Sensitivity of Giant Resonances Energies to Nuclear Matter Properties and the Equation of State

Shalom Shlomo
Texas A&M University

Outline

Introduction

1. Important task: Develop a modern Energy Density Functional (EDF), $E = E[\rho]$, with enhanced predictive power for properties of rare nuclei.
2. We start from EDF obtained from the Skyrme N-N interaction.
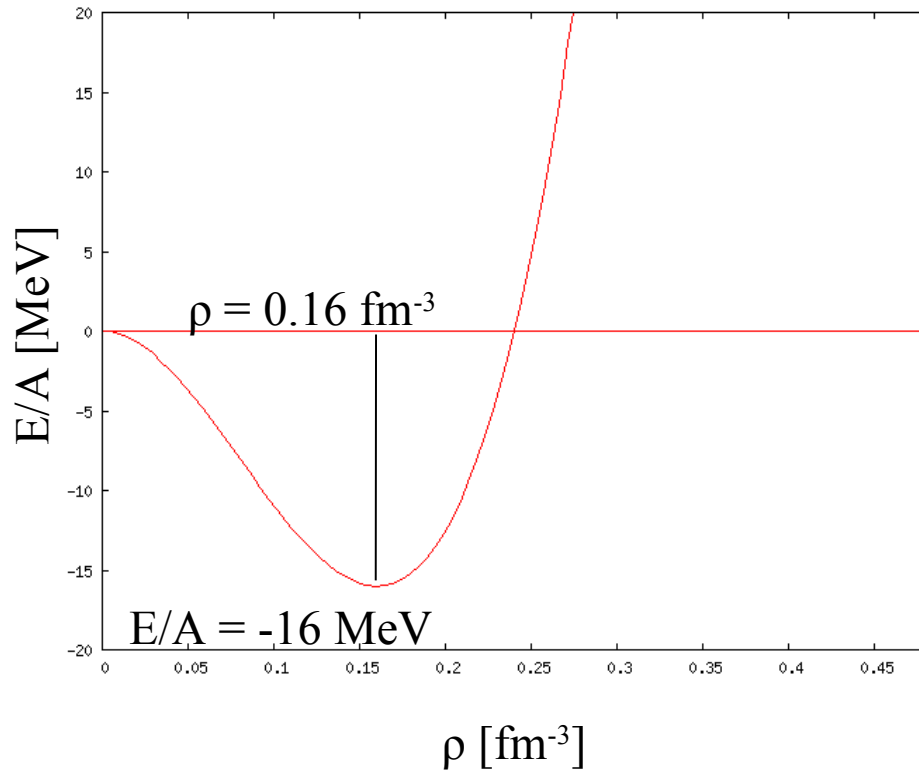3. The effective Skyrme interaction has been used in mean-field models for several decades. Many different parameterizations of the interaction have been realized to better reproduce nuclear masses, radii, and various other data. Today, there is more experimental data of nuclei far from the stability line. It is time to improve the parameters of Skyrme interactions. We fit our mean-field results to an extensive set of experimental data and obtain the parameters of the Skyrme type effective interaction for nuclei at and far from the stability line.
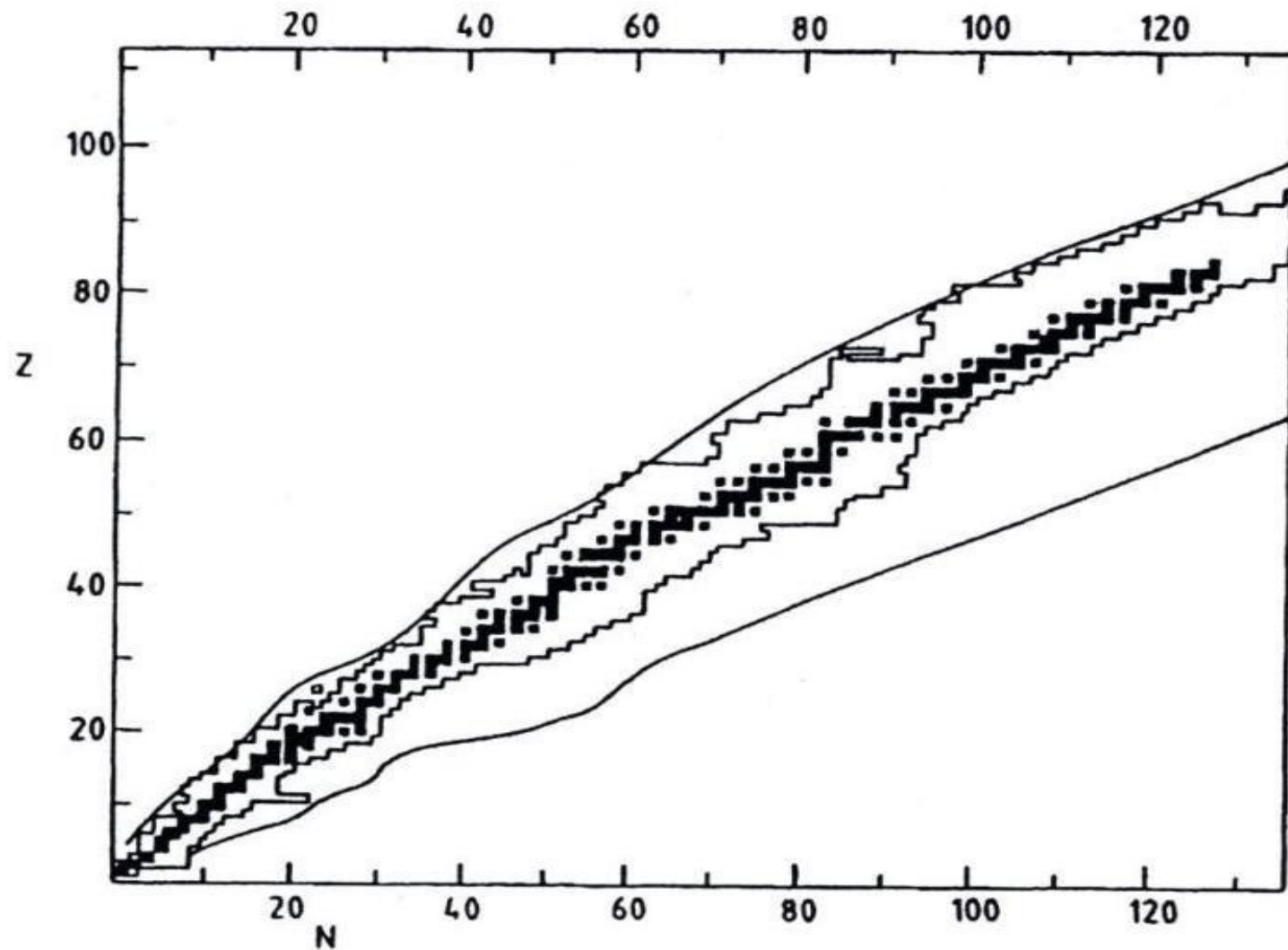
Map of the existing nuclei. The black squares in the central zone are stable nuclei, the broken inner lines show the status of known unstable nuclei as of 1986 and the outer lines are the assessed proton and neutron drip lines (Hansen 1991).

Equation of state and nuclear matter compressibility

The symmetric nuclear matter (N=Z and no Coulomb) incompressibility coefficient, *K,* is a important physical quantity in the study of nuclei, supernova collapse, neutron stars, and heavy-ion collisions, since it is directly related to the curvature of the nuclear matter (NM) equation of state (EOS), $E = E(\rho)$.

$$K = k_f^2 \frac{d^2(E/A)}{dk_f^2}\bigg|_{k_{fo}} = 9\rho^2 \frac{d^2(E/A)}{d\rho^2}\bigg|_{\rho_o}$$

$$E[\rho] = E[\rho_o] + \frac{1}{18} K \left(\frac{\rho-\rho_o}{\rho_o}\right)^2$$

$$E_{ANM}[\rho_o(\beta)] = E[\rho_o(\beta)] + \frac{1}{18} K(\rho_o(\beta)) \left(\frac{\rho-\rho_o(\beta)}{\rho_o(\beta)}\right)^2$$

$$E[\rho_o(\beta)] = E[\rho_o] + J\beta^2$$

$$K[\rho_o(\beta)] = K + K_{TV}\beta^2$$

$$E_{SYM}(\rho) = \frac{1}{8} \frac{d^2(E/A)}{dy^2}\bigg|_{\rho,y=1/2}$$

$$J = E_{SYM}[\rho_o]$$

$$\beta = (N-Z)/A \qquad y = Z/A$$

Modern Energy Density Functional

The total Hamiltonian of the nucleus

$$\hat{H}_{total} = T + V = \sum_{i=1}^{A} \frac{p_i^2}{2m_i} + \sum_{i \, ¿ \, ¿ \, ¿} ¿ \, ¿$$

where
$$V(\vec{r}_i, \vec{r}_j) = V_{ij}^{NN} + V_{ij}^{Coul} .$$

HF equations: minimize $E = \langle \Phi | \hat{H}_{total} | \Phi \rangle$

Within the HF approximation: the ground state wave function Φ

Skyrme interaction

For the nucleon-nucleon interaction $V(\vec{r}_i, \vec{r}_j) = V_{ij}^{NN} + V_{ij}^{Coul}$.

$$V_{ij}^{Coul} = -\frac{e^2}{4} \sum_{i,j=1}^{A} \frac{\tau_{ij}^2 + \tau_{ij}}{|\vec{r}_i - \vec{r}_j|}, \qquad \tau_{ij} = \tau_i + \tau_j$$

V_{ij}^{NN} we adopt the standard Skyrme type interaction

$$V_{ij}^{NN} = t_0(1 + x_0 P_{ij^\sigma})\delta(\vec{r}_i - \vec{r}_j) + \frac{1}{2} t_1(1 + x_1 P_{ij^\sigma})[\vec{k}_{ij}^2 \delta(\vec{r}_i - \vec{r}_j) + \delta(\vec{r}_i - \vec{r}_j)\vec{k}_{ij}^2] +$$

$$t_2(1 + x_2 P_{ij^\sigma})\vec{k}_{ij}\delta(\vec{r}_i - \vec{r}_j)\vec{k}_{ij} + \frac{1}{6} t_3(1 + x_3 P_{ij^\sigma})\rho^\alpha\left(\frac{\vec{r}_i + \vec{r}_j}{2}\right)\delta(\vec{r}_i - \vec{r}_j) +$$

$$iW_0\vec{k}_{ij}\delta(\vec{r}_i - \vec{r}_j)(\vec{\sigma}_i + \vec{\sigma}_j)\vec{k}_{ij},$$

t_i, x_i, α, W_0 are 10 Skyrme parameters.

The total energy

$$E = \langle \Phi | \hat{H}_{total} | \Phi \rangle = \langle \Phi | T + V_{Coulomb} + V_{12} | \Phi \rangle = \int H(\vec{r}) d\vec{r}$$

where

$$H(\vec{r}) = H_{Kinetic}(\vec{r}) + H_{Coulomb}(\vec{r}) + H_{Skyrme}(\vec{r})$$

$$H_{Kinetic}(\vec{r}) = \frac{\hbar^2}{2m_p} \tau_p(\vec{r}) + \frac{\hbar^2}{2m_n} \tau_n(\vec{r})$$

$$H_{Coulomb}(\vec{r}) = \frac{e^2}{2} \left[\rho_{ch}(\vec{r}) \int \frac{\rho_{ch}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d\vec{r}\,' - \int \frac{|\rho_{ch}(\vec{r}, \vec{r}\,')|^2}{|\vec{r} - \vec{r}\,'|} d\vec{r}\,' \right]$$

$$H_{Skyrme}(\vec{r}) = H_0 + H_3 + H_{eff} + H_{fin} + H_{so} + H_{sg}$$

$$\mathcal{H}_0 = \frac{1}{4}t_0 \left[(2 + x_0)\rho^2 - (2x_0 + 1)(\rho_p^2 + \rho_n^2) \right],$$

$$\mathcal{H}_3 = \frac{1}{24}t_3\rho^\alpha \left[(2 + x_3)\rho^2 - (2x_3 + 1)(\rho_p^2 + \rho_n^2) \right],$$

$$\mathcal{H}_{\text{eff}} = \frac{1}{8} \left[t_1(2 + x_1) + t_2(2 + x_2) \right] \tau\rho + \frac{1}{8} \left[t_2(2x_2 + 1) - t_1(2x_1 + 1) \right] (\tau_p\rho_p + \tau_n\rho_n),$$

$$
\begin{aligned}
\mathcal{H}_{\text{fin}} &= \frac{1}{32} \left[3t_1(2 + x_1) - t_2(2 + x_2) \right] (\nabla\rho)^2 \\
&\quad - \frac{1}{32} \left[3t_1(2x_1 + 1) + t_2(2x_2 + 1) \right] \left[(\vec{\nabla}\rho_p)^2 + (\vec{\nabla}\rho_n)^2 \right], \\
\mathcal{H}_{\text{so}} &= \frac{W_0}{2} \left[\mathbf{J} \cdot \nabla\rho + \mathbf{J}_p \cdot \nabla\rho_p + \mathbf{J}_n \cdot \nabla\rho_n \right], \\
\mathcal{H}_{\text{sg}} &= -\frac{1}{16}(t_1 x_1 + t_2 x_2)\mathbf{J}^2 + \frac{1}{16}(t_1 - t_2) \left[\mathbf{J}_p^2 + \mathbf{J}_n^2 \right].
\end{aligned}
$$

After carrying out the minimization of energy, we obtain the HF equations:

$$\frac{\hbar^2}{2\, m^{¿}_{\tau}(r)}\; ¿\; ¿\; ¿$$

¿

where $m^{¿}_{\tau}(r)$, $U_{\tau}(r)$, and $W_{\tau}(r)$ are the effective mass, the potential and the spin orbit potential. They are given in terms of the Skyrme parameters and the nuclear densities.

Simulated Annealing Method (SAM)

The SAM is a method for optimization problems of large scale, in particular, where a desired global extremum is hidden among many local extrema.

We use the SAM to determine the values of the Skyrme parameters by searching the global minimum for the chi-square function

$$\chi^2 = \frac{1}{N_d - N_p} \sum_{i=1}^{N_d} \left(\frac{M_i^{\exp} - M_i^{th}}{\sigma_i} \right)^2$$

N_d is the number of experimental data points.

N_p is the number of parameters to be fitted.

$M_i^{\exp}$ and M_i^{th} are the experimental and the corresponding theoretical values of the physical quantities.

σ_i is the adopted uncertainty.

Implementing the SAM to search the global minimum of χ^2 function:

1. t_i, x_i, α, W_0 are written in term of $B/A, K_{nm}, \rho_{nm}, \ldots$

2. Define $\vec{v}\left(B/A, K_{nm}, \rho_{nm}, m*/m, E_s, J, L, \kappa, G_0', W_0\right)$

3. Calculate χ^2_{old} for a given set of experimental data and the corresponding HF results (using an initial guess for Skyrme parameters).

    .

4. Determine a new set of Skyrme parameters by the following steps:

    + Use a random number to select a component v_r of vector $\vec{v}$

    + Use another random number η to get a new value of v_r

$$v_r \rightarrow v_r + d\eta$$

    + Use this modified vector $\vec{v}$ to generate a new set of Skyrme parameters.

5. Go back to HF and calculate χ^2_{new}

6. The new set of Skyrme parameters is accepted only if

$$P(\chi^2) = \exp\left(\frac{\chi^2_{old} - \chi^2_{new}}{T}\right) > \beta$$

$$0 < \beta < 1$$

7. Starting with an initial value of $T = T_i$, we repeat steps 4 - 6 for a large number of loops.

8. Reduce the parameter T as $E[\rho_i(\beta)] = E[\rho_i] + \gamma \beta^2$ and repeat steps 1 – 7.

9. Repeat this until hopefully reaching global minimum of χ^2

Fitted data

- The binding energies for 14 nuclei ranging from normal to the exotic (proton or neutron) ones: ^{16}O, $\underline{^{24}\mathbf{O}}$, $\underline{^{34}\mathbf{Si}}$, ^{40}Ca, ^{48}Ca, $\underline{^{48}\mathbf{Ni}}$, ^{56}Ni, ^{68}Ni, ^{78}Ni, ^{88}Sr, ^{90}Zr, $\underline{^{100}\mathbf{Sn}}$, ^{132}Sn, and ^{208}Pb.

  - Charge rms radii for 7 nuclei: ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, $\underline{^{88}\mathbf{Sr}}$, ^{90}Zr, ^{208}Pb.

- **The spin-orbit splittings for 2p proton and neutron orbits for** ^{56}**Ni**
$$\varepsilon(2p_{1/2}) - \varepsilon(2p_{3/2}) = 1.88 \text{ MeV (neutron)}$$
$$\varepsilon(2p_{1/2}) - \varepsilon(2p_{3/2}) = 1.83 \text{ MeV (proton).}$$

  - Rms radii for the valence neutron:

  in the 1$d_{5/2}$ orbit for ^{17}**O** $\quad r_n(1d_{5/2})=3.36\,fm$

  in the 1$f_{7/2}$ orbit for ^{41}**Ca** $\quad r_n(1f_{7/2})=3.99\,fm$

  - The breathing mode energy for 4 nuclei: ^{90}**Zr (17.81 MeV),** ^{116}**Sn (15.9 MeV),** ^{144}**Sm (15.25 MeV), and** ^{208}**Pb (14.18 MeV).**

Constraints

1. The critical density $\quad 2\rho_0 < \rho_{cr} < 3\rho_0$

$$V_{p-h}^{Landau} = \sum_l \left(F_l + F_l'\tau_1\tau_2 + G_l\sigma_1\sigma_2 + G_l'\sigma_1\sigma_2\tau_1\tau_2 \right)\delta\left(\vec{r}_1 - \vec{r}_2\right)$$

Landau stability condition: $\quad F_l, F_l', G_l, G_l' > -(2l+1)$

Example: $\quad K = 6\dfrac{\hbar^2 k_F^2 \left(1 + F_0\right)}{2m} \Big/ \left(1 + F_1/3\right)$

2. The Landau parameter $\quad G_0'\quad$ should be positive at $\quad \rho = \rho_0$

3. The quantity $\quad P = 3\rho\dfrac{dS}{d\rho}\quad$ must be positive for densities up to $\quad 3\rho_0$

4. The IVGDR enhancement factor $\quad 0.25 < \kappa < 0.5$

$$\int ES_{L=1}^{T=1}(E)\,dE = \frac{\hbar^2}{2m}\frac{NZ}{A}(1+\kappa)$$

	v	v_0	v_1	d
B/A (MeV)	16.0	17.0	15.0	0.4
K_{nm} (MeV)	230.0	200.0	300.0	20.0
ρ_{nm} (fm^{-3})	0.160	0.150	0.170	0.005
m*/m	0.70	0.60	0.90	0.04
E_s (MeV)	18.0	17.0	19.0	0.3
J (MeV)	32.0	25.0	40.0	4.0
L (MeV)	47.0	20.0	80.0	10.0
Kappa	0.25	0.1	0.5	0.1
G'$_0$	0.08	0.00	0.40	0.10
W$_0$ (MeV fm^5)	120.0	100.0	150.0	5.0

Values of the Skyrme parameters and the corresponding physical quantities of nuclear matter for the KDE0 and KDE0v1 and KDEX interactions.

| Parameter	KDE0	KDE0v1	KDEX
t_0 (MeV fm^3)	-2526.5110	-2553.0843	-1419.8304
t_1 (MeV fm^5)	430.9418	411.6963	309.1373
t_2 (MeV fm^5)	-398.3775	-419.8712	-172.9562
t_3 (MeVfm$^{3(1+\alpha)}$)	14235.5193	14603.6069	10465.3523
x_0	0.7583	0.6483	0.1474
x_1	-0.3087	-0.3472	-0.0853
x_2	-0.9495	-0.9268	-0.6144
x_3	1.1445	0.9475	0.0220
W_0(MeV fm^5)	128.9649	124.4100	98.8973
α	0.1676	0.1673	0.4989
B/A (MeV)	16.11	16.23	15.96
K (MeV)	228.82	227.54	274.20
ρ_0 (fm^{-3})	0.161	0.165	0.155
m*/m	0.72	0.74	0.81
J (MeV)	33.00	34.58	32.76
L (MeV)	45.22	54.69	63.70
κ	0.30	0.23	0.33
G'_0	0.05	0.00	0.41

HF results for the total binding energy B (in MeV) and charge rms radii r_{ch} (in fm) and the corresponding deviations from the experimental values $\Delta B = B^{exp} - B^{th}$ and $\Delta r_{ch} = r_{ch}^{exp} - r_{ch}^{th}$, for several nuclei

Nuclei	B^{exp}	$\Delta B = B^{exp} - B^{th}$		r^{exp}	$\Delta r_{ch} = r_{ch}^{exp} - r_{ch}^{th}$	
		KDE0	KDE		KDE0	KDE
^{16}O	127.620	0.394	1.011	2.730	-0.041	-0.039
^{24}O	168.384	-0.581	0.370			
^{34}Si	283.427	-0.656	0.060			
^{40}Ca	342.050	0.005	0.252	3.49	0.000	0.011
^{48}Ca	415.990	0.188	1.165	3.480	-0.021	-0.008
^{48}Ni	347.136	-1.437	-3.67			
^{56}Ni	483.991	1.091	1.106	3.750	-0.018	0.000
^{68}Ni	590.408	0.169	0.539			
^{78}Ni	641.940	-0.252	0.763			
^{88}Sr	768.468	0.826	1.132	4.219	-0.002	0.019
^{90}Zr	783.892	-0.127	-0.200	4.258	-0.008	0.013
^{100}Sn	824.800	-3.664	-4.928			
^{132}Sn	1102.850	-0.422	-0.314			
^{208}Pb	1636.430	0.945	-0.338	5.500	0.011	0.041

Recently,

M. Dutra, O. Lourenço, J. S. Sá Martins, A. Delfino, J. R. Stone, and P. D. Stevenson, Phys. Rev. C **85**, 035201 (2012); P. D. Stevenson, P. M. Goddard, J. R. Stone and M. Dutra, ArXiv:1210.1592,

analyzed 240 Skyrme interaction parameter sets, published in the literature, for their ability to pass constraints relating to current experimental data. Requiring a good fit to: (i) properties of nuclear matter close to the saturation density, such as the incompressibility coefficient, density dependence of the symmetry energy and effective mass; (ii) properties of finite nuclei, such as binding energy, radii and fission barriers: and (iii) the observational data on neutron stars, in particular, the maximum mass of neutron stars, only the KDE0v1 interaction of passes the test. It is interesting to note that the data on neutron stars and fission barriers of nuclei were not included in the fit resulting with the parameters of the KDE0v1 interaction.
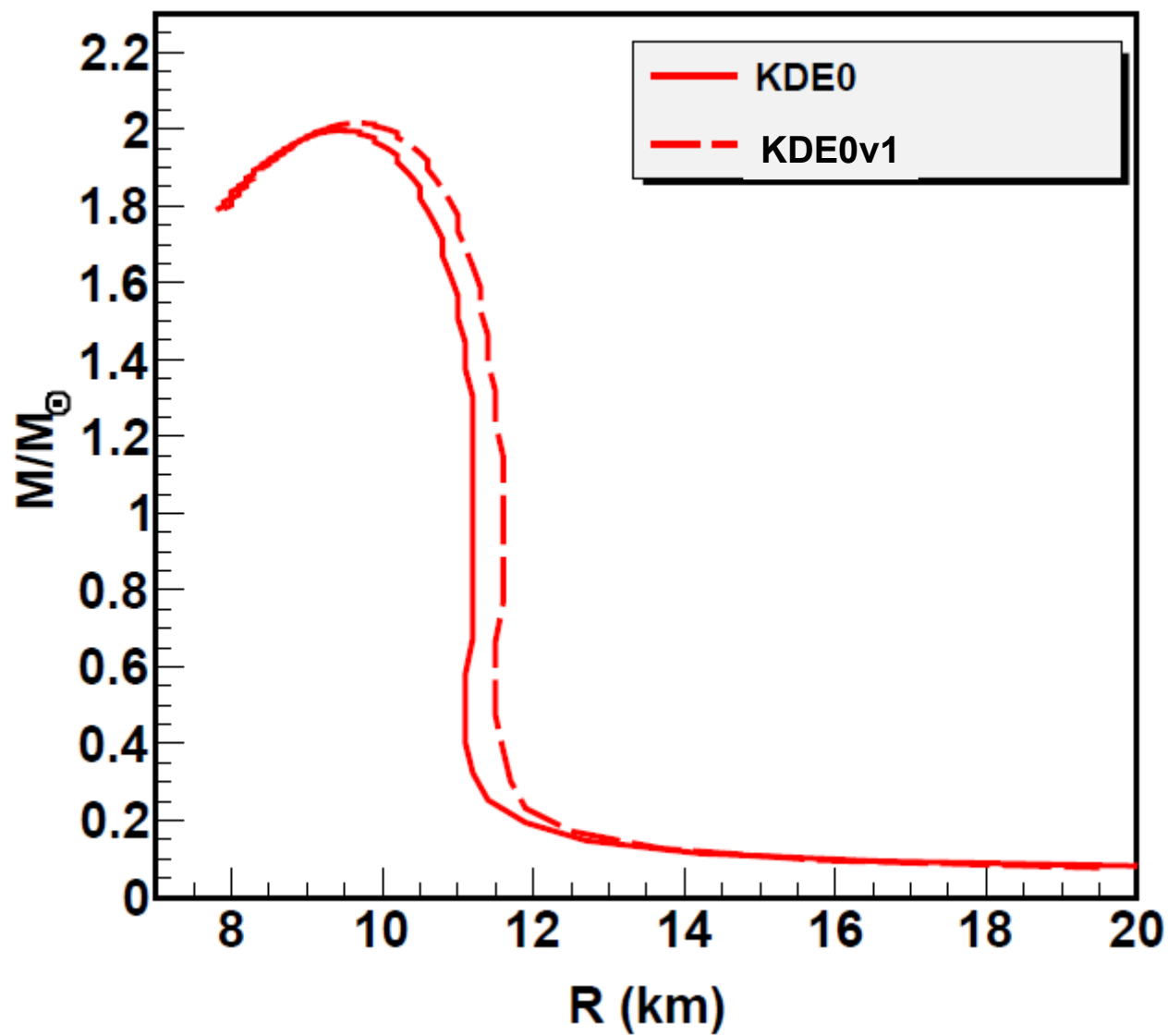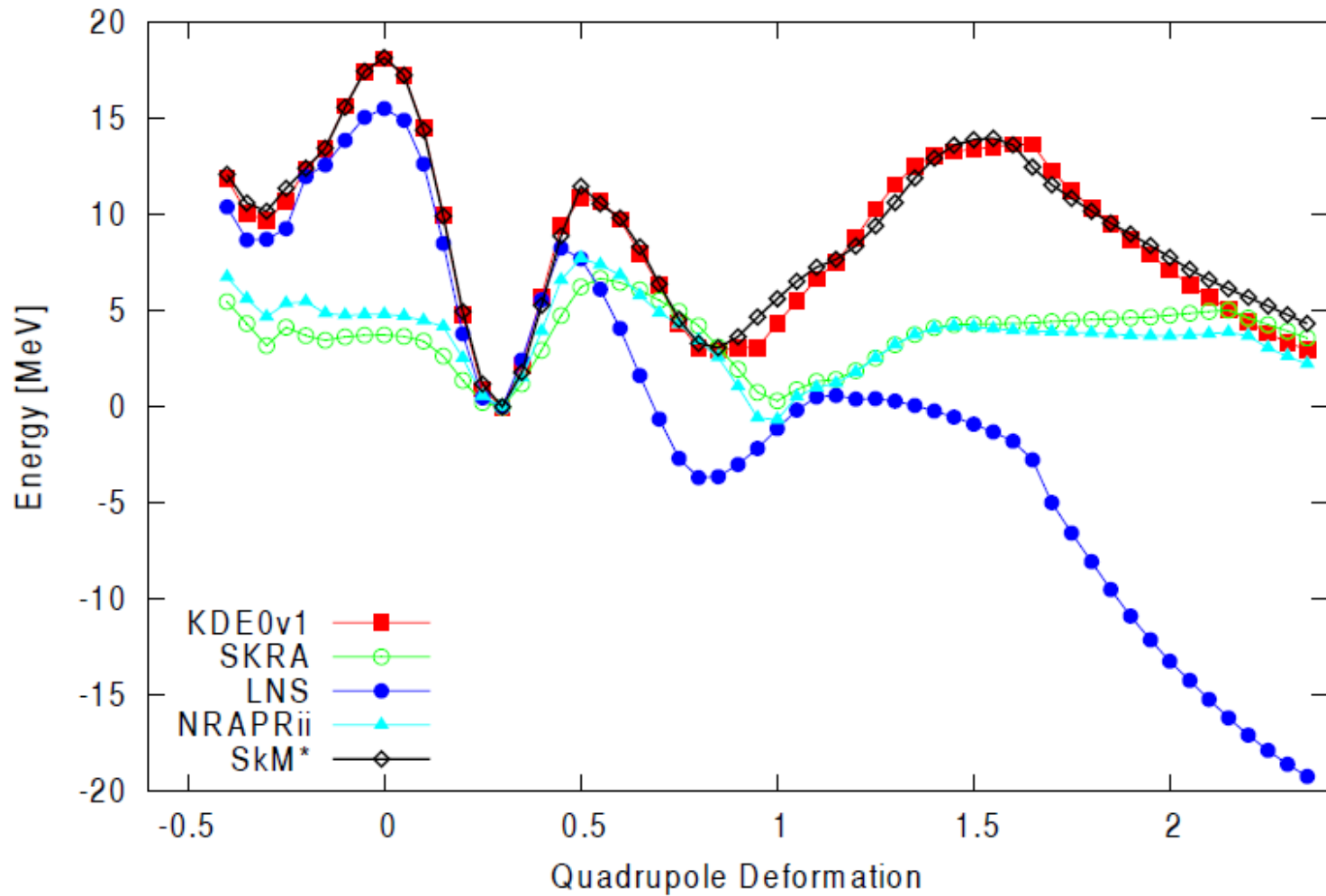
FIGURE 2. Fission barrier in ^{240}Pu for the subset of forces, compared with SkM*

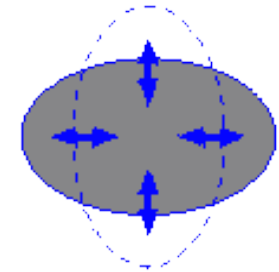P. D. Stevenson, P. M. Goddard, J. R. Stone and M. Dutra, ArXiv:1210.1592,
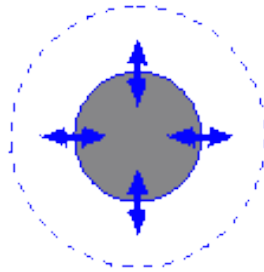
Macroscopic picture of giant resonances

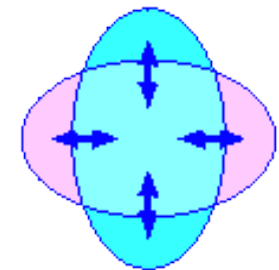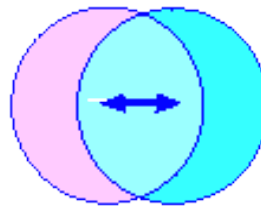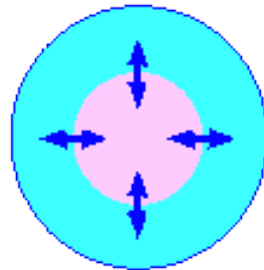monopole   dipole   quadrupole
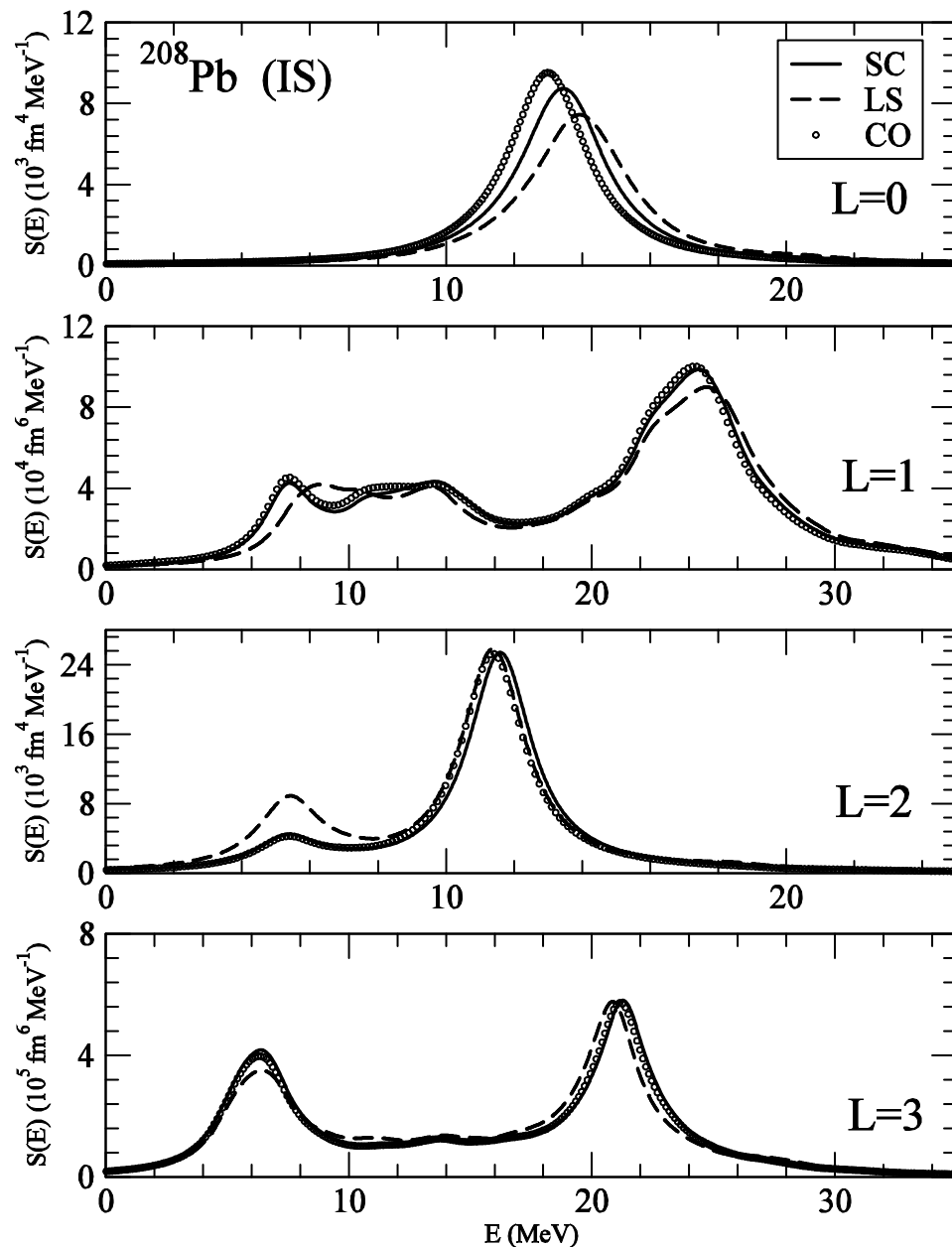
isoscalar
($\Delta T=0$)

isovector
($\Delta T=1$)

L = 0   L = 1   L = 2

Hartree-Fock (HF) - Random Phase Approximation (RPA)

In fully self-consistent calculations:

1. Assume a form for the Skyrme parametrization (δ-type).

2. Carry out HF calculations for ground states and determine the Skyrme parameters by a fit to binding energies and radii.

3. Determine the residual p-h interaction $\quad V_{\mathsf{php'h'}} = \dfrac{\delta^2 E[\rho]}{\delta \rho_{\mathsf{ph}} \delta \rho_{\mathsf{p'h'}}}$

4. Carry out RPA calculations of strength function, transition density etc.

Isoscalar strength functions of ^{208}Pb for L = 0 - 3 multipolarities are displayed. The SC (full line) corresponds to the fully self-consistent calculation where LS (dashed line) and CO (open circle) represent the calculations without the ph spin-orbit and Coulomb interaction in the RPA, respectively. The Skyrme interaction SGII [Phys. Lett. B **106**, 379 (1981)] was used.

Fully self-consistent HF-RPA results for ISGMR centroid energy (in MeV) with the Skyrme interaction SK255, SGII and KDE0 are compared with the RRPA results using the NL3 interaction. Note the corresponding values of the nuclear matter incompressibility, K, and the symmetry energy , J, coefficients. ω_1-ω_2 is the range of excitation energy. The experimental data are from TAMU.

Nucleus	ω_1-ω_2	Expt.	NL3	SK255	SGII	KDE0
^{90}Zr	0-60		18.7	18.9	17.9	18.0
	10-35	17.81±0.30		18.9	17.9	18.0
^{116}Sn	0-60		17.1	17.3	16.4	16.6
	10-35	15.85±0.20		17.3	16.4	16.6
^{144}Sm	0-60		16.1	16.2	15.3	15.5
	10-35	15.40±0.40		16.2	15.2	15.5
^{208}Pb	0-60		14.2	14.3	13.6	13.8
	10-35	13.96±0.30		14.4	13.6	13.8
K (MeV)			272	255	215	229
J (MeV)			37.4	37.4	26.8	33.0

ISGDR $\quad f = \left(r^3 - \frac{5}{3} \langle r^2 \rangle r \right) Y_{1M}$

SL1 interaction, K = 230 MeV, E_α = 240 MeV
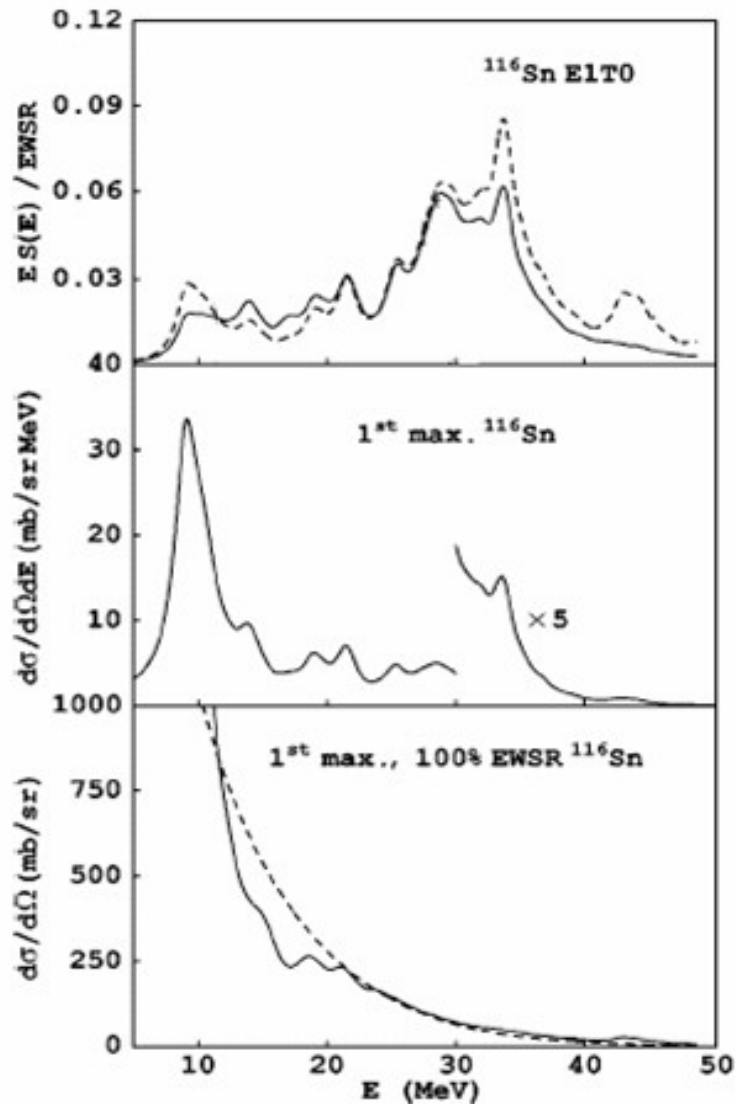
Reconstruction of the ISGDR EWSR in ^{116}Sn from the inelastic α-particle cross sections. The middle panel: maximum double differential cross section obtained from ρ_t (RPA). The lower panel: maximum cross section obtained with ρ_{coll} (dashed line) and ρ_t (solid line) normalized to 100% of the EWSR. Upper panel: The solid line (calculated using RPA) and the dashed line are the ratios of the middle panel curve with the solid and dashed lines of the lower panel, respectively.

$$\rho_{coll} = \left[10r + \left(3r^2 - \frac{5}{3} \langle r^2 \rangle \right) \frac{d\rho_0}{dr} \right] \rho_0(r)$$

HF based RPA calculations with 33 Skyrme interactions

We have carried out calculations of centroid energies, E_{CEN}, of the isoscalar (T = 0) and isovector (T = 1) giant resonances of multipolarities L = 0-3 in ^{40,48}Ca, ^{68}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb, within the fully self-consistent Hartree-Fock (HF)-based random phase approximation (RPA) theory, using 33 different Skyrme-type effective nucleon-nucleon interactions of standard form commonly adopted in the literature. We also study the sensitivity of E_{CEN} to physical properties of nuclear matter (NM), such as the effective mass m*/m, nuclear matter incompressibility coefficient K_{NM}, enhancement coefficient κ of the energy weighted sum rule for the isovector giant dipole resonance and symmetry energy at saturation density, associated with the Skyrme interactions used in the calculations. We deduced constraints for the values of the NM properties, by comparing the calculated values of E_{CEN} to experimental data.

Skyrme interactions

- 33 different Skyrme interactions

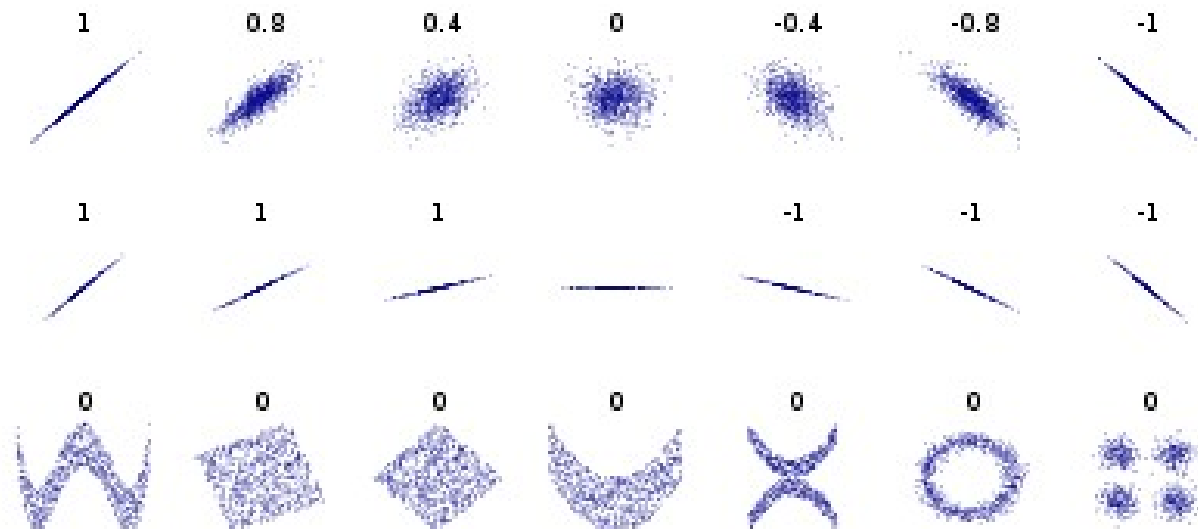- Cover wide range of nuclear matter properties
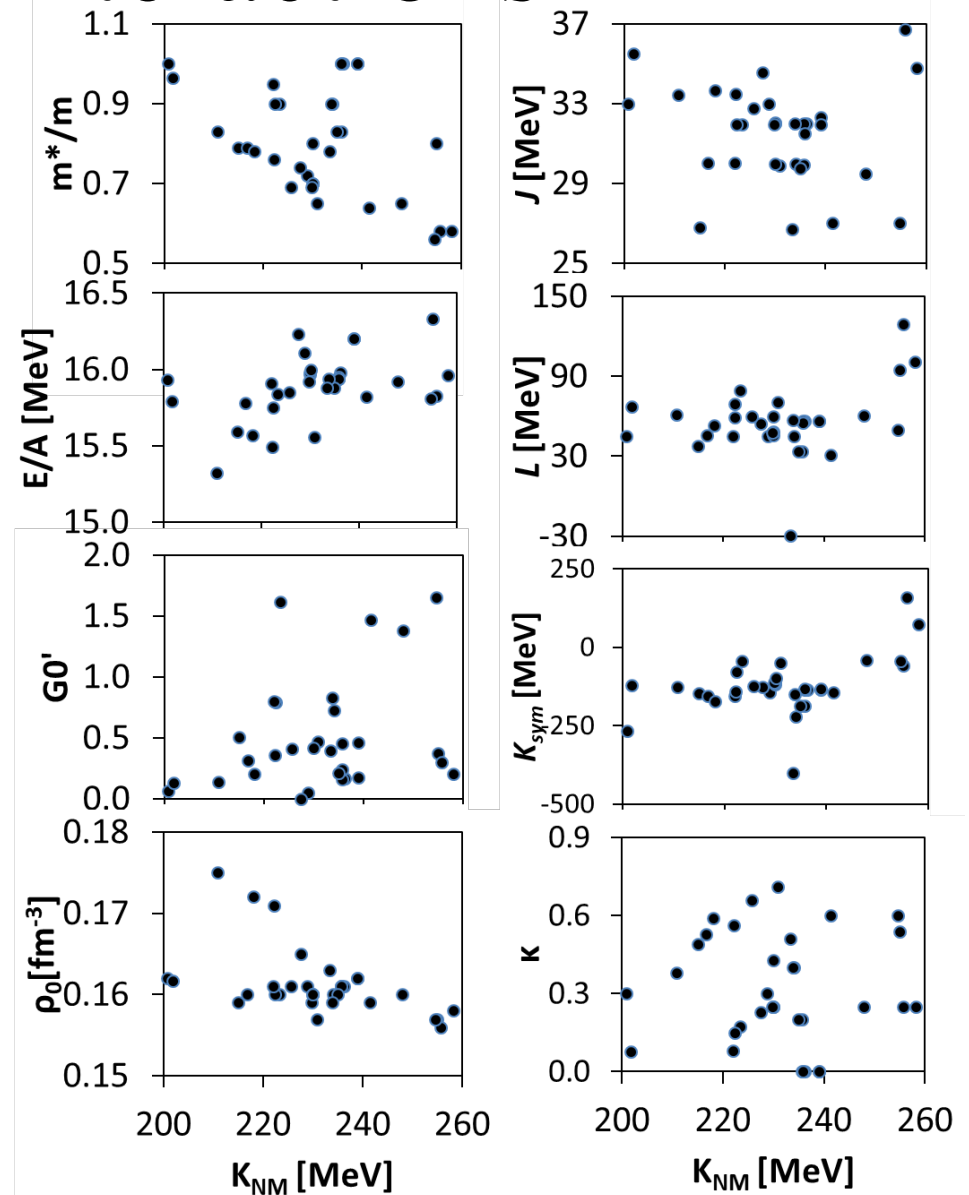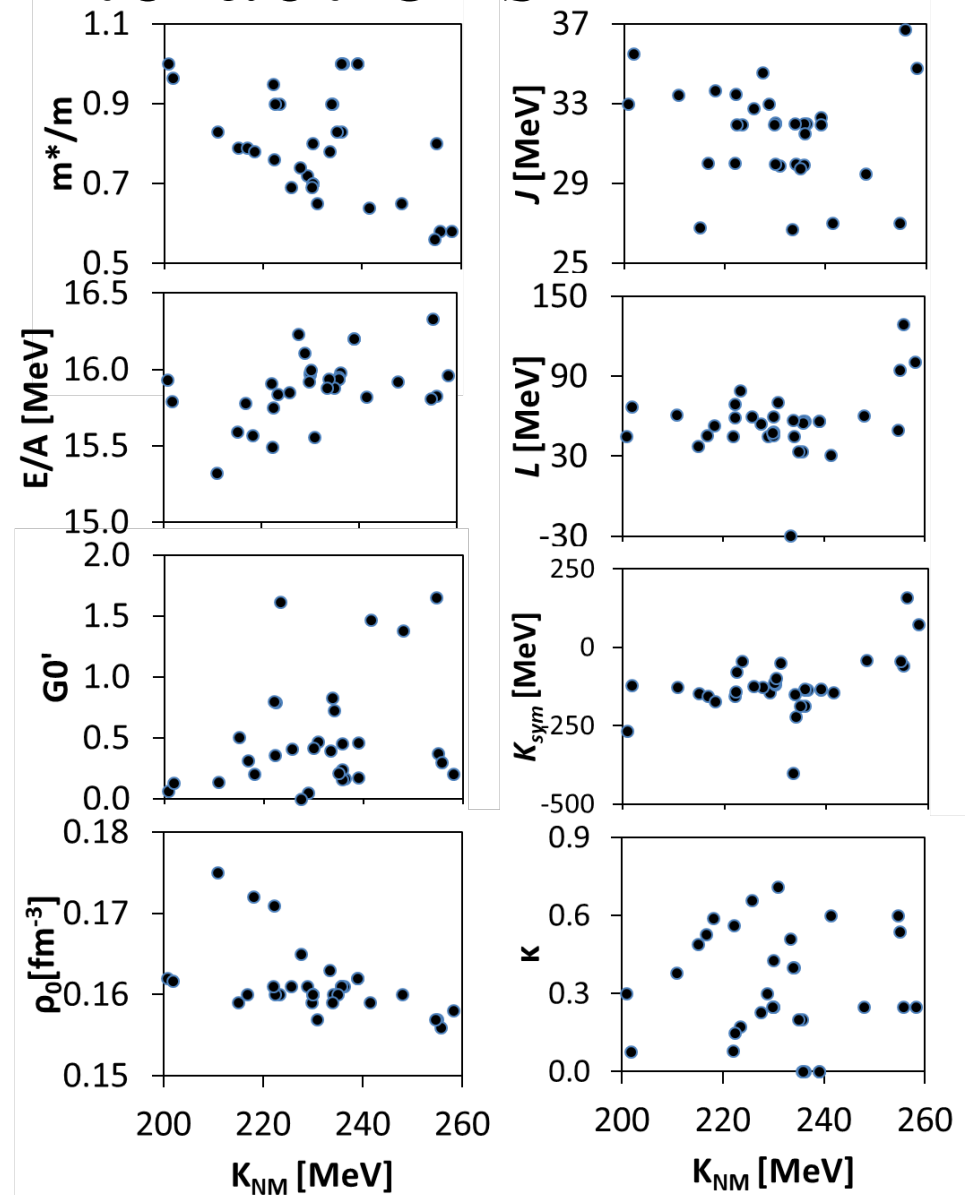
Pearson Linear Correlation Coefficient

$$C = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

$\bar{x}$ and $\bar{y}$ are the averages of x and y and the sum runs over all interactions ($n = 33$)

Skyrme interactions

- 33 different Skyrme interactions
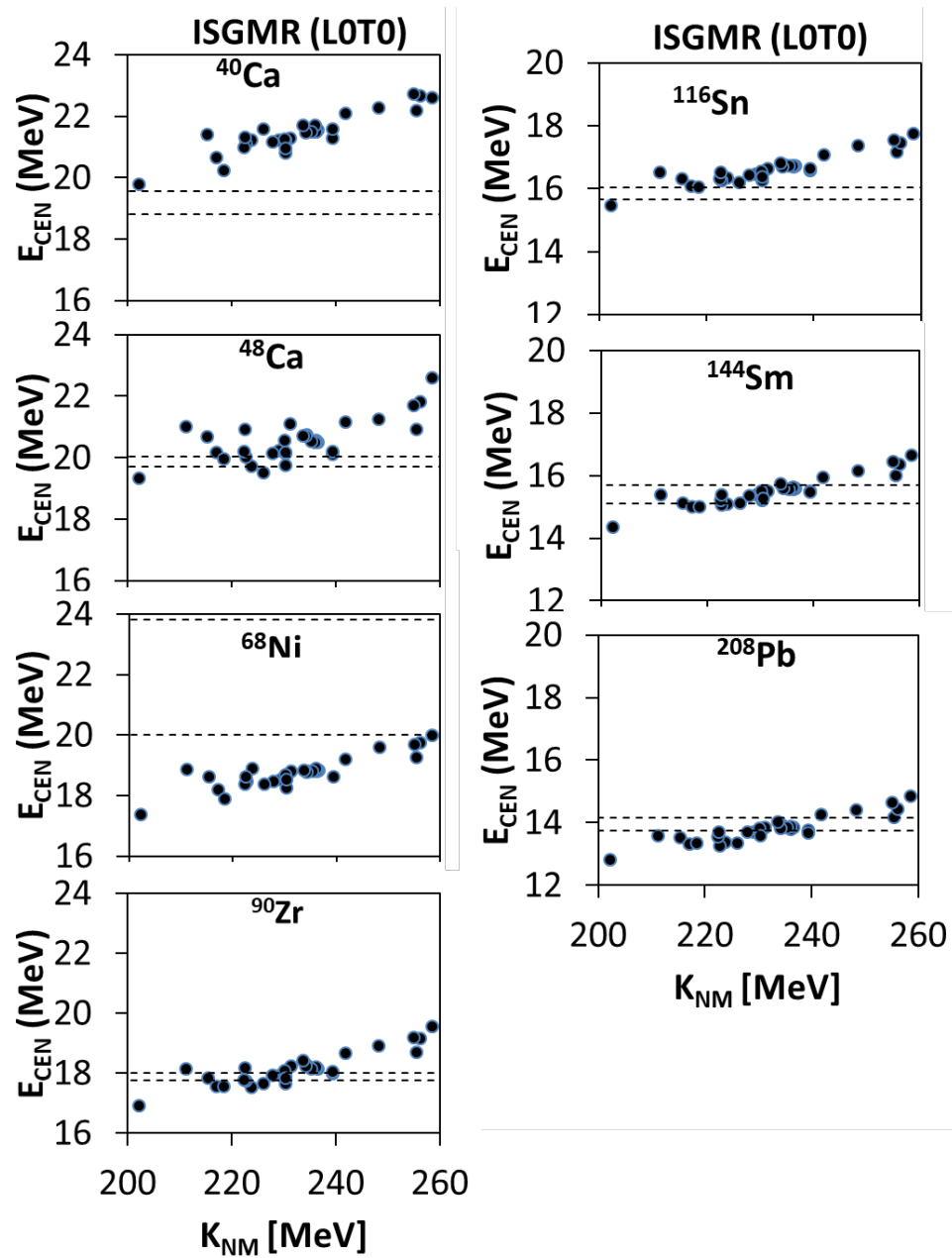
- Cover wide range of nuclear matter properties

Pearson linear correlation cofficient between nuclear matter properties

	K_{NM}	J	L	K_{sym}	m^*/m	κ
K_{NM}	1.00	0.03	0.30	0.43	-0.37	-0.02
J	0.03	1.00	0.72	0.49	0.07	-0.24
L	0.30	0.72	1.00	0.91	-0.15	-0.13
K_{sym}	0.43	0.49	0.91	1.00	-0.41	-0.08
m^*/m	-0.37	0.07	-0.15	-0.41	1.00	-0.63
κ	-0.02	-0.24	-0.13	-0.08	-0.63	1.00

Pearson liner correlation coefficient between GR energies and NM properties

	K_{NM}	J	L	K_{sym}	m*/m	κ
ISGMR	0.87	-0.10	0.25	0.45	-0.51	0.13
ISGDR	0.52	-0.10	0.13	0.36	-0.88	0.55
ISGQR	0.41	-0.09	0.15	0.41	-0.93	0.54
ISGOR	0.42	-0.10	0.15	0.43	-0.96	0.56
IVGMR	0.23	-0.26	-0.12	0.00	-0.70	0.86
IVGDR	0.05	-0.37	-0.42	-0.30	-0.60	0.84
IVGQR	0.18	-0.35	-0.29	-0.13	-0.74	0.80
IVGOR	0.25	-0.32	-0.19	0.02	-0.83	0.81

ISGMR (L0T0)

^{40}Ca

^{48}Ca

^{68}Ni

^{90}Zr

ISGMR (L0T0)

^{116}Sn

^{144}Sm

^{208}Pb

E_{CEN} (MeV)

K_{NM} [MeV]

Isoscalar Giant Monopole

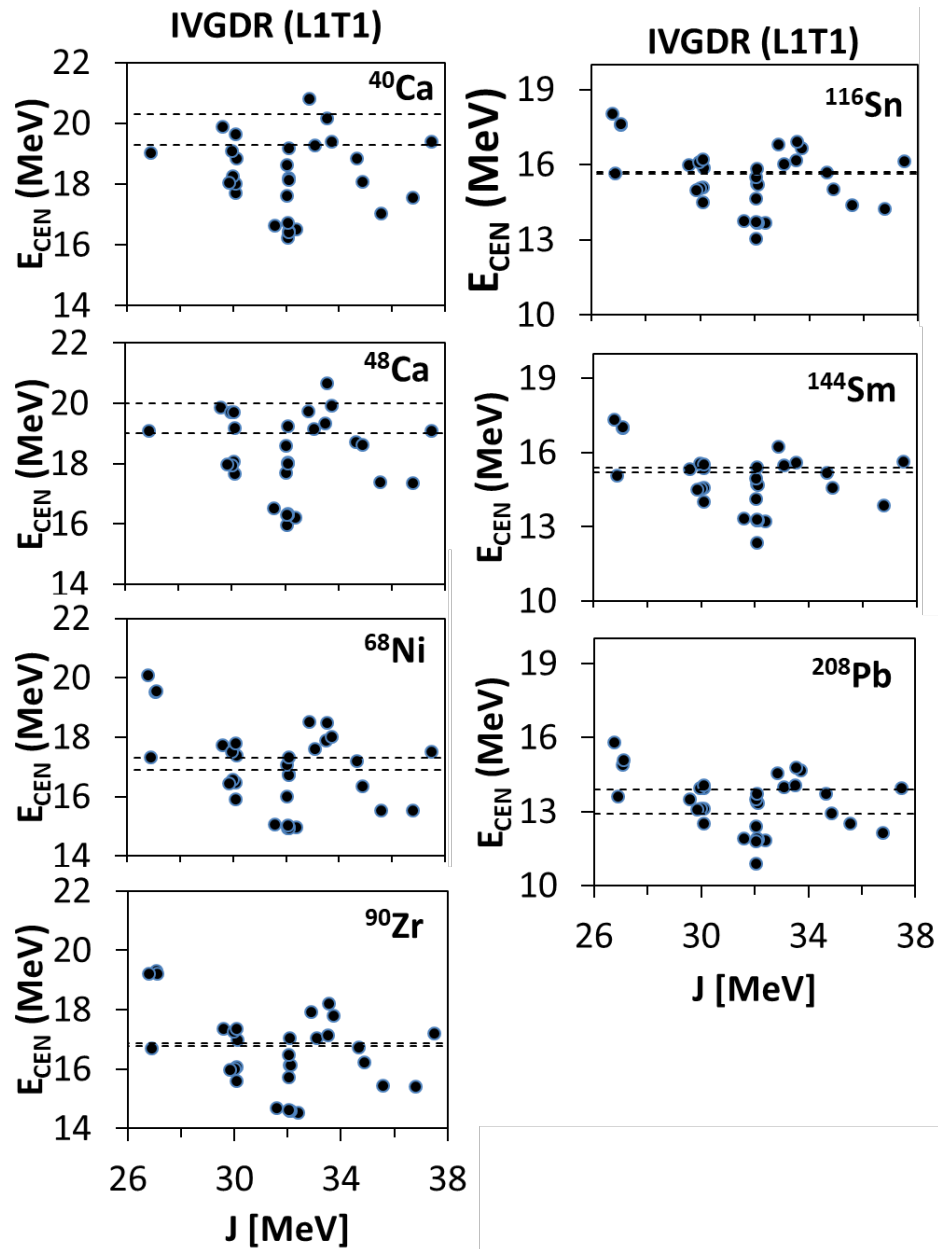K_{NM} = 210 to 240 MeV

ISGQR (L2T0)

**Isoscalar Giant Quadrupole**

m*/m = 0.70 to 0.90
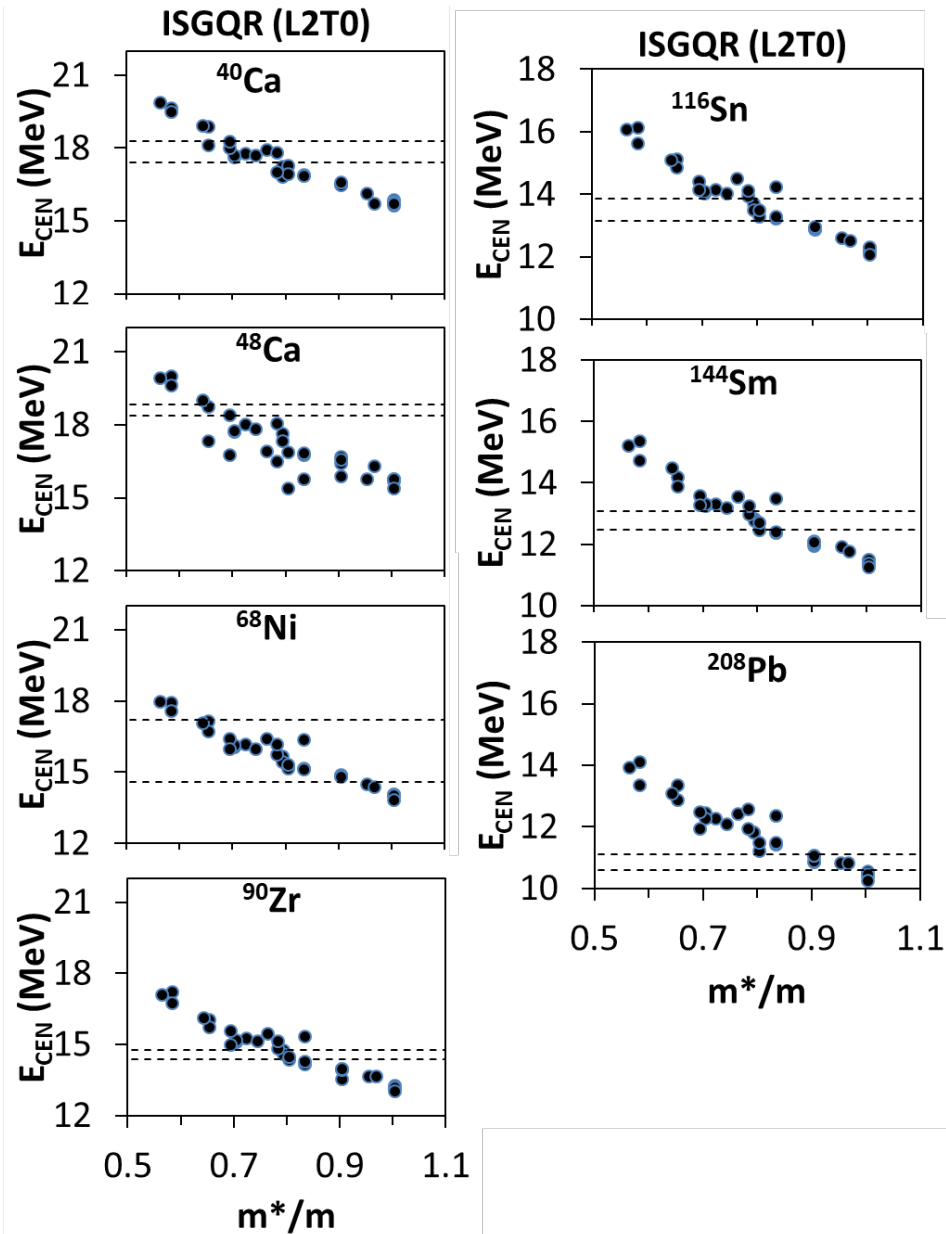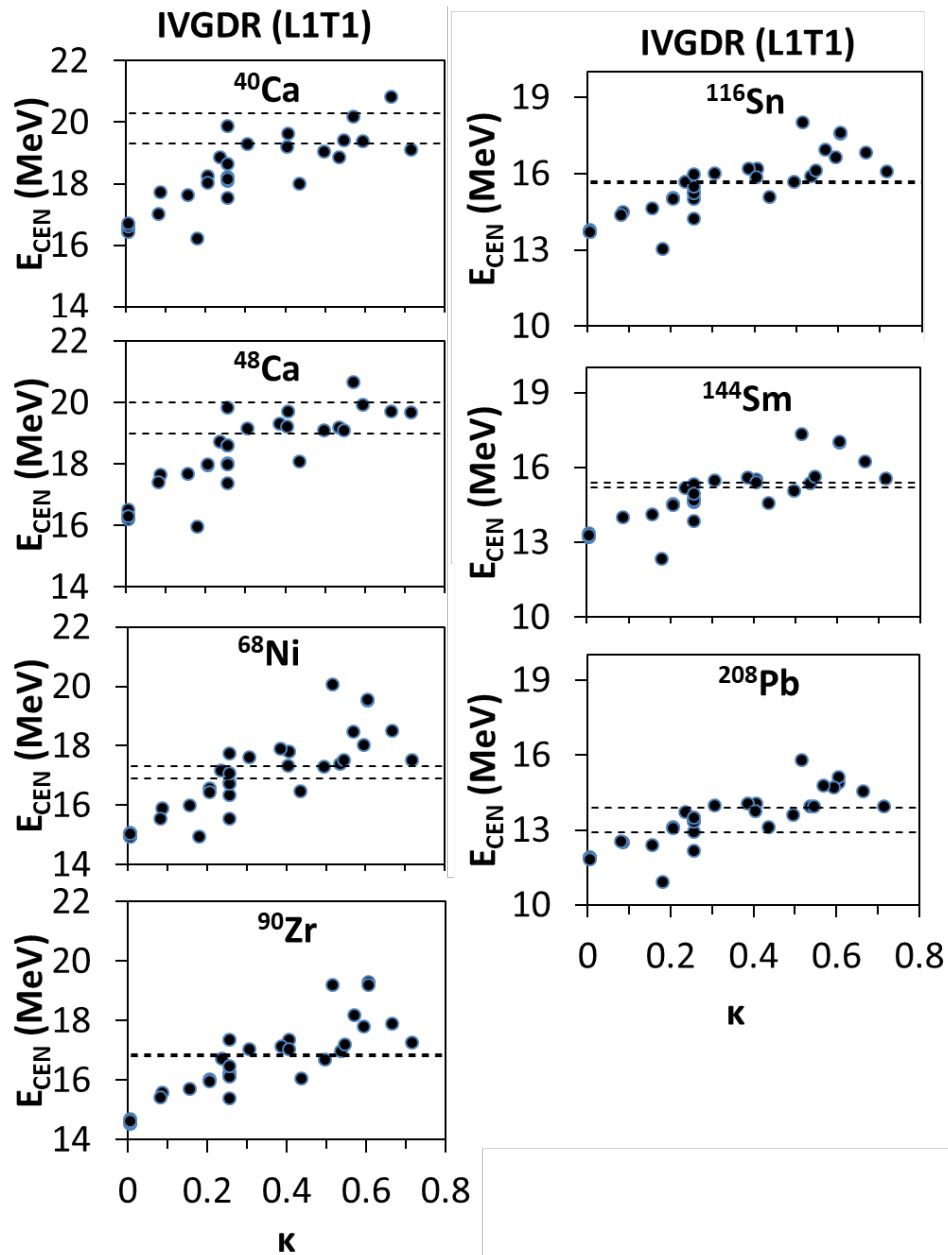
Isovector Giant Dipole

- Calculated Centroid energies (circles)
- Experimental data (dotted lines)

- Strong correlation with the enhancement factor κ (C~0.84)
- κ =0.25-0.70

Isovector Giant Dipole

- Calculated Centroid energies (circles)
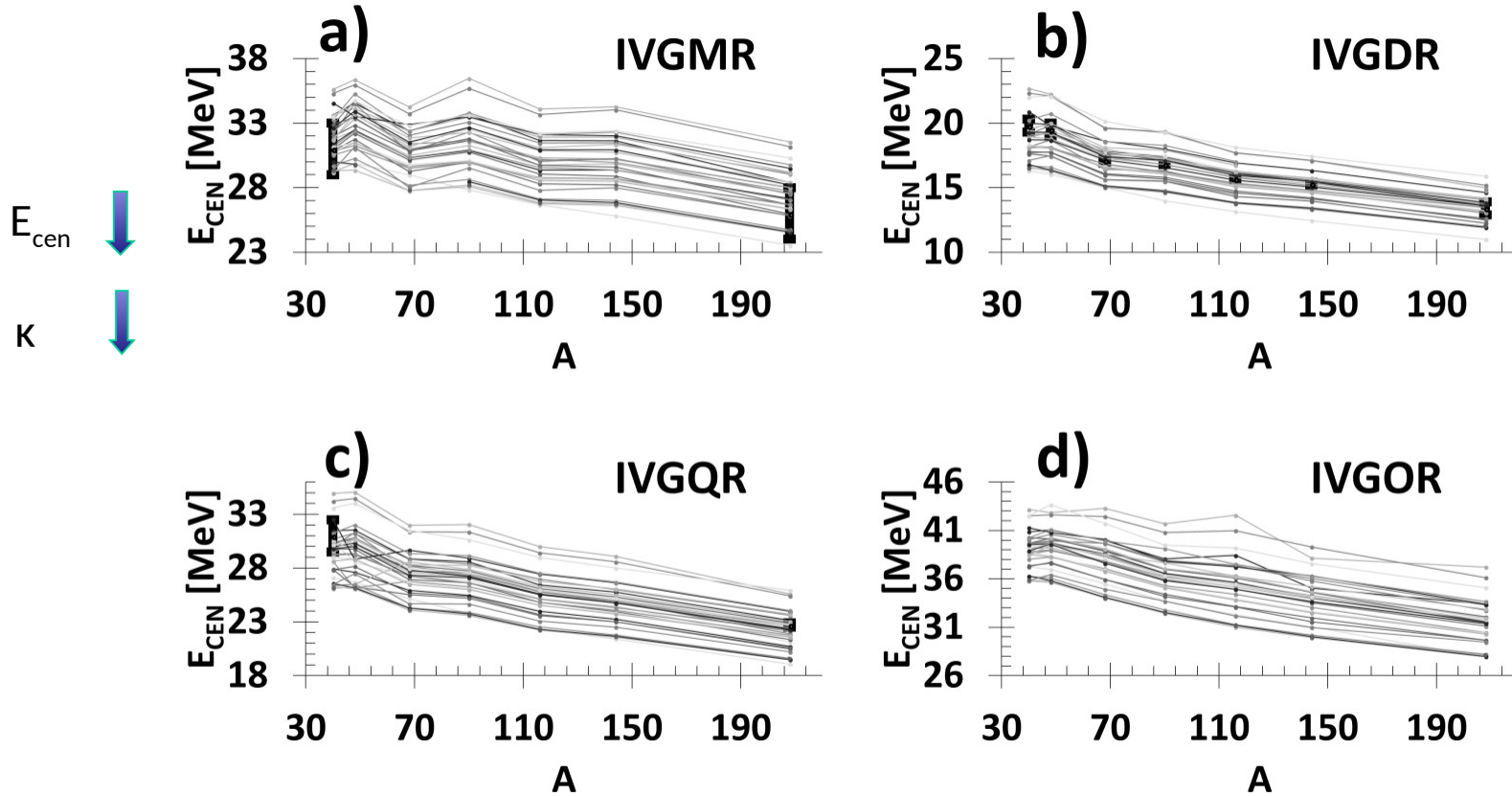- Experimental data (dotted lines)

- No correlation with J!

^{40,48}Ca, ^{68}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb: centroid energies
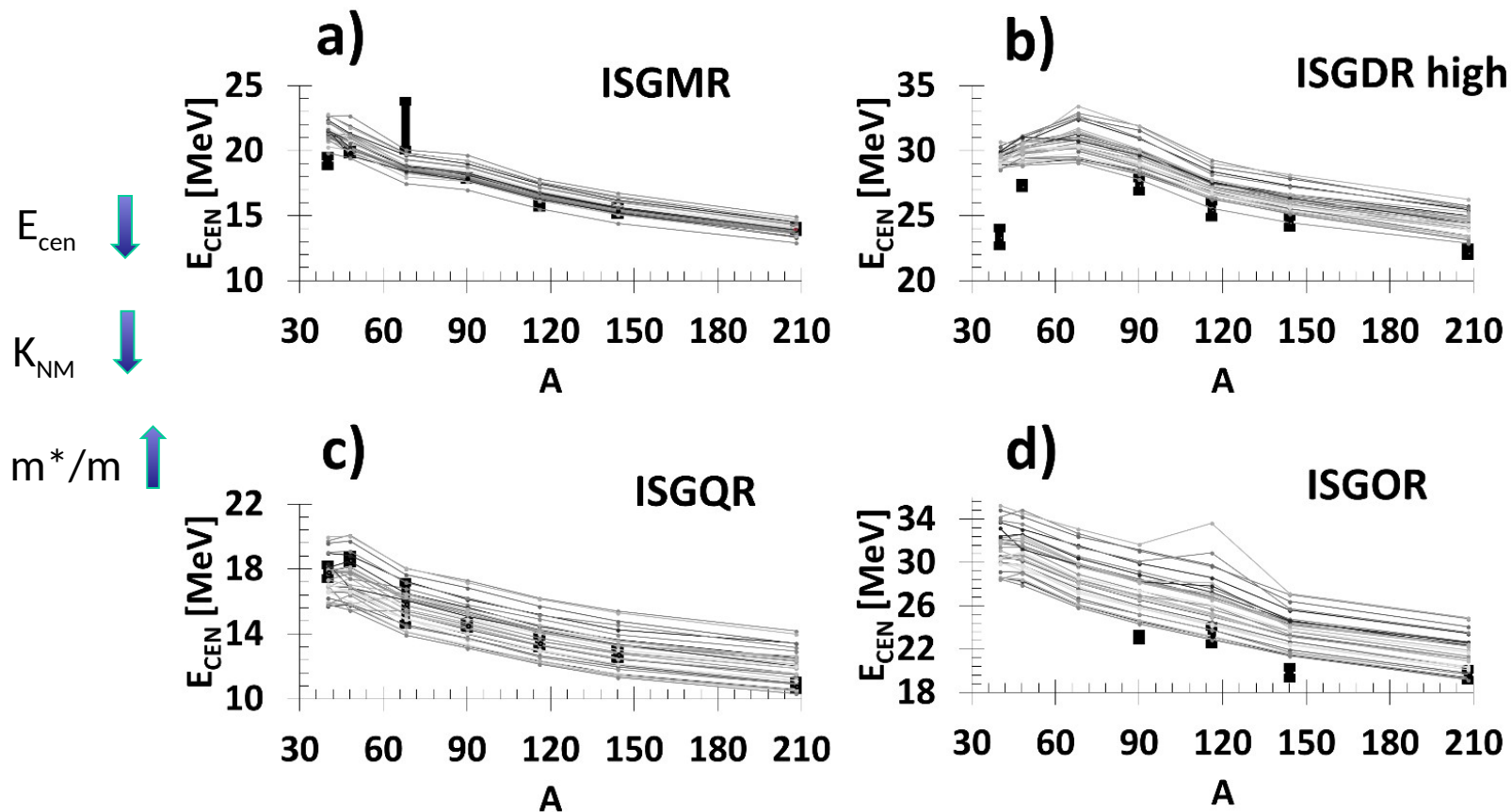
Centroid energy [MeV] plotted against the mass A of each nucleus.

- Theoretical calculations shown as dots connected by lines to guide the eye.
- Experimental error bars shown as solid vertical lines.

^{40,48}Ca, ^{68}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb: centroid energies

Centroid energy [MeV] plotted against the mass A of each nucleus.

- Theoretical calculations shown as dots connected by lines to guide the eye.
- Experimental error bars shown as solid vertical lines.
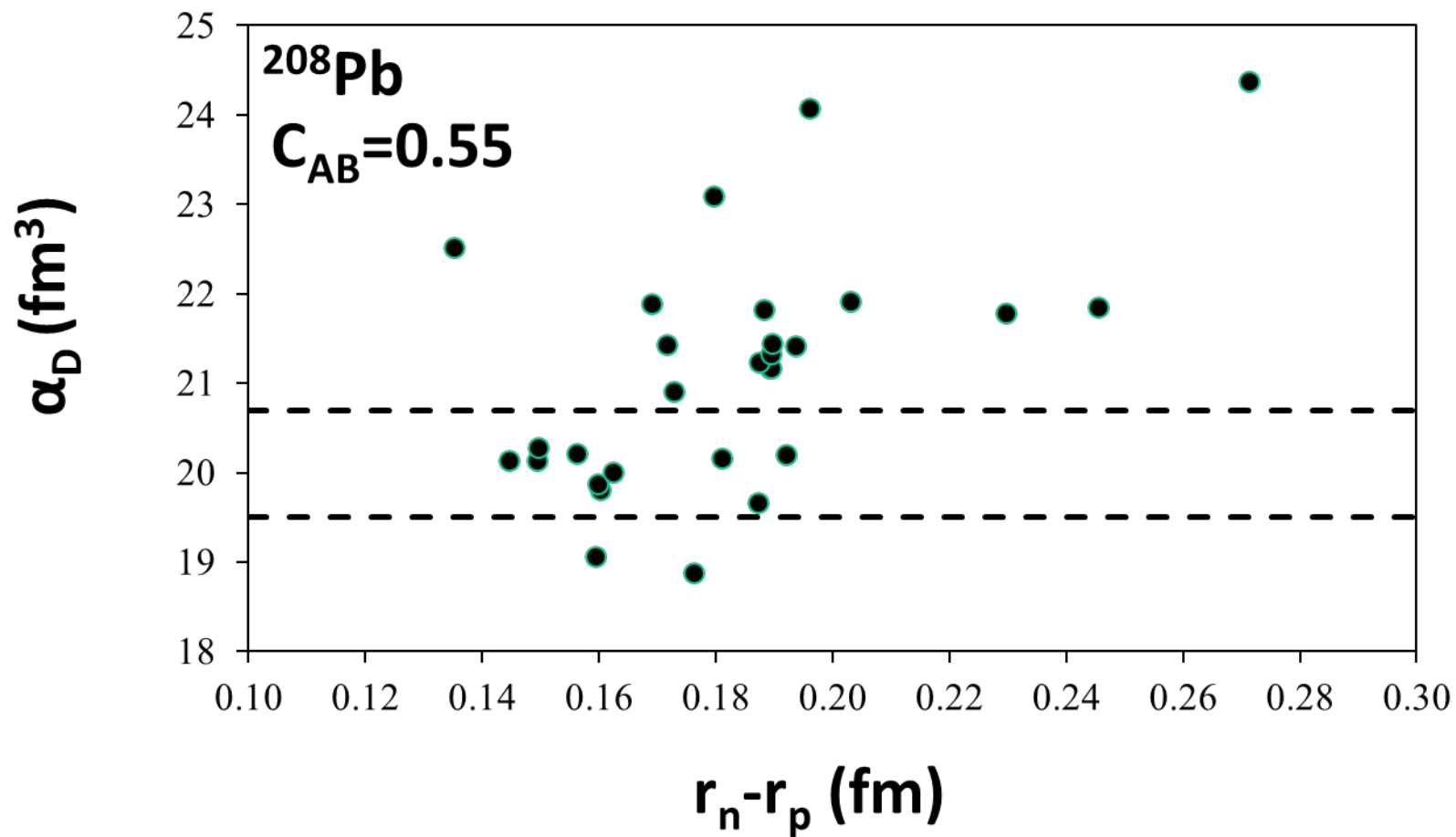
Conclusions

1) We have developed a new EDFs based on Skyrme type interaction (KDE0, KDE0v1,...) applicable to properties of rare nuclei and neutron stars.

2) Considering the calculated HF-based RPA results for the E_{CEN} for the ISGMR, ISGQR, and IVGDR of ^{40,48}Ca, ^{68}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm and ^{208}Pb, carried out with commonly used 33 Skyrme interactions, we obtained good agreement with the experimental data for some interactions. Comparing the calculated E_{CEN} to the experimental results we find that:

- Strong correlations exist between the calculated centroid energies E_{CEN} of the isoscalar giant monopole resonance (ISGMR) and the nuclear matter (NM) incompressibility coefficient, K_{NM}, leading to the value of $K_{NM} = 210$ to 240 MeV.

- Strong correlations exist between the centroid energy of the isoscalar giant quadrupole resonance (ISGQR) and the effective mass, leading to an accepted value for the NM effective mass in the range of $m^*/m = 0.70$ to 0.90.

- Strong correlations exist between the energy of the isovector giant dipole resonance (IVGDR) and the enhancement coefficient κ for the EWSR, leading to an accepted value in the range of $\kappa = 0.25$ to 0.70.

- No correlations exist between the centroid energy E_{CEN} of the IVGDR and the symmetry energy coefficient J, or its first derivative L and its second derivative K_{sym}, associated with the density dependence of the symmetry energy. These results, which contradict statements in the literature, can be understood by noting that the value of E_{CEN} of the IVGDR also depends on other NM quantities, such as m^*/m, which have different values for the different interactions used in the calculations.

- We note that these constraints on the values of K_{NM}, m^*/m and κ can be used, together with additional data on neutron rich and proton rich nuclei, for further improvement in determining a modern energy density functional.

Acknowledgments

Work done with:

B. K. Agrawal

M. Anders

G. Bonasera

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