

Clusters in Nuclear Matter: from Nuclei in the Laboratory to Stars in the Cosmos

Stefan Typel



Light Clusters in Nuclei and Nuclear Matter: Nuclear Structure and Decay, Heavy-Ion Collisions, and Astrophysics

ECT*

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DFG Deutsche
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- ▶ **Introduction**
 - ▶ correlations and clusters in nuclei and strongly interacting matter
- ▶ **Theoretical Description**
 - ▶ approaches for different ranges in baryon density n :
very low ($n \ll n_{\text{sat}}$) / intermediate ($n \lesssim n_{\text{sat}}$) / high ($n \gtrsim n_{\text{sat}}$) densities
- ▶ **Generalized Relativistic Density Functional**
 - ▶ basic features
 - ▶ application to compact star matter
- ▶ **Experimental Tests**
 - ▶ cluster emission in heavy-ion collisions
 - ▶ α -particles at the surface of heavy nuclei
- ▶ **Correlations above Nuclear Saturation Density**
- ▶ **Application to Astrophysics**
- ▶ **Conclusions**

Introduction

Correlations and Clusters



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- ▶ nucleons bound in nuclei
 - ▶ light nuclei
 - ▶ cluster formation important for structure
 - ▶ state dependent
 - ▶ heavy nuclei
 - ▶ role of clusters less clear for structure
 - ▶ cluster formation in α decay
 - ▶ widely discussed during this workshop

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 - ▶ widely discussed during this workshop
- ▶ **strongly interacting matter**
 - ▶ change of chemical composition and thermodynamic properties
 - ▶ equation of state/phase diagram
 - ⇒ description of astrophysical objects
(neutron stars, their mergers, core-collapse supernovae)
 - ▶ simulation of heavy-ion collisions

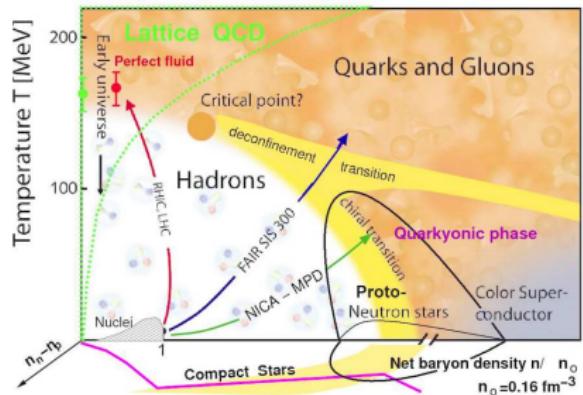
Description of Strongly Interacting Matter



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► short-range nuclear interaction

- ▶ many-body correlations of nucleons
⇒ formation of bound states (clusters)
- ▶ competition of bulk/surface/Coulomb contributions and thermodynamic conditions (temperature, density, isospin asymmetry)
- ▶ effects of medium on cluster formation



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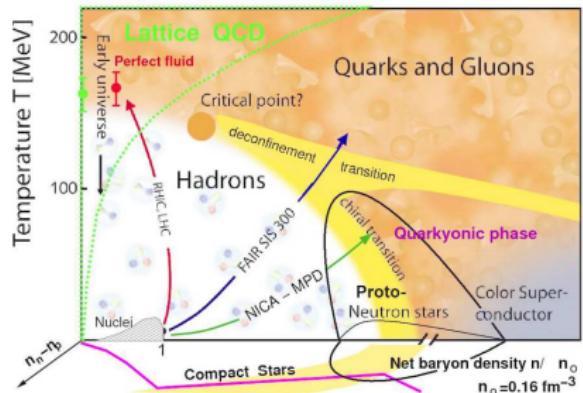
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important distinction

- ▶ nuclear/baryonic matter
(only nucleons/baryons,
no Coulomb interaction)
 - ▶ compact star matter
(with leptons and Coulomb interaction, charge-neutral system)
- ⇒ different properties and phase diagrams



Theoretical Description

► ab-initio approaches

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(e.g. fitted to nucleon-nucleon scattering data)
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- ▶ effective in-medium interaction
- ▶ quasiparticles

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► different strategies

- ▶ microscopic: clusters as many-body correlations
(two-body ok, anything beyond more difficult)
⇒ physical picture
- ▶ semi-microscopic: clusters as explicit degrees of freedom
(no real limitation on cluster size)
⇒ chemical picture

Description at Very Low Densities – Finite Temperatures

► exact limit \Rightarrow virial equation of state (VEOS)

(E. Beth and G. Uhlenbeck, Physica 3(1936) 729, Physica 4 (1937) 915;

C. J. Horowitz and A. Schwenk, NPA 776 (2006) 55)

► expansion of pressure in powers of fugacities $z_i = \exp(\mu_i / T)$

$$p = TV \left(\sum_i \frac{g_i}{\lambda_i^3} z_i + \sum_{ij} \frac{b_{ij}}{\lambda_i^{3/2} \lambda_j^{3/2}} z_i z_j + \dots \right) \quad \text{with thermal wavelength} \quad \lambda_i = [2\pi/(m_i T)]^{1/2}$$

and virial coefficients $g_i, b_{ij}, \dots \Rightarrow$ limitation $n_i \lambda_i^{-3} \ll 1$

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► only two-body correlations relevant at lowest densities, encoded in

$$b_{ij} = \frac{1 + \delta_{ij}}{2} \frac{\lambda_i^{3/2} \lambda_j^{3/2}}{\lambda_{ij}^3} \int dE \exp\left(-\frac{E}{T}\right) D_{ij}(E) \pm \delta_{ij} \frac{g_i}{2^{5/2}} \quad \lambda_{ij} = \{2\pi/[(m_i + m_j)T]\}^{1/2}$$

$$\text{with 'density of states'} \quad D_{ij}(E) = \sum_k g_k^{(ij)} \delta(E - E_k^{(ik)}) + \sum_l \frac{g_l^{(ij)}}{\pi} \frac{d\delta_l^{(ij)}}{dE}$$

\Rightarrow contribution from ground state and continuum,

depends only on experimental data: binding energies $E_k^{(ik)}$, phase shifts $\delta_l^{(ij)}$

(not independent! Levinson theorem)

Description at Low Densities – Finite Temperatures

- ▶ **simplification of VEOS**

⇒ **nuclear statistical equilibrium (NSE)**

- ▶ consider nucleons and all nuclei (ground and excited states)
- ▶ no contributions from continuum, no explicit interaction

Description at Low Densities – Finite Temperatures



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► extension of VEOS

⇒ generalized Beth-Uhlenbeck approach

(G. Röpke, L. Münchow, and H. Schulz, NPA 379 (1982) 536,
M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. 202 (1990) 57,
G. Röpke, N.-U. Bastian et al., NPA 897 (2013) 70,
N.-U. Bastian et al., Universe 4 (2018) 67)

- ▶ quantum statistical description with thermodynamic Green's functions or self-consistent Φ -derivable approach
- ▶ part of interaction included in self-energies of quasiparticles
- ▶ modified second virial coefficient, dependence on cluster momentum

⇒ suppression of cluster formation with increasing density

Description at Low Densities – Zero-Temperature Limit I



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► pure neutron matter

exact limit \Rightarrow Lee-Yang type expansion

(T. D. Lee and C. N. Yang, Phys. Rev. 105 (1957) 1119,

H.-W. Hammer and R. J. Furnstahl, NPA 678 (2000) 277)

$$\frac{E}{N} = \frac{3}{5} \frac{k_n^2}{2m_n} \left[1 + \frac{10}{9\pi} \zeta + \frac{4}{21\pi^2} (11 - 2 \ln 2) \zeta^2 + \dots \right]$$

$\zeta = a_{nn} k_n$ with s-wave scattering length a_{nn}

and Fermi momentum k_n

\Rightarrow small radius of convergence ($a_{nn} \approx -18.8$ fm)

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► nuclear matter

condensation of (bosonic) clusters expected,
does not stop at α condensation

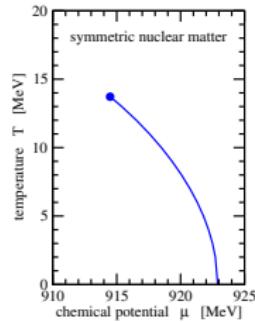
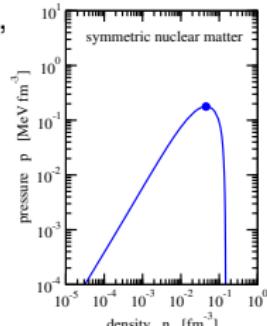
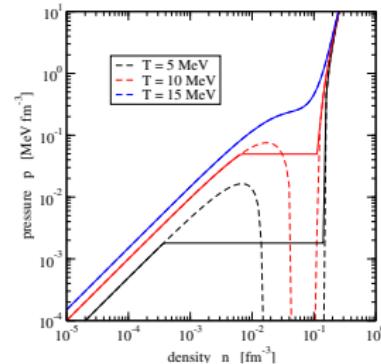
\Rightarrow increase of cluster size

(no Coulomb interaction \rightarrow no size limit)

\Rightarrow coexistence of low-/high-density phases

('liquid-gas phase transition')

\Rightarrow effect on symmetry energy



Description at Low Densities – Zero-Temperature Limit II



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► nuclear symmetry energy

- standard definition from expansion of energy per baryon

$$E(n, \alpha) = E_0(n) + E_{\text{sym}}(n)\alpha^2 + \dots$$

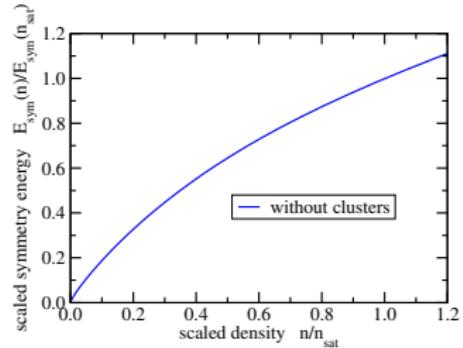
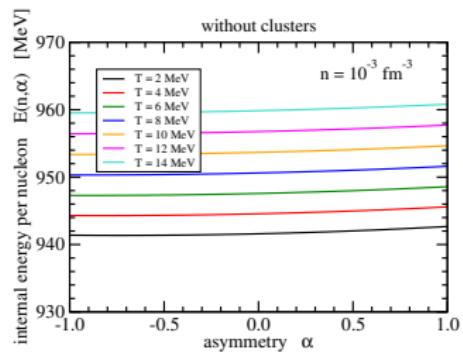
with asymmetry $\alpha = (n_n - n_p)/n$

and density $n = n_n + n_p$

$$\Rightarrow E_{\text{sym}}(n) = \frac{1}{2} \left. \frac{d^2 E(n, \alpha)}{d\alpha^2} \right|_{\alpha=0}$$

- uniform matter without clusters:

$$\lim_{n \rightarrow 0} E_{\text{sym}}(n) = 0$$



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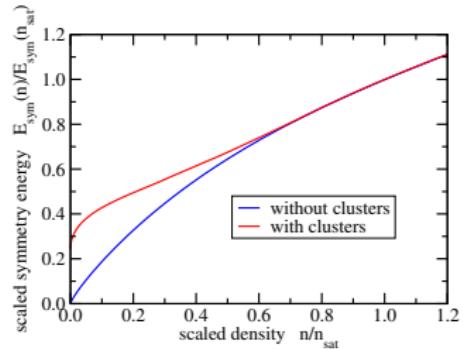
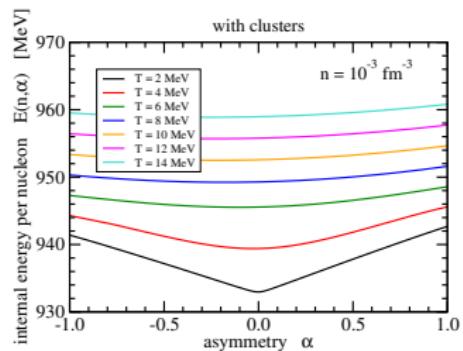
- models with clusters:

standard definition diverges for $T \rightarrow 0$

\Rightarrow modified definition

$$E_{\text{sym}}(n) = [E(n, 1) - 2E(n, 0) + E(n, -1)]/2$$

$\Rightarrow E_{\text{sym}}$ finite at zero density



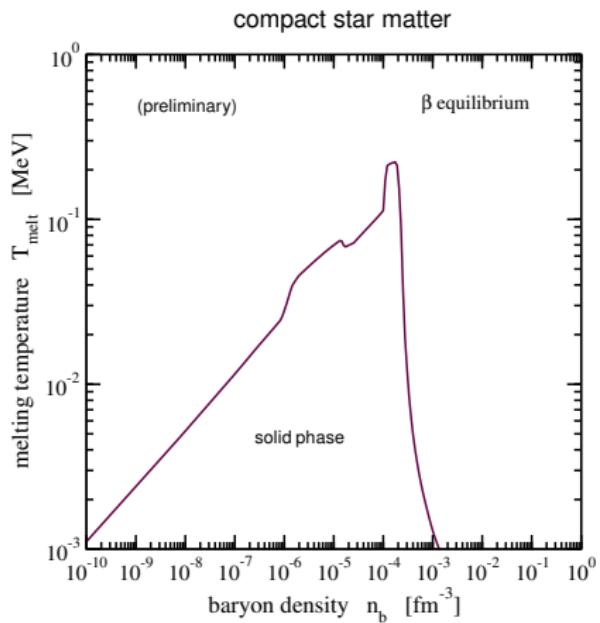
Description at Low Densities – Zero-Temperature Limit III

► compact star matter

- charge neutral system (nucleons + leptons) in β equilibrium
- phase transition to solid crystal (Coulomb correlations essential), driven by plasma parameter

$$\Gamma = Z_{\text{ion}}^{5/3} e^2 / (a_e T) \approx 175$$

with $a_e = [3n_e/(4\pi)]^{1/3}$



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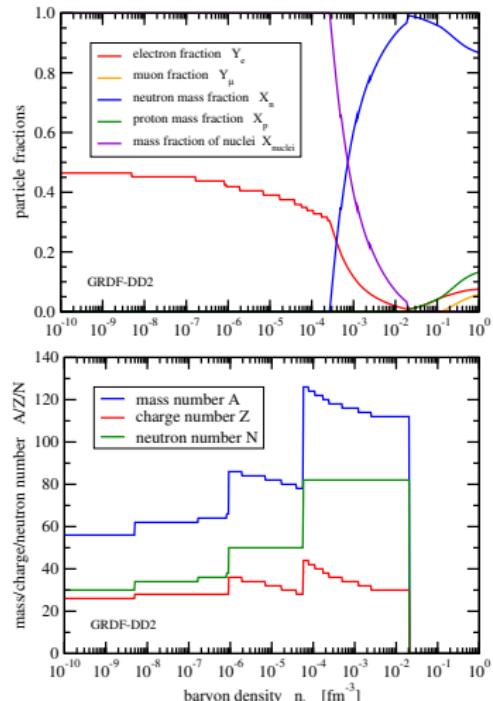
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⇒ formation of neutron star crust

- sequence of ions, phase transitions
- more neutron rich at higher densities
- neutron drip density
- pasta phases before transition to uniform matter



Description at Intermediate Densities

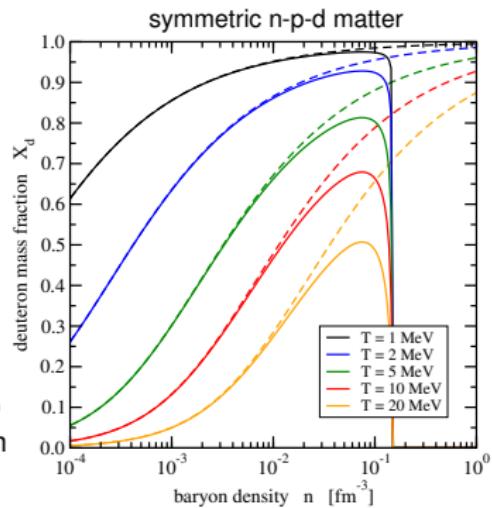


- ▶ when baryon density n approaches n_{sat}
 - ⇒ dissolution of clusters and
 - ⇒ transition to homogeneous matter expected
- ▶ not realized in standard VEOS or NSE

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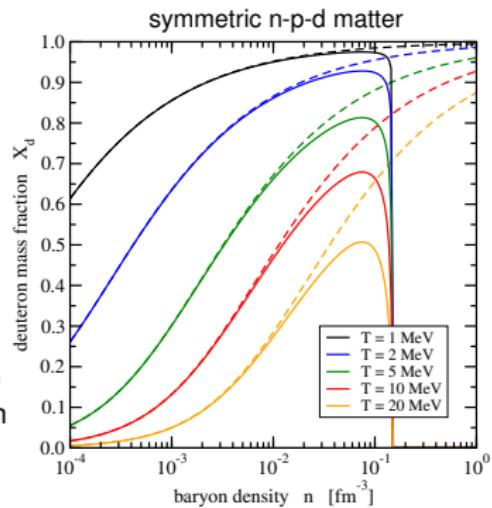
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- ▶ different theoretical approaches
 - ▶ geometric picture (finite size of particles)
 - ⇒ **excluded-volume mechanism**
 - ▶ applications to compact star matter
(M. Hempel and J. Schaffner-Bielich, NPA 837 (2010) 210;
S. Banik et al., ApJ. Suppl. 214 (2014) 22;
T. Fischer et al., EPJA 50 (2014) 46; M. Hempel, PRC 91 (2015) 055897)
 - ▶ generalized formulation, different interpretation
(S. Typel, EPJA 52 (2016) 16)



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 - ▶ generalized formulation, different interpretation
(S. Typel, EPJ A 52 (2016) 16)
 - ▶ medium modification of cluster properties
 - ⇒ **mass shifts**
 - ▶ action of Pauli principle ⇒ blocking of states
 - ▶ density, temperature, momentum dependence



Description at High Densities – Zero-Temperature Limit



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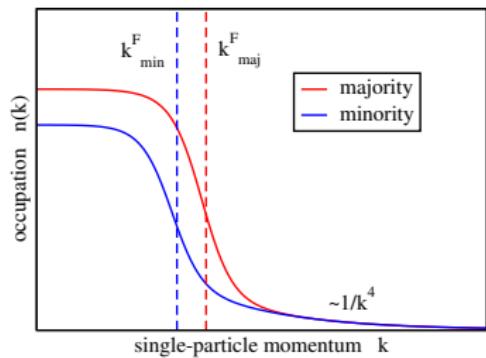
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- ▶ **energy density functionals**
 - ▶ mixture of baryons as quasiparticles
 - ▶ no explicit correlations between baryons
 - ⇒ ideal mixture of Fermion gases
 - ⇒ step function in single-particle momentum distributions at zero temperature

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experiments: nucleon knockout from nuclei in inelastic electron scattering
(O. Hen et al. (CLAS Collaboration), Science 346 (2014) 614)
⇒ no sharp cut-off, high-momentum tail

Generalized Relativistic Density Functional

Energy Density Functionals



- ▶ various types
 - ▶ nonrelativistic (e.g. Skyrme, Gogny) or relativistic/covariant
 - ▶ often derived from mean-field models in different approximations (Hartree, Hartree-Fock, Hartree-Fock-Bogoliubov)

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- ▶ usually only nucleons as degrees of freedom
- ▶ **here: generalized relativistic density functional (GRDF)**
 - ▶ nucleons, clusters (= many-body correlations) and mesons as degrees of freedom in grand-canonical ensemble
 - ▶ minimal coupling of nucleons (free or bound) to mesons
 - ▶ quasiparticles with effective mass $m_i^* = m_i - S_i$ and effective chemical potential $\mu_i^* = \mu_i - V_i$
 - ▶ effective interaction by meson exchange with density dependent couplings
 \Rightarrow vector (V_i) and scalar (S_i) potentials
 - ▶ applications:
EoS tables for astrophysical simulations (`compose.obspm.fr`),
structure of heavy nuclei

Generalized Relativistic Density Functional



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- ▶ generalisation of relativistic mean field model
 - ▶ density dependent meson-nucleon couplings, parametrisation DD2



- ▶ **generalisation of relativistic mean field model**
 - ▶ density dependent meson-nucleon couplings, parametrisation DD2
- ▶ **extended set of particle species**
 - ▶ nucleons, electrons, muons, photons, hyperons (optional), ...
 - ▶ light nuclei (^2H , ^3H , ^3He , ^4He) and heavy nuclei ($A > 4$)
 - ▶ binding energies from mass tables
 - ⇒ shell effects included, full distribution, not only average heavy nucleus
 - ▶ two-nucleon scattering states
 - ⇒ consistency with virial EoS at low densities
- ▶ **excited states of nuclei**

temperature dependent degeneracy factors with density of states



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temperature dependent degeneracy factors with density of states
- ▶ **medium dependence of particle properties**

quasiparticles with mass shifts (coupling to mesons, effective Pauli principle)

(S. Typel et al., Phys. Rev. C 81 (2010) 015803; M. D. Voskresenskaya et al., Nucl. Phys. A 887 (2012) 42;
M. Hempel et al., Phys. Rev. C 91 (2015) 045805; S. Typel, arXiv:1504.01571; H. Pais et al., arXiv:1612.07022;
H. Pais et al. Nuovo Cim. C 39 (2016) 393; S. Typel, J. Phys. G 45 (2018) 114001)

- ▶ **concept applies to composite particles: clusters**
 - ▶ light and heavy nuclei
 - ▶ nucleon-nucleon correlations in continuum
 - ⇒ medium dependent resonances
- ▶ **effective change of masses/binding energies**

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► **effective change of masses/binding energies**

► **two major contributions** $\Delta m_i = \Delta m_i^{\text{strong}} + \Delta m_i^{\text{Coul}}$

- strong shift $\Delta m_i^{\text{strong}} = \Delta m_i^{\text{meson}} + \Delta m_i^{\text{Pauli}}$
 - effects of strong interaction (coupling to mesons)
 - Pauli exclusion principle: blocking of states in the medium
⇒ reduction of binding energies
 - ⇒ cluster dissolution at high densities: Mott effect
 - ⇒ replaces traditional excluded-volume mechanism

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 - ⇒ reduction of binding energies
 - ⇒ cluster dissolution at high densities: Mott effect
 - ⇒ replaces traditional excluded-volume mechanism
 - ▶ electromagnetic shift Δm_i^{Coul} (in compact star matter)
 - ▶ electron screening of Coulomb field
 - ⇒ increase of binding energies

Mass Shifts II

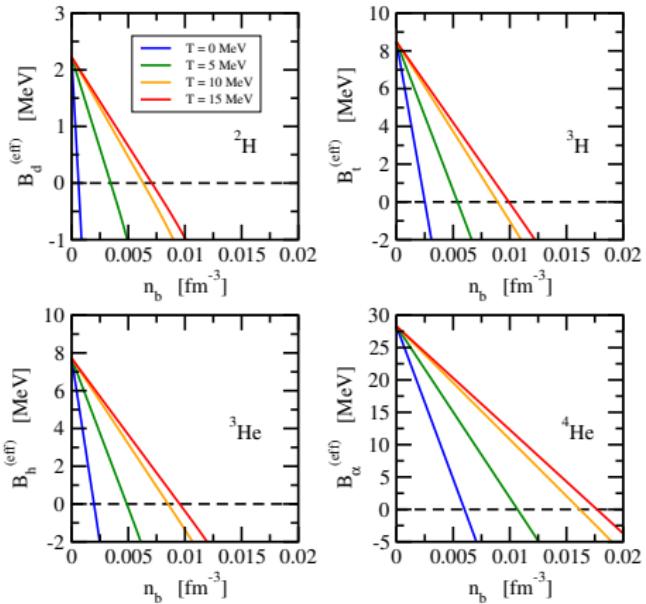
- ▶ light nuclei and NN scattering states

- ▶ parametrization from G. Röpke

simplified and modified for high densities and temperatures

- ▶ scattering states:
mass shifts as for deuteron

$$\text{effective binding energies } B_i^{(\text{eff})} = B_i^{(0)} - \Delta m_i^{\text{Pauli}}$$



Mass Shifts II



- ▶ light nuclei and NN scattering states

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- ▶ dependence of $\Delta m_i^{\text{Pauli}}$ on temperature T and effective density

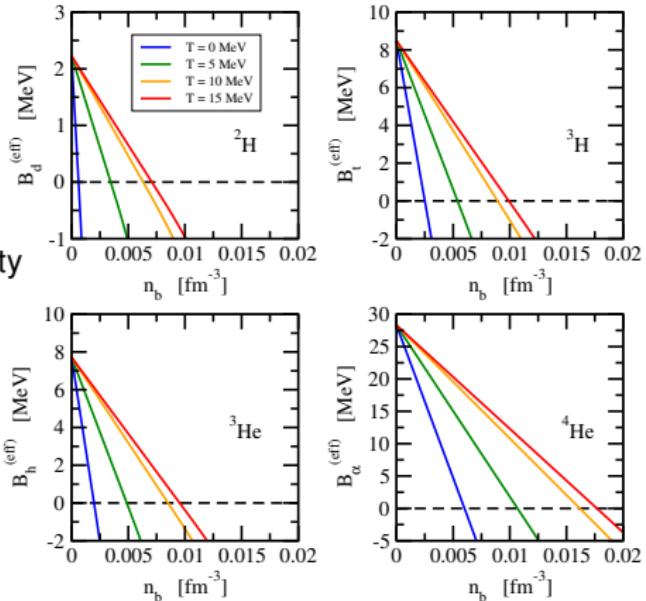
$$n_i^{\text{eff}} = \frac{2}{A_i} [Z_i Y_q + N_i(1 - Y_q)] n_b$$

⇒ asymmetry of medium

- ▶ Δm_i^{Coul} in Wigner-Seitz approximation

- ▶ full coupling of nucleons in clusters to meson fields

effective binding energies $B_i^{(\text{eff})} = B_i^{(0)} - \Delta m_i^{\text{Pauli}}$



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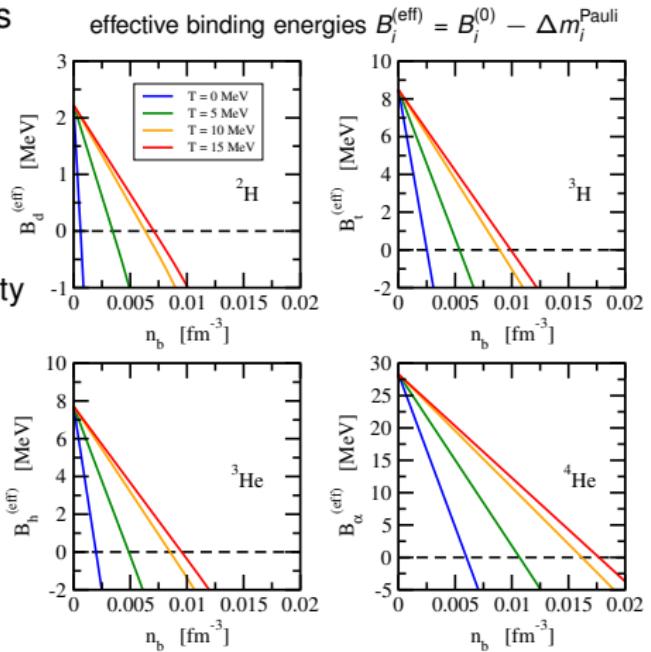
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- ▶ scattering states:
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- ▶ dependence of $\Delta m_i^{\text{Pauli}}$ on temperature T and effective density

$$n_i^{\text{eff}} = \frac{2}{A_j} [Z_i Y_q + N_i(1 - Y_q)] n_b$$

⇒ asymmetry of medium

- ▶ Δm_i^{Coul} in Wigner-Seitz approximation
- ▶ full coupling of nucleons in clusters to meson fields
- ▶ heavy nuclei
 - ▶ heuristic parametrization



- ▶ charge neutral system
- ▶ β equilibrium
 - ⇒ determines hadronic charge fraction
(= electron fraction if no muons)

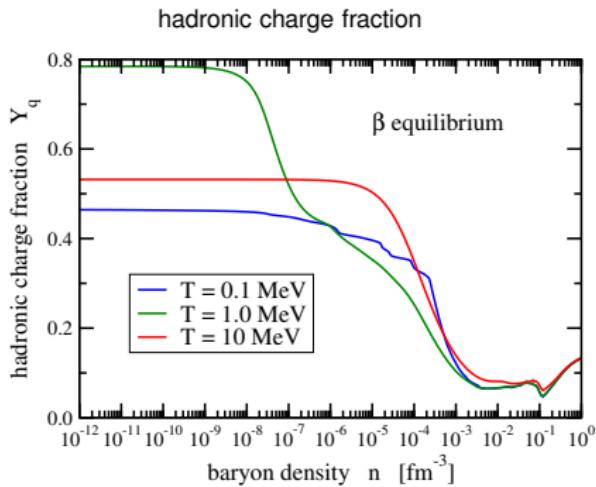
- ▶ charge neutral system
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- ▶ binding energies of nuclei from tables
 - ▶ AME2016
 - (M. Wang et al., Chin. Phys. C 41 (2017) 030003)
 - ▶ extension with DZ31 masses
 - (J. Duflo and A.P. Zuker, Phys. Rev. C 52 (1995) R23)

Compact Star Matter



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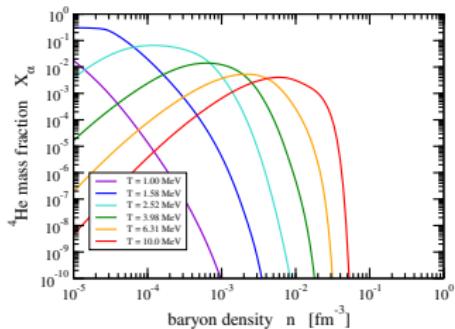
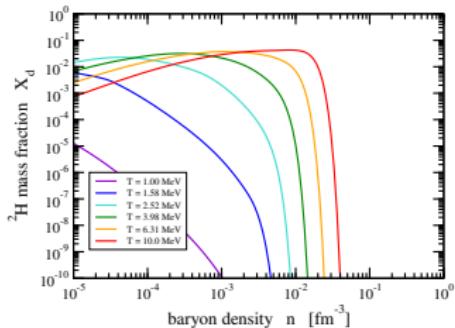
- ▶ charge neutral system
- ▶ β equilibrium
 - determines hadronic charge fraction (= electron fraction if no muons)
- ▶ binding energies of nuclei from tables
 - ▶ AME2016
(M. Wang et al., Chin. Phys. C 41 (2017) 030003)
 - ▶ extension with DZ31 masses
(J. Duflo and A.P. Zuker, Phys. Rev. C 52 (1995) R23)
- ▶ neutronisation with increasing baryon density



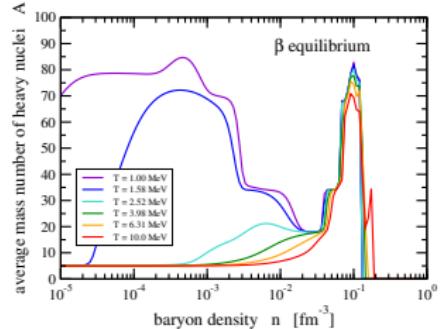
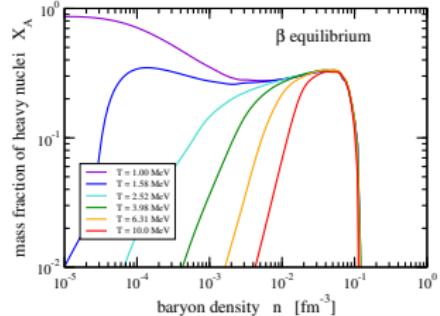
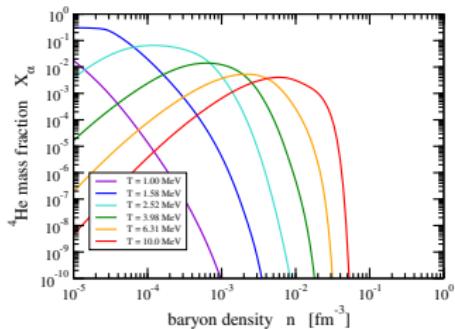
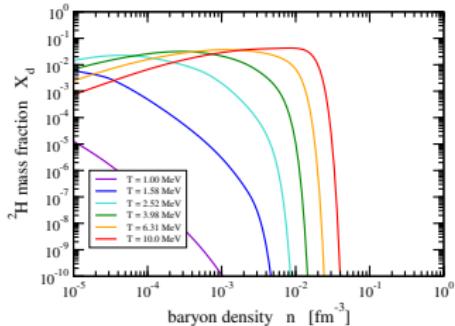
Compact Star Matter Light and Heavy Clusters



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Compact Star Matter Light and Heavy Clusters



Low-Density Limit

- ▶ finite temperatures and very low densities:
EoS determined by two-body correlations
- ▶ theoretical benchmark: **virial equation of state**
 - ▶ expansion in powers of fugacities
 - ▶ two-body correlations encoded
in second virial coefficient
 - ▶ depends only on experimental data
(phase shifts, binding energies)

(E. Beth and G. Uhlenbeck, Physica 3(1936) 729; Physica 4 (1937) 915,

C. J. Horowitz and A. Schwenk, NPA 776 (2006) 55)

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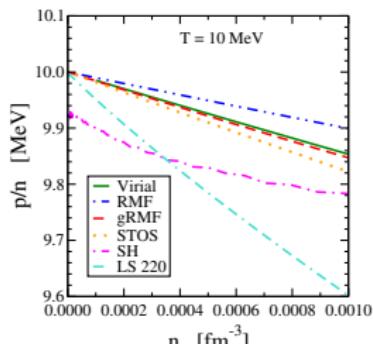
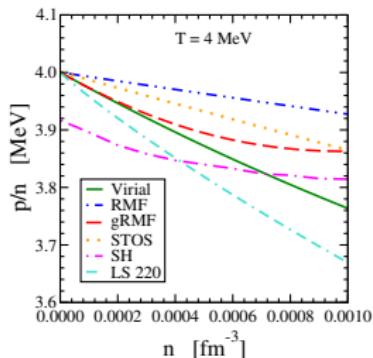


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C. J. Horowitz and A. Schwenk, NPA 776 (2006) 55)

- ▶ treatment in generalized relativistic density functional with two-body states in continuum as explicit degrees of freedom

(M. D. Voskresenskaya and S. Typel, NPA 887 (2012) 42)



Experimental Tests

emission of light nuclei

- ▶ determination of density and temperature of source
 - S. Kowalski et al. PRC 75 (2007) 014601
 - J. Natowitz et al. PRL 104 (2010) 202501
 - R. Wada et al. PRC 85 (2012) 064618
- ▶ thermodynamic conditions as in neutrinosphere of core-collapse supernovae

Light Clusters in Heavy-Ion Collisions

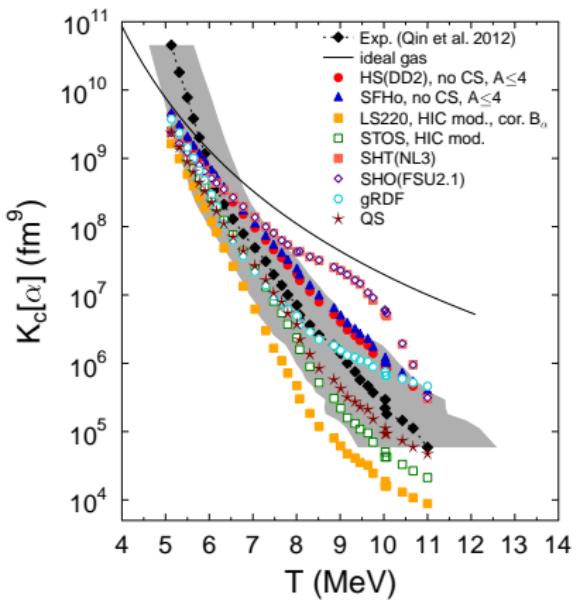
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- ▶ thermodynamic conditions as in neutrinosphere of core-collapse supernovae
- ▶ particle yields ⇒ chemical equilibrium constants

$$K_c[i] = n_i / (n_p^{Z_i} n_n^{N_i})$$

L. Qin et al., PRL 108 (2012) 172701

- ▶ mixture of ideal gases not sufficient



M. Hempel, K. Hagel, J. Natowitz, G. Röpke, S. Typel,

PRC C 91 (2015) 045805

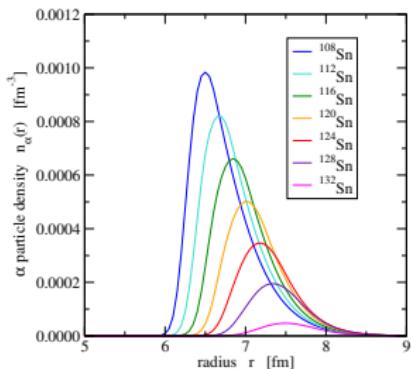
Application to Heavy Nuclei

- extension of GRDF to zero temperature

(S. Typel, PRC 89 (2014) 064321)

- only with α cluster \Rightarrow explicit α -particle wave function
- chain of Sn nuclei: α particles located at surface
- reduced probability of α occurrence with increasing neutron excess

(consistent with trend of α particle reduced widths in $(d, {}^6\text{Li})$ pickup reactions on Sn nuclei, A. A. Cowley, Phys. Rev. C 93 (2016) 054329)

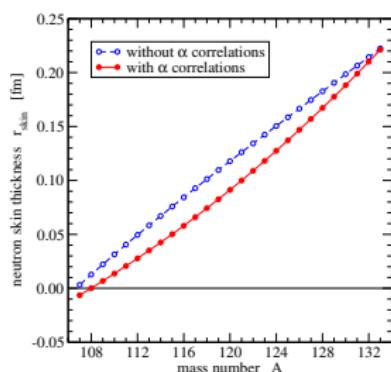
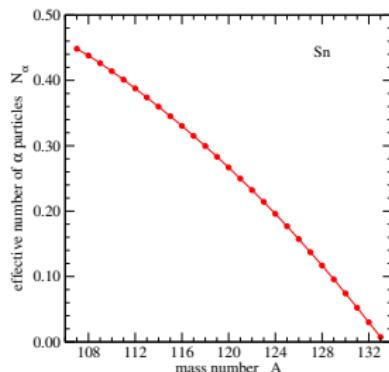
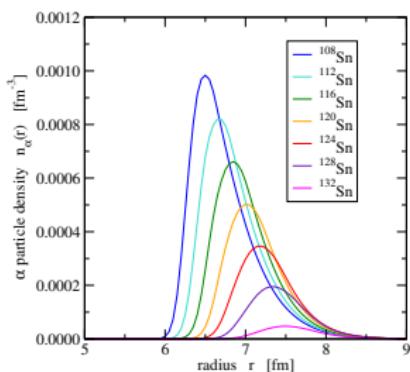


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- reduction of neutron skin thickness, effect depends on symmetry energy



Study of Correlations at Nuclear Surface I

► quasifree ($p,p\alpha$) knockout reactions on Sn nuclei

- experimental signatures:
 - dependence of cross sections
on neutron excess
 - localisation of α particles at surface
 \Rightarrow broad momentum distribution

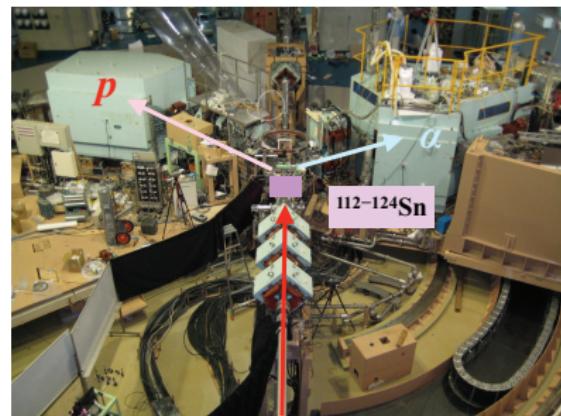
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► experiments at RCNP, Osaka (E461)

- targets: stable $^{112-124}\text{Sn}$ nuclei
- beam: 392 MeV protons, 100 pnA
- proton detection: Grand Raiden
- α detection: LAS
- first experiment (June 2015): failure of some detectors
- second experiment (February 2018): successful



Study of Correlations at Nuclear Surface II



► quasifree ($p, p\alpha$) knockout reactions on Sn nuclei

► experiment

- spectrometer setting: $\theta_{\text{lab}}(p) = 45.3 \text{ deg}$, $\theta_{\text{lab}}(\alpha) = 60 \text{ deg}$
- momentum coverage: $Q_\alpha \leq 80 \text{ MeV}/c$
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- theory
 - distorted-wave eikonal model
in impulse approximation
⇒ factorization of cross section
 - α particle distribution from GRDF
 - proton optical potential from
global Dirac phenomenology
(S. Hama et al., Phys. Rev. C 41 (1990) 2737)
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(K. Yoshida et al., Phys. Rev. C 94 (2016) 044604)
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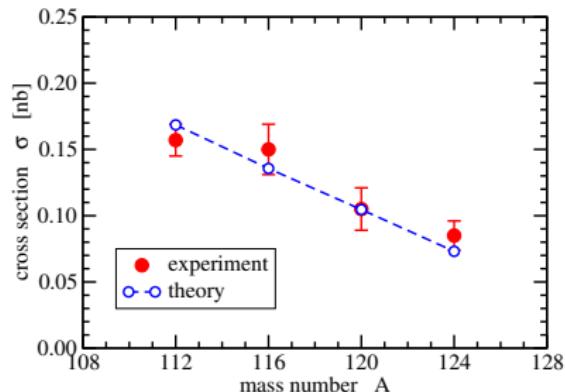
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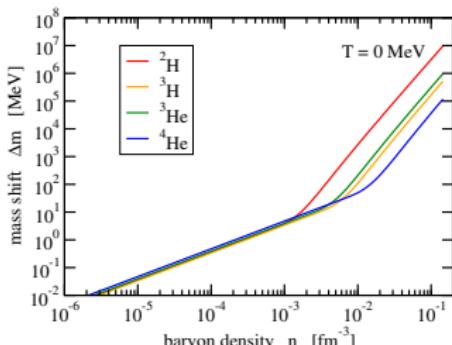
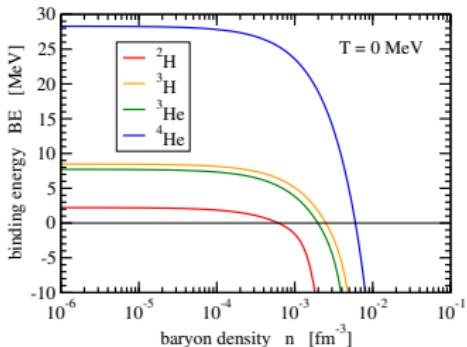
Correlations above Nuclear Saturation Density

Correlations Below and Above Saturation Density I



► choice of density dependence of cluster mass shifts in GRDF

- low densities: linear in n as given by parametrisation of G. Röpke
- higher densities (above Mott density): steeper function ($\propto n^3$, artificial) to avoid reappearance of clusters



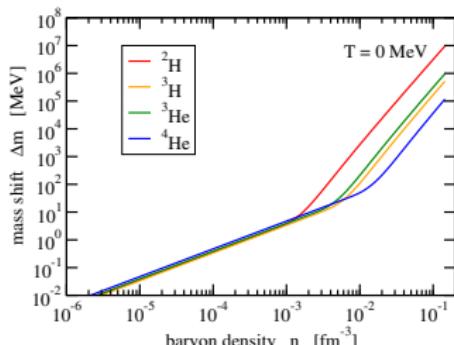
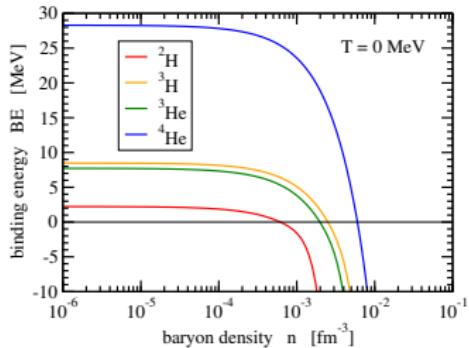
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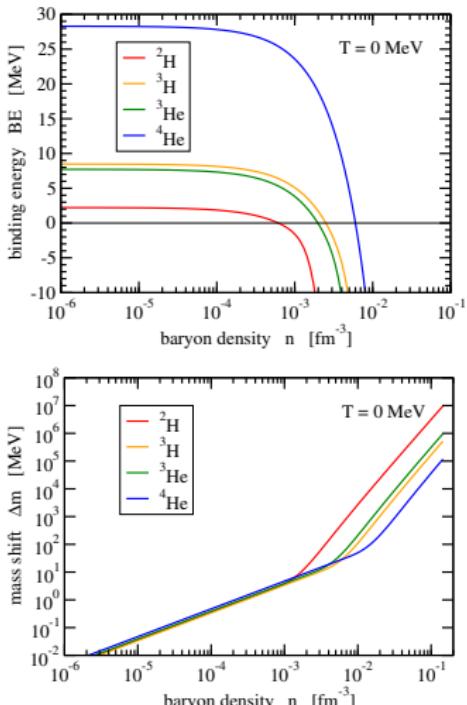
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- ▶ representation of many-body correlations above saturation density ?



Correlations Below and Above Saturation Density II



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- ▶ clusters as effective many-body correlations
 - ▶ internal motion of nucleons in cluster
⇒ tail in single-nucleon momentum distributions

Correlations Below and Above Saturation Density II



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⇒ smaller Δm_i for larger p_{cm}
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Correlations Below and Above Saturation Density II

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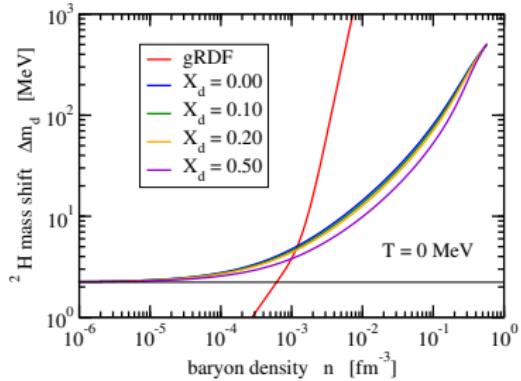
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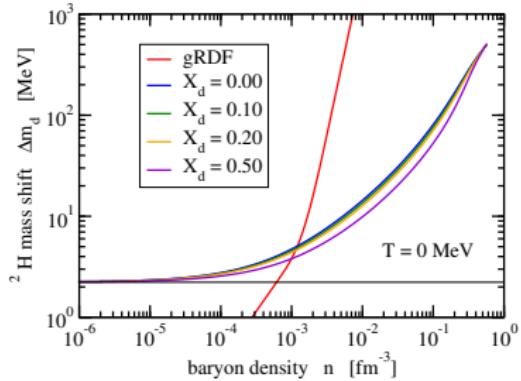
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\Rightarrow revision of functional form of cluster mass shifts



Application to Astrophysics

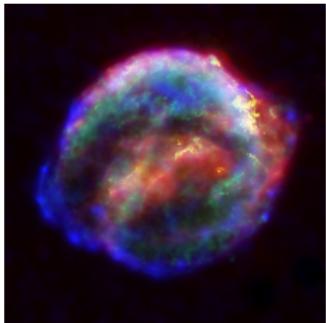
Equation of State (EoS) and Astrophysics I

essential ingredient in astrophysical model calculations

- ▶ static properties of **neutron stars**
- ▶ dynamical evolution of **core-collapse supernovae, neutron star mergers**
- ▶ conditions for **nucleosynthesis**
- ▶ energetics, **chemical composition**, transport properties



X-ray: NASA/CXC/J.Hester (ASU)
Optical: NASA/ESA/J.Hester & A.Loll (ASU)



NASA/ESA/R.Sankrit & W.Blair (Johns Hopkins Univ.)

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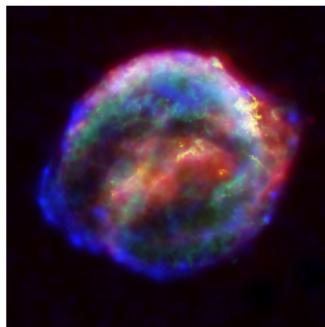
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timescale of reactions \ll timescale of system evolution

- ▶ **equilibrium** (thermal, chemical, ...)
- ▶ application of **EoS** reasonable



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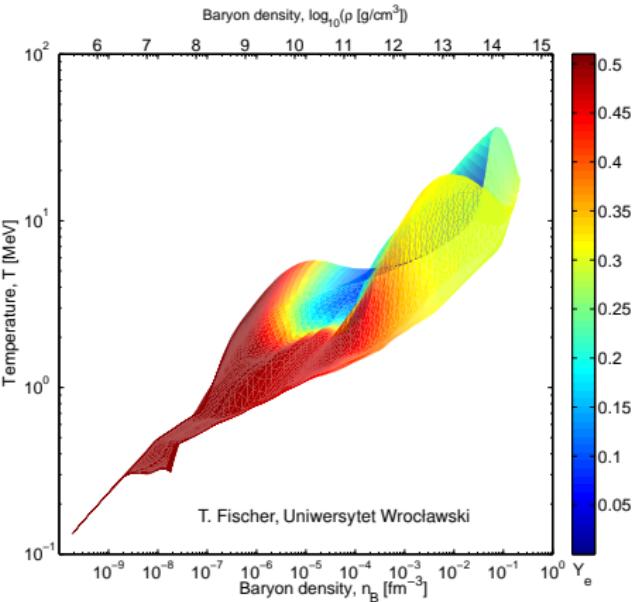


NASA/ESA/R.Sankrit & W.Blair (Johns Hopkins Univ.)

wide range of thermodynamic variables

- ▶ **temperature T**
- ▶ **baryon density n_b**
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simulation of core-collapse supernova



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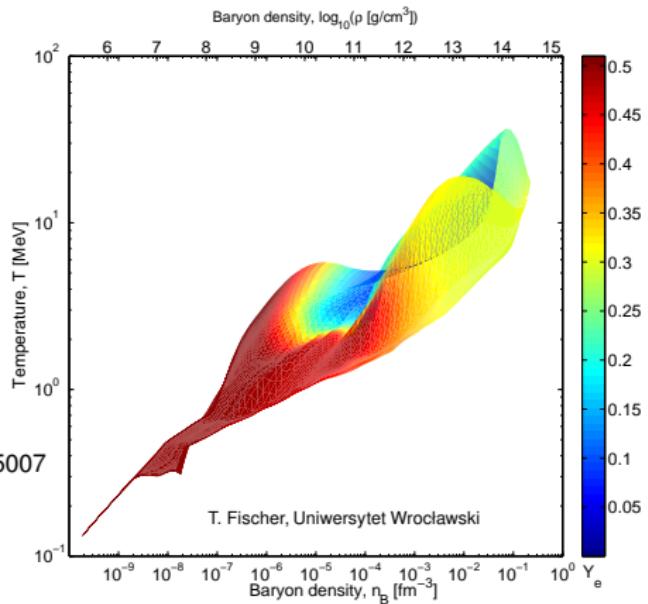
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⇒ **global, multi-purpose EoS required**

EoS database: compose.obspm.fr

EoS review: M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007

simulation of core-collapse supernova



Conclusions

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- ▶ modify thermodynamic properties and chemical composition of strongly interacting matter
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- ▶ extension of relativistic mean-field model
- ▶ density dependent couplings with well constrained parametrisation DD2
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- ▶ cluster mass shifts \Rightarrow dissolution at high densities
- ▶ strong constraints below saturation density
- ▶ applications: equation of state, nuclear structure

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► matter above saturation density

- ▶ no correlations in present GRDF
- ▶ revision of cluster mass shifts