Theoretical Investigation of α-like Quasimolecules in Heavy Nuclei

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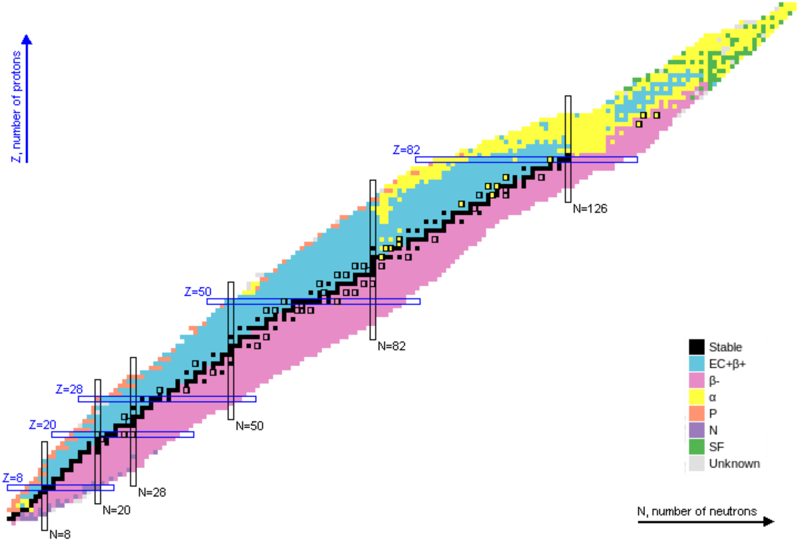
Summary of our study

- an α-nucleus quasimolecular potential can be constructed from experimental decay widths and a realistic α-daughter interaction given by independent scattering data.
- the parameters of this potential allow one to predict the position of excited resonant states, which can in principle be detected in excitation response functions and prove the existence of α-molecules in heavy nuclei.

α-decay phenomenology I



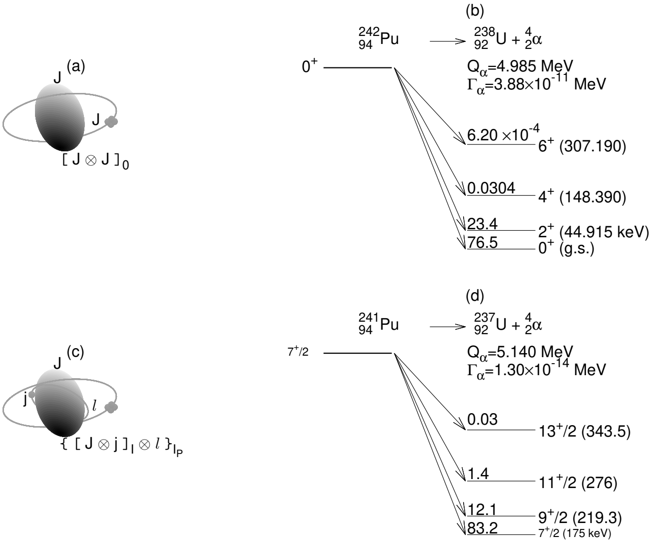
α-decay phenomenology[1] II

g.s. $\to$ g.s.	transitions
even-even	149
even-odd	72
odd-even	67
odd-odd	50
total	338

g.s. $\to$ ex.s.	transitions
even-even	238
odd-mass favored	130
odd-mass unfavored	333
total	701

[1] Data from www.nndc.bnl.gov/ensdf/. For analysis, see D.S. Delion and A. Dumitrescu, Phys. Rev. C **92** (2), 021303 (2015).

α-decay phenomenology[2] III



[2] D.S. Delion, Zhongzhou Ren, A.D., Dongdong Ni, J. Phys. G **45**, 5 (2018).

α-decay phenomenology IV

- The α-decay process

$$P\left(I_P\right) \to D(I) + \alpha(\ell)$$

can be described by a separable wave function depending on the degrees of the freedom of the daughter nucleus ξ_D and the relative distance $\mathbf{R}$ between the fragments

$$\Psi_{I_P,M_P}(\xi_D,\mathbf{R}) = \sum_{c=1}^{N} \frac{\psi_c(R)}{R} \mathcal{Y}_c(\xi_D,\hat{R})$$

- The core-angular harmonic describes the angular relative motion of the fragments

$$\mathcal{Y}_c(\xi_D,\hat{R}) = \left[\Phi_I(\xi_D) \otimes Y_\ell(\hat{R})\right]_{I_P,M_P}$$

α-decay phenomenology V

- The α–daughter dynamics is governed by a stationary Schrödinger equation[3]

$$H\Psi_{I_P,M_P}(\xi_D,\mathbf{R}) = Q_\alpha \Psi_{I_P,M_P}(\xi_D,\mathbf{R})$$

with a Hamiltonian containing three components: the kinetic energy, the structure term of the daughter nucleus and the α–daughter interaction

$$H = -\frac{\hbar^2}{2\mu}\nabla^2_{\mathbf{R}} + H_D(\xi_D) + V(\xi_D,\mathbf{R})$$

- The interaction consists of a spherically-symmetric term and a deformed component

$$V(\xi_D,\mathbf{R}) = V_0(R) + V_d(\xi_D,\mathbf{R})$$

[3] due to the fact that $T_\alpha \in \left[1\mu s, 10^9 y\right]$.

α-decay phenomenology VI

The coupled-channels equation

$$\frac{d^2\psi_c(R)}{d\rho_c^2} = \sum_{c'=1}^{N} A_{cc'}(R)\psi_{c'}(R), \;\; c = 1, ..., N \;,$$

is given in terms of the coupling matrix

$$\begin{aligned} A_{cc'}(R) &= \left[\frac{L_c(L_c+1)}{\rho_c^2} + \frac{V_0(R)}{Q_\alpha - E_c} - 1\right]\delta_{cc'} \\ &+ \frac{\langle \mathcal{Y}_c | V_d(\xi_D, \mathbf{R}) | \mathcal{Y}_{c'} \rangle}{Q_\alpha - E_c}, \end{aligned}$$

and radial parameters

$$\rho_c = \kappa_c R, \qquad \kappa_c = \sqrt{\frac{2\mu(Q_\alpha - E_c)}{\hbar^2}}.$$

The Double Folding Method[4]

- The interaction is estimated by integrating a nucleon-nucleon force ν over the densities of the two fragments $\rho_{D,\alpha}$:

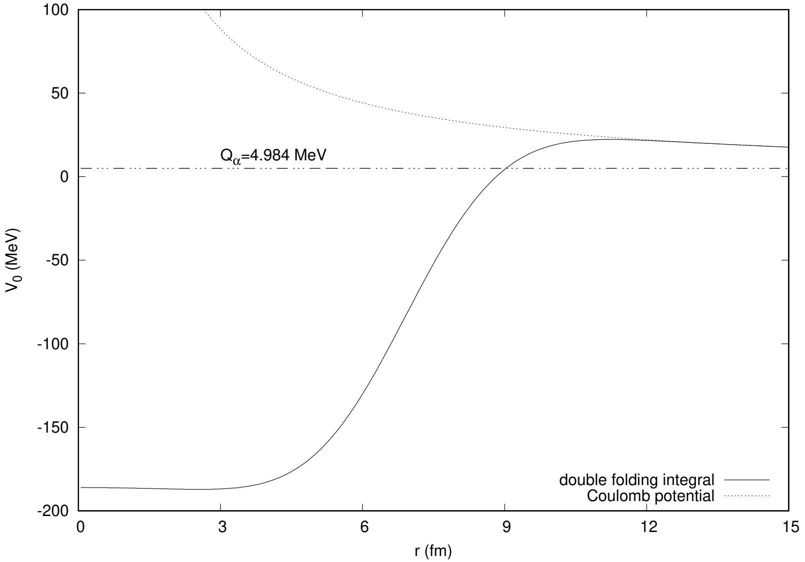
$$V\left(\xi_D, \mathbf{R}\right) = \int d\mathbf{r}_D \int d\mathbf{r}_\alpha \rho_D\left(\mathbf{r}_D\right) \rho_\alpha\left(\mathbf{r}_\alpha\right) \nu\left(\mathbf{R} + \mathbf{r}_D - \mathbf{r}_\alpha\right)$$

- The deformed component follows from a multipole-multipole expansion of the nuclear densities:

$$V_d\left(\xi_D, \mathbf{R}\right) = \sum_{\lambda>0} V_\lambda\left(R\right) \mathcal{Y}_\lambda\left(\xi_D, \xi_\alpha\right)$$

[4] G. Bertsch *et al.*, Nucl. Phys. A **284**, 399 (1977).
F. Cârstoiu, R. J. Lombard, Ann. Phys. **217**, 279 (1992).

The M3Y Double Folding Potential for ${}^{242}_{94}\mathrm{Pu} \rightarrow {}^{238}_{92}\mathrm{U} + {}^{4}_{2}\alpha$



The Monopole Component

$$
\begin{aligned}
V_0(R) &= \overline{V}_0(R),\ \ R > R_m \\
&= v_0 + \frac{1}{2}\hbar\omega_0\beta_0\left(R - R_{min}\right)^2,\ \ R < R_m
\end{aligned}
$$

- $\overline{V}_0$ is obtained through the double folding integration of the nucleon-nucleon M3Y plus Coulomb force.
- β_0 is the harmonic oscillator parameter for the monopole component.
- v_0 is the minimum of the oscillator potential; it is determined from the matching condition.
- ω_0 is the oscillator frequency, satisfying $Q_\alpha - v_0 \sim \frac{1}{2}\hbar\omega_0$.

The Wave Functions

$$\psi_c^{(ext)}(R) = \sum_{a=1}^{N} \mathcal{H}_{ca}^{+}(R)\sqrt{\frac{\Gamma_a}{\hbar v_a}}, \; R > R_m$$

$$\psi_c^{(int)}(X) = A_c\sqrt{\frac{1}{N}\sqrt{\frac{\beta_c}{\pi}}}e^{-\beta_c\frac{X^2}{2}}, \; X = R - R_{min}, \; R < R_m$$

- $\mathcal{H}_{ca}^{+}(R)$ are outgoing Coulomb-Hankel asymptotics.
- Γ_a & v_a are the decay width and particle velocity of channel a.
- β_c and A_c are the channel harmonic oscillator parameter and amplitude.
- for the monopole channel:

$$\beta_0 = \frac{1}{b_0^2} = \frac{m_\alpha \omega_0}{\hbar} = f\hbar\omega_0$$

$$f = \frac{m_\alpha c^2}{(\hbar c^2)} = 0.096\mathrm{MeV}^{-1}\mathrm{fm}^{-2}$$

Spectroscopic Factors

- Due to the external components being much smaller than the internal ones in the internal region $R \in [0, R_m]$, one has

$$S_\alpha = \int |\Psi(\mathbf{R})|^2 d\mathbf{R} \approx \sum_c A_c^2 = \sum_c S_c$$

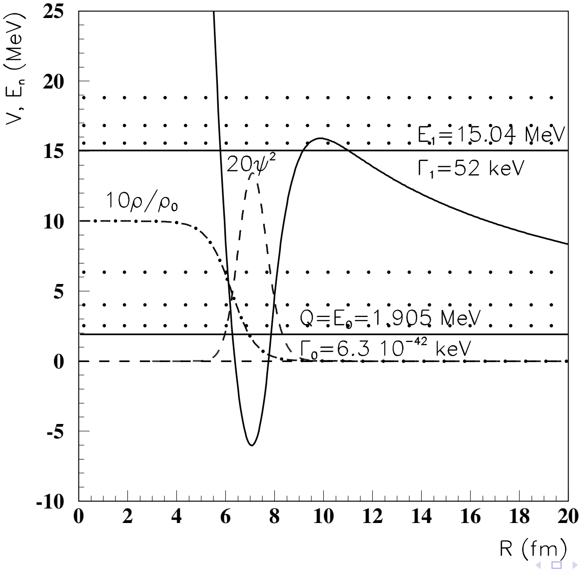
Matching Conditions

- The coefficients v_0, β_0 and R_{min} are obtained from the matching relations:

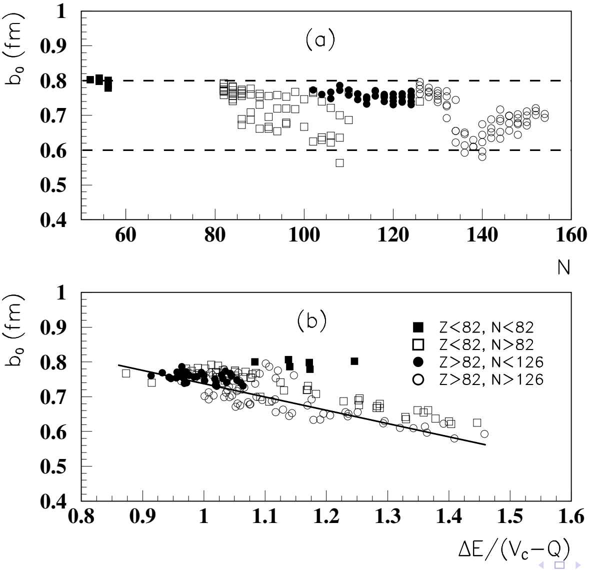
$$\begin{aligned} v_0 + \frac{1}{2f}\beta_0^2 X_m^2 &= \bar{V}_0(R_m)\ , \quad X_m = R_m - R_{min} \\ \frac{1}{f}\beta_0^2 X_m &= \bar{V}_0'(R_m) \\ \ln'\psi_0^{(int)}(X_m) &= -\beta_0 X_m = \ln'\psi_0^{(ext)}(R_m). \end{aligned}$$

- With Q_α fixed from the experiment and $R_m = R_{int}$, one ensures the existence of a narrow resonance corresponding to the first eigenvalue of the harmonic oscillator pocket.

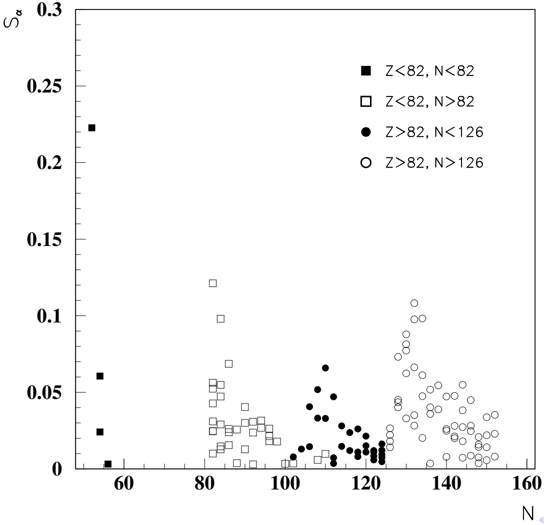
The $\alpha - Ce^{140}$ molecular potential



Systematics of the monopole length parameter



Systematics of the spectroscopic factor S_α versus neutron number



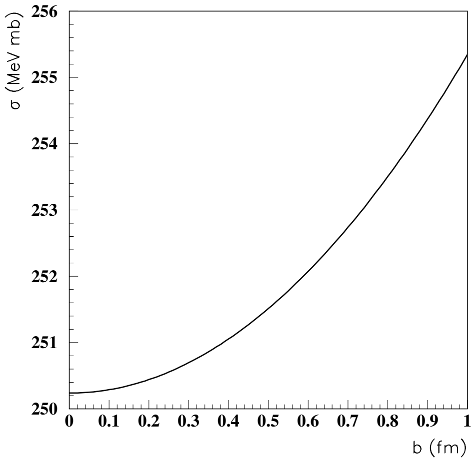
α photoabsorbtion[5]

- Energy-integrated cross section for the dipole excitation of a harmonic oscillator:

$$
\begin{aligned}
\sigma &= \sigma_0 \left(1 + \frac{b^2}{R_{min}^2}\right) \\
\sigma_0 &= \frac{8\pi^3}{3} \frac{\hbar c}{m_\alpha c^2}\, e_\alpha^2 = 252.123 \text{ MeV mb}
\end{aligned}
$$

[5] V.V. Baran and D.S. Delion, J. Phys. G **45**, 035106 (2018).

Absorption energy-integrated cross section versus the monopole length parameter for $R_{min} = 7\ \mathrm{fm}$



Conclusions[6]

- The shape of the α-clusters on the nuclear surface can be determined in a stable way by using decay widths as input data. A molecular potential was set up, with the equilibrium radius slightly larger than the Mott transition point from nucleonic to the α-cluster phase in finite nuclei.
- The first excited vibrational resonant state is close to the Coulomb barrier for nuclei with $b_0 > 0.75\ \mathrm{fm}$ and its rotational band can in principle be evidenced as a structure of maxima in the α-particle scattering cross section. The associated ALAS phenomenon diminishes due to the hindrance of the α-exchange effects.
- The dipole excitation of this resonance by γ-rays in odd-mass emitters would provide additional proof for the existence of α molecules in heavy nuclei.

[6] D.S. Delion, A.D., V.V. Baran, Phys. Rev. C **97**, 6, 064303 (2018).

Thank you!