \\ \left. + \sum_{B=N, \Lambda, \dots} d_B (3 - N_s) \int \frac{dp p^2}{2\pi^2} \frac{m_B}{E_B(p)} [f_B^+(E_B(p)) + f_B^-(E_B(p))] \right] \\ - \sum_{G=\pi, K, \eta, \eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp \frac{p^2}{E_G(p)} f_G(E_G(p)).$$

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle ; \quad r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu)$$

$$r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1}$$

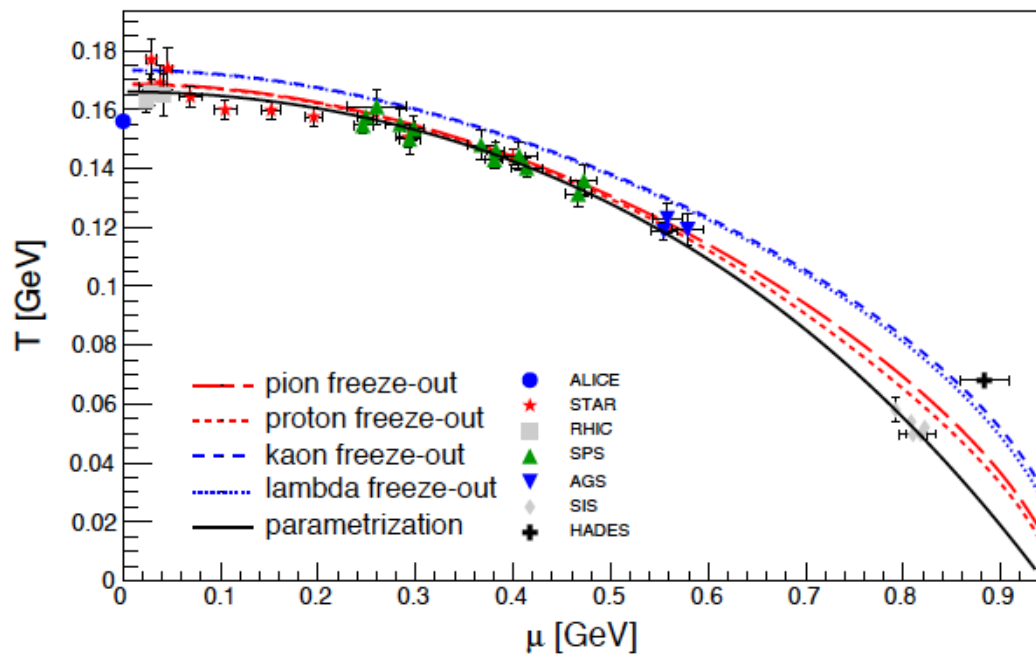
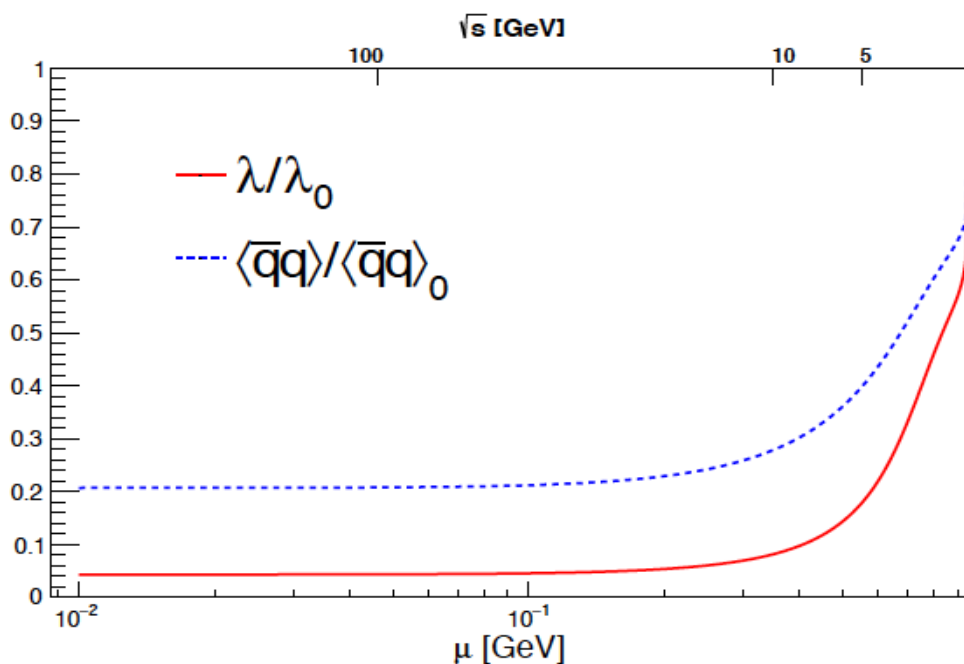
The factor  $a$  stands for the inverse system size in the formula

$$\tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu)$$

for the 3D expansion time scale assuming entropy conservation

# Mott-Anderson localization model for $K^+/\pi^+$ “horn”

DB, J. Jankowski, M. Naskret, arxiv:1705.00169; arxiv:1501.01599



Full HRG model condensate;

J. Jankowski et al., Phys. Rev. D (2013)

$$\langle \bar{q}q \rangle_{T,\mu} = \langle \bar{q}q \rangle_{T,\mu}^{MF} + \sum_{h=M,B} \frac{\sigma_q^h}{m_q} n_h(T, \mu),$$

$$n_h(T, \mu) = \frac{d_h}{2\pi^2} \int_0^\infty dk k^2 \frac{m_h}{E_h} \frac{1}{e^{(E_h - \mu_h)/T} \mp 1}.$$

$$\tau_{\text{coll},i}^{-1}(T, \mu) = \sum_j \sigma_{ij} v n_j(T, \mu); \quad \sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

$$\langle r_\pi^2 \rangle_{T,\mu} \simeq \frac{3}{4\pi^2} f_\pi^{-2}(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1}$$

$$\langle r_K^2 \rangle_{T,\mu} \simeq \frac{3M_K^2}{\pi^2 (m_q + m_s)} |\langle \bar{q}q \rangle_{T,\mu} + \langle \bar{s}s \rangle_{T,\mu}|^{-1}$$

The factor  $a$  stands for the inverse system size in the formula

$$\tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu)$$

for the 3D expansion time scale assuming entropy conservation



## Nuclear to quark-matter transition in the string-flip model

C. J. Horowitz

*Nuclear Theory Center and Department of Physics, Indiana University, Bloomington, Indiana 47405*

J. Piekarewicz

*Supercomputer Computations Research Institute, Florida State University, Tallahassee, Florida 32306*

(Received 20 May 1991)

$$\begin{aligned}
 H &= \frac{1}{2}P_1^2 + \frac{1}{2}P_2^2 + \frac{1}{2}P_3^2 + v(x_{12}) + v(x_{23}) + v(x_{31}) \\
 &= -\frac{1}{2}\partial_1^2 - \frac{1}{2}\partial_2^2 - \frac{1}{2}\partial_3^2 + \frac{1}{2}(x_1 - x_2)^2 + \frac{1}{2}(x_2 - x_3)^2 \\
 &\quad + \frac{1}{2}(x_3 - x_1)^2.
 \end{aligned}
 \tag{3.1}$$

$$\psi_0(x_1, x_2, x_3) = x_{12}x_{23}x_{31} e^{-\lambda_0(x_{12}^2 + x_{23}^2 + x_{31}^2)^{1/2}}, \tag{3.2}$$

$$v(x_{p_1 p_2 p_3}) = \frac{1}{2}(x_{p_1} - x_{p_2})^2 + \frac{1}{2}(x_{p_2} - x_{p_3})^2 + \frac{1}{2}(x_{p_3} - x_{p_1})^2.$$

$$V(x) = \min_{[P]} V_P(x), \quad V_P(x) \equiv \sum_{i=1}^A v(x_{p_i^1 p_i^2 p_i^3}).$$

$$E_{\text{HF}}(\rho) = T_{\text{FG}}(\rho) + V_{\text{HF}}(\rho) \simeq \frac{\pi^2 \rho^2}{6} + \frac{1}{4\rho^2}. \tag{4.1}$$

Transition from ideal gas of isolated hadrons to  
 A Fermi gas of quarks = quark matter

