# Light clusters in nuclei and nuclear matter: Nuclear structure and decay, heavy ion collisions, and astrophysics 

Trento, September 02-06, 2019

## Main Topics

- Cluster models, structure of light nuclei, cluster quantum phase transition
- Reaction theory, $\alpha$ decay of heavy and superheavy nuclei
- $\quad$ Clustering in nuclear matter and consequences for thermodynamic properties
- Heavy-ion collisions and clustering in nonequilibrium systems, transport codes
- Astrophysical consequences of clustering


## Confirmed Speakers

M. Barbui (TAMU), R. Bougault (LPC Caen), J. Cseh (INR Debrecen), D.-S. Delion (NIPNE Bucharest), B. Doenigus (U Frankfurt), T. Fischer (U Wroclaw), Y. Funaki (Kanto Gakuin University), F. Iachello (Yale), M. Ito (Kansai University), M. Itoh (Tohoku University), Y. Kanada-En'yo (Kyoto University), K. Kato (U Sapporo), T. Kawabata (Osaka University), T. Kokalova (U Birmingham), M. Lyu (Osaka University), M. Mirea (IFIN-HH), S. Mrowczynski (NCBJ Warsaw), J. Natowicz (TAMU), T. Neff (GSI Darmstadt), H. Pais (U Coimbra), C. Providencia (U Coimbra),
Z. Ren (Tongji Shangai), S. Shlomo (TAMU), M. Szala (Goethe University Frankfurt), A. Tohsaki (RCNP Osaka), S. Typel (TU Darmstadt), D. Vretenar (University of Zagreb), H. Wolter (LMU Munich), C. Xu (Nanjing University), T. Yamada (Kanto Gakuin University), P. Zarubin (JINR Dubna),
B. Zhou (Hokkaido University)

## Organizers

Gerd Roepke (Universität Rostock), Peter Schuck (Université Paris-Sud - IN2P3/CNRS - Orsay), David Blaschke (University of Wroclaw and JINR Dubna), Masaaki Kimura (Hokkaido University, Sapporo), Hisashi Horiuchi (RCNP - Osaka University)

Director of the ECT*: Professor Jochen Wambach (ECT*)

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## Preliminary Program

(status: 22.08.2019)

|  | Monday, 2.9. | Tuesday, 3.9. | Wednesday, 4.9. | Thursday, 5.9. | Friday, 6.9. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 9:00-9:45 | Registr. 8:30-9:30 Opening 9:30-9:45 | Delion | Natowitz | Doenigus | Typel |
| 9:45-10:30 | Kanada-En’yo | Ren | Wolter | Mrowczynski | Fischer |
| 10:30-11:00 | Coffee Break |  |  |  |  |
| 11:00-11:30 | Ito | Iachello | Gauthier | Szala | Shlomo |
| 11:30-12:00 | Lyu | Xu | Barbui | Pysz | Bastian |
| 12:00-12:30 | Vretenar | Bai | Pais | Cseh | Samarin/Closing |
| 12:30-14:30 | Lunch Break |  |  |  |  |
| 14:30-15:15 | Neff | Funaki | Zarubin/Zarubina | Mirea |  |
| 15:15-16:00 | Kokalova | Kawabata | Zhou | Itoh (30') <br> Baran (30') |  |
| 16:00-16:30 | Coffee Break |  |  |  |  |
| 16:30-17:00 | Han | Santa Rita | Yamada | Dumitrescu |  |
| 17:00-17:30 | Fujikawa | Shneydman | Gallmeister | Lasseri |  |
| 17:30-18:00 | Kimura | Liu | Kozhevnikova |  |  |
| 18:00-18:30 | Discussion (Kimura) | Discussion (Schuck) | Discussion (Röpke) | Discussion (Blaschke) |  |
| 18:30-... | Welcome dinner |  |  |  |  |
| 19:30-... |  |  |  | Social Dinner |  |


| 1 | Bai | Some Progress on the New Double-Folding Potential and Alpha-Alpha Elastic Scattering |
| :---: | :---: | :---: |
| 2 | Baran | Description of nuclei in terms of pairs and quartets |
| 3 | Barbui | Experimental search for states analogous to the $\wedge\{12\} \mathrm{C}$ Hoyle state in heavier nuclei |
| 4 | Bastian | From nuclear clusters to composite hadrons |
| 5 | Blaschke | Discussion; Clusters in HIC and astrophysics |
| 7 | Delion | Alpha decay versus alpha clustering |
| 8 | Dönigus | Cluster production in HIC at ALICE and STAR |
| 9 | Dumitrescu | Investigation of $\alpha$-like Quasimolecules in Heavy Nuclei |
| 10 | Fischer | Weak reactions with light clusters in simulations of core-collapse supernovae |
| 11 | Fujikawa | Search for alpha-condensed state in 20Ne |
| 12 | Funaki | Alpha cluster condensates and dynamics of cluster formation |
| 13 | Gallmeister | Nucleosynthesis in heavy-ion collisions at the LHC via the Saha equation |
| 14 | Gauthier | A Nucleation Model Analysis of Neck emission yields in Heavy Ion-Reactions |
| 49 | Han | Investigation of the 14C $+\alpha$ molecular configuration in 180 by means of transfer and sequential decay reaction |
| 15 | Iachello | Symmetry approach to clustering in nuclei |
| 16 | Ito | Isoscalar transitions and alpha cluster structures |
| 17 | Itoh | Measurement of four-alpha decays near the four-alpha threshold energy in 160 |
| 18 | Kanada-En'yo | Cluster excited states probed by alpha and proton inelastic scattering |
| 20 | Kawabata | Gamma decay width of the 3-1 state in 12C, and recent activities of the Kyoto-Osaka group |
| 21 | Kimura | Discussion; Shape of light clustered nuclei |
| 22 | Kokalova | Theoretical challenges from an experimentalist's point of view. |
| 23 | Kozhevnikova | Production of light clusters in generator THESEUS |
| 24 | Lasseri | Quantum localisation and dilution effects on clusterisation |
| 25 | Liu | Preliminary Experimental Results of the Molecular States in 16C |
| 26 | Lyu | Ab initio study of high-momentum physics in s-shell nuclei |
| 27 | Mirea | Fine structure of alpha decay from the time dependent pairing equations |
| 28 | Mrowczynski | Thermal vs. coalescence model and production of light nuclei in relativistic heavy-ion collisions |
| 29 | Natowitz | Laboratory Studies of Dilute Nuclear Matter |
| 30 | Neff | The role of clustering in structure and reactions of light nuclei |
| 31 | Pais | Light clusters and pasta phases in warm stellar matter |
| 32 | Pysz | Production of H and He isotopes in pA collision at a few GeV beam energy |
| 33 | Ren | Calculation on alpha-decay half-lives of heavy nuclei |
| 34 | Röpke | Discussion; Correlations in nuclear systems |
| 35 | Samarin | Application of Feynman's Continual Integrals and Hyperspherical Functions to the Cluster Structure of Light Nuclei |
| 36 | Santa Rita | Advances towards the measurement of gamma transitions in clustered states of 160 |
| 37 | Schuck | Discussion;Life time of alpha GDR state in alpha cluster states; family of Hoyle states in 12C |
| 38 | Shlomo | Sensitivity of giant resonances energies to nuclear matter properties and the equation of state |
| 39 | Shneydman | Nuclear reflection-asymmetry in cluster approach |
| 40 | Szala | Light nuclei formation in heavy ion collisions measured with HADES |
| 41 | Typel | Clusters in nuclear matter: from nuclei in the laboratory to stars in the cosmos |
| 42 | Vretenar | Localization and clustering in atomic nuclei |
| 43 | Wolter | Light cluster production in intermediate energy heavy-ion collisions |
| 44 | Xu | Alpha-cluster formation and decay in the quartetting wave function approach |
| 45 | Yamada | Nuclear matter calculation with the tensor optimized Fermi sphere method |
| 46 | Zarubin | Nuclear clustering studied inside fragmentation cone of relativistic nuclei |
| 47 | Zarubina | Imaging of dissociation of relativistic nuclei in nuclear track emulsion |
| 48 | Zhou | Microscopic description of multi-clusters in light nuclei |

## Freeze-out temperatures and densities



## Nuclear matter phase diagram

## Core collapse supernovae

Relevant Parameters:

- density:
$10^{-9} \lesssim \varrho / \varrho_{\text {sat }} \lesssim 10$
with nuclear saturation density
$\varrho_{\text {sat }} \approx 2.5 \cdot 10^{14} \mathrm{~g} / \mathrm{cm}^{3}$
$\left(n_{\text {sat }}=\varrho_{\text {sat }} / m_{n} \approx 0.15 \mathrm{fm}^{-3}\right)$
- temperature:
$0 \mathrm{MeV} \leq k_{B} T \lesssim 50 \mathrm{MeV}$

$$
\left(\hat{=} 5.8 \cdot 10^{11} \mathrm{~K}\right)
$$

- electron fraction:
$0 \leq Y_{e} \lesssim 0.6$
T. Fischer, GSI Darmstadt



## Correlations in nuclear systems

- Kimura: Clusters?
- Correlations in many-particle systems, finite density?
- Classical - quantum: antisymmetrization
- Bound states, continuum correlations
- Spectral function - quasiparticle concept
- Pairing: $n_{\text {up }} n_{\text {down }}, p_{\text {up }} p_{\text {down }}, n_{\text {up }} p_{\text {down }}$
- Quartetting: $\mathrm{n}_{\mathrm{up}} \mathrm{n}_{\text {down }} \mathrm{p}_{\text {up }} \mathrm{p}_{\text {down }}$
- Correlations in thermodynamic equilibrium
- Center-of-mass motion, intrinsic motion are separated
- Inhomogeneous systems
- Time-dependent processes


## Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$
\begin{gathered}
\left(\frac{p_{1}^{2}}{2 m_{1}}+\Delta_{1}+\frac{p_{2}^{2}}{2 m_{2}}+\Delta_{2}\right) \Psi_{d, P}\left(p_{1}, p_{2}\right)+\sum_{p_{1}^{\prime}, p_{2}^{\prime}}\left(1-f_{p_{1}}-f_{p_{2}}\right) V\left(p_{1}, p_{2} ; p_{1}^{\prime}, p_{2}{ }^{\prime}\right) \Psi_{d, P}\left(p_{1}^{\prime}, p_{2}{ }^{\prime}\right) \\
\text { Add self-energy } \quad \text { Pauli-blocking }=E_{d, P} \Psi_{d, P}\left(p_{1}, p_{2}\right)
\end{gathered}
$$

Fermi distribution function

$$
f_{p}=\left[e^{\left(p^{2} / 2 m-\mu\right) / k_{B} T}+1\right]^{-1}
$$

Correlated medium?

Thouless criterion

$$
E_{d}(T, \mu)=2 \mu
$$

BEC-BCS crossover:
Alm et al.,1993

## Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$
\begin{array}{ll}
\left(\left[E^{H F}\left(p_{1}\right)+E^{H F}\left(p_{2}\right)+E^{H F}\left(p_{3}\right)+E^{H F}\left(p_{4}\right)\right]\right) \Psi_{n, P}\left(p_{1}, p_{2}, p_{3}, p_{4}\right) \\
+\sum_{p_{1}^{\prime}, p_{2}^{\prime}}\left(1-f_{p_{1}}-f_{p_{2}}\right) V\left(p_{1}, p_{2} ; p_{1}^{\prime}, p_{2}^{\prime}\right) \Psi_{n, P}\left(p_{1}^{\prime}, p_{2}^{\prime}, p_{3}, p_{4}\right) \\
+\{\text { permutations }\} & \\
=E_{n, P} \Psi_{n, P}\left(p_{1}, p_{2}, p_{3}, p_{4}\right) & \text { Thouless criterion } \\
& \text { for quantum condensate: } \\
& \mathrm{E}_{\mathrm{n}, \mathrm{P}=0}(\mathrm{~T}, \mu)=4 \mu
\end{array}
$$

## Composition of dense nuclear matter

$$
\begin{aligned}
n_{p}\left(T, \mu_{p}, \mu_{n}\right) & =\frac{1}{V} \sum_{A, \nu, K} Z_{A} f_{A}\left\{E_{A, \nu K}-Z_{A} \mu_{p}-\left(A-Z_{A}\right) \mu_{n}\right\} \\
n_{n}\left(T, \mu_{p}, \mu_{n}\right) & =\frac{1}{V} \sum_{A, \nu, K}\left(A-Z_{A}\right) f_{A}\left\{E_{A, \nu K}-Z_{A} \mu_{p}-\left(A-Z_{A}\right) \mu_{n}\right\}
\end{aligned}
$$

mass number $A$
charge $Z_{A}$
energy $E_{A, v, K}$
$v$ : internal quantum number

$$
f_{A(z)}=\frac{1}{\exp (z / T)-(-1)^{A}}
$$

- Medium effects: correct behavior near saturation self-energy and Pauli blocking shifts of binding energies, Coulomb corrections due to screening (Wigner-Seitz, Debye)


## Asymmetric nuclear light clusters in supernova matter



Figure 1. Upper three panels, from left ro right: temperature $T$ (in MeV ), log of density $\rho$ (in $\mathrm{g} \cdot \mathrm{cm}^{-3}$ ) and electron fraction $Y_{\mathrm{e}}$ as a functions of mass coordinate $m$. Lower panel: mass fractions of of nuclei $X_{i}$ as a function of $m$. The black dashed line marked $X_{\mathrm{Z}>2}$ shows the total mass fraction of elements with $Z>2$. EoS is pure NSE.


Figure 7. Upper three panels, from left ro right: temperature $T$ (in MeV ), $\log$ of density $\rho$ (in $\mathrm{g} \cdot \mathrm{cm}^{-3}$ ) and electron fraction $Y_{\mathrm{e}}$ as a functions of mass coordinate $m$. Lower panel: mass fractions $X_{i}$ of of hydrogen and helium isotopes as a function of $m$. The black dashed line marked $X_{Z>2}$ shows the total mass fraction of all rest nuclei. Stellar profile corresponds to 200 ms after bounce approximately, calculations according to modified HS EoS.
A. V. Yudin, M. Hempel, S. I. Blinnikov, D. K. Nadyozhin, I. V. Panov, Monthly Notices of the Royal Astronomical Society 483, 5426 (2019)

## Light unstable clusters


arXiv:1812.09494

MNRAS 000, 1-9 (2018)


Asymmetric Nuclear Light Clusters In Supernova Matter
A. V. Yudin, ${ }^{1 \star}$ M. Hempel, ${ }^{2}$ S. I. Blinnikov, ${ }^{1}$ D. K. Nadyozhin, ${ }^{1,3}$ I. V. Panov ${ }^{1,3}$

## Core-collapse supernovae



Density.
electron fraction, and
temperature profile
of a 15 solar mass supernova at 150 ms after core bounce as function of the radius.

Influence of cluster formation on neutrino emission in the cooling region and on neutrino absorption in the heating region?
K.Sumiyoshi et al.,

Astrophys.J. 629, 922 (2005)

## Composition of supernova core



Mass fraction $X$ of light clusters for a post-bounce supernova core
K.Sumiyoshi,
G. R.,

PRC 77,
055804 (2008)

## EOS: continuum contributions

Partial density of channel A,c at P (for instance, ${ }^{3} \mathrm{~S}_{1}=\mathrm{d}$ ):

$$
z_{A, c}^{\text {part }}\left(\mathbf{P} ; T, \mu_{n}, \mu_{p}\right)=e^{\left(N \mu_{n}+Z \mu_{p}\right) / T}\left\{\sum_{\nu_{c}}^{\text {bound }} g_{A, \nu_{c}} e^{-E_{A, \nu_{c}}(\mathbf{P}) / T} \Theta\left[-E_{A, \nu_{c}}(\mathbf{P})+E_{A, c}^{\text {cont }}(\mathbf{P})\right]+z_{A, c}^{\text {cont }}(\mathbf{P})\right\}
$$

separation: bound state part - continuum part?

$$
\begin{aligned}
& z_{c}^{\mathrm{part}}\left(\mathbf{P} ; T, n_{B}, Y_{p}\right)=e^{\left[N \mu_{n}+Z \mu_{p}-N E_{n}\left(\mathbf{P} / A ; T, n_{B}, Y_{p}\right)-Z E_{p}\left(\mathbf{P} / A ; T, n_{B}, Y_{p}\right)\right] / T} \\
& \times g_{c}\left\{\left[e^{-E_{c}^{\mathrm{intr}}\left(\mathbf{P} ; T, n_{B}, Y_{p}\right) / T}-1\right] \Theta\left[-E_{c}^{\mathrm{intr}}\left(\mathbf{P} ; T, n_{B}, Y_{p}\right)\right]+v_{c}\left(\mathbf{P} ; T, n_{B}, Y_{p}\right)\right\}
\end{aligned}
$$

parametrization (d - like):

$$
\begin{aligned}
& v_{c}\left(\mathbf{P}=0 ; T, n_{B}, Y_{p}\right) \approx\left[1.24+\left(\frac{1}{v_{T_{I}=0}(T)}-1.24\right) e^{\gamma_{c} n_{B} / T}\right]^{-1} . \\
& v_{d}^{0}(T)=v_{T_{I}=0}^{0}(T) \approx 0.30857+0.65327 e^{-0.102424 T / \mathrm{MeV}}
\end{aligned}
$$

## Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_{d}^{0} / T}-1+\frac{1}{\pi T} \int_{0}^{\infty} d E e^{-E / T}\left\{\delta_{c}(E)-\frac{1}{2} \sin \left[2 \delta_{c}(E)\right]\right\}$


Tamm-Dancoff

## Deuteron-like scattering phase shifts

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Tamm-Dancoff
deuteron bound state -2.2 MeV

## $\alpha-n$ scattering phase shifts



Fig. 2. (Color online.) The phase shifts for elastic neutron-alpha scattering $\delta_{\mathrm{L}_{J}}(E)$ versus laboratory energy $E$. As discussed in the text, the solid lines are from Arndt and Roper [37] and the symbols are from Amos and Karataglidis [38]. For clarity, we do not show the F -waves included in our results for $b_{\alpha n}$.

## $\alpha-\alpha$ scattering phase shifts


C.J.Horowitz, A.Schwenk, Nucl. Phys. A 776, 55 (2006)

## ${ }^{212 P o: ~} \alpha$ on top of ${ }^{208 P b}$

Problem:
decay half-life too short
Finite system:
discrete spectrum of energy levels.
Relax the
Thomas-Fermi rule $\mu_{4}=\mathrm{E}_{\text {tunneling }}$

Discrete spectrum: shell effects


## Clusters in an external potential

c. o. m. coordinate $R$, relative coordinates $s_{j}$

$$
\Psi\left(\mathbf{R}, \mathbf{s}_{j}\right)=\varphi^{\operatorname{intr}}\left(\mathbf{s}_{j}, \mathbf{R}\right) \Phi(\mathbf{R})
$$

$$
\text { normalization } \quad \int d R|\Phi(\mathbf{R})|^{2}=1 \quad \int d s_{j}\left|\varphi^{\mathrm{intr}}\left(\mathbf{s}_{j}, \mathbf{R}\right)\right|^{2}=1
$$

Wave equation for the c.o.m. motion

$$
\begin{aligned}
& -\frac{\hbar^{2}}{2 A m} \nabla_{R}^{2} \Phi(\mathbf{R})-\frac{\hbar^{2}}{A m} \int d s_{j} \varphi^{\text {intr }, *}\left(\mathbf{s}_{j}, \mathbf{R}\right)\left[\nabla_{R} \varphi^{\operatorname{intr}}\left(\mathbf{s}_{j}, \mathbf{R}\right)\right]\left[\nabla_{R} \Phi(\mathbf{R})\right] \\
& -\frac{\hbar^{2}}{2 A m} \int d s_{j} \varphi^{\operatorname{intr}, *}\left(\mathbf{s}_{j}, \mathbf{R}\right)\left[\nabla_{R}^{2} \varphi^{\operatorname{intr}}\left(\mathbf{s}_{j}, \mathbf{R}\right)\right] \Phi(\mathbf{R})+\int d R^{\prime} W\left(\mathbf{R}, \mathbf{R}^{\prime}\right) \Phi\left(\mathbf{R}^{\prime}\right)=E \Phi(\mathbf{R})
\end{aligned}
$$

c.o.m. effective potential

$$
W\left(\mathbf{R}, \mathbf{R}^{\prime}\right)=\int d s_{j} d s_{j}^{\prime} \varphi^{\operatorname{intr}, *}\left(\mathbf{s}_{j}, \mathbf{R}\right)\left[T\left[\nabla_{s_{j}}\right] \delta\left(\mathbf{R}-\mathbf{R}^{\prime}\right) \delta\left(\mathbf{s}_{j}-\mathbf{s}_{j}^{\prime}\right)+V\left(\mathbf{R}, \mathbf{s}_{j} ; \mathbf{R}^{\prime}, \mathbf{s}_{j}^{\prime}\right)\right] \varphi^{\mathrm{intr}}\left(\mathbf{s}_{j}^{\prime}, \mathbf{R}^{\prime}\right)
$$

Wave equation for the intrinsic motion

$$
\begin{aligned}
& -\frac{\hbar^{2}}{A m} \Phi^{*}(\mathbf{R})\left[\nabla_{R} \Phi(\mathbf{R})\right]\left[\nabla_{R} \varphi^{\operatorname{intr}}\left(\mathbf{s}_{j}, \mathbf{R}\right)\right]-\frac{\hbar^{2}}{2 A m}|\Phi(\mathbf{R})|^{2} \nabla_{R}^{2} \varphi^{\mathrm{intr}}\left(\mathbf{s}_{j}, \mathbf{R}\right) \\
& +\int d R^{\prime} d s_{j}^{\prime} \Phi^{*}(\mathbf{R})\left[T\left[\nabla_{s_{j}}\right] \delta\left(\mathbf{R}-\mathbf{R}^{\prime}\right) \delta\left(\mathbf{s}_{j}-\mathbf{s}_{j}^{\prime}\right)+V\left(\mathbf{R}, \mathbf{s}_{j} ; \mathbf{R}^{\prime}, \mathbf{s}_{j}^{\prime}\right)\right] \Phi\left(\mathbf{R}^{\prime}\right) \varphi^{\operatorname{intr}}\left(\mathbf{s}_{j}^{\prime}, \mathbf{R}^{\prime}\right)=F(\mathbf{R}) \varphi^{\mathrm{intr}}\left(\mathbf{s}_{j}, \mathbf{R}\right)
\end{aligned}
$$

## Quantum condensate: quartetting

Ideal Bose condensate : $\quad|0\rangle=b_{0}^{\dagger} b_{0}^{\dagger} \cdots b_{0}^{\dagger} \mid$ vac $\rangle$
$\alpha$-particle condensate : $\quad\left|\Phi_{\alpha c}\right\rangle=C_{\alpha}^{\dagger} C_{\alpha}^{\dagger} \cdots C_{\alpha}^{\dagger} \mid$ vac $\rangle$

In $r$-space :

$$
\left\langle\vec{r}_{1}, \vec{r}_{2}, \cdots, \vec{r}_{4 n} \mid \Phi_{\alpha \mathrm{C}}\right\rangle=\mathcal{A}\left\{\Phi\left(\vec{r}_{1}, \vec{r}_{2}, \vec{r}_{3}, \vec{r}_{4}\right) \Phi\left(\vec{r}_{5}, \vec{r}_{6}, \vec{r}_{7}, \vec{r}_{8}\right) \cdots \Phi\left(\vec{r}_{4 n-3}, \vec{r}_{4 n-2}, \vec{r}_{4 n-1}, \vec{r}_{4 n}\right)\right\}
$$

In comparison with pairing :

$$
\left\langle\vec{r}_{1}, \vec{r}_{2}, \cdots \mid \mathrm{BCS}\right\rangle=\mathcal{A}\left\{\Phi\left(\vec{r}_{1}, \vec{r}_{2}\right) \Phi\left(\vec{r}_{3}, \vec{r}_{4}\right) \cdots\right\}
$$

