Advances in the description of pairing and quarteting correlations in nuclear systems

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IFIN-HH, Romania

Light clusters in nuclei and nuclear matter,
September 4th 2019

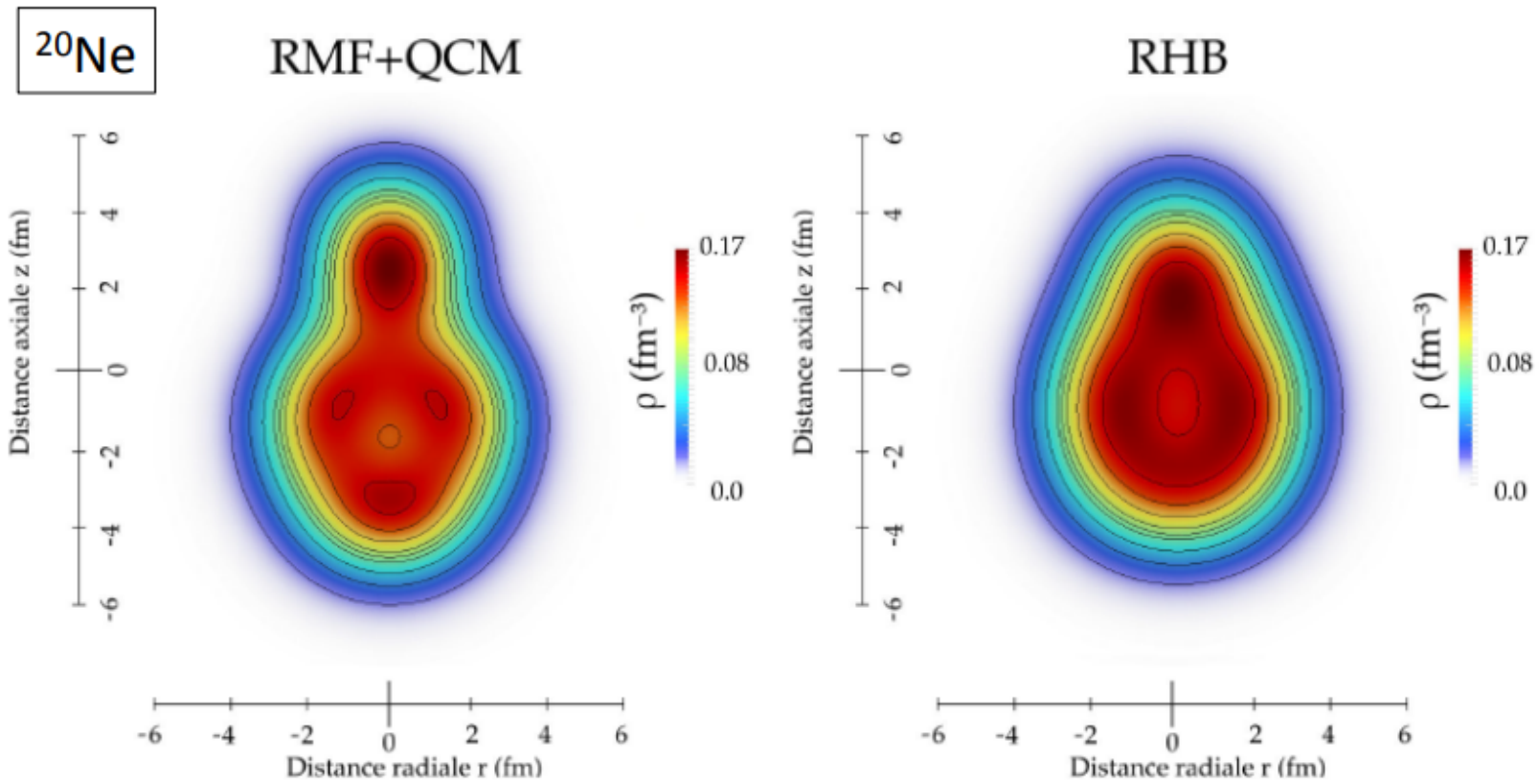
Approximation methods for pair and quartet correlations

- **Analytical approach for PBCS and QCM**
  Baran, Delion, "Analytical approach for the Quartet Condensation Model", Phys. Rev. C 99, 031303(R) (2019)

- **Bosonic picture for PBCS and QCM**
  Baran, Delion, "Unified description of pairing and quarteting correlations within the particle-hole-boson approach", Phys. Rev. C 99, 064311 (2019)

- **New disentanglement approach**
  Baran, Delion, Dolteanu, "Disentangling the pair and quartet condensates", arXiv:1905.06639, referee acceptance for PRC.

Effect of pairing and quarteting on clustering



- Pairing acts against clustering (smearing of the density)
- Quarteting is compatible with clustering

E. Khan, 2018 European Nuclear Physics Conference

Pairing Hamiltonian

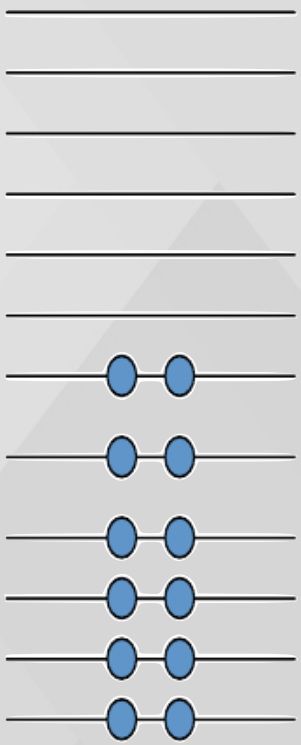
Model:

- N_{lev} doubly degenerate levels
- single particle energies ϵ_i
- Hamiltonian:

$$H = \sum_{i=1}^{N_{\text{lev}}} \epsilon_i N_i + \sum_{i,j=1}^{N_{\text{lev}}} V_{ij} P_i^\dagger P_j \ ,$$

where the pair operator is:

$$P_i^\dagger = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger \ .$$



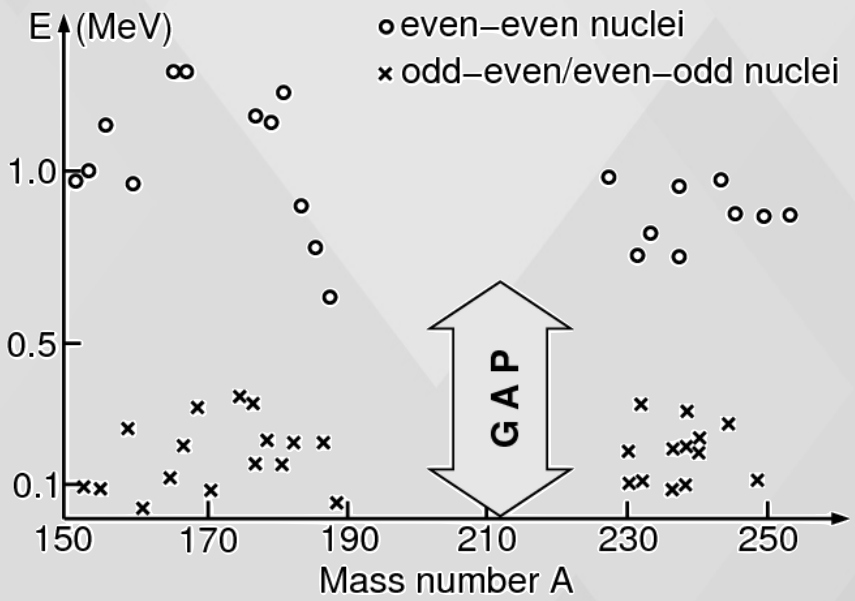
The non-interacting ground state $|\text{HF}\rangle$

Number projected BCS

The ground state is taken to be the PBCS condensate of n_p pairs,

$$|PBCS\rangle = \left(\Gamma^\dagger(x)\right)^{n_p}|0\rangle = \left(\sum_{i=1}^{N_{\text{lev}}} x_i P_i^\dagger\right)^{n_p}|0\rangle \ ,$$

obtained from $|BCS\rangle = \exp(\sum_i \frac{v_i}{u_i} P_i^\dagger)|0\rangle = \sum_n \frac{1}{n!}\left(\Gamma^\dagger(v/u)\right)^n|0\rangle$.



Bohr, Mottelson, Pines 1958

Isovector pairing

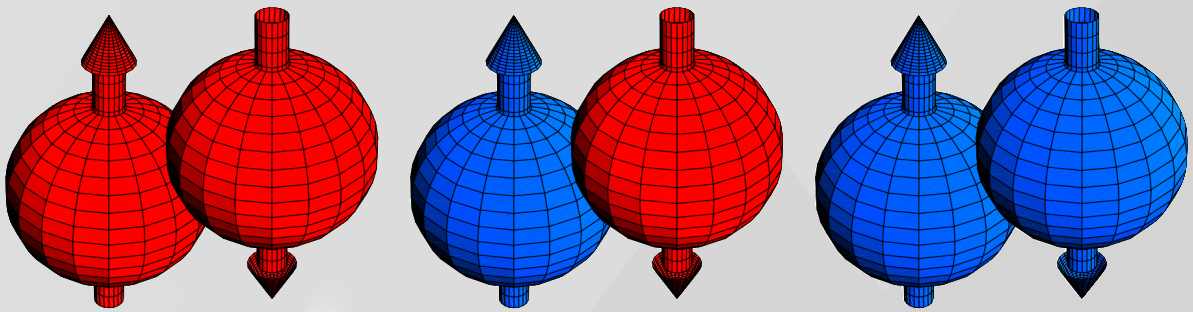
$$H = \sum_{i=1}^{N_{\rm lev}} \epsilon_i \left(N_{i,\pi} + N_{i,\nu}\right) + \sum_{\tau=0,\pm1} \sum_{i,j=1}^{N_{\rm lev}} V_{ij} P^\dagger_{i,\tau} P_{j,\tau} \ ,$$

In this case, we may construct collective $\pi\pi$, $\nu\nu$ and $\pi\nu$ Cooper pairs

$$\Gamma^\dagger_\tau(x) \equiv \sum_{i=1}^{N_{\rm lev}} x_i P^\dagger_{\tau,i} \quad , \quad \tau = \pm1, 0 \ ,$$

where the individual pairs are:

$$P_1 = \pi_\uparrow \pi_\downarrow \quad , \quad P_0 = \frac{1}{\sqrt{2}} \left(\pi_\uparrow \nu_\downarrow - \nu_\uparrow \pi_\downarrow\right) \quad , \quad P_{-1} = \nu_\uparrow \nu_\downarrow$$



Quartet Condensation Model

Collective quartet operator:

$$Q^\dagger(x) \equiv \left[\Gamma^\dagger\Gamma^\dagger\right]^{T=0}_{S=0} \equiv 2\Gamma^\dagger_1\Gamma^\dagger_{-1} - \left(\Gamma^\dagger_0\right)^2 .$$

The ground state is as a "condensate" of such α-like quartets

$$|\Psi_q(x)\rangle = \left[Q^\dagger(x)\right]^q|0\rangle ,$$

"*The term condensate has here the same meaning as in the case of pair condensate: a state obtained by acting many times with the same operator on a vacuum state.*" [D. Negrea, P. Buganu, D. Gambacurta and N. Sandulescu, PRC 98, 064319 (2018)]

- N. Sandulescu, D. Negrea, J. Dukelsky, C. W. Johnson, Phys. Rev. C **85**, 061303(R) (2012).
- D. Negrea, *Proton-neutron correlations in atomic nuclei*, Ph.D. thesis, 2013
- N. Sandulescu, D. Negrea, C. W. Johnson, Phys. Rev. C **86**, 041302(R) (2012).
- D. Negrea, N. Sandulescu, Phys. Rev. C **90**, 024322 (2014).
- N. Sandulescu, D. Negrea, D. Gambacurta, Phys. Lett. B, **751**, 348 (2015).
- D. Negrea, N. Sandulescu, D. Gambacurta, Prog. Theor. Exp. Phys. 073D05 (2017).

Finally we would like to make a few clarifying comments relative to the quartet condensation model which we have used in our thesis. Here the name " quartet condensate" is used in the same sense as it is used "pair condensate" in BCS theory, namely as a state formed by applying many times the same quartet operator. Since the quartet operator is not a boson, the quartet condensate is not a bosonic condensate. In addition, should be kept in mind that the alpha-like quartet is not describing an alpha particle (^{4}He) localized in the space. Alpha-like quartet means here a four-body structure of two neutrons and two protons correlated in spin and isospin and not necessarily in coordinate space.

	SM	QCM	PBCS1	PBCS0
^{20}Ne	9.173	9.170 (0.033%)	8.385 (8.590%)	7.413 (19.187%)
^{24}Mg	14.460	14.436 (0.166%)	13.250 (8.368%)	11.801 (18.389%)
^{28}Si	15.787	15.728 (0.374%)	14.531 (7.956%)	13.102 (17.008%)
^{32}S	15.844	15.795 (0.309%)	14.908 (5.908%)	13.881 (12.389%)
^{44}Ti	5.973	5.964 (0.151%)	5.487 (8.134%)	4.912 (17.763%)
^{48}Cr	9.593	9.569 (0.250%)	8.799 (8.277%)	7.885 (17.805%)
^{52}Fe	10.768	10.710 (0.539%)	9.815 (8.850%)	8.585 (20.273%)
^{104}Te	3.831	3.829 (0.052%)	3.607 (5.847%)	3.356 (12.399%)
^{108}Xe	6.752	6.696 (0.829%)	6.311 (6.531%)	5.877 (12.959%)
^{112}Ba	8.680	8.593 (1.002%)	8.101 (6.670%)	13.064 (13.064%)

Daniel Negrea, PhD Thesis, Correlations dappariement proton-neutron dans le noyau atomique, University of Bucharest and University Paris-Sud XI, 2013

Young criterion for quarteting

"4-body density matrix indicates long-range correlations of condensate type"

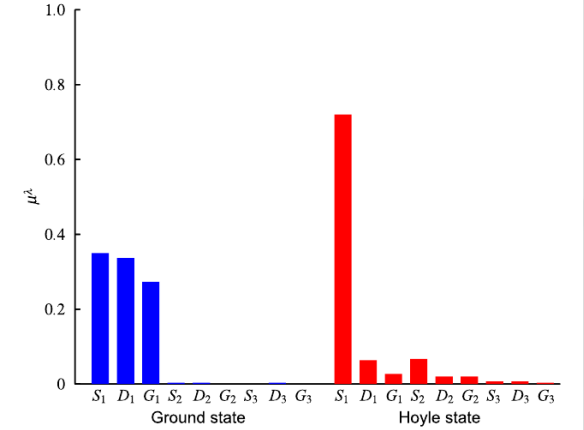
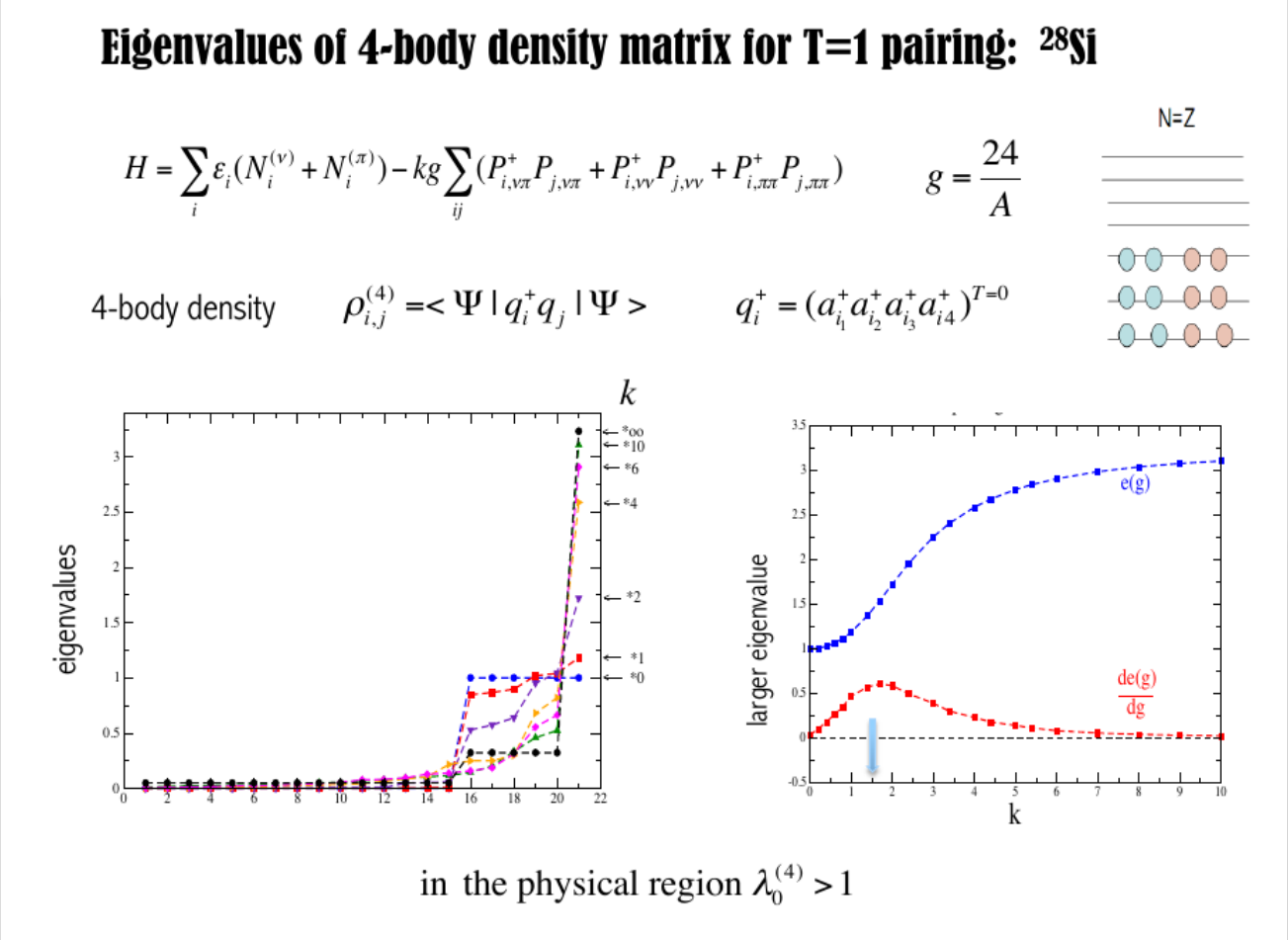


Figure 13. Occupation of the single α orbitals of the Hoyle state of ^{12}C compared with the ground state.

Phys. Scr. 91 (2016) 123001

- Low density?



M. Sambataro, N. Sandulescu, "Recent advances on proton-neutron pairing and quartet correlations in nuclei" workshop (2018); to be published.

Solving QCM

We need the minimum of

$$E(x) \equiv \frac{\langle \Psi_q(x)|H|\Psi_q(x)\rangle}{\langle \Psi_q(x)|\Psi_q(x)\rangle} \ .$$

- Recurrence relations method:
  - work in the basis $|n\rangle \equiv |n_1 n_2 n_3\rangle \equiv \left(\Gamma_1^\dagger\right)^{n_1} \left(\Gamma_{-1}^\dagger\right)^{n_2} \left(\Gamma_0^\dagger\right)^{n_3} |0\rangle$
  - derive the recurrence relations of the matrix elements of the operators $P_\tau^\dagger P_\sigma$,P, N_τ, T_τ.
  - implement the recurrence relations numerically.

- Symbolic computation $\rightarrow$ analytical formulas.

Cadabra2: Harmonic Oscillator Tutorial

```
\vacR::LaTeXForm("|0\rangle").
\vacL::LaTeXForm("\langle 0|").
{\dagger}::Symbol.
{a^{\dagger},a,N,\vacR,\vacL}::NonCommuting;
```

Attached property NonCommuting to $\left[a^\dagger,\ a,\ N,\ |0\rangle,\ \langle 0|\right]$.

```
rules:={
N a^{\dagger}-> a^{\dagger} N+a^{\dagger},
a a^{\dagger} -> a^{\dagger} a +1,
N \vacR -> 0,
a \vacR -> 0,

\vacL \vacR ->1};
```

$$\left[Na^\dagger \to a^\dagger N + a^\dagger,\ aa^\dagger \to a^\dagger a + 1,\ N|0\rangle \to 0,\ a|0\rangle \to 0,\ \langle 0||0\rangle \to 1\right]$$

```
ex:=\vacL a a a  N  a^{\dagger} a^{\dagger} a^{\dagger} \vacR;
```

$$\langle 0|aaaNa^\dagger a^\dagger a^\dagger|0\rangle$$

```
converge(ex):
 substitute(ex,rules,repeat=True)
 distribute(_)
;
```

$$18$$

SO(5) algebra for $P^\dagger = c^\dagger c^\dagger$, $N = c^\dagger c$, $T = \pi^\dagger \nu$ or $\nu^\dagger \pi$

$$\left[P_{i,0}, P^\dagger_{j,0}\right] = \delta_{ij}\left(1 - \frac{1}{2}N_{i,0}\right)$$

$$\left[P_{i,1}, P^\dagger_{j,1}\right] = \delta_{ij}\left(1 - N_{i,1}\right)$$

$$\left[P_{i,-1}, P^\dagger_{j,-1}\right] = \delta_{ij}\left(1 - N_{i,-1}\right)$$

$$\left[P_{i,1}, P^\dagger_{j,-1}\right] = 0$$

$$\left[P_{i,0}, P^\dagger_{j,1}\right] = \delta_{ij} T_{i,1}$$

$$\left[P_{i,0}, P^\dagger_{j,-1}\right] = -\delta_{ij} T_{i,-1}$$

$$\left[N_{i,0}, P^\dagger_{j,\tau}\right] = 2\delta_{ij} P^\dagger_{j,\tau}$$

$$\left[N_{i,\pm 1}, P^\dagger_{j,\mp 1}\right] = 0$$

$$\left[N_{i,\pm 1}, P^\dagger_{j,\pm 1}\right] = 2\delta_{ij} P^\dagger_{j,\pm 1}$$

$$\left[N_{i,\pm 1}, P^\dagger_{j,0}\right] = \delta_{ij} P^\dagger_{j,0}$$

$$\left[T_{i,\pm 1}, P^\dagger_{j,0}\right] = \mp\delta_{ij} P^\dagger_{j,\pm 1}$$

$$\left[T_{i,1}, P^\dagger_{j,-1}\right] = -\delta_{ij} P^\dagger_{j,0}$$

$$\left[T_{i,-1}, P^\dagger_{j,1}\right] = \delta_{ij} P^\dagger_{j,0}$$

Analytical QCM relations

The norms of the quartet states and the Hamiltonian averages

$$\langle \Psi_q(x) | \Psi_q(x) \rangle = \mathcal{N}_q(x) \ ,$$
$$\langle \Psi_q(x) | H | \Psi_q(x) \rangle = E_q(x) + v_q(x) \ .$$

They are polynomial functions of the mixing amplitudes of degree $4q$, expressed in terms of

$$\Sigma_\alpha = \sum_{i=1}^{N_{\rm lev}} x_i^\alpha \ , \qquad \mathcal{E}_\alpha = \sum_{i=1}^{N_{\rm lev}} \epsilon_i \ x_i^\alpha \ ,$$
$$\mathcal{V}_{\alpha\beta} = \sum_{i,j=1}^{N_{\rm lev}} V_{ij} \ x_i^\alpha \ x_j^\beta , \quad \mathcal{U}_\alpha = \sum_{i=1}^{N_{\rm lev}} V_{ii} \ x_i^\alpha \ ,$$

Analytical QCM relations

For $q = 1$, we obtain

$$\begin{aligned} \mathcal{N}_1 &= 3\big(2\ \Sigma_2{}^2 + \ \Sigma_4\big)\ , \\ E_1 &= 12\,(2\ \mathcal{E}_2\ \Sigma_2 + \ \mathcal{E}_4)\ , \\ v_1 &= 3\ (4\ \Sigma_2\ \mathcal{V}_{1,1} + 4\ \mathcal{V}_{1,3} + \mathcal{U}_4)\ . \end{aligned}$$

- These formulas may be employed to compute directly the ground state correlations in $N = Z$ nuclei.
- Particle-hole symmetry: a system with q quartets may be mapped to an equivalent system with $N_{lev} - q$ quartet holes.

QCM Analytical relations

This approach presents a twofold benefit:

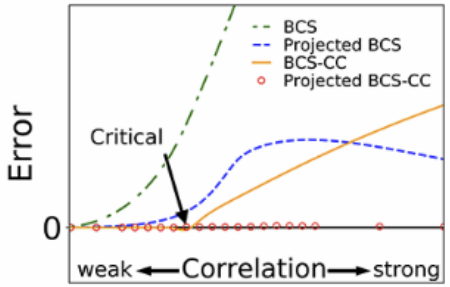
- on the numerical side, the computational time may be significantly reduced.
- on the implementation side the effort is made negligible (copy and paste).

Many body approximations

1. tested against exactly solvable scenarios.
2. applied then to realistic scenarios.

For pairing:

- RPA J. Dukelsky, G. G. Dussel, J. C. Hirsch, and P. Schuck, Nucl. Phys. A 714, 63 (2003).
- coupled clusters T. M. Henderson et al, PRC 89, 054305 (2014); Y. Qiu et al, PRC 99, 044301 (2019)



EDITORS' SUGGESTION

Particle-number projected Bogoliubov-coupled-cluster theory: Application to the pairing Hamiltonian

A many-body formalism is developed to consistently combine particle-number projection techniques with Bogoliubov coupled-cluster theory. Applied to the Richardson pairing Hamiltonian, the method produces highly accurate solutions over the complete range of pairing strengths (from weak to strong correlations). The next step will be to apply the method to realistic nuclear Hamiltonians and treat similarly the angular-momentum breaking and restoration.

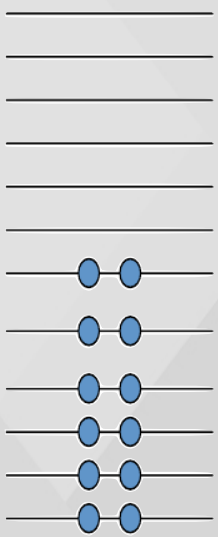
Y. Qiu, T. M. Henderson, T. Duguet, and G. E. Scuseria
Phys. Rev. C **99**, 044301 (2019)

Particle-hole approach

[J. Dukelsky et al., PRC 93, 034313 (2016)]

Instead of expressing the $|PBCS\rangle$ state with respect to the $|0\rangle$ vacuum, we may find an equivalent form involving the Hartree-Fock state

$$|\mathrm{HF}\rangle = \left(\prod_{i=1}^{n_p} P_i^\dagger\right)|0\rangle \ .$$



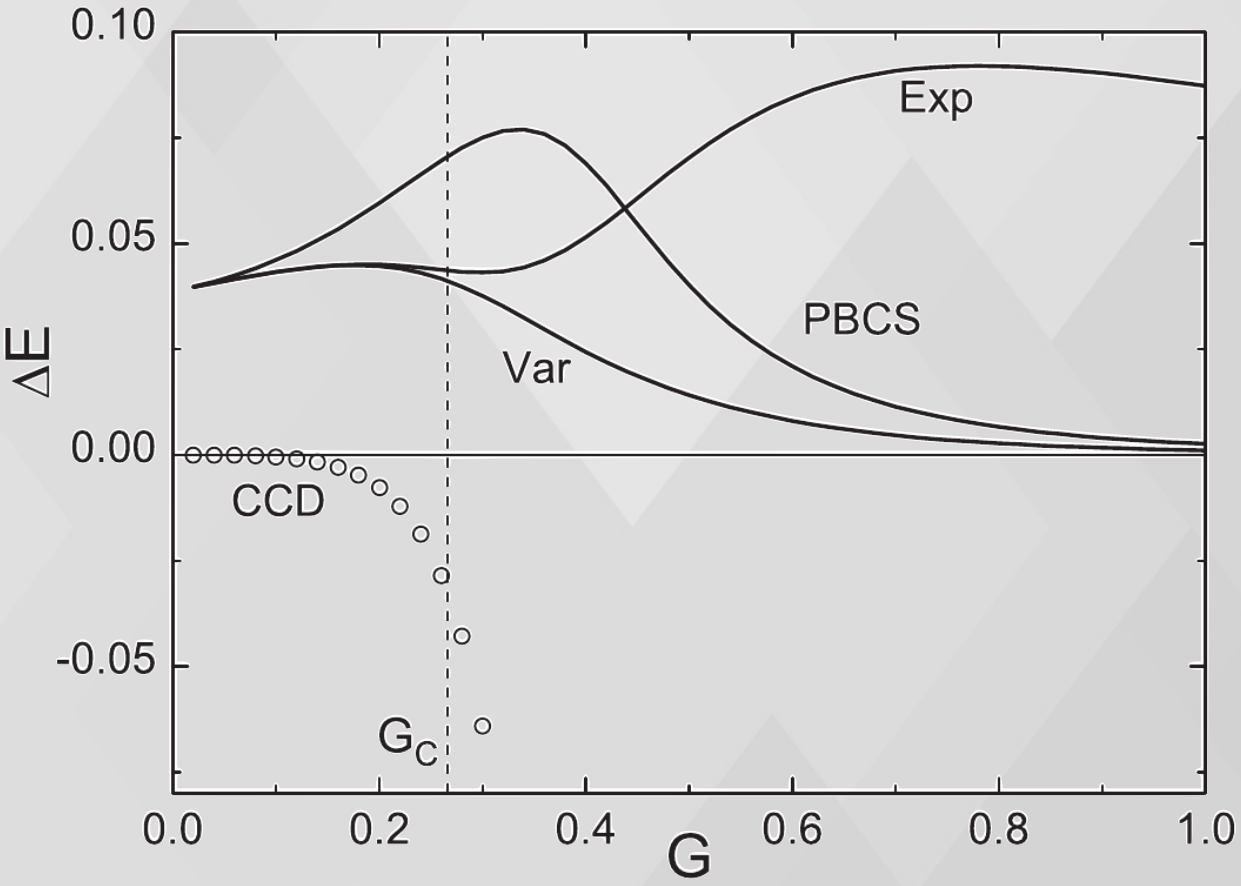
To this end, we first decompose the coherent pair on components below and above the Fermi level as follows

$$\Gamma^\dagger(x) = \sum_{i=1}^{n_p} x_i P_i^\dagger + \sum_{i=n_p+1}^{N_{\mathrm{lev}}} x_i P_i^\dagger \equiv \Gamma_h^\dagger(x) + \Gamma_p^\dagger(x)$$

Particle-hole reformulation of PBCS

[J. Dukelsky et al., PRC 93, 034313 (2016)]

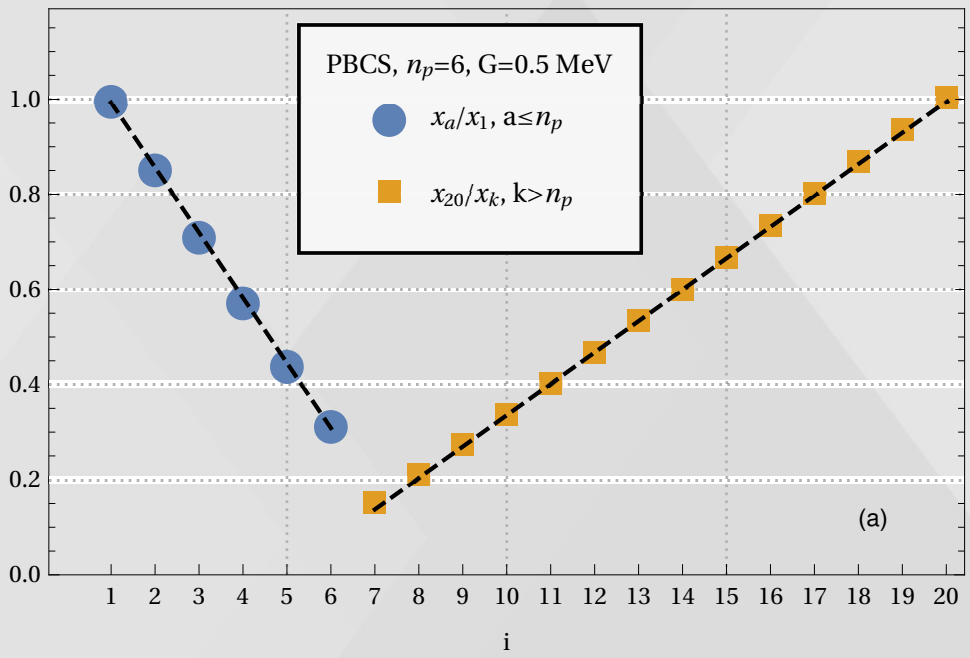
$$|PBCS\rangle = \left(\Gamma^{\dagger}(x)\right)^{n_p}|0\rangle \sim \sum_{n=0}^{n_p} \frac{1}{(n!)^2}\left(\Gamma_p^{\dagger}(x)\,\Gamma_h\left(\frac{1}{x}\right)\right)^n |\mathrm{HF}\rangle$$

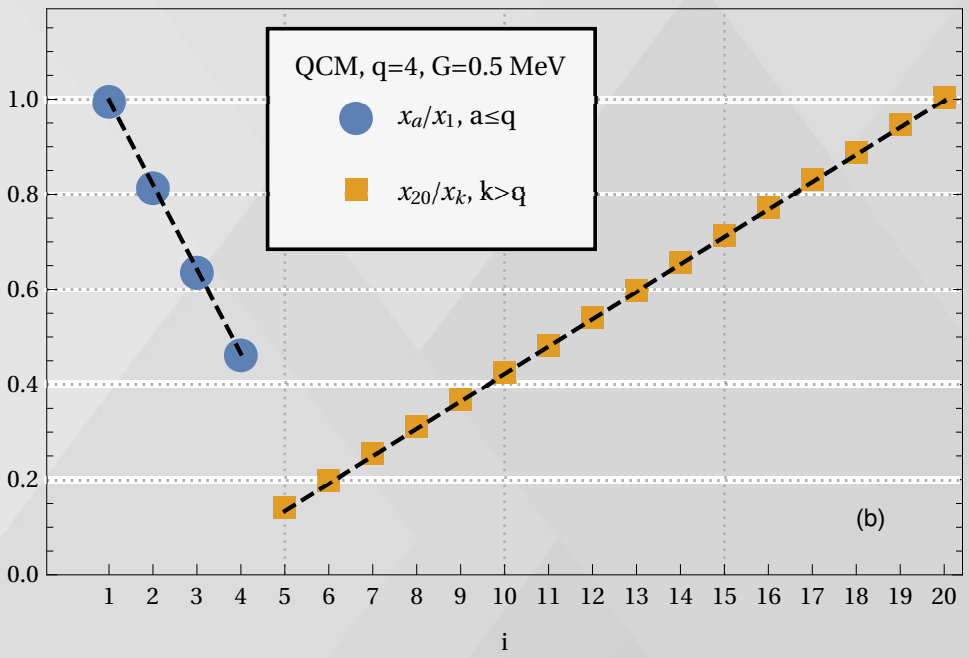


Particle-hole approach

V.V. Baran, D.S. Delion, PRC 99, 064311 (2019)

$$|PBCS\rangle = \left(\Gamma^\dagger(x)\right)^{n_p}|0\rangle \sim \sum_{n=0}^{n_p}\frac{1}{(n!)^2}\left(\Gamma_p^\dagger(x)\,\Gamma_h\left(\frac{1}{x}\right)\right)^n|\mathrm{HF}\rangle$$





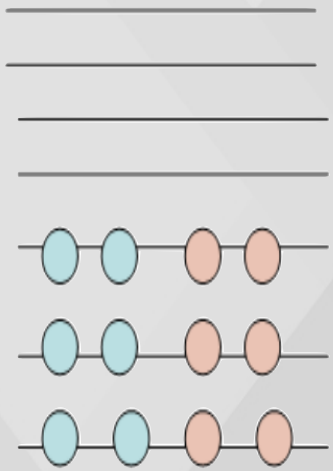
$$\Gamma^\dagger = \sum_{a=1}^{N_F} x_a P_a^\dagger + \sum_{k=N_F+1}^{N_{\mathrm{lev}}} x_k P_k^\dagger$$

Particle-hole reformulation of QCM

V.V. Baran, D.S. Delion, PRC 99, 064311 (2019)

Hartree-Fock state for a proton-neutron system:

$$|\mathrm{HF}\rangle = \left(\prod_{i=1}^{q} P^{\dagger}_{1,i} P^{\dagger}_{-1,i} \right) |0\rangle \ .$$



The coherent pairs may be decomposed on components below and above the Fermi level

$$\Gamma^{\dagger}_{\tau}(x) = \sum_{i=1}^{q} x_i P^{\dagger}_{\tau,i} + \sum_{i=q+1}^{N_{\mathrm{lev}}} x_i P^{\dagger}_{\tau,i} \equiv \Gamma^{\dagger}_{\tau,h}(x) + \Gamma^{\dagger}_{\tau,p}(x) \ .$$

Particle-hole reformulation of QCM

The coherent pairs may be decomposed on components below and above the Fermi level as

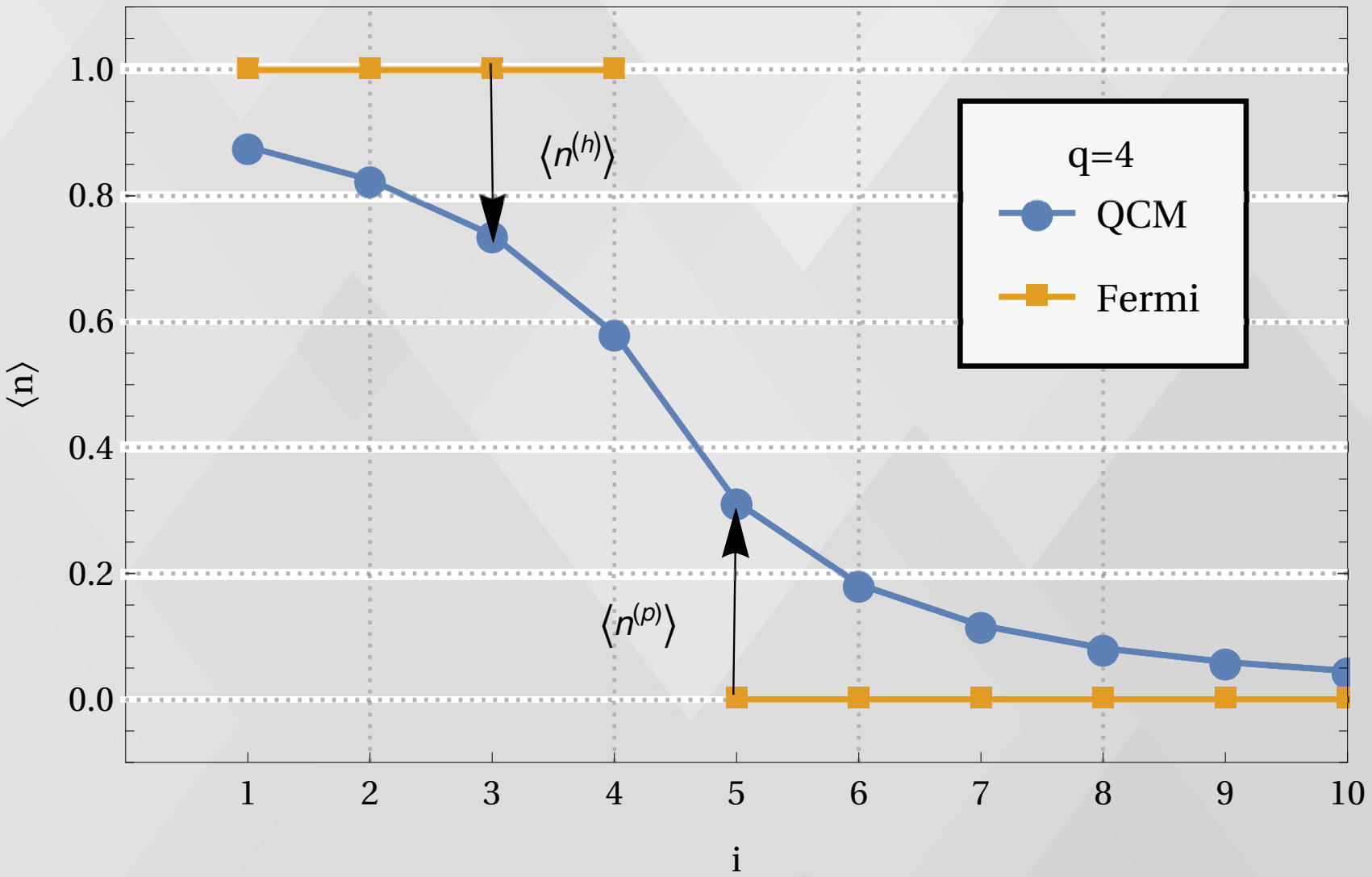
$$\Gamma^\dagger_\tau(x) = \Gamma^\dagger_{\tau,h}(x) + \Gamma^\dagger_{\tau,p}(x) \ .$$

As a consequence, the collective quartet decomposes as follows

$$\begin{aligned} Q^\dagger(x) &= 2\Gamma^\dagger_1\Gamma^\dagger_{-1} - \left(\Gamma^\dagger_0\right)^2 \\ &\equiv Q^\dagger_h(x) + Q^\dagger_p(x) + 2\left[\Gamma^\dagger_p(x)\Gamma^\dagger_h(x)\right] \ . \end{aligned}$$

$$|\Psi_q\rangle \sim \sum_{a=0}^{q}\sum_{b=0}^{q}\lambda_{ab}\left(Q^\dagger_p(x)Q_h\left(\frac{1}{x}\right)\right)^a\left[\Gamma^\dagger_p(x)\Gamma_h\left(\frac{1}{x}\right)\right]^b|\mathrm{HF}\rangle$$

Particle and hole degrees of freedom



Average level occupation $\langle n \rangle$ vs state number i.

Particle hole bosons

Boson mapping:

$$
\begin{aligned}
P_i^\dagger &\to p_i^\dagger \ , \\
\tilde{P}_a^\dagger &\to h_a^\dagger \ , \\
\tilde{N}_a &\to \mathcal{N}_a \ , \\
N_i &\to \mathcal{N}_i \ , \\
|\mathrm{HF}\rangle &\to |0) \ ,
\end{aligned}
$$

where $p_i|0) = 0$ and $h_a|0) = 0$.

Boson algebra:

$$
\begin{aligned}
\left[p_i, p_j^\dagger\right] &= \delta_{ij}\pi_j \ , \\
\left[h_a, h_b^\dagger\right] &= \delta_{ab}\eta_b \ , \\
\left[p_i, h_j^\dagger\right] &= 0 \ , \\
\left[\mathcal{N}_i, p_j^\dagger\right] &= 2\delta_{ij}p_j^\dagger \ , \\
\left[\mathcal{N}_a, h_b^\dagger\right] &= 2\delta_{ab}h_b^\dagger \ ,
\end{aligned}
$$

where the coefficients π_i and η_j are c-numbers and all other commutators vanish.

Particle hole bosons

We also define the corresponding collective bosons

$$\mathcal{H}^\dagger(y) \equiv \sum_{a=1}^{n_p} y_a h_a^\dagger \ , \ \mathcal{P}^\dagger(x) \equiv \sum_{i=n_p+1}^{N_{\text{lev}}} x_i p_i^\dagger \ .$$

The bosonic ground state has same form as the fermionic PBCS condensate

$$|\psi(x,y)\rangle \equiv \sqrt{\chi} \sum_{n=0}^{n_p} \frac{1}{(n!)^2} \left(\mathcal{P}^\dagger(x)\ \mathcal{H}^\dagger(y)\right)^n |0) \ ,$$

where χ is a normalization constant.

Particle hole bosons

The ground state energy corresponds to the minimum of the energy function

$$E(x,y) \equiv \frac{\langle \psi(x,y)|H_b|\psi(x,y)\rangle}{\langle \psi(x,y)|\psi(x,y)\rangle} \ .$$

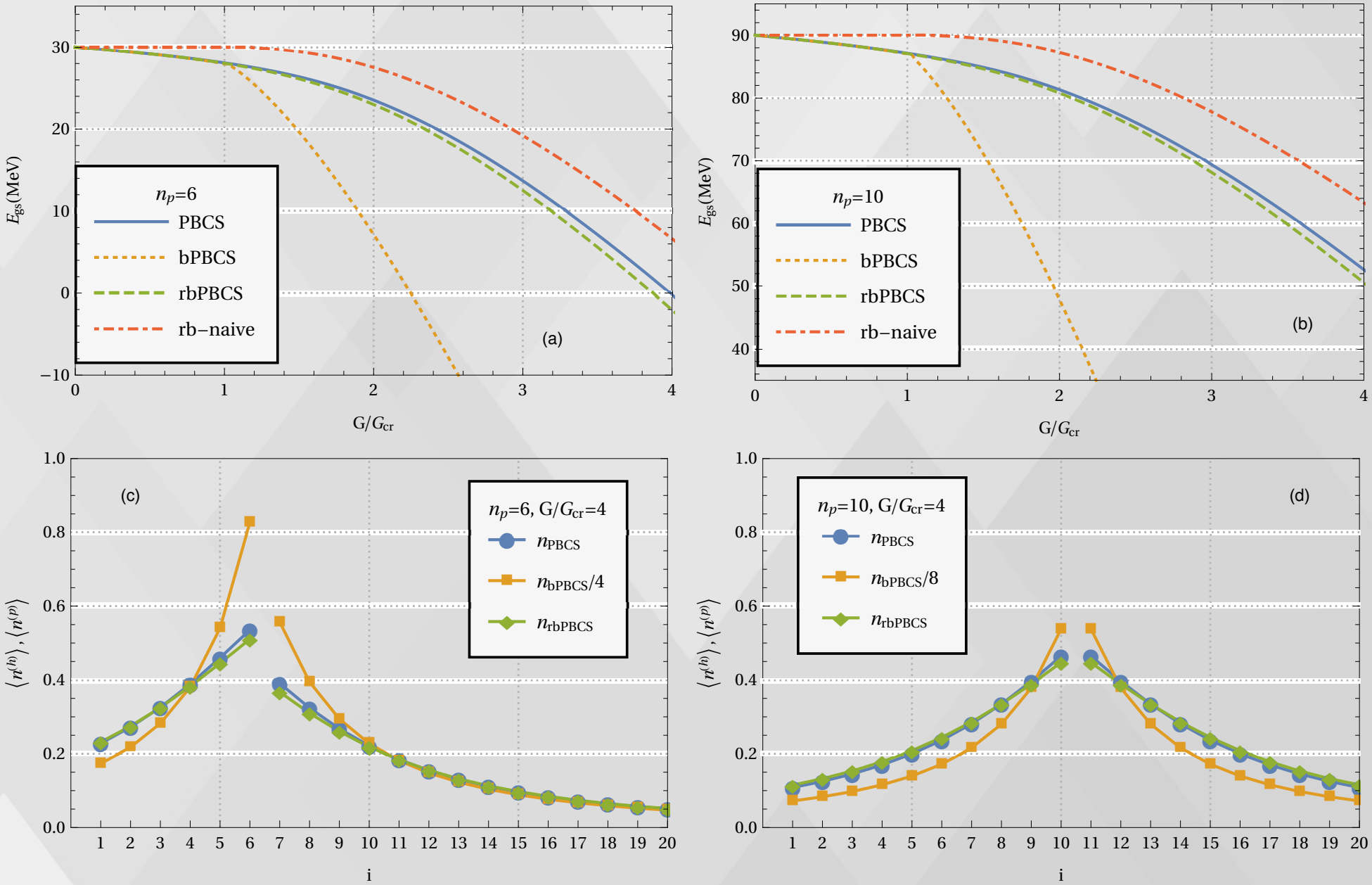
We will analyze two choices for the commutator coefficients

1. pure bosonic case: $\left[p_i, p_j^\dagger\right] = \delta_{ij}, \left[h_a, h_b^\dagger\right] = \delta_{ab}$.
2. renormalized bosonic case:

$$\left[p_i, p_j^\dagger\right] = \delta_{ij}\left(1 - \frac{1}{2}\langle \mathcal{N}_j\rangle\right) \ ,$$

$$\left[h_a, h_b^\dagger\right] = \delta_{ab}\left(1 - \frac{1}{2}\langle \mathcal{N}_a\rangle\right) \ .$$

Projected BCS case



QCM case

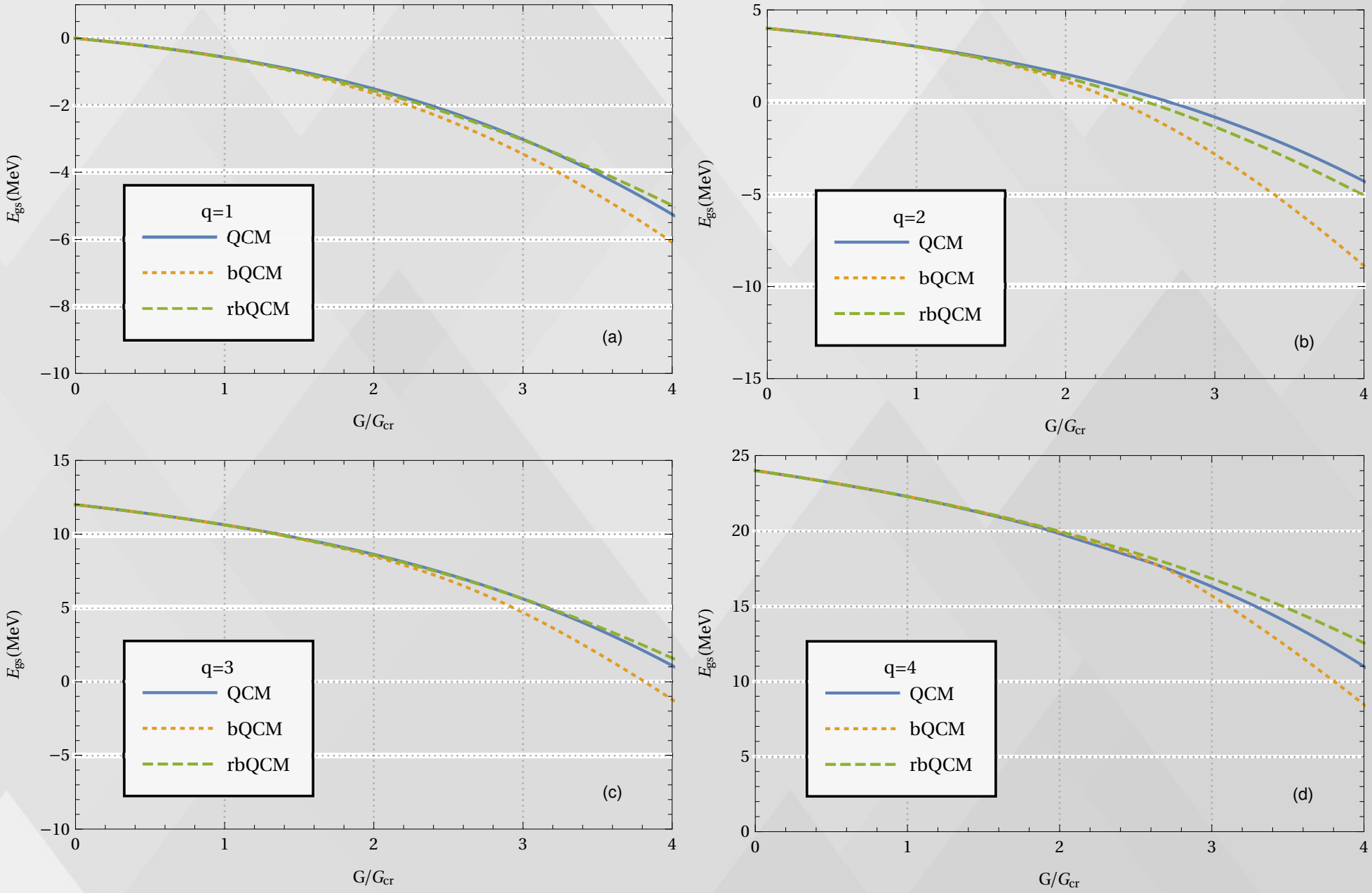
Each projection of the triplet of pair operators translates into a corresponding boson

$$P^{\dagger}_{\tau,i} \rightarrow p^{\dagger}_{\tau,i} \ , \ \tilde{P}^{\dagger}_{\tau,a} \rightarrow h^{\dagger}_{\tau,a} \ ,$$

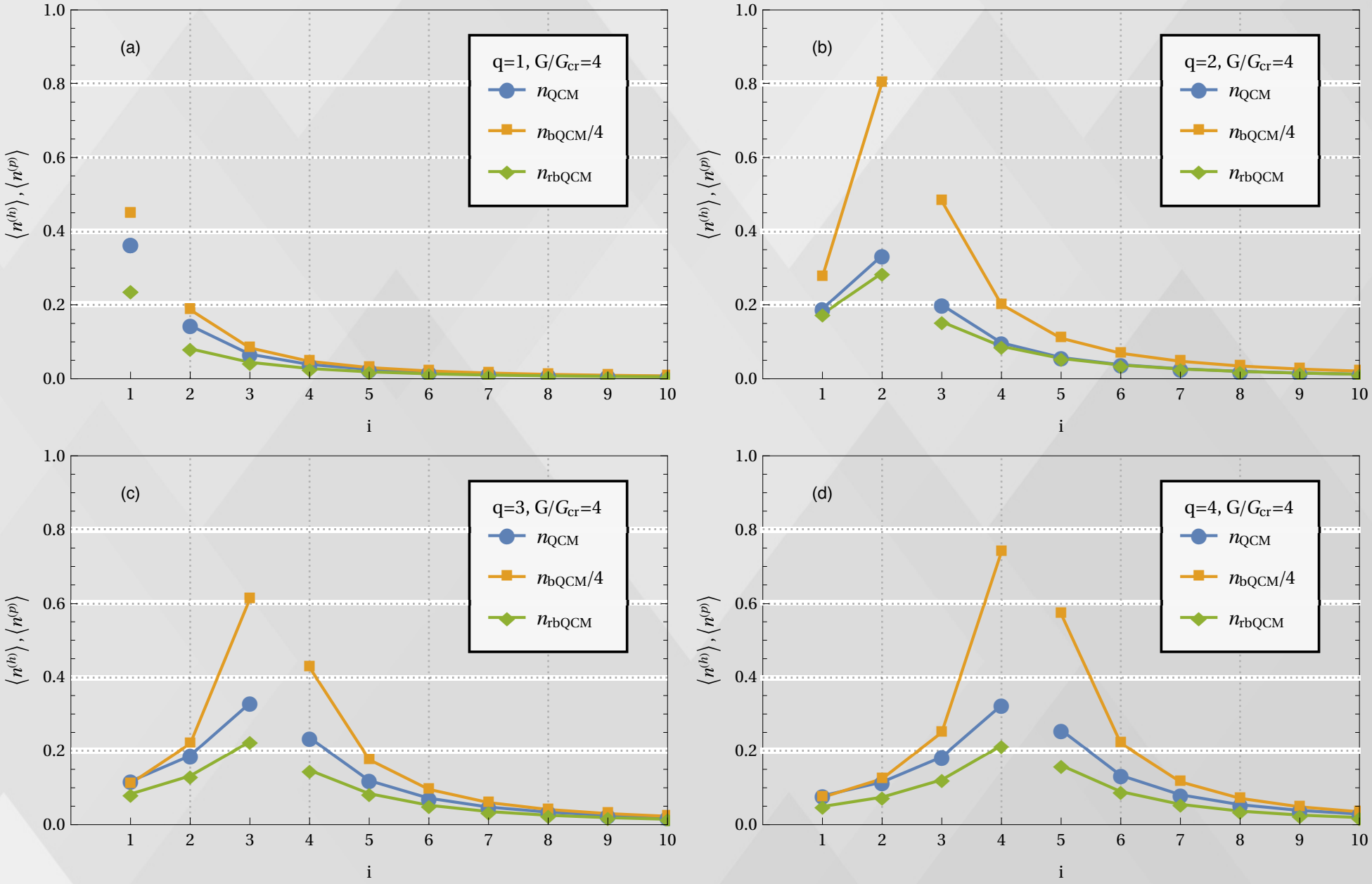
where we consider bosonic pairs of different isospin projection to commute:

$$\left[p_{\tau,i}, p^{\dagger}_{\sigma,j}\right] = \delta_{\tau\sigma}\delta_{ij}\pi_j \ , \ \left[h_{\tau,a}, h^{\dagger}_{\sigma,b}\right] = \delta_{\tau\sigma}\delta_{ab}\eta_b$$

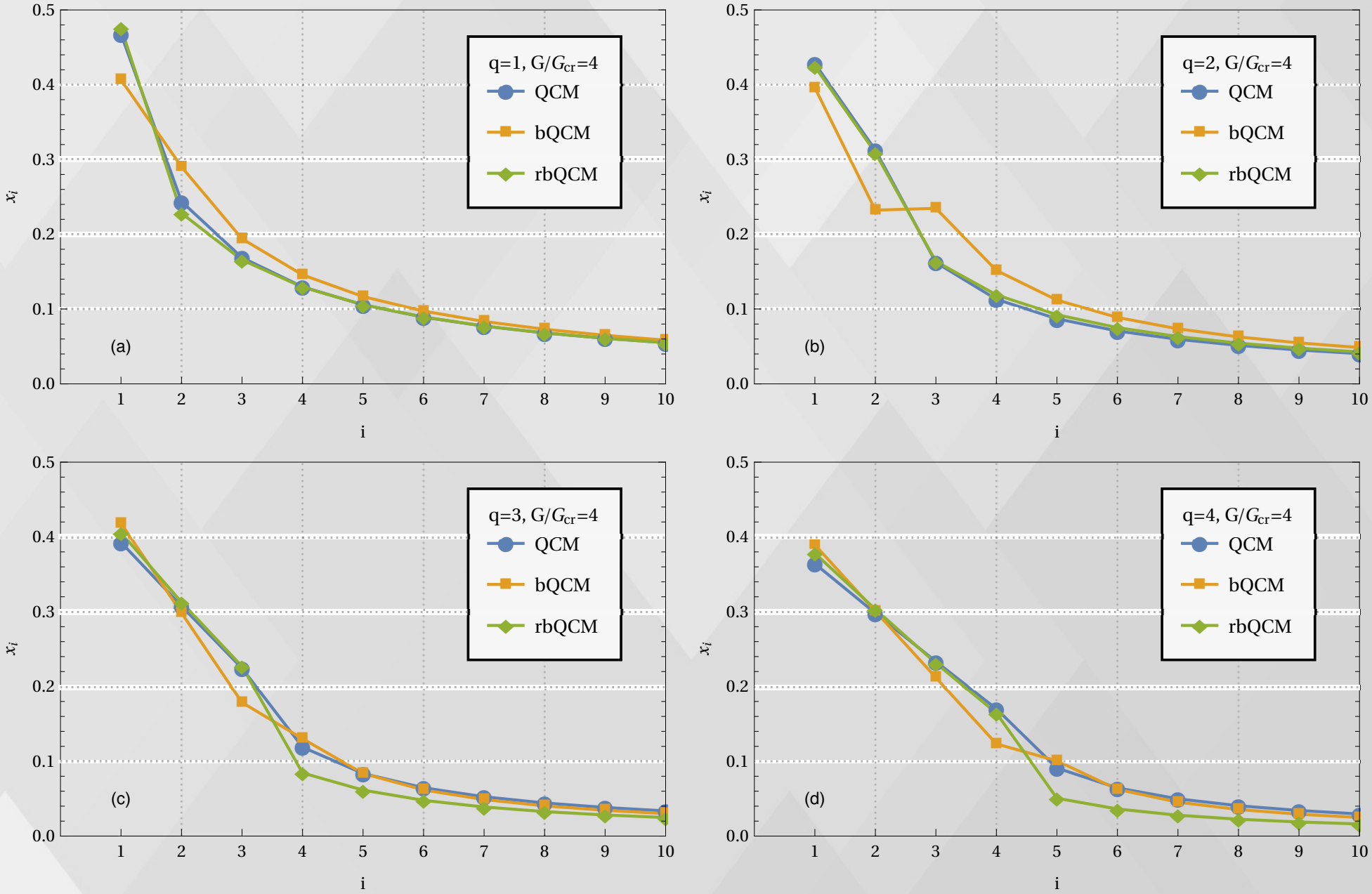
Quartet case: ground state energy



Quartet case: particle and hole average level occupancies



Quartet case: mixing amplitudes



Particle-hole bosons

- Quartet condensate state as a particle-hole expansion: quartet-quartet excitations and coupled pair excitations.
- Bosonic approximation for pair and quartet condensates.
- The particle-hole expansion of the pair and quartet condensates contains a lot of information about the pure fermionic correlations in the ground state.

Disentangling the pair and quartet condensates

[V.V. Baran, D.S. Delion, arxiv: 1905.06639]

For $N > Z$ nuclei, $|QCM\rangle = \left[Q^{\dagger}(x)\right]^{n_q} \left[\Gamma^{\dagger}_{\nu\nu}(y)\right]^{n_p} |0\rangle$.

- How much do the quartet structures contribute to the total amount of correlations?
- How much do the neutron pairs contribute?
- What about the quartet-pair correlations?

Disentangling the pair and quartet condensates

Isovector pairing in a degenerate shell with $N > Z$:

$$E_{\text{deg}}(n_q, n_p; \Omega) = G\left[(2n_q + n_p)\left(\Omega + \frac{3}{2} - \frac{2n_q + n_p}{2}\right) - \frac{1}{2}n_p(n_p + 1)\right]$$

We evaluate this expression for two subsystems,

- one of quartets,
- one of neutron pairs,

each living in a space whose degeneracy is reduced by the Pauli blocking of the other:

$$\begin{aligned} E_{\text{deg}}^{q-p} =& E_{\text{deg}}(n_q, n_p; \Omega) - E_{\text{deg}}(n_q, 0; \Omega - n_p) - E_{\text{deg}}(0, n_p; \Omega - n_q) \\ =& \, G n_p n_q \ . \end{aligned}$$

ADC

Within the Analytical Disentangled Condensate model we use

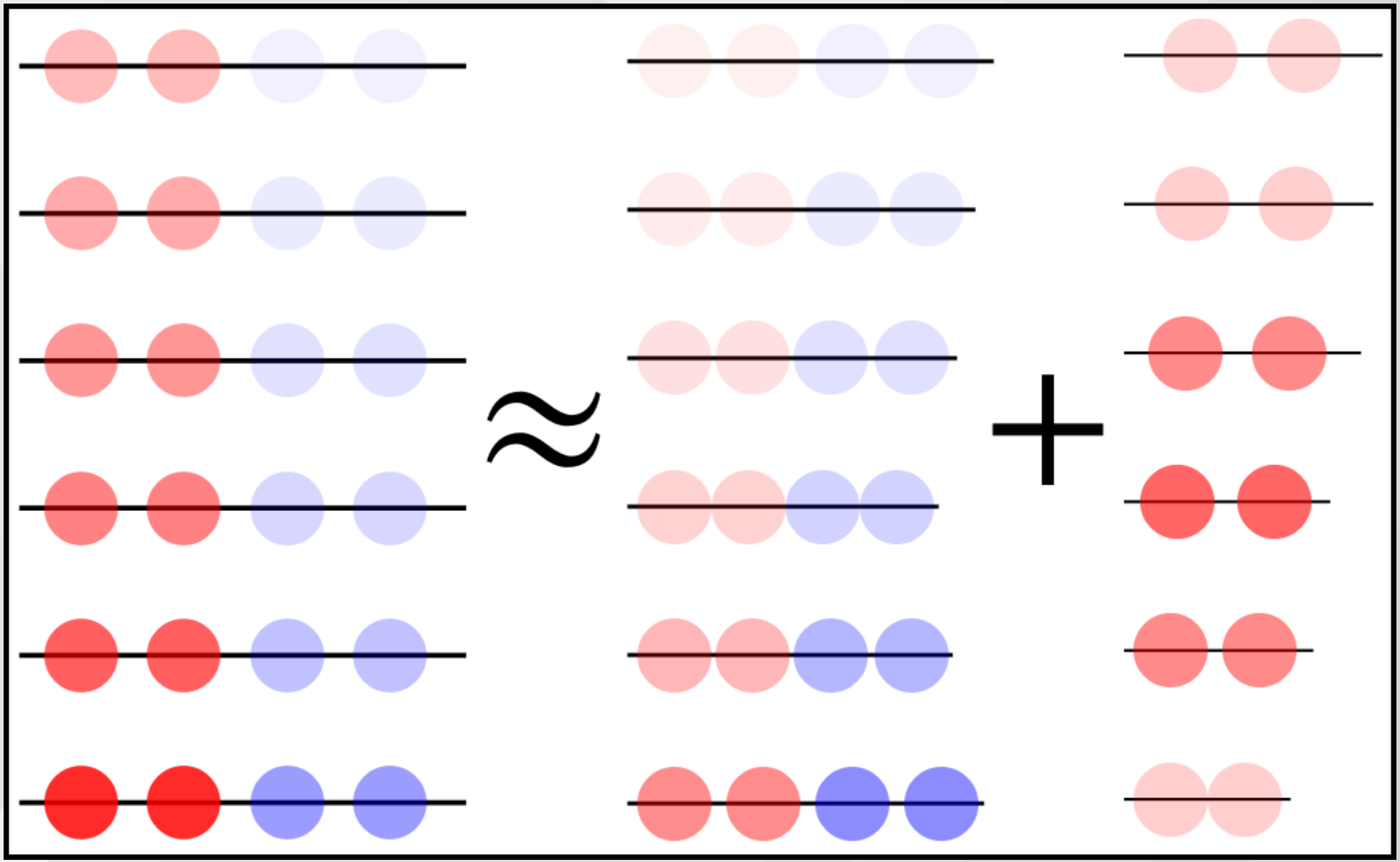
$$\mathcal{H}_{ADC}(x,y) = \mathcal{H}^{(QCM)}_{n_q}[x;\Omega^{(q)}(x,y)] + \mathcal{H}^{(PBCS)}_{n_p}[y;\Omega^{(p)}(x,y)]$$

with the Pauli blocking self-consistent conditions

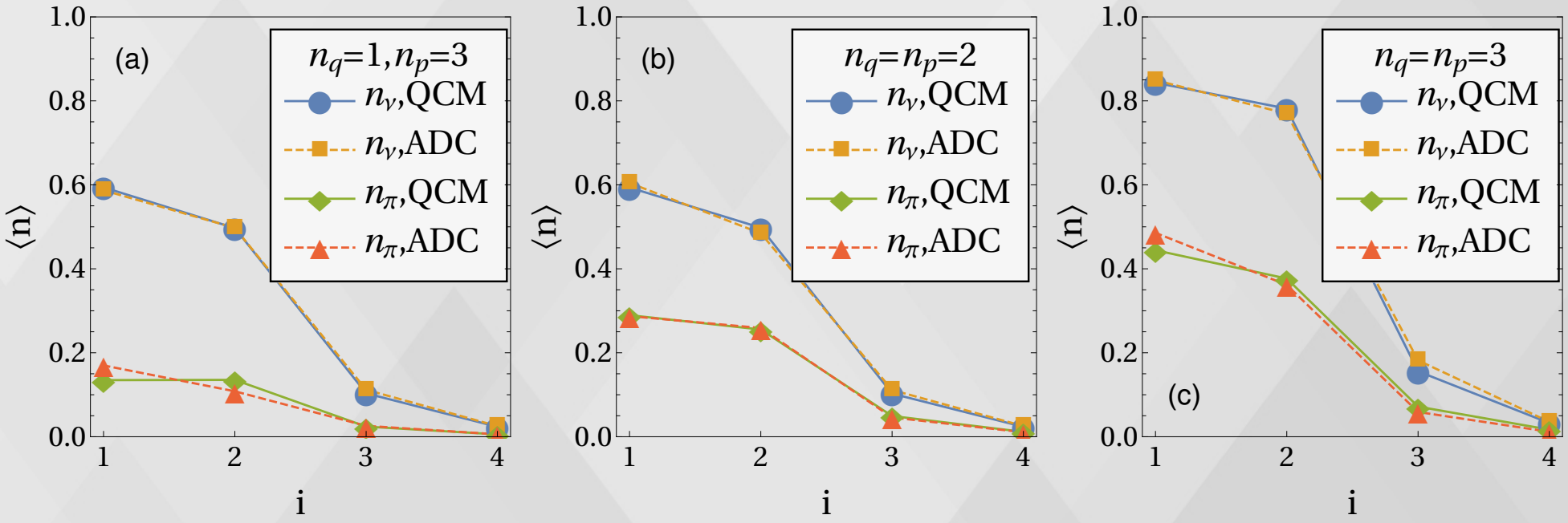
$$\Omega_i^{(q)} = 1 - \langle n_i \rangle^{(PBCS)} \ ,$$
$$\Omega_i^{(p)} = 1 - \langle n_i \rangle^{(QCM)} \ .$$

$\mathcal{H} = \langle H \rangle$ and $\langle n \rangle$ are defined by their analytical expressions which contain sums like $\sum_{i=1}^{N_{\rm lev}} \Omega_i x_i^4 y_i^2$.

ADC: opacity$\sim \langle n \rangle$, level line length $\sim \Omega_i$



Average level occupation fractions



- Average neutron and proton level occupations computed with the Bonn A isovector pairing interaction for the nuclei ^{110}Te (a),^{112}Xe (b) and ^{118}Ba (c).
- In the ADC approach, we identify the proton and neutron occupancies as $\langle n_{\pi,i}\rangle \equiv \langle n_i\rangle^{(QCM)}$ and, respectively, $\langle n_{\nu,i}\rangle \equiv \langle n_i\rangle^{(QCM)} + \langle n_i\rangle^{(PBCS)}$.

Correlation energies, $N > Z$

Table: Correlation energies in the rigourous approach E_c^{ex} and in the separable case E_c^{ADC}, together with the distribution of the correlation energy among quartets and pairs and the quartet-pair correlation energy $E_c^{q-p} = E_c^{\text{ex}} - E_c^{ADC}$, versus the number of quartets and pairs, for the nuclei in the *sdg* shell above ^{100}Sn with the Bonn interaction. All energies are expressed in MeV.

Nucleus	n_q	n_p	E_c^{ex}	E_c^{ADC}	E_c^q	E_c^p	E_c^{q-p}	$\frac{E_c^{q-p}}{n_q \cdot n_p}$
^{108}Te	1	2	5.96	5.37	2.81	2.56	0.59	0.295
^{110}Te	1	3	6.64	5.77	2.29	3.48	0.87	0.29
^{116}Te	1	6	5.27	3.92	1.19	2.73	1.35	0.23
^{112}Xe	2	2	8.24	7.10	4.68	2.42	1.14	0.285
^{114}Xe	2	3	8.40	6.73	3.67	3.06	1.67	0.28
^{118}Ba	3	3	9.10	6.72	4.16	2.56	2.28	0.26
^{130}Sm	6	3	9.22	7.73	5.37	2.36	1.49	0.08

Disentangling the quartet condensates

Isovector pairing in a degenerate shell with $N = Z$:

$$\begin{aligned} E_{\text{deg}}^{q-q} &= E_{\text{deg}}(n_q, 0; \Omega) - n_q E_{\text{deg}}[1, 0; \Omega - (n_q - 1)] \\ &= 0 \ . \end{aligned}$$

The quartets are actually *noninteracting* in the sense of the ADC, as they only feel each other's presence through Pauli blocking (for the considered schematic model).

Correlation energies, $N = Z$

- Energy function $\mathcal{H}_{ADC}(x) = n_q \mathcal{H}^{(QCM)}_{n_q=1}[x; \Omega(x)]$
- Effective degeneracies $\Omega_i = 1 - (n_q - 1)\langle n_i \rangle^{(n_q=1)}$

Table: Same as before, but for symmetric systems made out of just quartets, with no excess neutron pairs: correlation energies in the rigourous approach E_c^{ex} and in the separable case E_c^{ADC}, together with the quartet-quartet correlation energy $E_c^{q-q} = E_c^{\text{ex}} - E_c^{ADC}$, versus the number of quartets, for nuclei above ^{100}Sn.

n_q		E_c^{ex}	E_c^{ADC}	E_c^{q-q}	$\frac{E_c^{q-q}}{n_q(n_q-1)/2}$
2	^{108}Xe	6.73	6.65	0.08	0.08
3	^{112}Ba	8.63	8.41	0.22	0.07
4	^{116}Ce	10.33	9.90	0.43	0.07
5	^{120}Nd	10.99	10.30	0.69	0.07
6	^{124}Sm	10.52	9.57	0.95	0.06

Conclusions

- The basic philosophy of the ADC may be extended beyond its current application to paired systems.
- The approach can be applied to any many-body system which exhibits correlated structures that may be treated as quasi-elementary degrees of freedom, by considering the analytical continuation of the basic operator algebra to fractional degeneracies.
- Real space ADC implementation for clusters?

Thank you!