From nuclear clusters to composite hadrons

A unified equation of state from the cluster virial expansion
within the generalized Beth-Uhlenbeck approach

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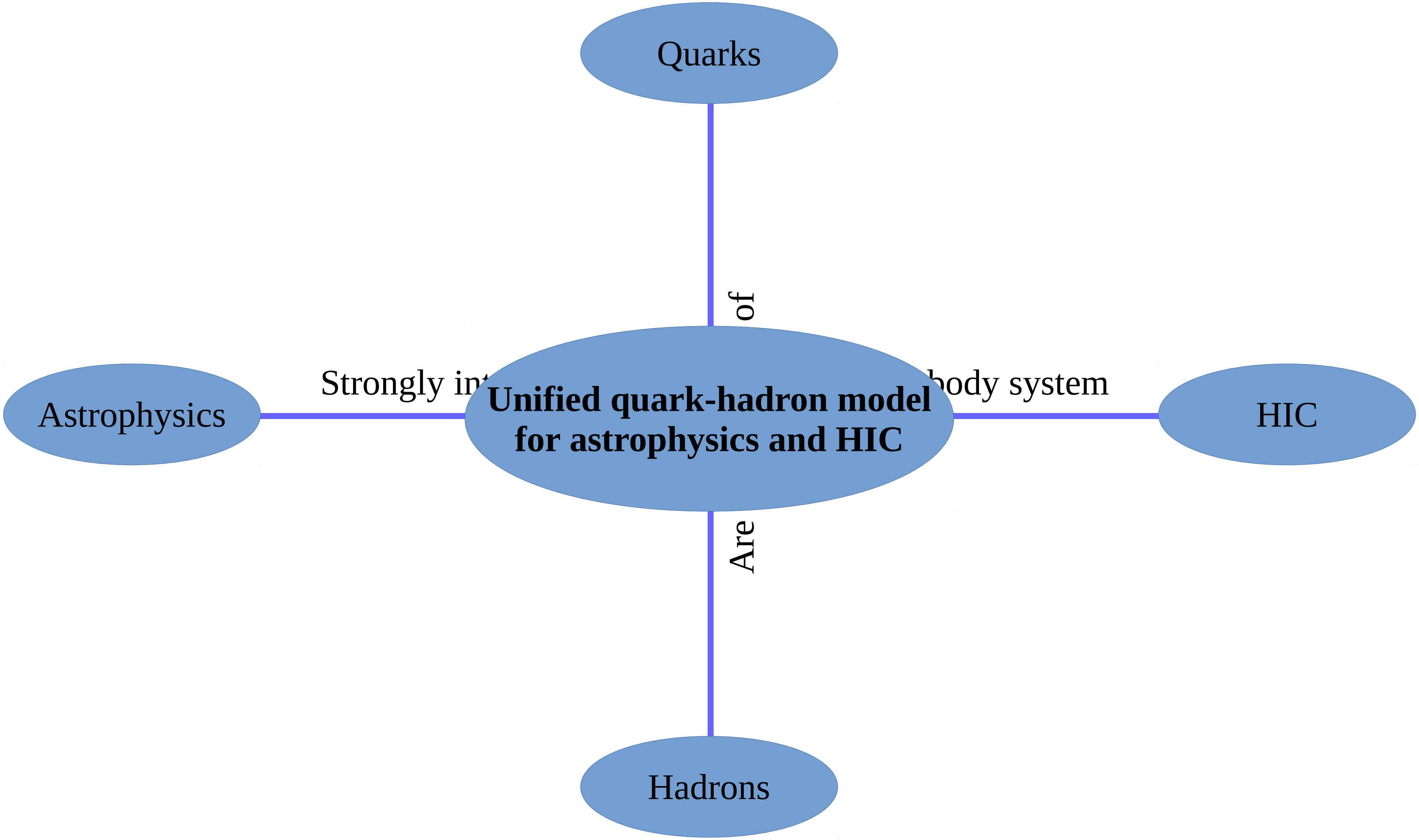
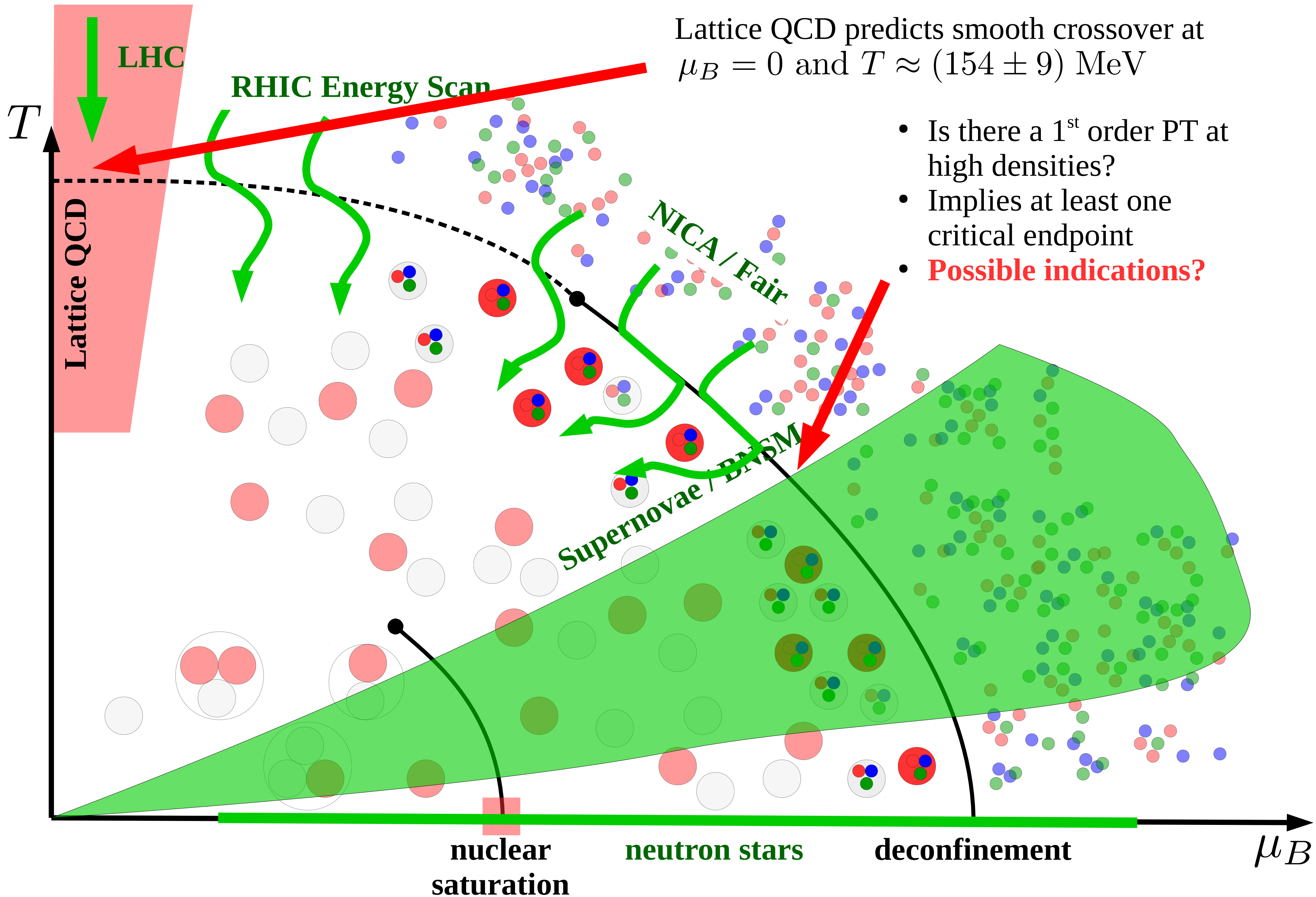
Unified equation of state



Possibility of 1st order PT at high densities

Lattice QCD predicts smooth crossover at $\mu_B = 0$ and $T \approx (154 \pm 9)$ MeV

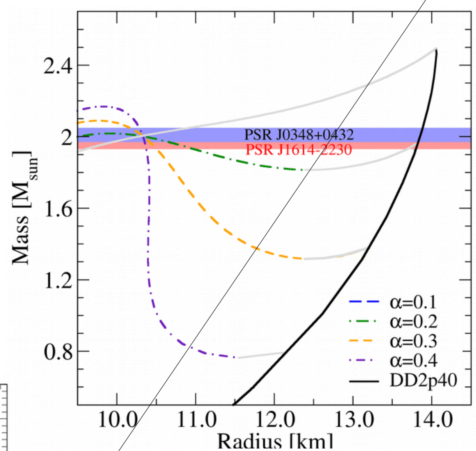
- Is there a 1st order PT at high densities?
- Implies at least one critical endpoint
- **Possible indications?**



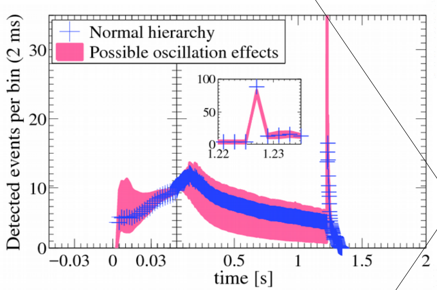
Outline

Possible signals of 1st – order phase transitions.

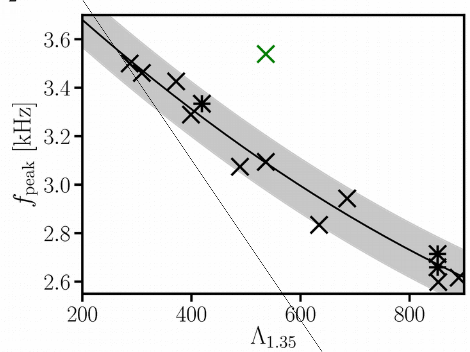
Neutron star configurations

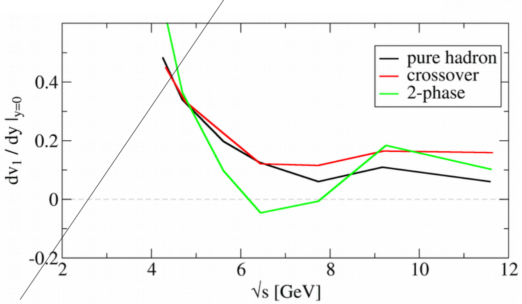


Supernova explosions of 50Ms stars



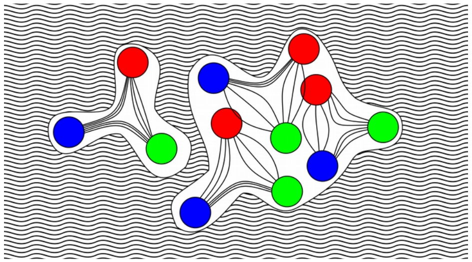
Binary neutron star mergers





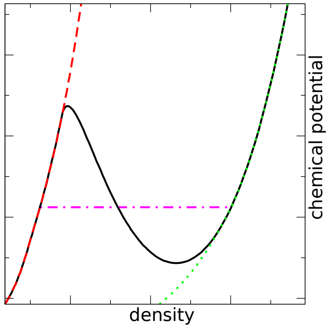
Heavy-Ion Collisions

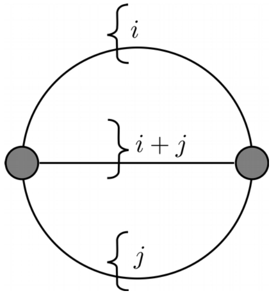
Unified description of the equation of state.



density functional theory

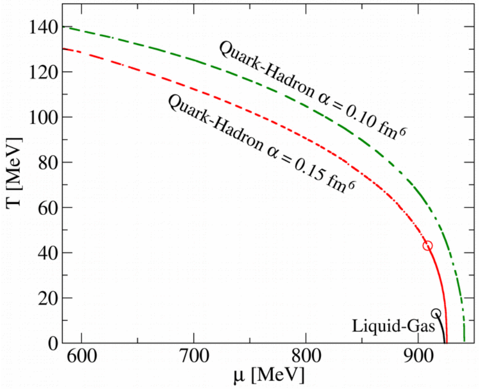
construction of phase transitions





Phi-derivable formalism

current status



Relativistic density functionals

Starting with free fermion Lagrangian plus an interaction term, which depends on quark currents

$$\mathcal{L}_{\text{eff}} = \underbrace{\bar{q}\left(\imath\gamma^\mu\partial_\mu - m\right)q}_{\mathcal{L}_{\text{free}}} - U(\bar{q}q, \bar{q}\gamma^\mu q)$$

Mean field → linear dependence of U on densities is important! → expansion around expectation values

$$U(\bar{q}q, \bar{q}\gamma^\mu q) = U(n_{\text{S}}, n_{\text{V}}) + \Sigma_{\text{S}}(\bar{q}q - n_{\text{S}}) + \Sigma_{\text{V}}(\bar{q}\gamma^\mu q - n_{\text{V}}) + \ldots$$

derivatives

$$\mathcal{L}_{\text{eff}} \approx \underbrace{\bar{q}\left(\gamma^\mu(\imath\partial_\mu - \Sigma_{\text{V}}) - (m + \Sigma_{\text{S}})\right)q}_{\mathcal{L}_{\text{quasi}}} - \Theta(n_{\text{S}}, n_{\text{V}})$$

$$P = g\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left[\ln(1 + e^{-\beta(\sqrt{p^2 - M^2} - \tilde{\mu})}) + \text{a.p.}\right] - \Theta$$

with

$$n_s = \langle\bar{q}q\rangle \;,\; n_v = \langle\bar{q}\gamma^0 q\rangle \qquad M = m + \Sigma_{\text{S}} \;,\; \tilde{\mu} = \mu - \Sigma_{\text{V}}$$

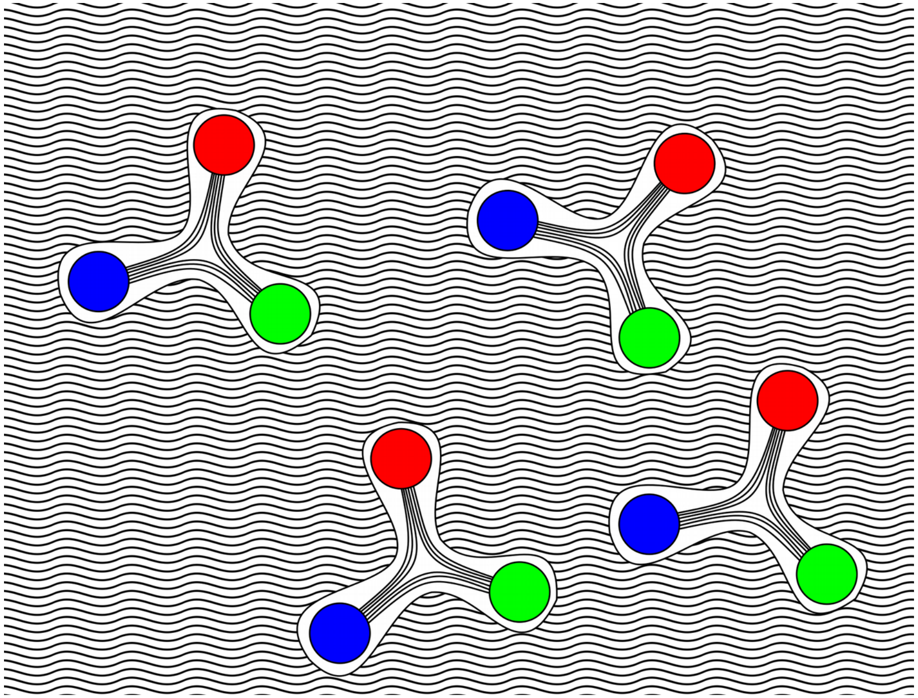
M. Kaltenborn, **NUFB**, D. Blaschke. Phys. Rev. D 2017, 96, 056024

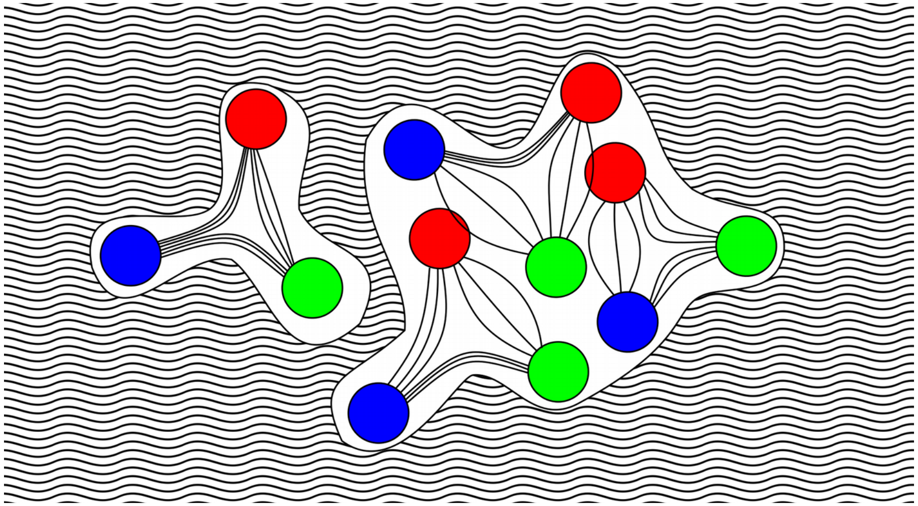
Density functional approach: Stringflip model

Low density

- Color field lines compressed by dual Meissner effect
- String-potential

$$V(r) = \sigma r \sim n^{-1/3}$$





High density

- Dual superconducting vacuum occupied by hadrons
- Pressure on field lines reduced
- Effective string-tension reduced

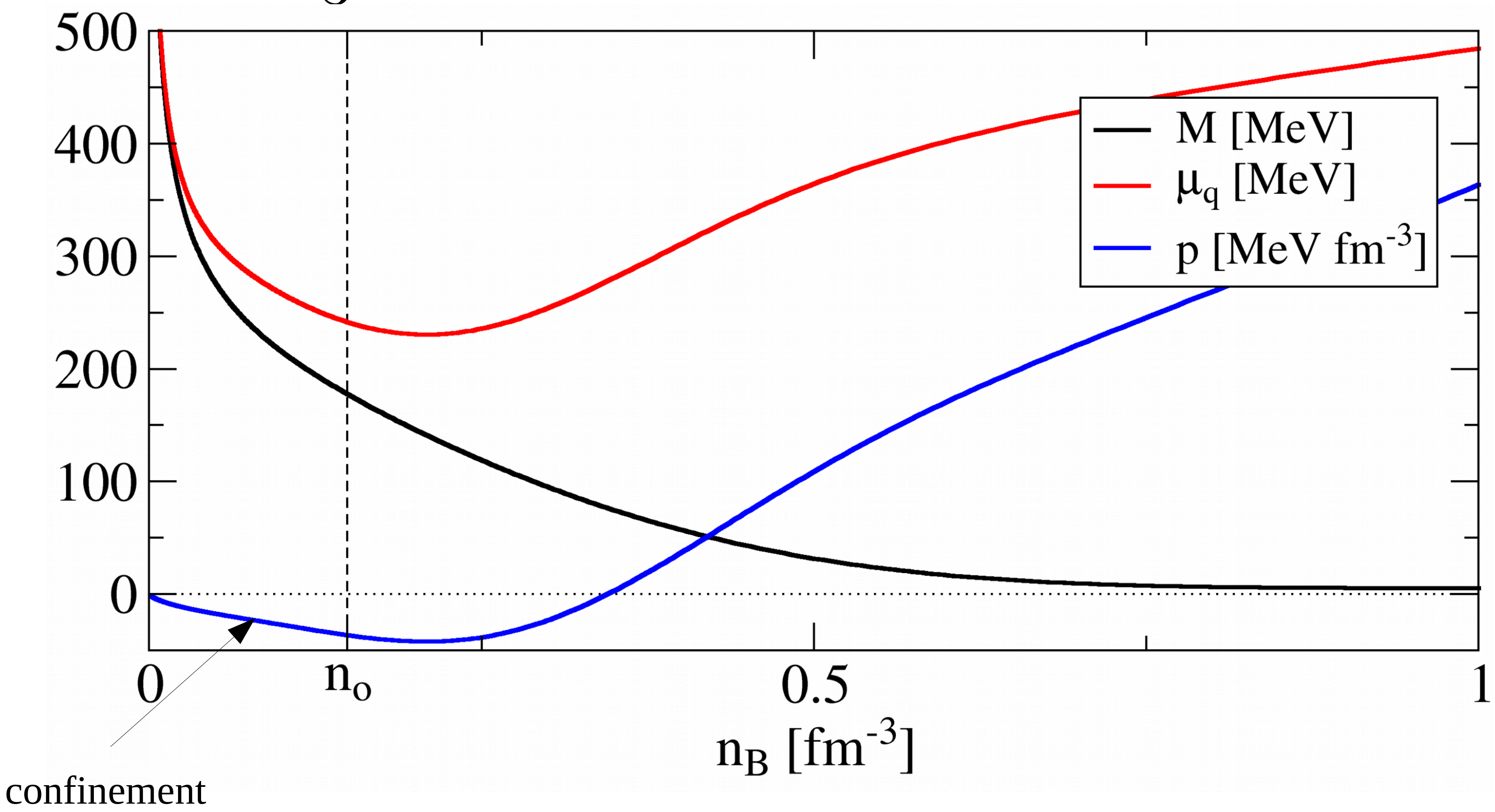
$$\sigma = \Phi \sigma_0$$

G. Ropke, et. al., Phys.Rev. D34 (1986) 3499-3513
M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

$$U^{\mathrm{SF}}(n_{\mathrm{S}}, n_{\mathrm{V}}) = D(n_{\mathrm{V}}) n_{\mathrm{S}}^{2/3}$$

Mean-field model

$$M_i = m_i + \frac{2}{3} D \cdot \left(n^{\mathrm{s}}\right)^{-1/3} \qquad D = D_0 \mathrm{e}^{-\alpha(n-n_0)^2}$$



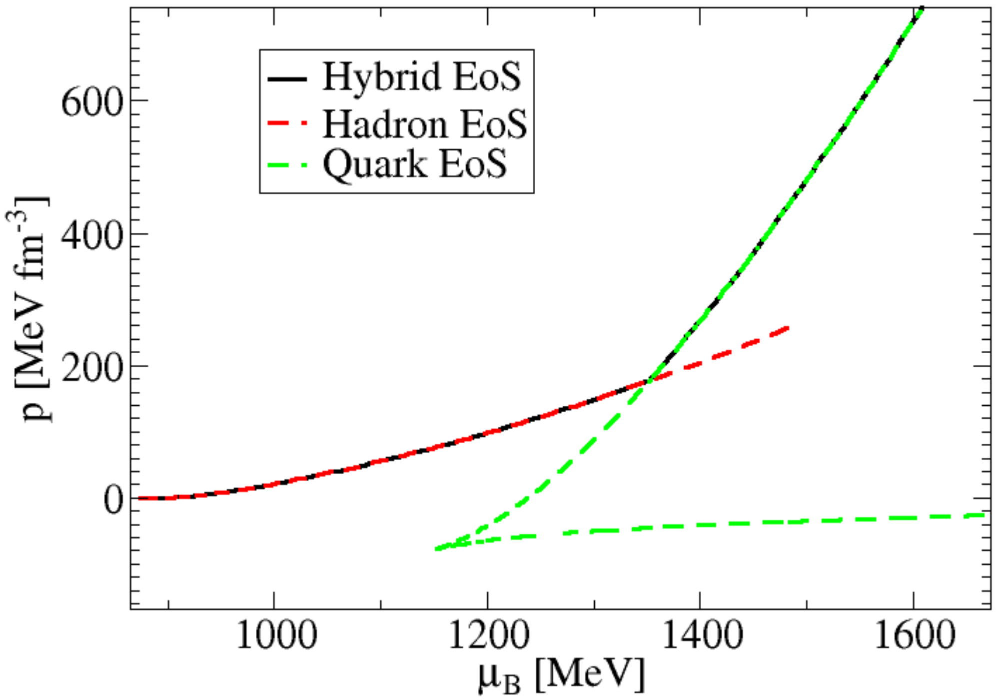
M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

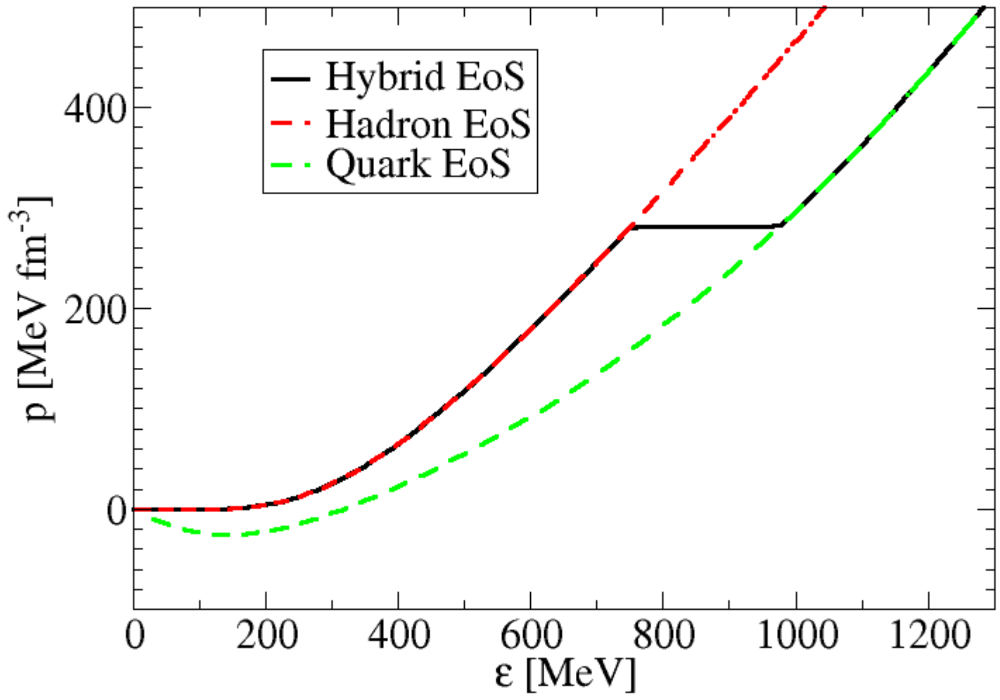
2-phase approach

old

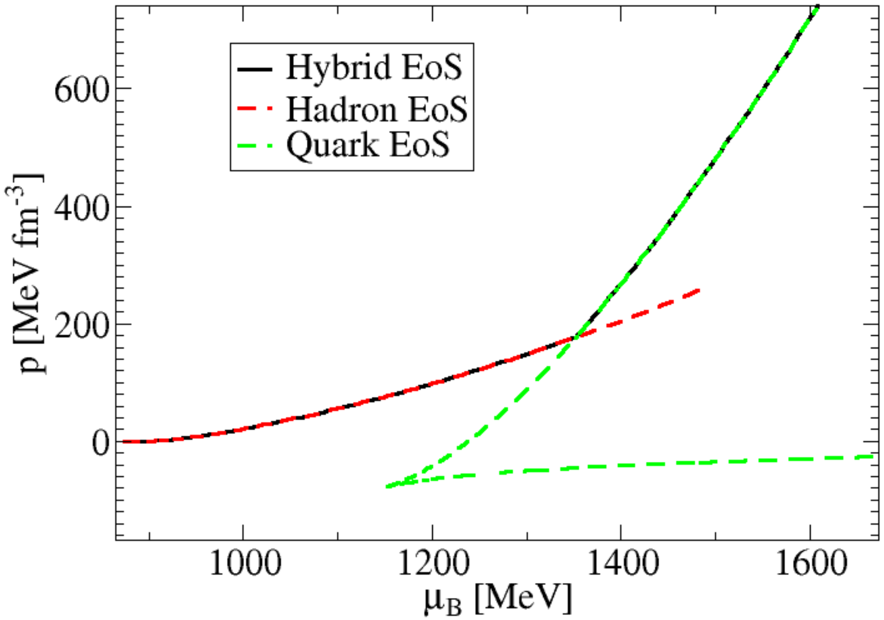
- Two independent models for hadrons and quarks
- Match while fulfilling Gibbs condition for thermal, mechanical and chemical phase equilibrium

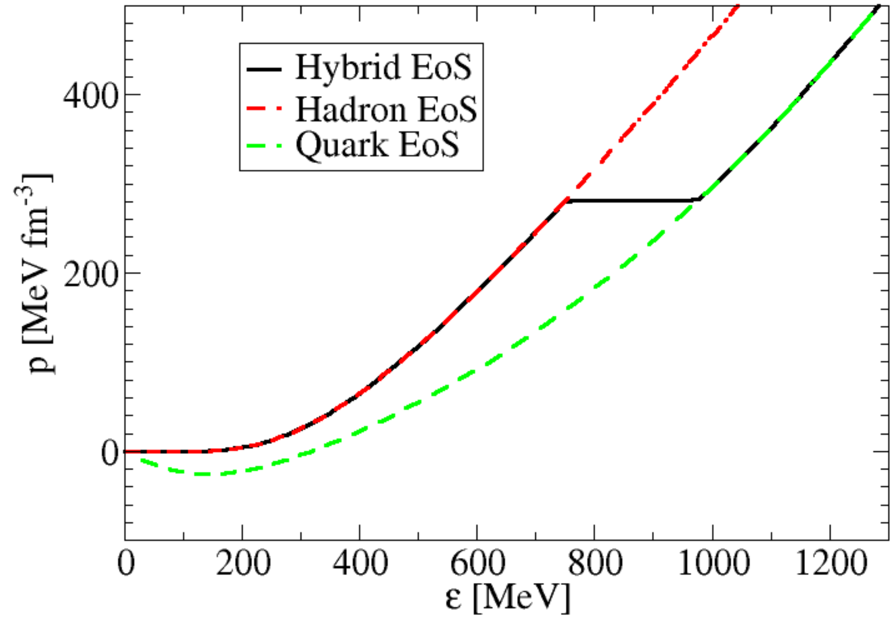
$$T^{\mathrm{H}} = T^{\mathrm{Q}} \quad p^{\mathrm{H}} = p^{\mathrm{Q}} \qquad \mu^{\mathrm{H}} = \mu^{\mathrm{Q}}$$

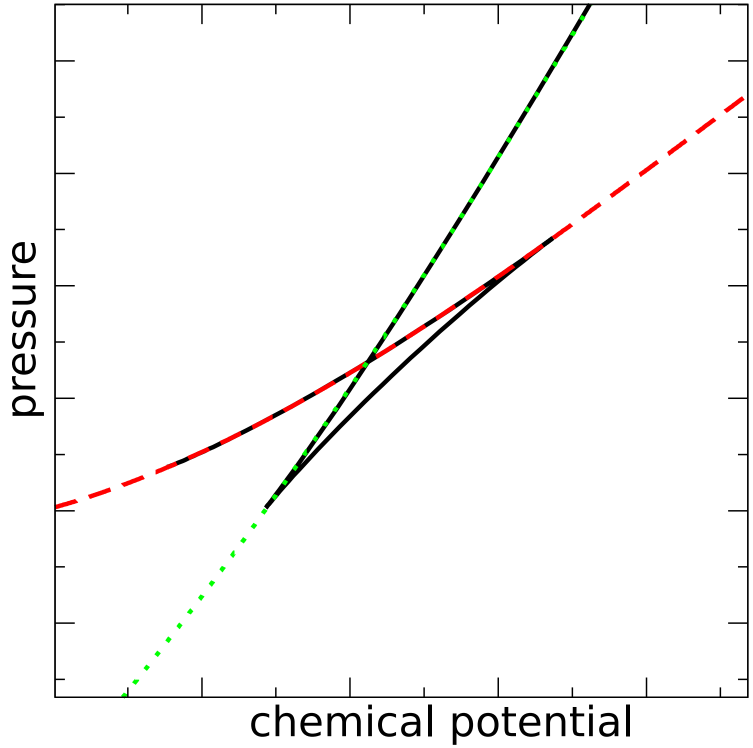


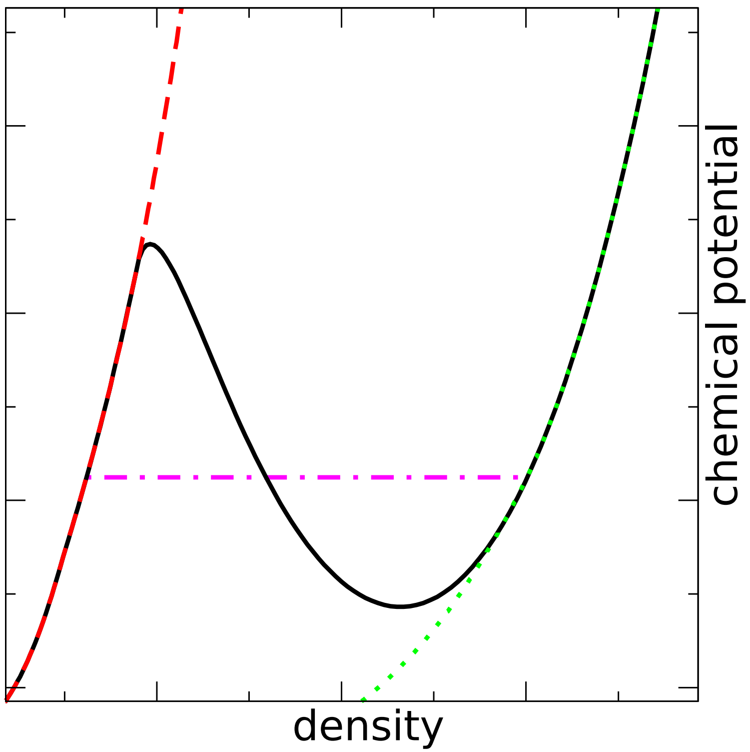


Two-phase approach vs van der Waals wiggle









Cluster expansion

Generating functional formalism by Baym and Kadanoff [1,2]

$$\Omega = -\mathrm{Tr}\ \ln(-G_1^{-1}) - \mathrm{Tr}\Sigma_1 G_1 + \Phi \qquad \text{With} \qquad \Sigma_1(1,1') = \frac{\delta\Phi}{\delta G_1(1,1')}.$$

Can be generalized for a consistent cluster expansion[3]

$$\Omega = \sum_{l=1}^{A} \Omega_l = \sum_{l=1}^{A} \left\{ c_l \left[\mathrm{Tr} \ln\left(-G_l^{-1}\right) + \mathrm{Tr}\left(\Sigma_l\ G_l\right)\right] + \sum_{\substack{i,j \\ i+j=l}} \Phi[G_i, G_j, G_{i+j}] \right\}$$

with

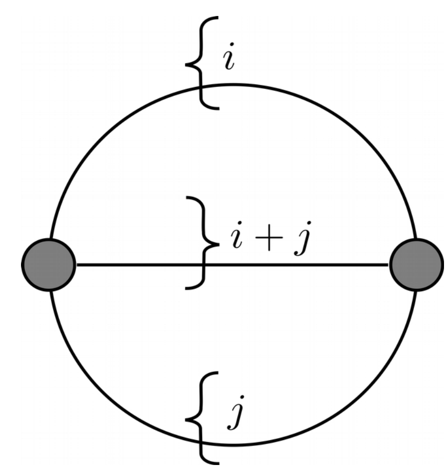
$$\Sigma_A(1\ldots A, 1'\ldots A', z_A) = \frac{\delta\Phi}{\delta G_A(1\ldots A, 1'\ldots A', z_A)}$$

Always sustains full Dyson equation and thermodynamic stability

$$G_A^{-1} = {G_A^0}^{-1} - \Sigma_A^{-1} \qquad\qquad \frac{\partial\Omega}{\partial G_A} = 0$$

Reduction on generalized sunset diagrams is recommended

$$\Phi[G_i, G_j, G_{i+j}] =$$

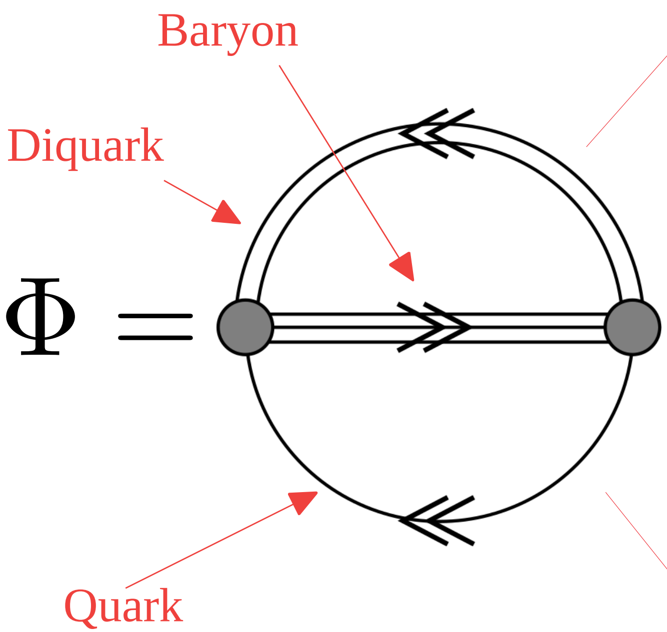


[1]Baym, G.; Kadanoff, L.P. Phys. Rev. 1961, 124, 287–299.
[2]Baym, G. Phys. Rev. 1962, 127, 1391–1401.
[3]**NUFB**, and others, Universe 2018, 4(6), 67

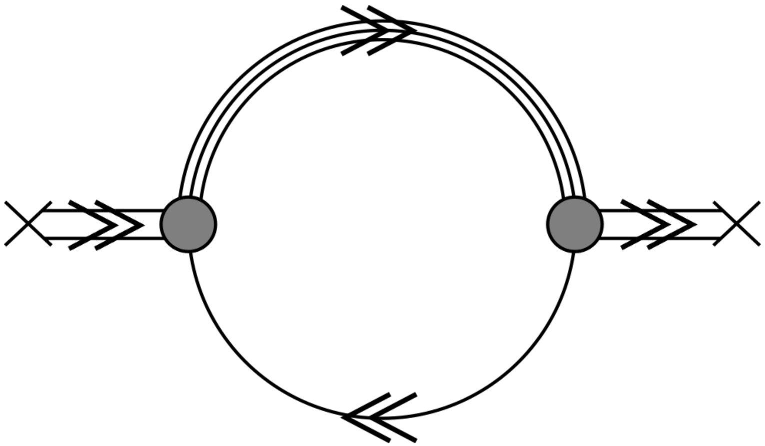
Self energy

$$\Sigma_A = \frac{\delta \Phi}{\delta G_A}$$

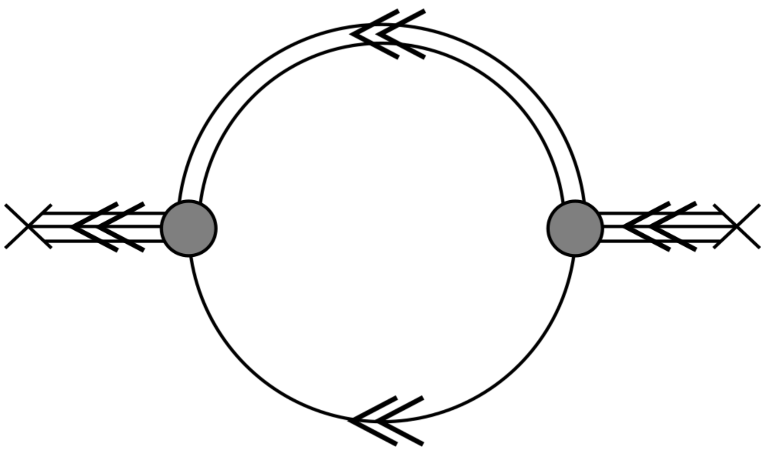
$$\Phi =$$



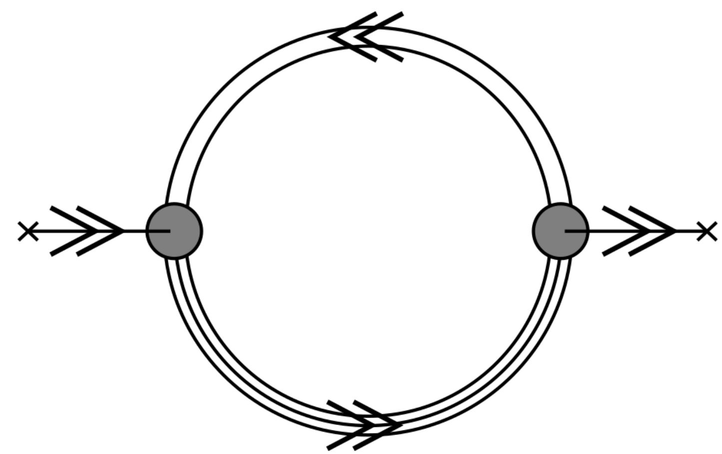
$$\Sigma_{\mathrm{D}} = \frac{\delta \Phi}{\delta G_{\mathrm{D}}} =$$



$$\Sigma_{\mathrm{B}} = \frac{\delta \Phi}{\delta G_{\mathrm{B}}} =$$



$$\Sigma_{\mathrm{Q}} = \frac{\delta \Phi}{\delta G_{\mathrm{Q}}} =$$



$$G_A^{-1} = {G_A^0}^{-1} - \Sigma_A^{-1}$$

Analogy to density functional approach

Phi-derivable approach

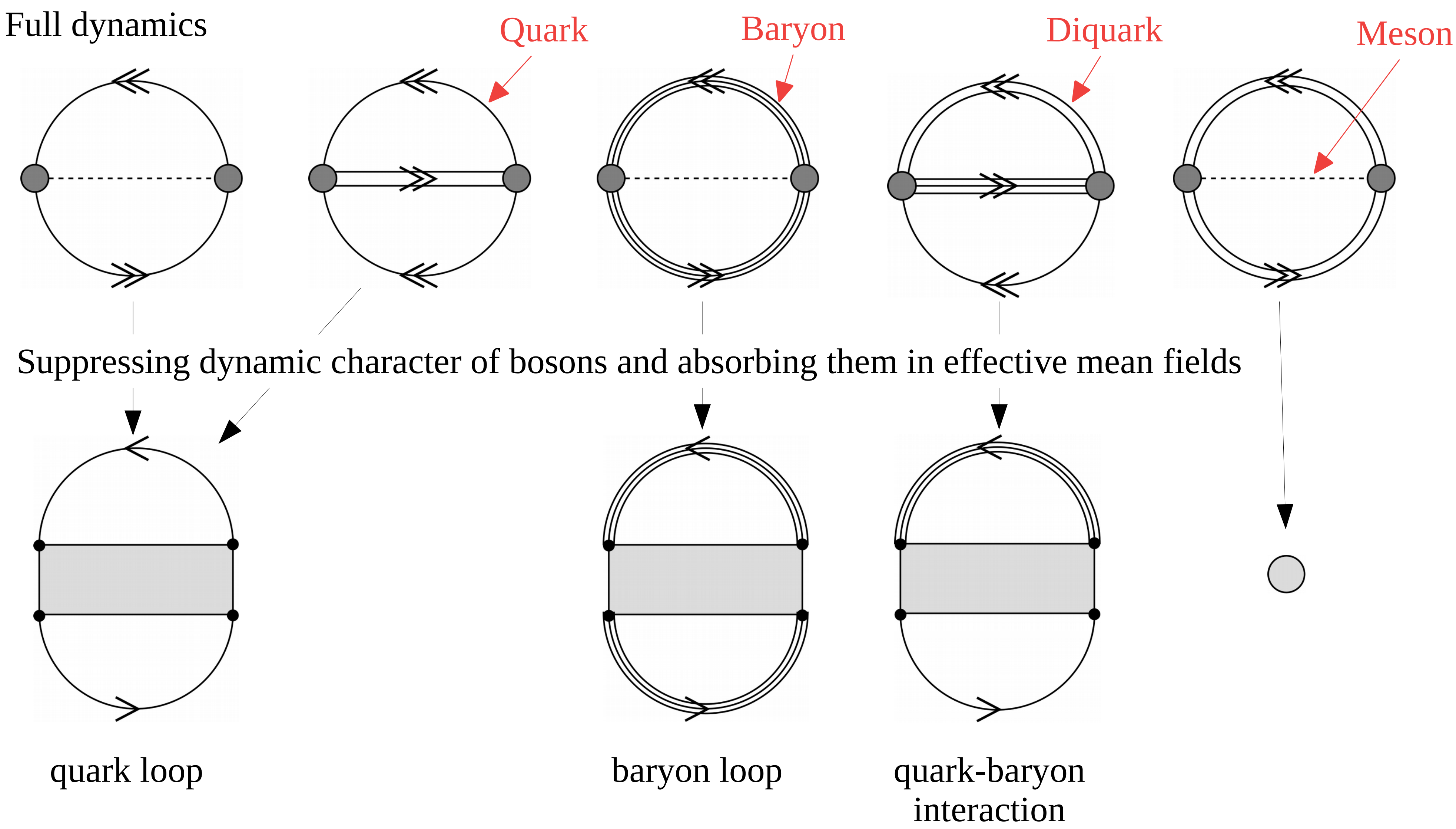
$$\Omega = -\mathrm{Tr}\ \ln(-G_1) - \mathrm{Tr}\Sigma_1 G_1 + \Phi[G_1]$$

Density functional approach

$$\Omega = \Omega^{\mathrm{quasi}} - n_{\mathrm{s}}\Sigma_{\mathrm{s}} - n_{\mathrm{v}}\Sigma_{\mathrm{v}} + U(n_{\mathrm{s}}, n_{\mathrm{v}})$$

First step: cluster expansion on basis of densities instead of Green functions (local limit)

The Quark-Diquark-Meson-Baryon Model



→ Real self energies → quasi particles

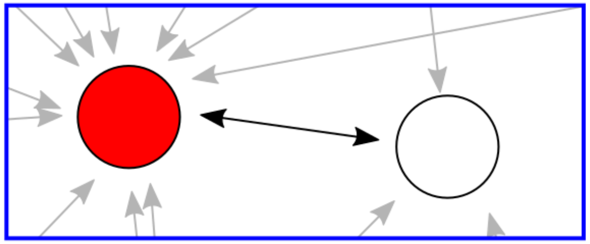
$$\delta := \arctan \frac{\mathrm{Im}\,\Sigma}{\mathrm{Re}\,\Sigma} = n\pi$$

Generalized Beth-Uhlenbeck

Cluster expansion

$$n_\mathrm{u} = n_\mathrm{u}^\mathrm{free} + 2n_\mathrm{p}^\mathrm{free} + 1n_\mathrm{n}^\mathrm{free}$$

$$n_\mathrm{d} = n_\mathrm{d}^\mathrm{free} + 1n_\mathrm{p}^\mathrm{free} + 2n_\mathrm{n}^\mathrm{free}$$



Chemical equilibrium

$$\mu_i = B_i \mu_\mathrm{B} + C_i \mu_\mathrm{C}$$

Generalized Beth-Uhlenbeck formula

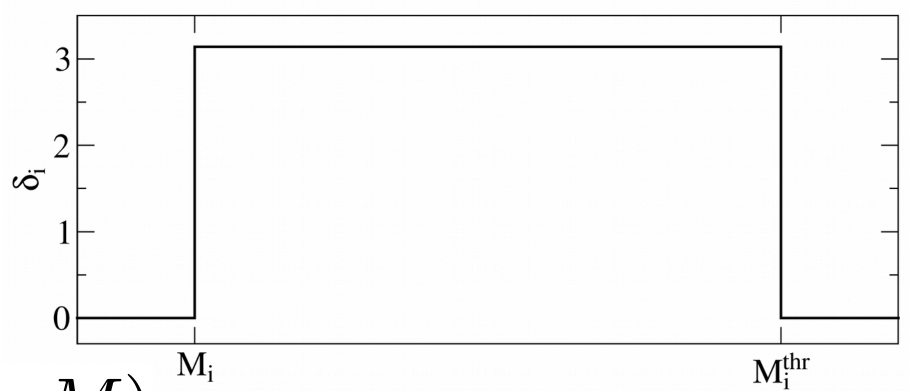
$$n_i^\mathrm{free} = g_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}E}{2\pi} f_i(E_i) 2\sin^2 \delta_i(E) \frac{\mathrm{d}\delta_i(E)}{\mathrm{d}E}$$

Substitution: $E_\mathrm{i} = \sqrt{p^2 + (m_i + S_i)^2} + V_i$

$$n_i^\mathrm{free} = g_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}M}{2\pi} f_i\left(\sqrt{p^2 + M^2} + V_i\right) 2\sin^2 \delta_i(M) \frac{\mathrm{d}\delta_i(M)}{\mathrm{d}M}$$

Analogy to density functional approach

$$n_i^{\mathrm{free}} = g_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}M}{2\pi} f_i\left(\sqrt{p^2+M^2}+V_i\right) 2\sin^2\delta_i(M) \frac{\mathrm{d}\delta_i(M)}{\mathrm{d}M}$$



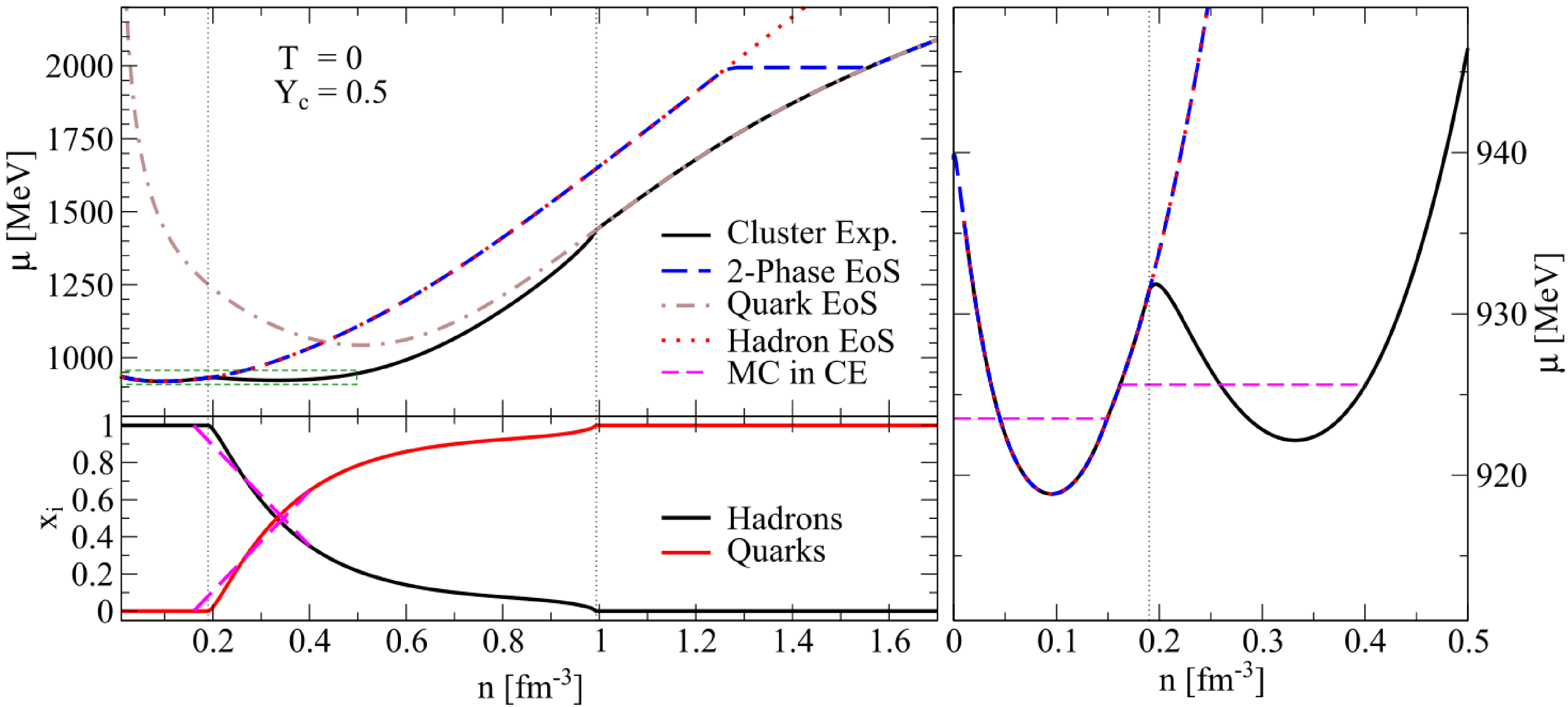
$$\delta_{i=\mathrm{u,d}}(M) = \pi\,\Theta(M-M_i)$$

$$\delta_{i=\mathrm{p,n}}(M) = \pi\,\Theta(M-M_i)\,\Theta(M_i^{\mathrm{th}}-M)$$

$$n_{i=\mathrm{p,n}} = g_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \Big[f_i(\sqrt{p^2+M_i^2}+V_i) - f_i(\sqrt{p^2+(M_i^{\mathrm{th}})^2}+V_i) \Big]\,\Theta(M_i^{\mathrm{thr}}-M_i)$$

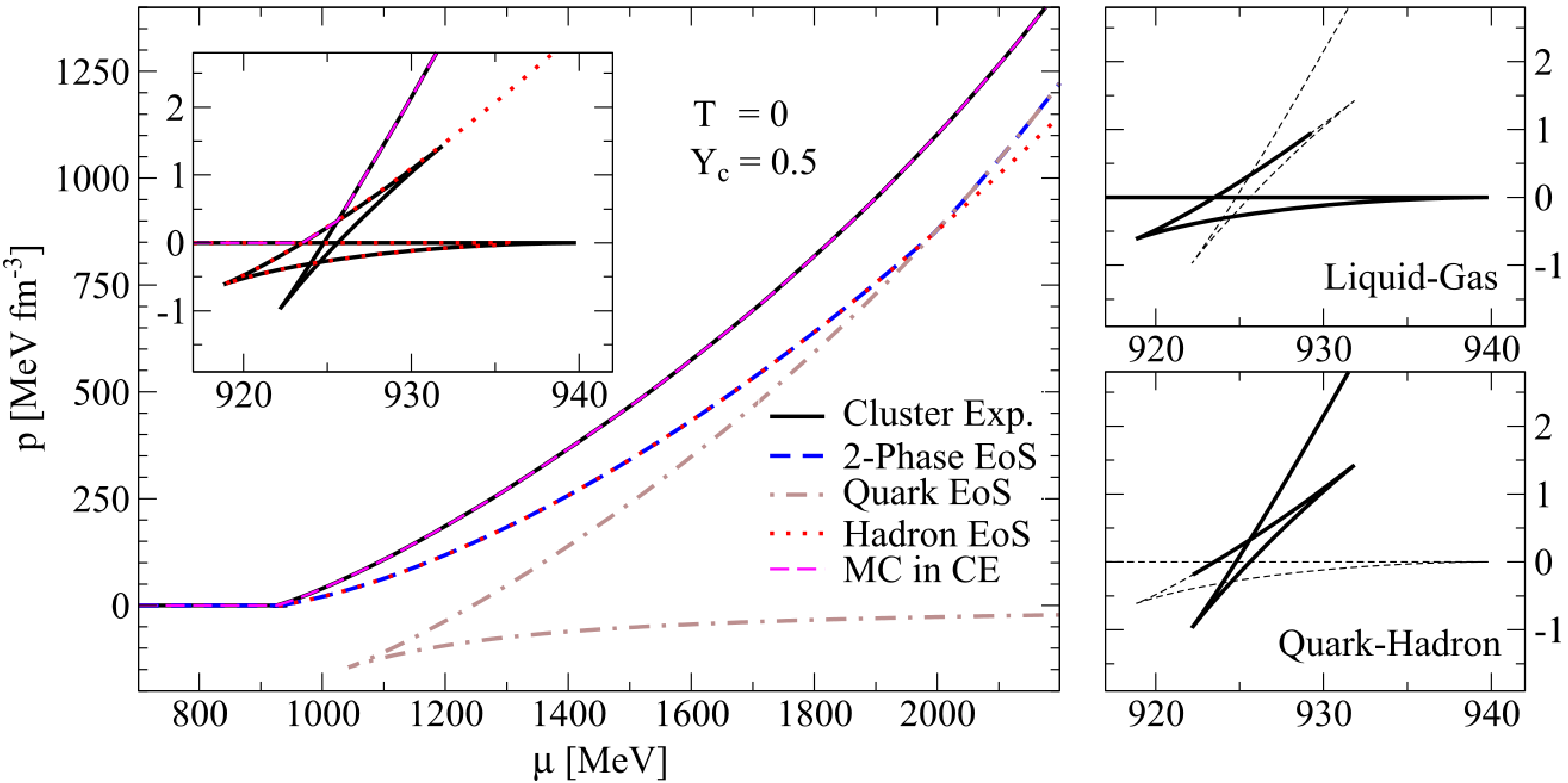
$$= \left(n_{\mathrm{N}}^{\mathrm{qu}} - n_{\mathrm{q}}^{\mathrm{thr}}\right)\Theta(M_i^{\mathrm{thr}}-M_i)$$

$$M_{\mathrm{p}}^{\mathrm{thr}} = 2M_{\mathrm{u}} + 1M_{\mathrm{d}}$$
$$M_{\mathrm{n}}^{\mathrm{thr}} = 1M_{\mathrm{u}} + 2M_{\mathrm{d}}$$



NUFB, D. Blaschke, arXiv:1812.11766

Cluster-expansion of Quarks



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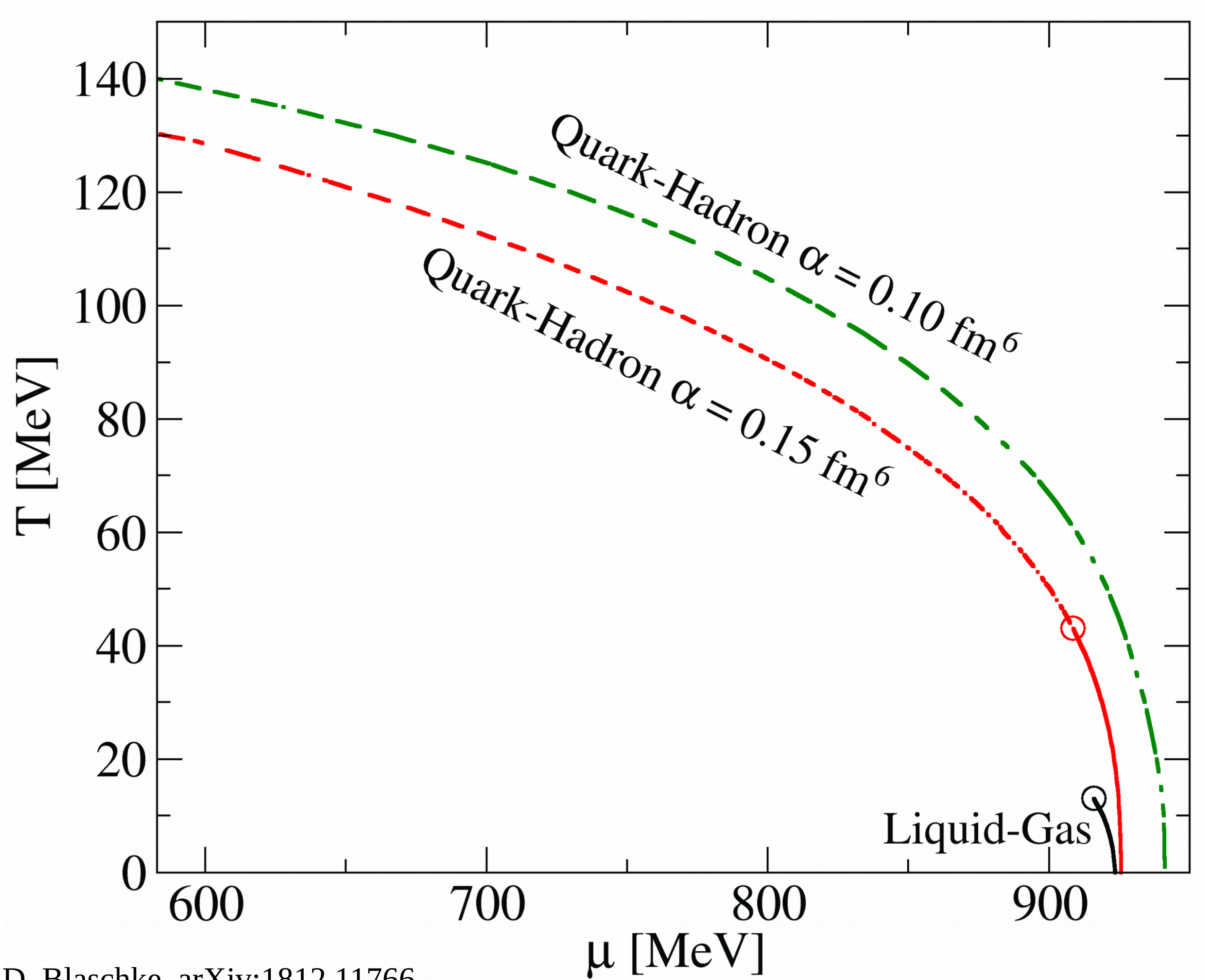

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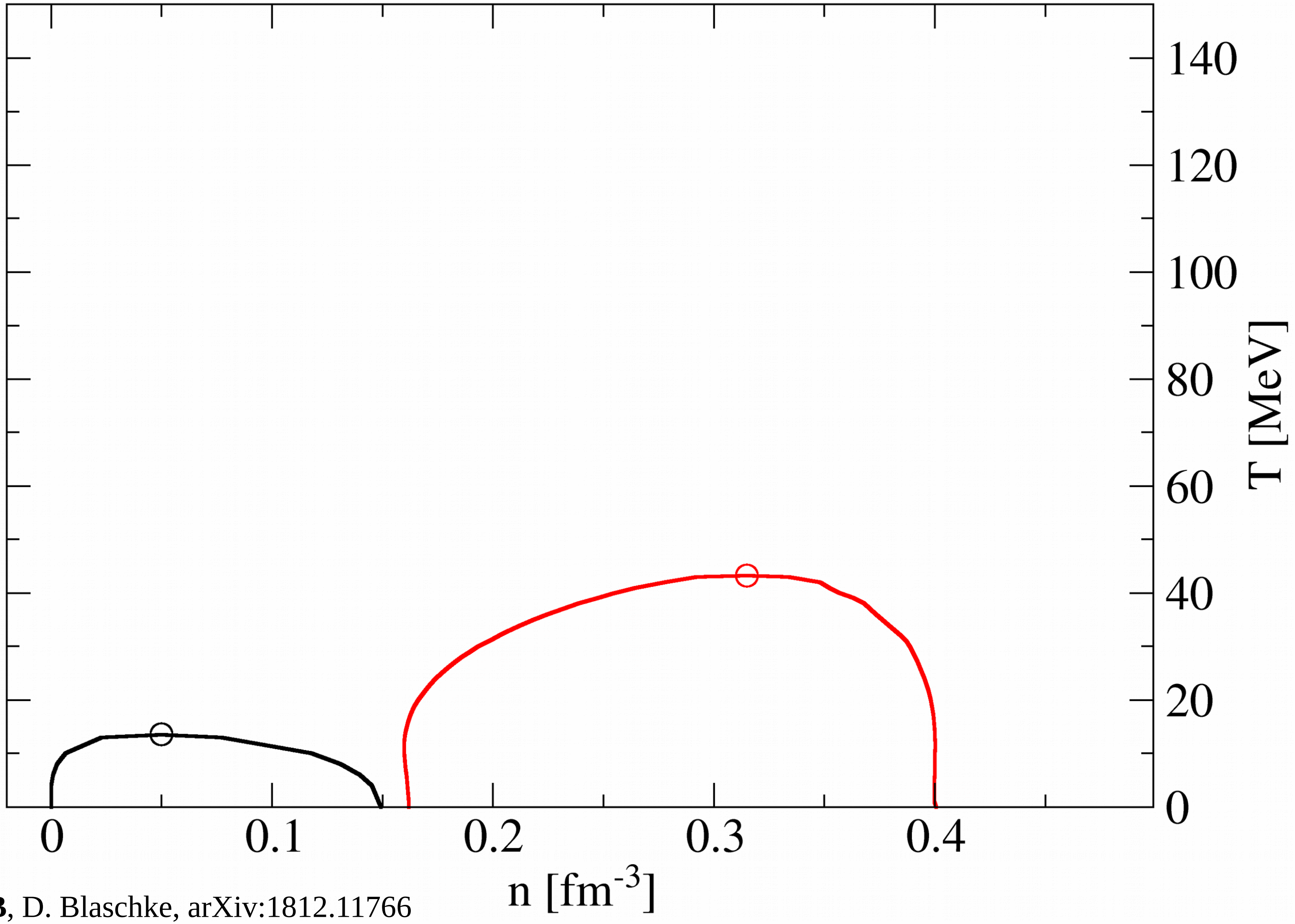

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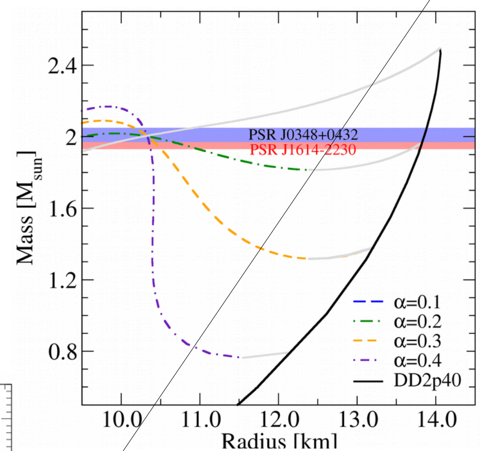
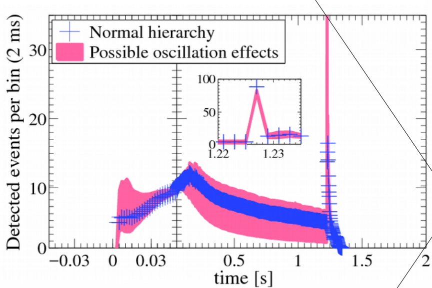
Possible signals of 1st – order phase transitions.

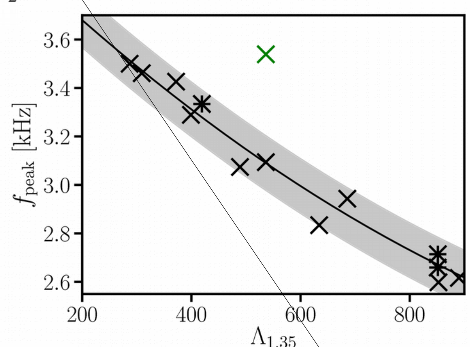
Neutron star configurations

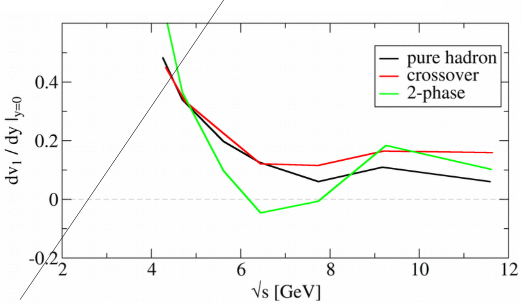




Supernova explosions of 50Ms stars

Binary neutron star mergers





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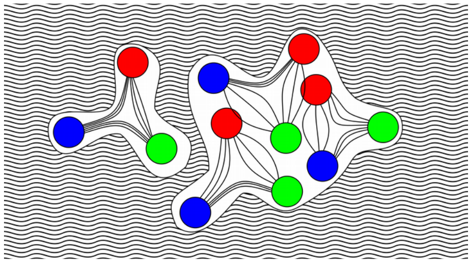
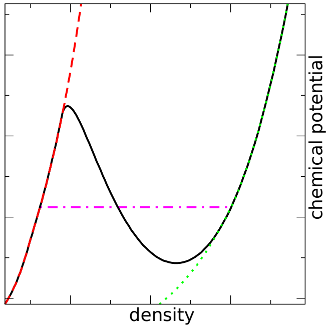
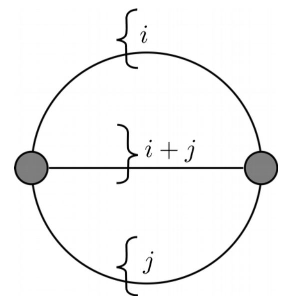
Unified description of the equation of state.



density functional theory

construction of phase transitions





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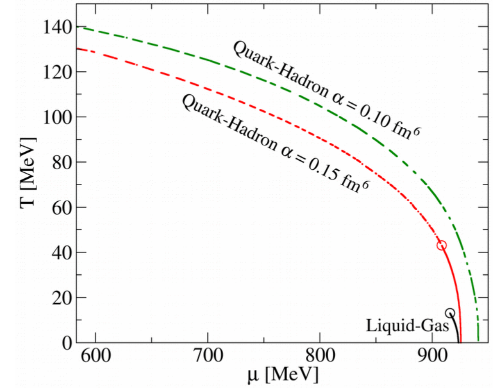

Last Slide

Conclusions

- Possible scenarios are explored in which a 1st order phase transition is detectable in
  - neutron star configurations
  - neutrino signals of supernova explosions
  - Gravitational wave signal of binary neutron star mergers
  - Flow data of heavy-ion collision experiments
- Astrophysical objects and HIC collisions are based on the same physics of strongly interacting many-particle systems
- Hadrons are bound states of quarks and should be treated as such
- A cluster virial expansion within the Beth-Uhlenbeck formalism can be derived from the PHI-derivable approach
- Initial reduction to mean field already results in a consistent description of Quark-Hadron phase transition

Outlook

- Density functional with chiral physics
- Reproduction of Lattice results
- Continuum contributions and substructure effects
- Cluster mean field

Thank you!

Collaboration

- Tobias Fischer, David Blaschke, Andreas Bauswein, Stefan Typel, Gerd Röpke, Yuri Ivanov, Diana Alvear Terrero