

Light clusters and pasta phases in warm stellar matter

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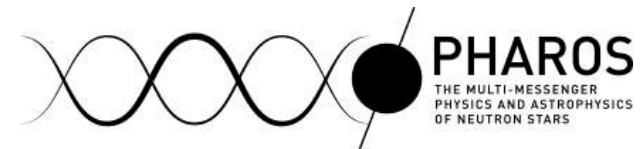
Light clusters in nuclei and nuclear matter: Nuclear structure and decay, heavy ion collision, and astrophysics

ECT*, Trento, September 4, 2019

Acknowledgments:

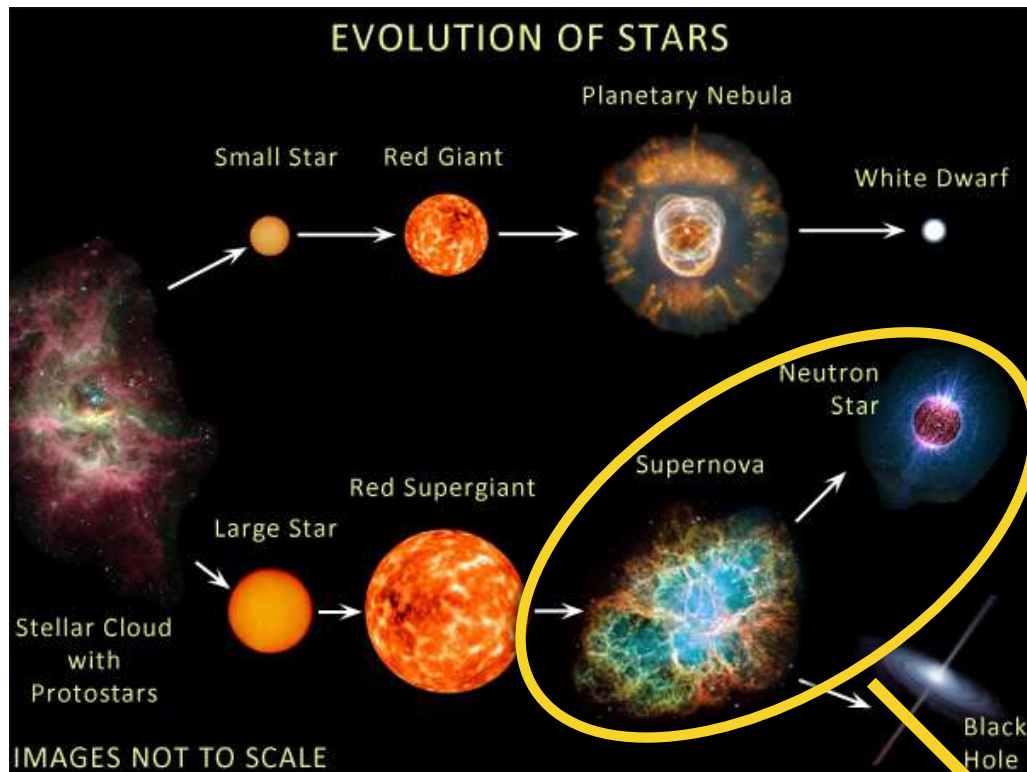
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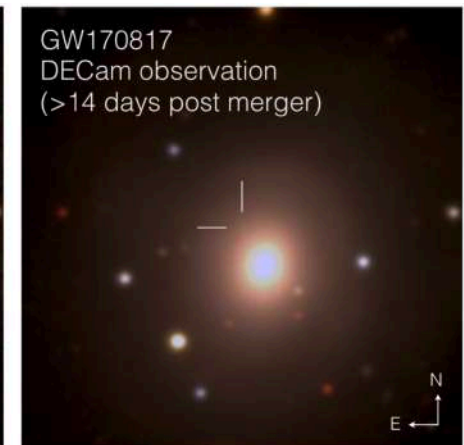
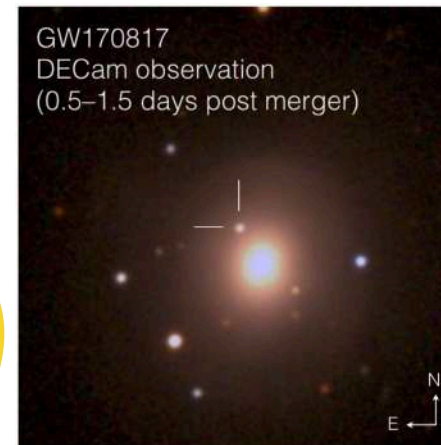
Where do these clusters form?

in <http://essayweb.net/astromy/blackhole.shtml>



in <https://www.ligo.org/detections/GW170817.php>

Credit: Soares-Santos et al. and DES Collab



NS mergers

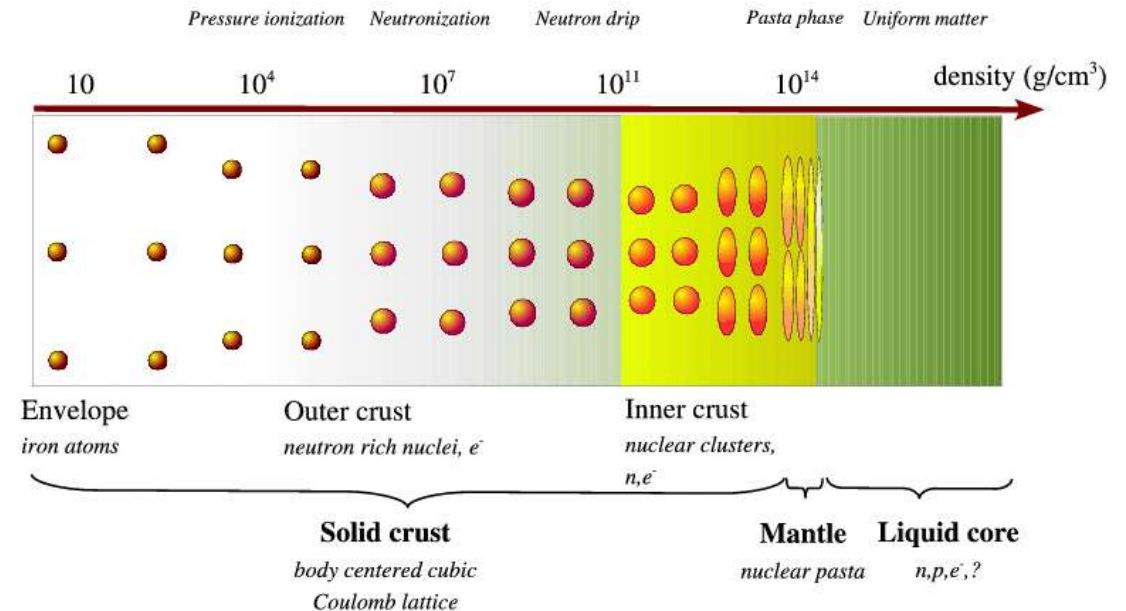
scenarios where light and heavy clusters are important:
supernovae, NS mergers, (crust of) neutron stars

Neutron stars

- Divided in 3 main layers:
- NS: catalized cold stellar matter:

1. Outer crust
2. Inner crust
3. Core

N. Chamel and P. Hansel,
Liv. Rev. Rel. 11, 10, 2008



- **Surface:** ^{56}Fe , $P=0$
- **Outer crust:** Neutron rich nuclei embedded in electron sea
- **Inner crust:** Above neutron drip density, nucleons form geometrical structures (non-spherical: pasta phases) embedded in neutron and electron background gas.
- **Core:** Uniform matter, in the centre exotic matter may exist.

How do these clusters affect the star?

- They influence supernova properties: the clusters can modify the neutrino transport, affecting the cooling of the proto-neutron star.
- These clusters may also affect the cooling of binary and accreting systems.
- Magnetars (neutron stars with very strong magnetic fields) may have an inner crust even more complex.

Supernova EoS with light clusters

- The SN EoS should incorporate: all relevant clusters, (mean-field) interaction between nucleons and clusters, and a suppression mechanism of clusters at high densities.
- Different methods: nuclear statistical equilibrium, quantum statistical approach, and
 - **RMF approach:** clusters as new degrees of freedom, with effective mass dependent on density.
 - **In-medium effects:** cluster interaction with medium described via the meson couplings, or effective mass shifts, or both
 - Constrains are needed to fix the couplings:
 - low densities:** Virial EoS
 - high densities:** cluster formation has been measured in HIC

Non-linear Walecka Model

mesons: mediation of nuclear force

$$\mathcal{L} = \sum_{i=p,n} \mathcal{L}_i + \mathcal{L}_e + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\omega\rho}$$

nucleons

electrons

mesons

non-linear mixing coupling

$$\mathcal{L}_i = \bar{\psi}_i [\gamma_\mu i D^\mu - M^*] \psi_i$$

$$\mathcal{L}_e = \bar{\psi}_e [\gamma_\mu (i \partial^\mu + e A^\mu) - m_e] \psi_e$$

$$\mathcal{L}_\sigma = \frac{1}{2} \left(\partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 - \frac{1}{3} \kappa \phi^3 - \frac{1}{12} \lambda \phi^4 \right)$$

$$\mathcal{L}_\omega = -\frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu + \frac{1}{4!} \xi g_v^4 (V_\mu V^\mu)^2$$

$$\mathcal{L}_\rho = -\frac{1}{4} \mathbf{B}_{\mu\nu} \cdot \mathbf{B}^{\mu\nu} + \frac{1}{2} m_\rho^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu$$

non-linear mixing coupling term:
responsible for density dependence of
E_{sym}

$$\mathcal{L}_{\omega\rho} = g_{\omega\rho} g_\rho^2 g_v^2 V_\mu V^\mu \mathbf{b}_\nu \cdot \mathbf{b}^\nu$$

Light clusters

- New degrees of freedom of the system.
- Interaction with the medium via the meson couplings.

$$\mathcal{L} = \sum_{j=t,h} \mathcal{L}_j + \mathcal{L}_\alpha + \mathcal{L}_d$$

with

$$\mathcal{L}_j = \bar{\psi} [\gamma_\mu i D_j^\mu - M_j^*] \psi, \quad i D_j^\mu = i \partial^\mu - g_{vj} \omega^\mu - \frac{g_\rho}{2} \tau_j \cdot \mathbf{b}^\mu$$

the vector cluster-meson coupling

$$g_{vj} = A_j g_v$$

for the fermions tritons and helions,
and for the bosons alphas and deuterons, we have:

$$\mathcal{L}_\alpha = \frac{1}{2} (i D_\alpha^\mu \phi_\alpha)^* (i D_{\mu\alpha} \phi_\alpha) - \frac{1}{2} \phi_\alpha^* (M_\alpha^*)^2 \phi_\alpha,$$

$$\mathcal{L}_d = \frac{1}{4} (i D_d^\mu \phi_d^\nu - i D_d^\nu \phi_d^\mu)^* (i D_{d\mu} \phi_{d\nu} - i D_{d\nu} \phi_{d\mu}) - \frac{1}{2} \phi_d^{\mu*} (M_d^*)^2 \phi_{d\mu}, \quad i D_j^\mu = i \partial^\mu - g_{vj} \omega^\mu$$

and

$$M_j^* = A_j m - g_{sj} \phi_0 - (B_j^0 + \delta B_j) \quad \text{with } j=t,h,d,4\text{He}$$

In-medium effects - g_{sj}

PRC 97, 045805 2018

- Binding energy of each cluster: $B_j = A_j m^* - M_j^*$, $j = d, t, h, \alpha$

with $m^* = m - g_s \phi_0$ the nucleon effective mass and

$$M_j^* = A_j m - g_{sj} \phi_0 - (B_j^0 + \delta B_j) \text{ the cluster effective mass.}$$

the scalar cluster-meson coupling

$$g_{sj} = x_{sj} A_j g_s$$

needs to be determined from exp. constraints

In-medium effects - δB_j

PRC 97, 045805 2018

- Binding energy of each cluster: $B_j = A_j m^* - M_j^*$, $j = d, t, h, \alpha$
with $m^* = m - g_s \phi_0$ the nucleon effective mass and

$$M_j^* = A_j m - g_{sj} \phi_0 - (B_j^0 + \delta B_j)$$
 the cluster effective mass.

binding energy shift

$$\delta B_j = \frac{Z_j}{\rho_0} (\epsilon_p^* - m \rho_p^*) + \frac{N_j}{\rho_0} (\epsilon_n^* - m \rho_n^*)$$

energetic counterpart of
classical ExV mechanism

$$\epsilon_j^* = \frac{1}{\pi^2} \int_0^{p_{F_j}(\text{gas})} p^2 e_j(p) (f_{j+}(p) + f_{j-}(p)) dp$$
$$\rho_j^* = \frac{1}{\pi^2} \int_0^{p_{F_j}(\text{gas})} p^2 (f_{j+}(p) + f_{j-}(p)) dp,$$

the energy states occupied by
the gas are excluded:
double counting avoided!

associated with the gas lowest
energy levels

Supernova EoS with light clusters

- The total baryonic density is defined as:

$$\rho = \rho_p + \rho_n + 4\rho_\alpha + 2\rho_d + 3\rho_h + 3\rho_t$$

- The global proton fraction as

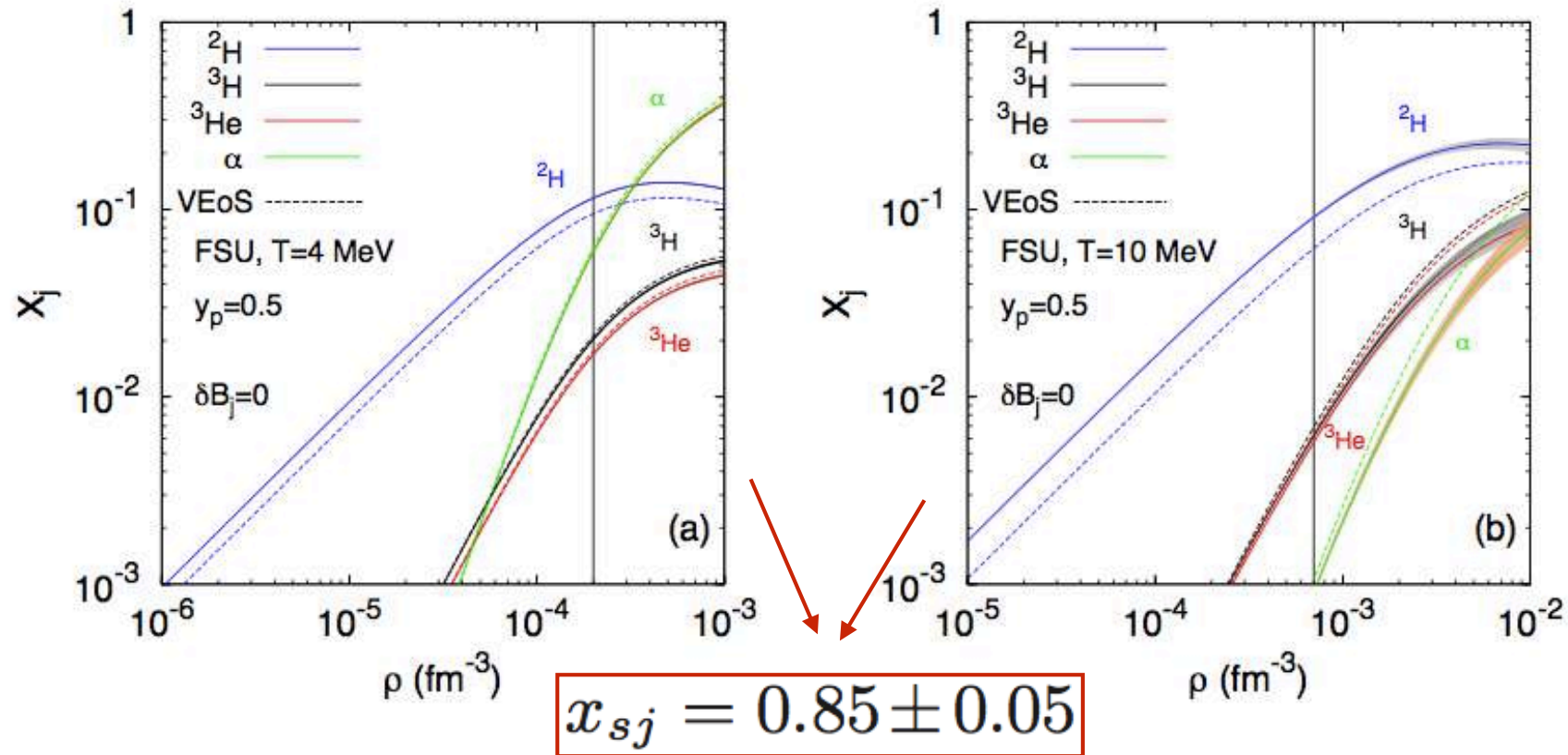
$$Y_p = y_p + \frac{1}{2}y_\alpha + \frac{1}{2}y_d + \frac{2}{3}y_h + \frac{1}{3}y_t$$

with $y_i = A_i(\rho_i/\rho)$ the mass fraction of cluster i.

- Charge neutrality must be imposed: $\rho_e = Y_p \rho$
- The light clusters are in chemical equilibrium, with the chemical potential of each cluster i defined as

$$\mu_i = N_i\mu_n + Z_i\mu_p$$

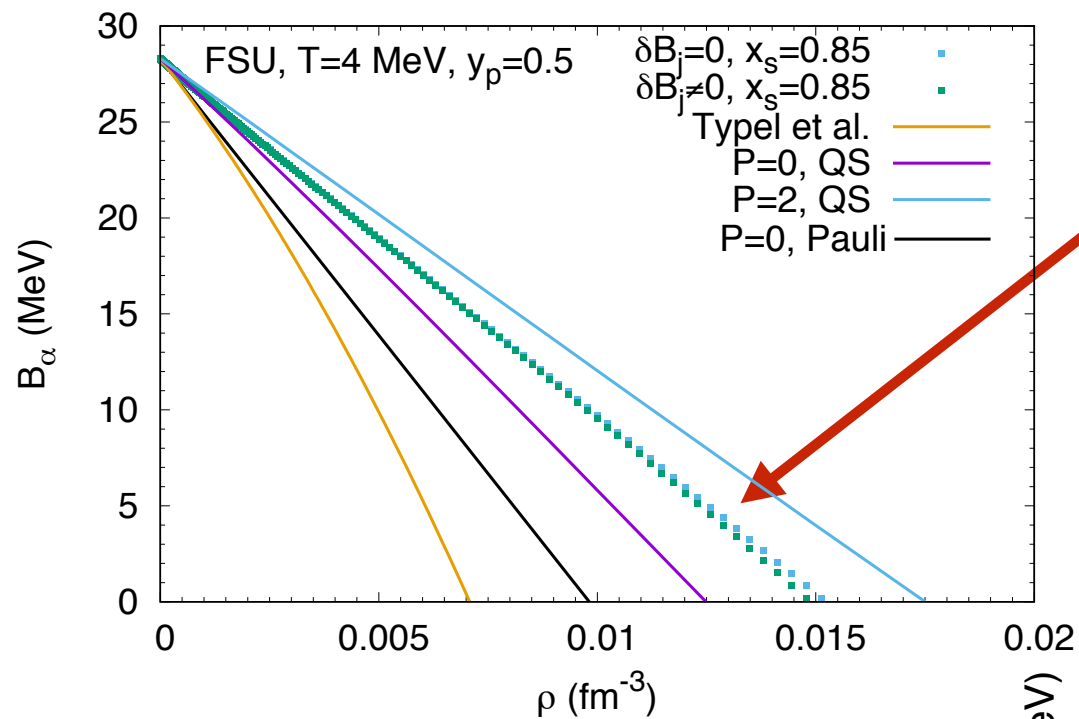
Determination of x_s : Virial EoS



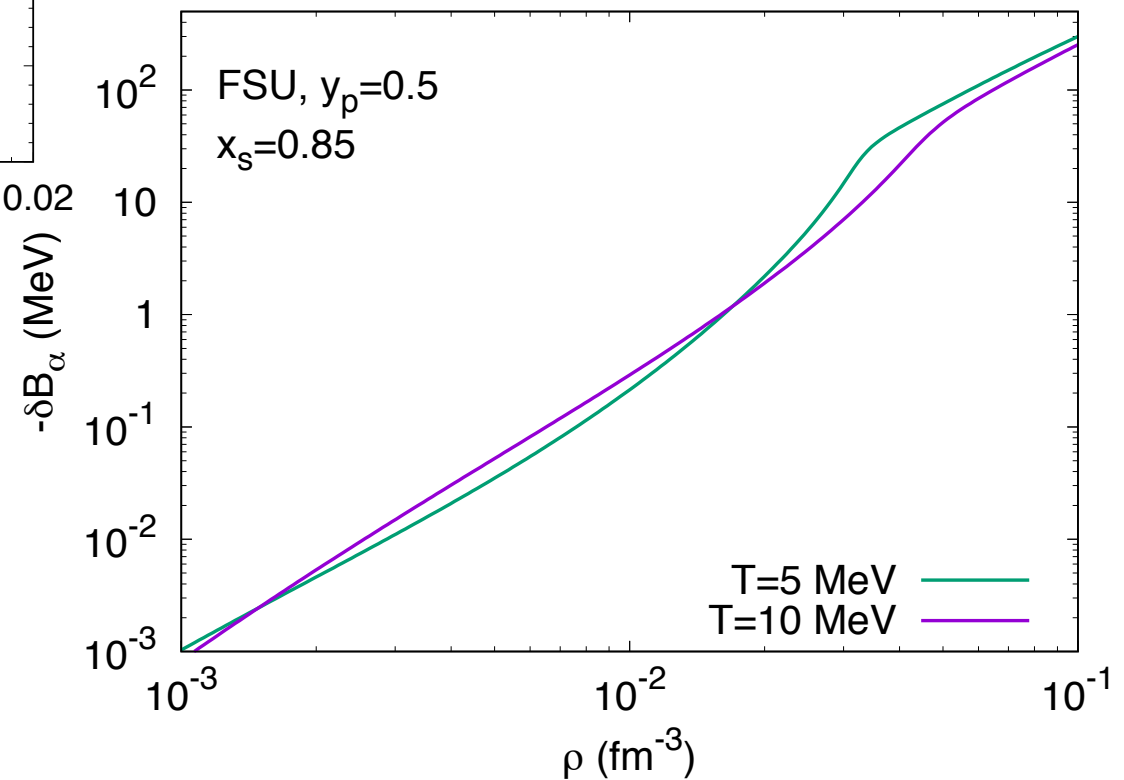
- **VEoS**: model-independent constraint, only depends on experimentally binding energies and scattering phase shifts.
- Provides correct zero-density limit for finite T EoS.
- Breaks down when interaction with particles becomes stronger:

δB_j takes action!!

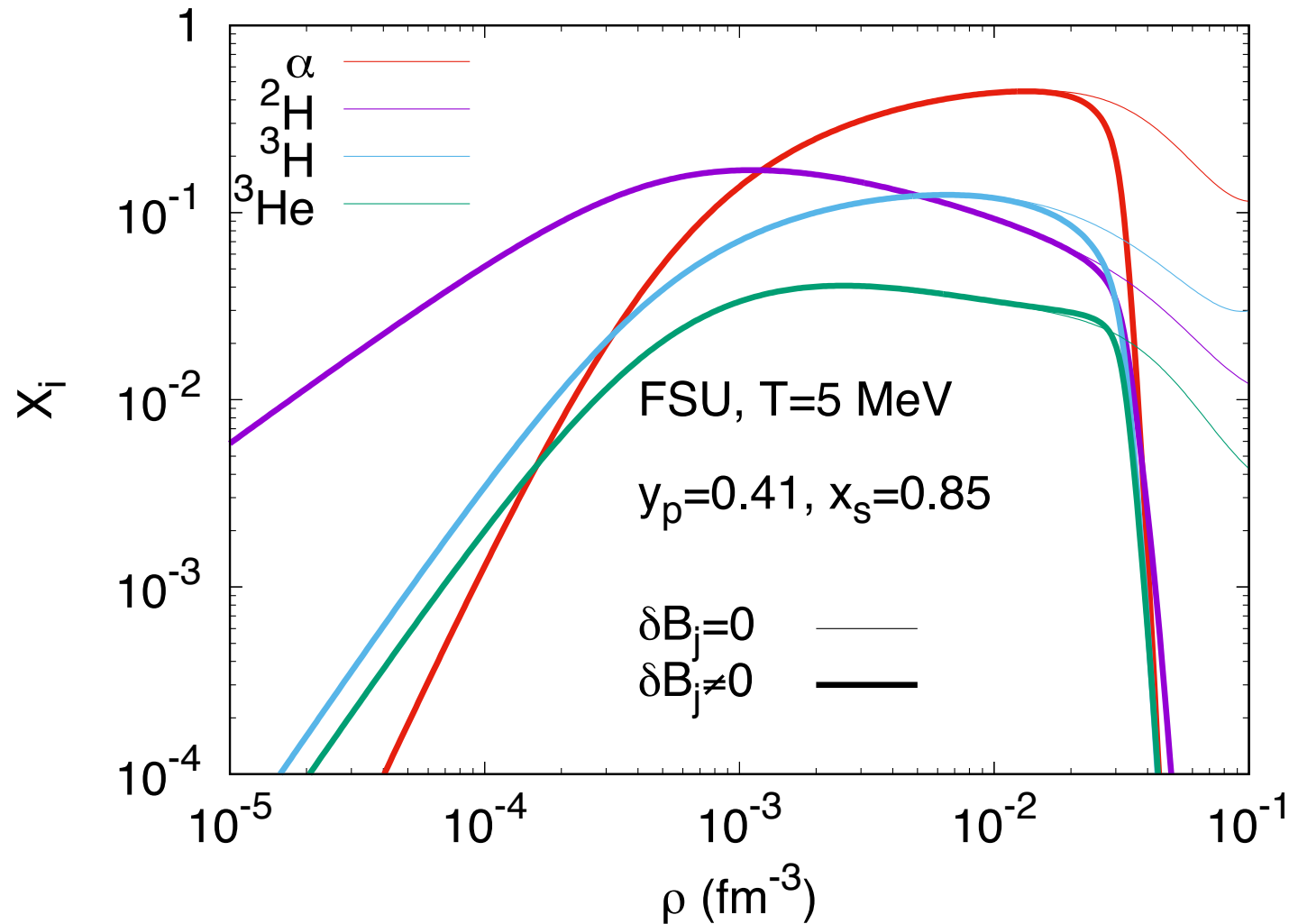
Contribution of δB_j



- δB_j completely negligible in the VEOs range of densities
- but rises fast for larger densities



Cluster fractions - effect of δB_j



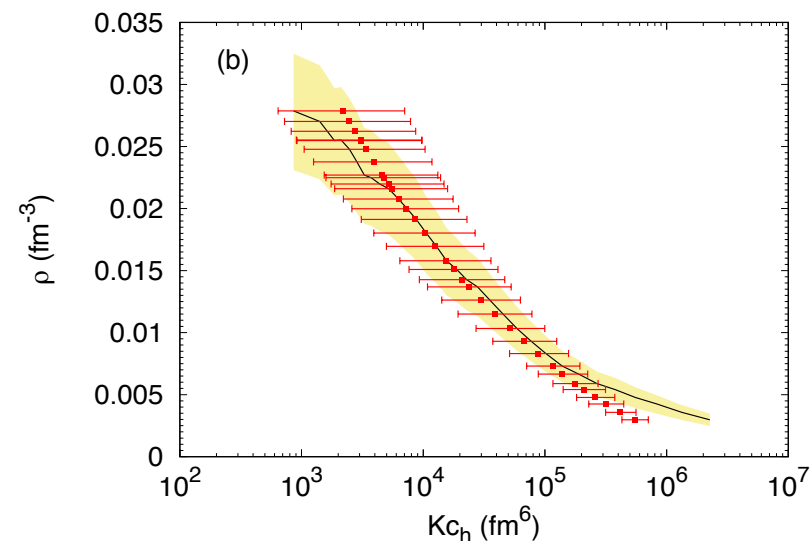
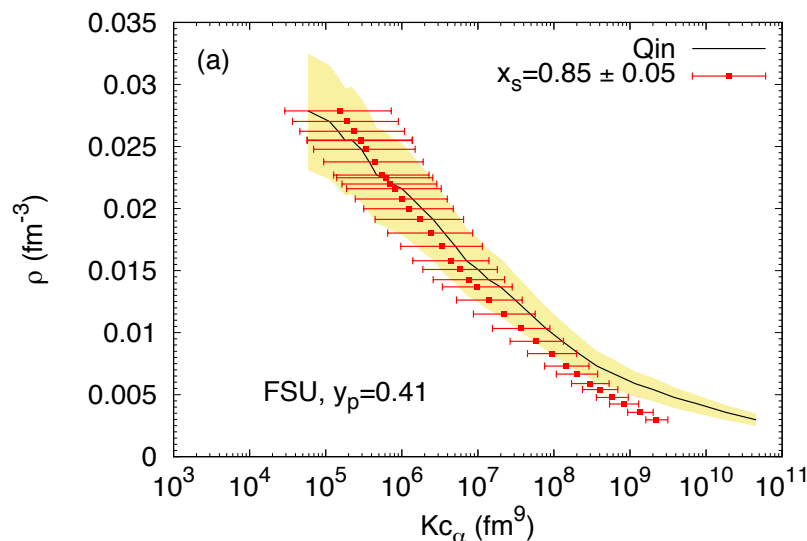
δB_j important for dissolution of clusters!

Equilibrium constants

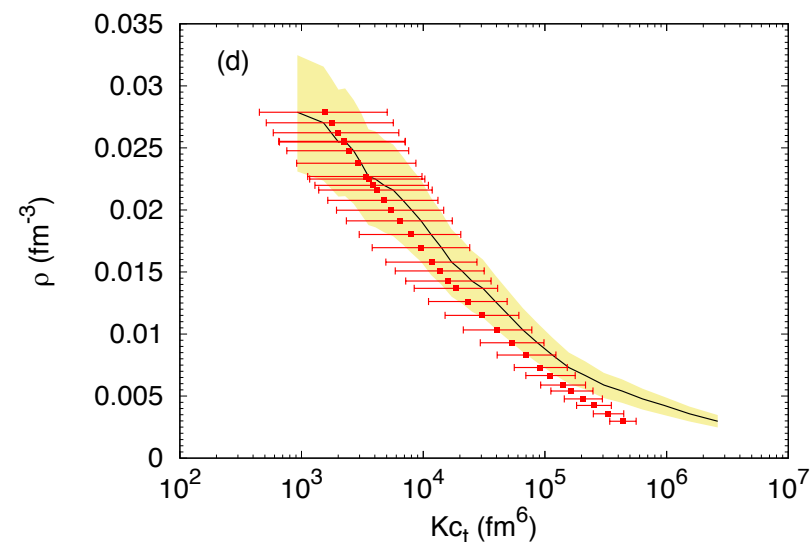
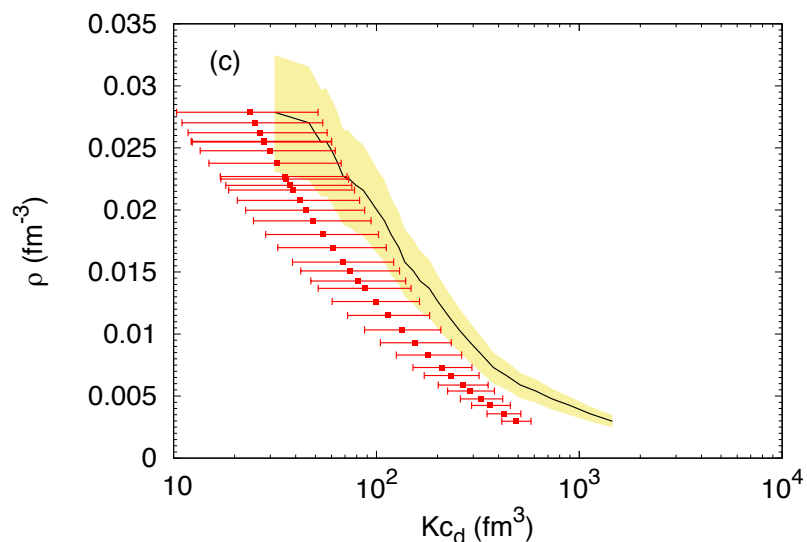
Qin et al, PRL 108, 172701 2012

- K_c calculated with data from HIC:

$$K_c[j] = \frac{\rho_j}{\rho_n^{N_j} \rho_p^{Z_j}}$$



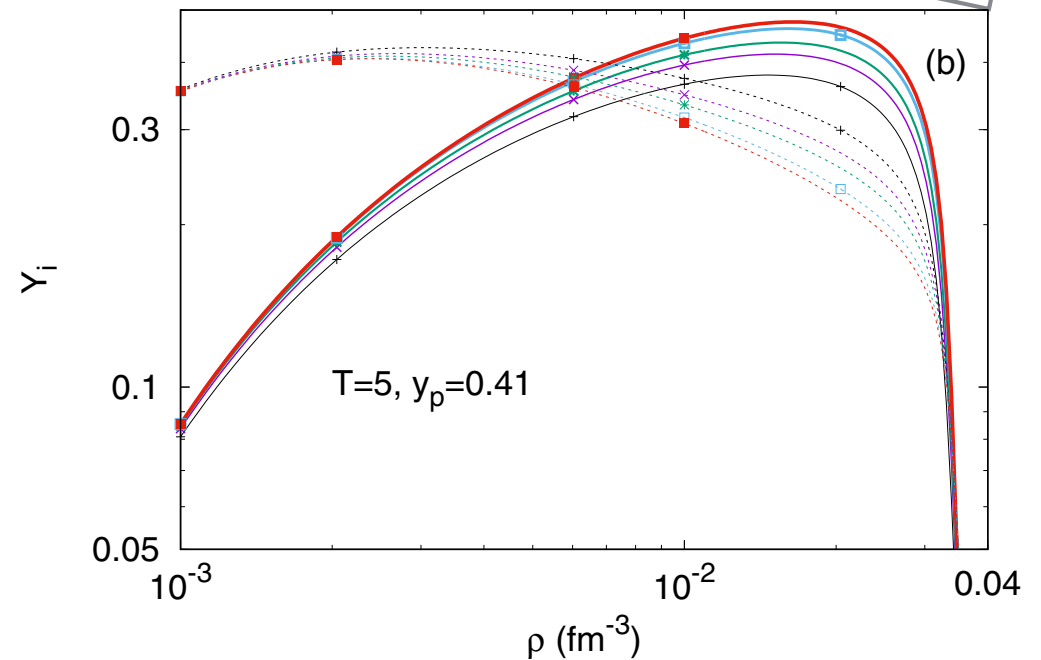
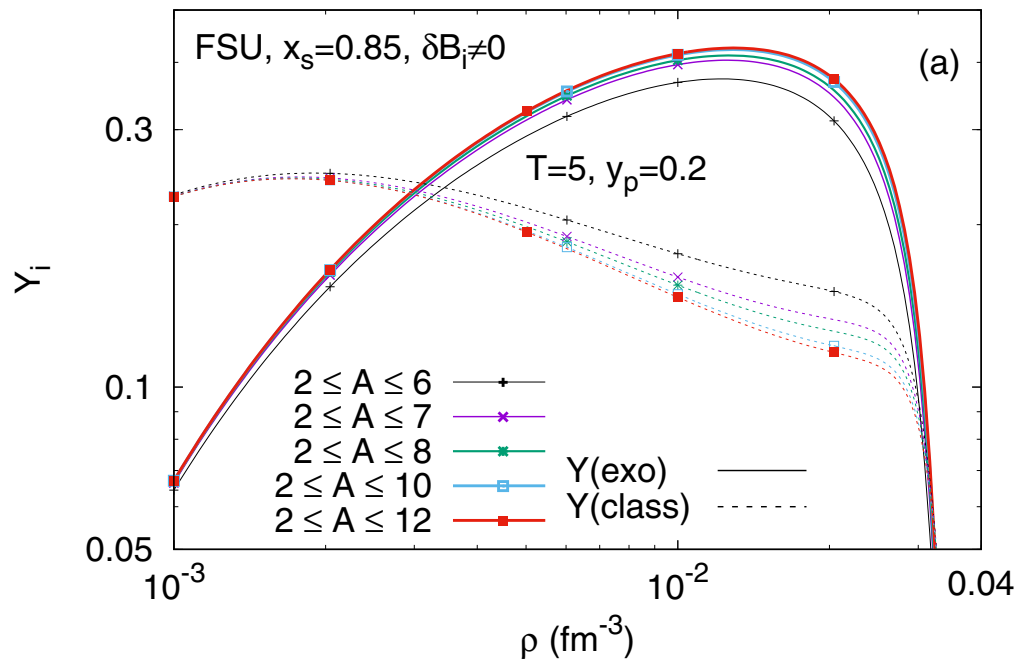
- Unique existing constraint on in-medium modifications of light clusters at finite T



- Our model describes quite well experimental data!

Light clusters: classical + exotic

PRC 99, 055806 2019



$$Y_{\text{light}} = \sum_{i=2}^{A_{\text{max}}} Y_i$$

$$Y_{\text{class}} = Y_d + Y_t + Y_h + Y_{\alpha}.$$

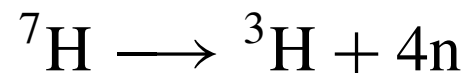
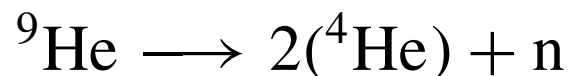
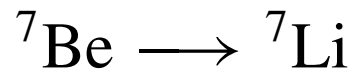
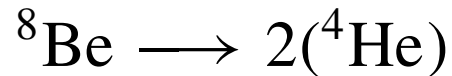
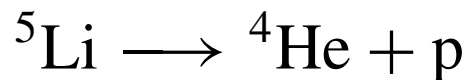
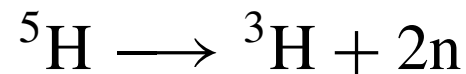
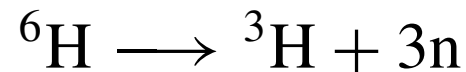
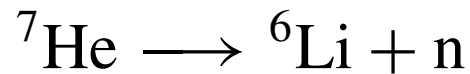
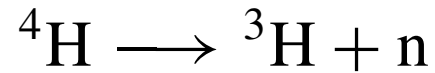
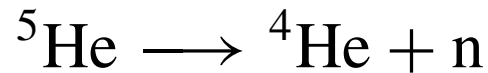
$$Y_{\text{exo}} = Y_{\text{light}} - Y_{\text{class}}$$

- The largest contribution of the exotic clusters occurs at the maximum of the distribution of the clusters.

Decays and effective densities

PRC 99, 055806 2019

- Decay modes:



- Effective densities:

$$\tilde{\rho}^4_{\text{He}} = \rho^4_{\text{He}} + \rho^5_{\text{He}} + \rho^5_{\text{Li}} + 2\rho^8_{\text{Be}} + 2\rho^9_{\text{He}}$$

$$\tilde{\rho}^3_{\text{H}} = \rho^3_{\text{H}} + \rho^4_{\text{H}} + \rho^5_{\text{H}} + \rho^6_{\text{H}} + \rho^7_{\text{H}}$$

$$\tilde{\rho}^6_{\text{Li}} = \rho^6_{\text{Li}} + \rho^7_{\text{He}}$$

$$\tilde{\rho}^7_{\text{Li}} = \rho^7_{\text{Li}} + \rho^7_{\text{Be}}$$

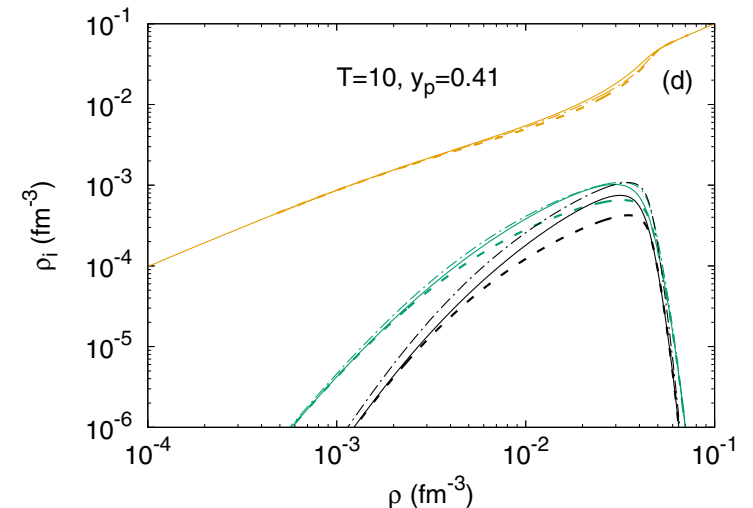
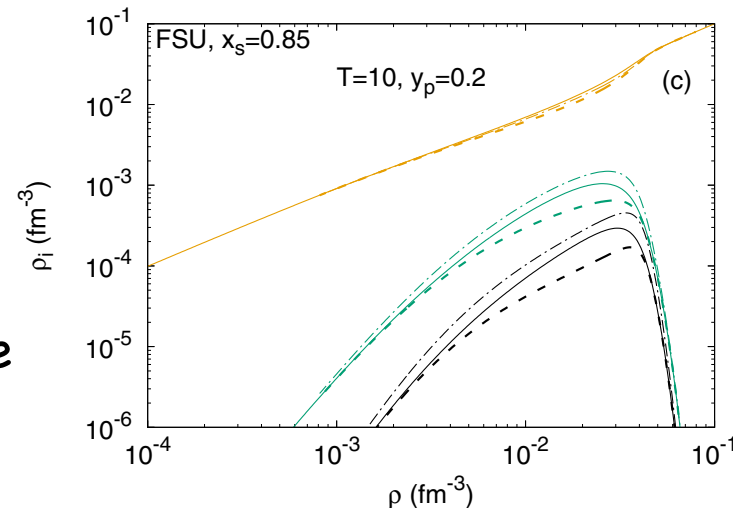
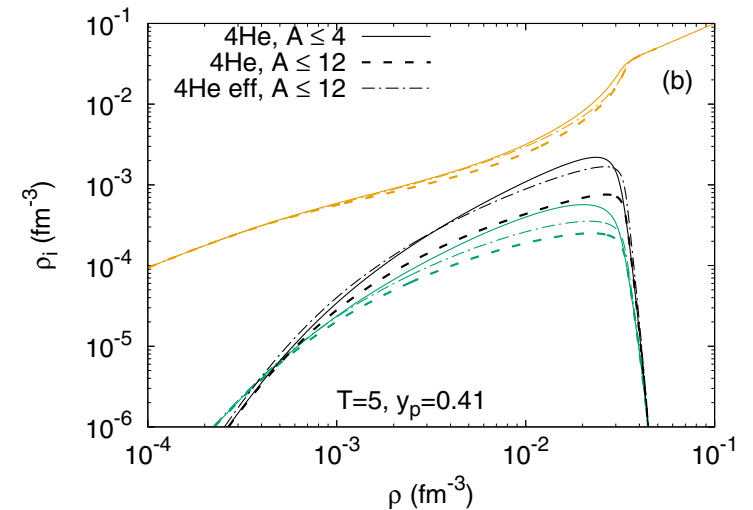
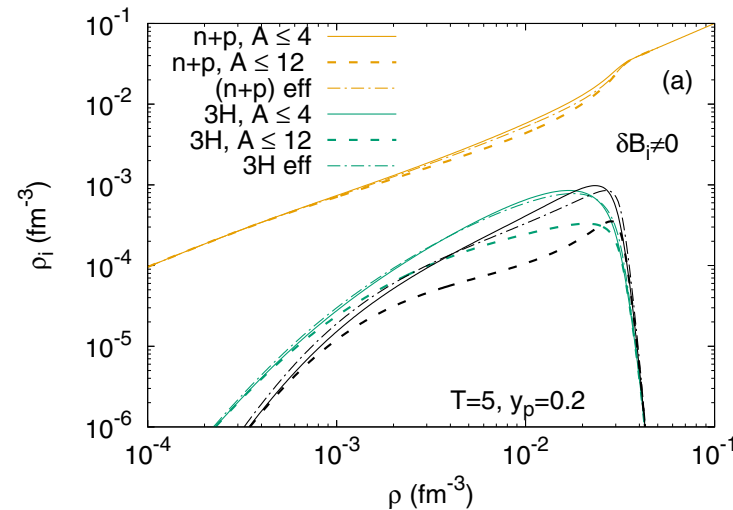
$$\begin{aligned}\tilde{\rho}_n = & \rho_n + \rho^5_{\text{He}} + \rho^4_{\text{H}} + \rho^7_{\text{He}} + 3\rho^6_{\text{H}} \\ & + 2\rho^5_{\text{H}} + \rho^9_{\text{He}} + 4\rho^7_{\text{H}}\end{aligned}$$

$$\tilde{\rho}_p = \rho_p + \rho^5_{\text{Li}}$$

Decays and effective densities

PRC 99, 055806 2019

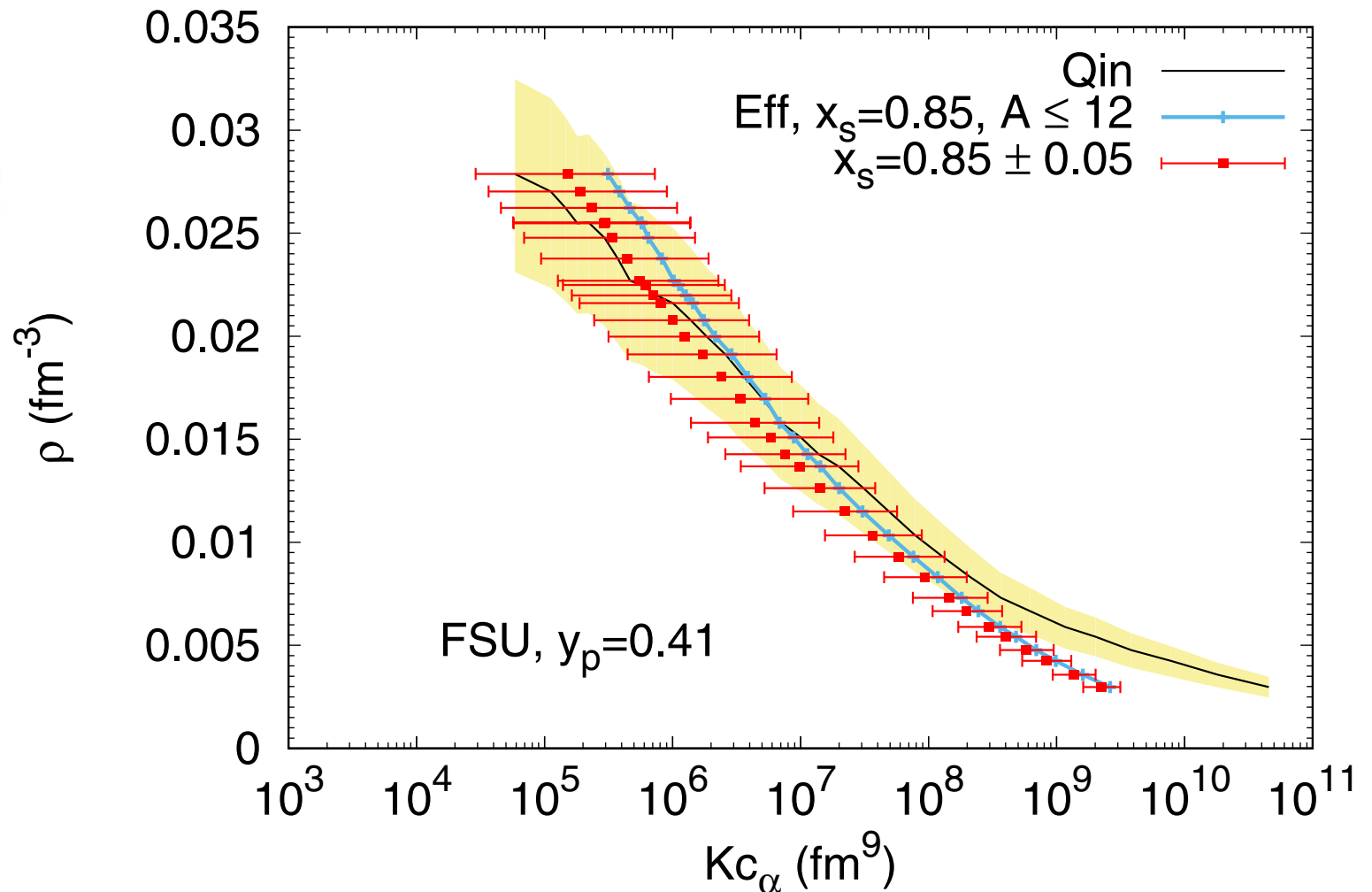
- Exotic clusters are non-negligible at intermediate densities.
- For low T , at the peak of the distribution: the mass fractions without exotic are more abundant.
- For high T , the opposite happens: there is an increase of the effective.



Equilibrium constants with exotic clusters

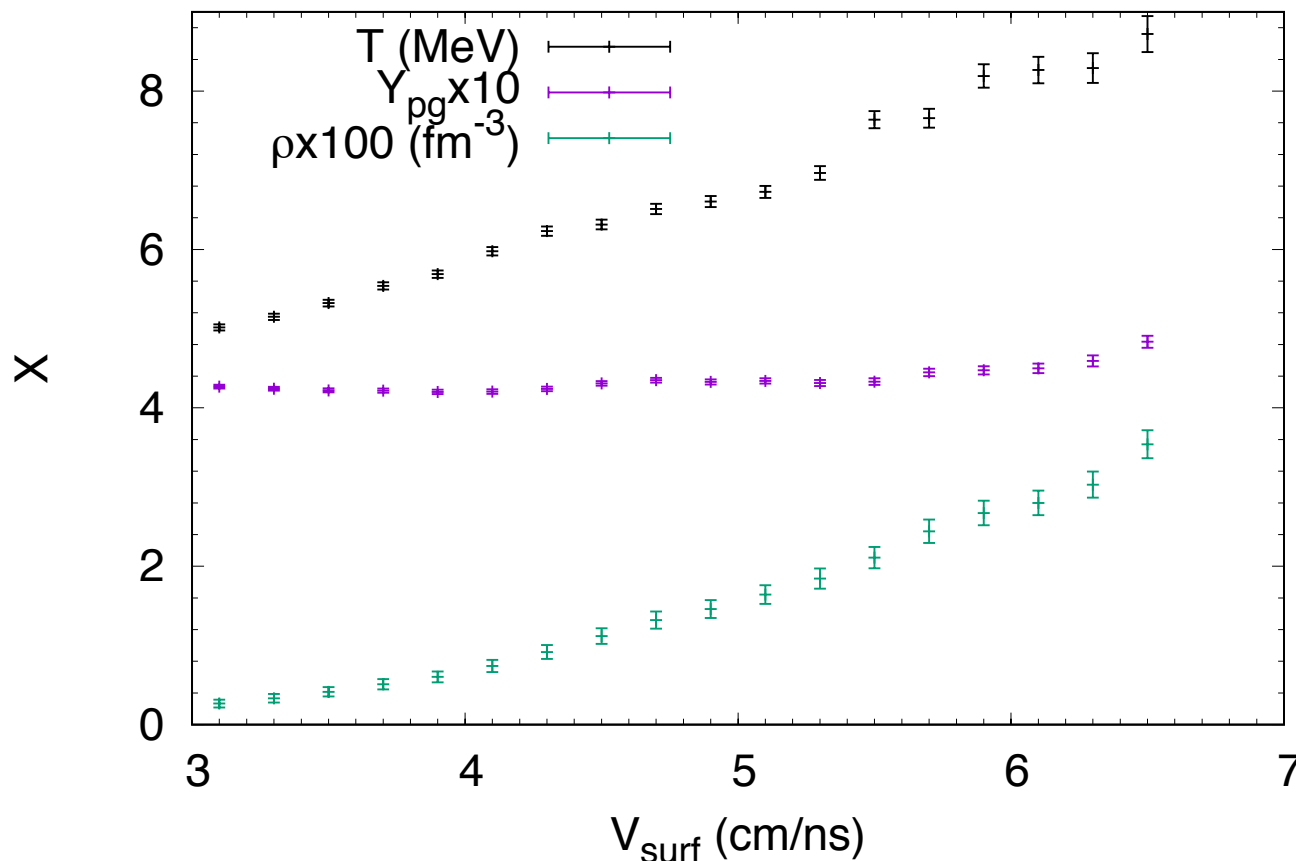
$$K_c[j] = \frac{\rho_j}{\rho_n^{N_j} \rho_p^{Z_j}}$$

- Contribution of exotic clusters are non-negligible at higher densities.



Experimental chemical equilibrium constants with INDRA data

- Experimental data includes 4He , 3He , 3H , 2H , and 6He .
- 3 experimental systems: $^{136}\text{Xe}+^{124}\text{Sn}$, $^{124}\text{Xe}+^{124}\text{Sn}$, and $^{124}\text{Xe}+^{112}\text{Sn}$.



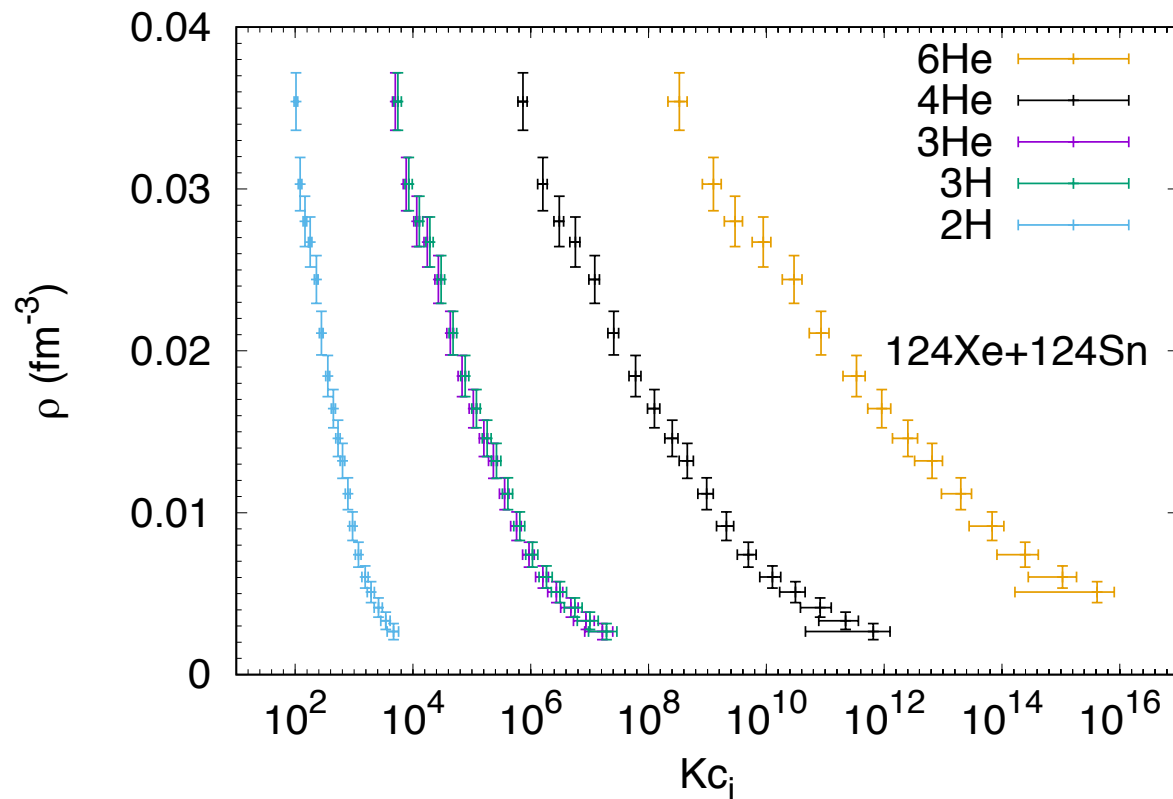
R. Bougault et al, for the INDRA collab, submitted to J. Phys. G (2019)

- V_{surf} is the velocity of the emitted particles at the nuclear surface, so fastest particles correspond to earliest emission times.

- The temperature, proton fraction and density as a function of V_{surf} , for the intermediate mass system.

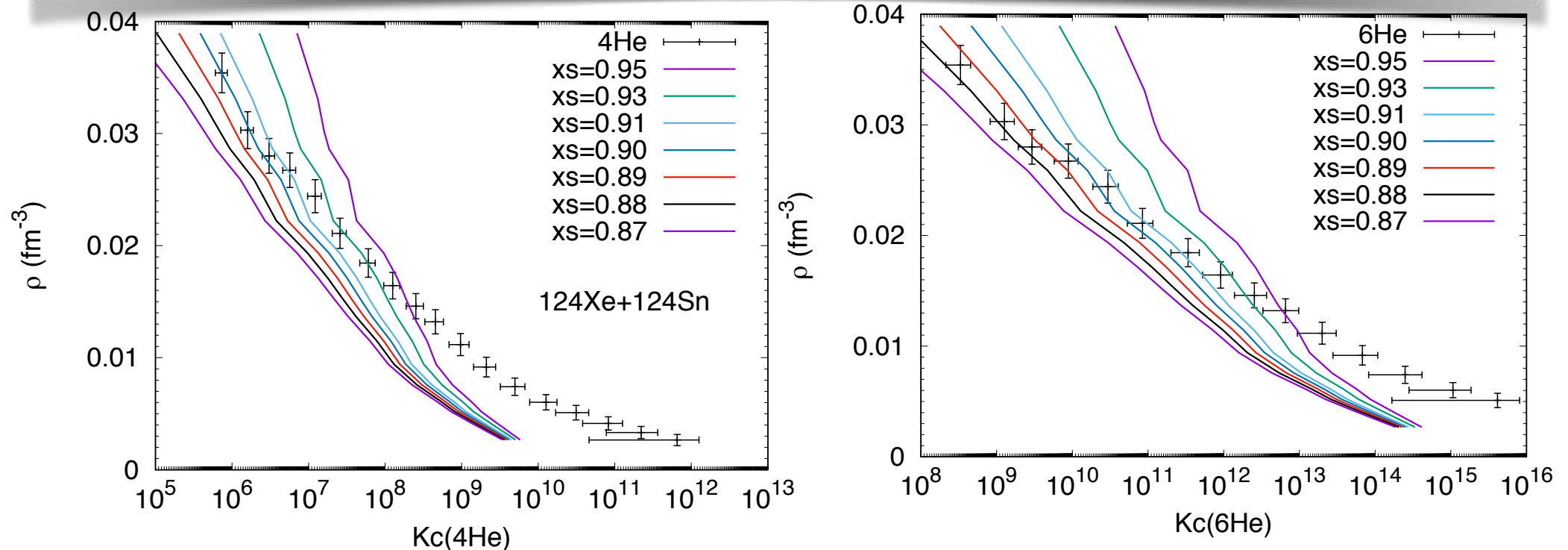
Experimental chemical equilibrium constants with INDRA data

R. Bougault et al, for the INDRA collab, submitted to J. Phys. G (2019)



- Here the chemical equilibrium constants are for the intermediate-mass system, $124\text{Xe}+124\text{Sn}$.

Experimental chemical equilibrium constants with INDRA data

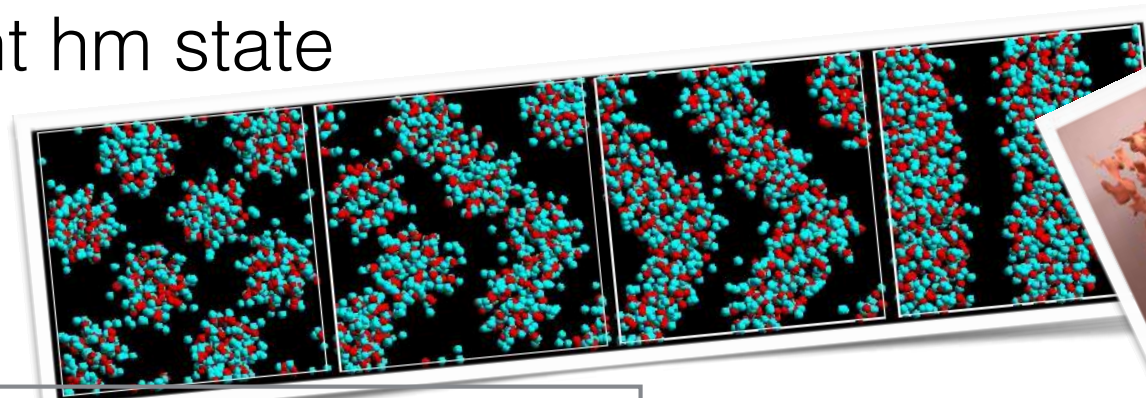


- When we apply our model we need a higher x_s to fit this data, 0.88 (6He)-0.91 (4He), as compared to Qin data, that prefers 0.85.
- The low-density region is very underestimated, and the high-density region is well reproduced.
- The Qin fit is slightly better, but qualitatively it is the same.
 - More work is needed to understand this behaviour.

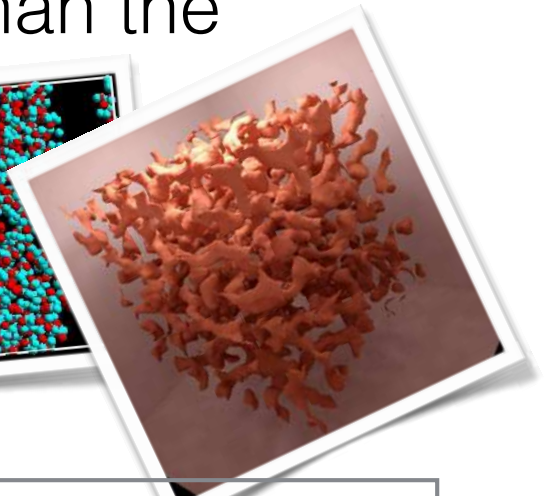
The pasta phases

- Competition between Coulomb and nuclear forces leads to frustrated system
- Geometrical structures, the **pasta phases**, evolve with density until they melt $\rightarrow$ **crust-core transition**
- Criterium: pasta free energy must be lower than the correspondent hm state

QMD calculations:

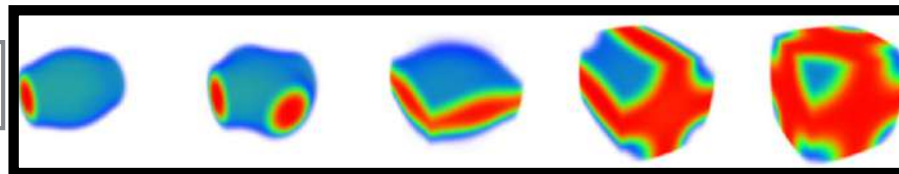


G. Watanabe *et al*, PRL 103, 121101, 2009



C. J. Horowitz *et al*, PRC 70, 065806, 2004

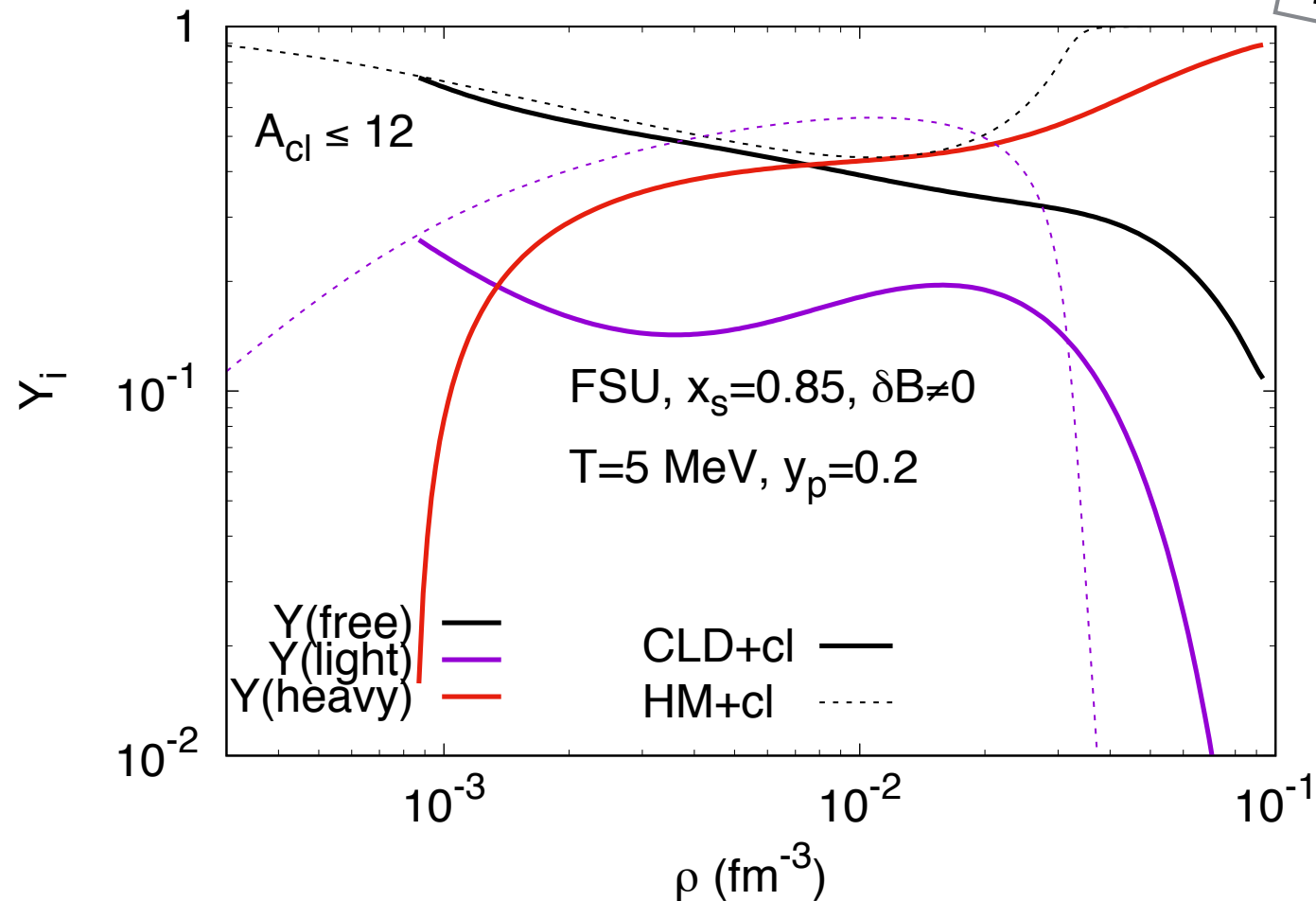
3D-SHF calculation:



Pais and Stone, PRL 109, 151101, 2012

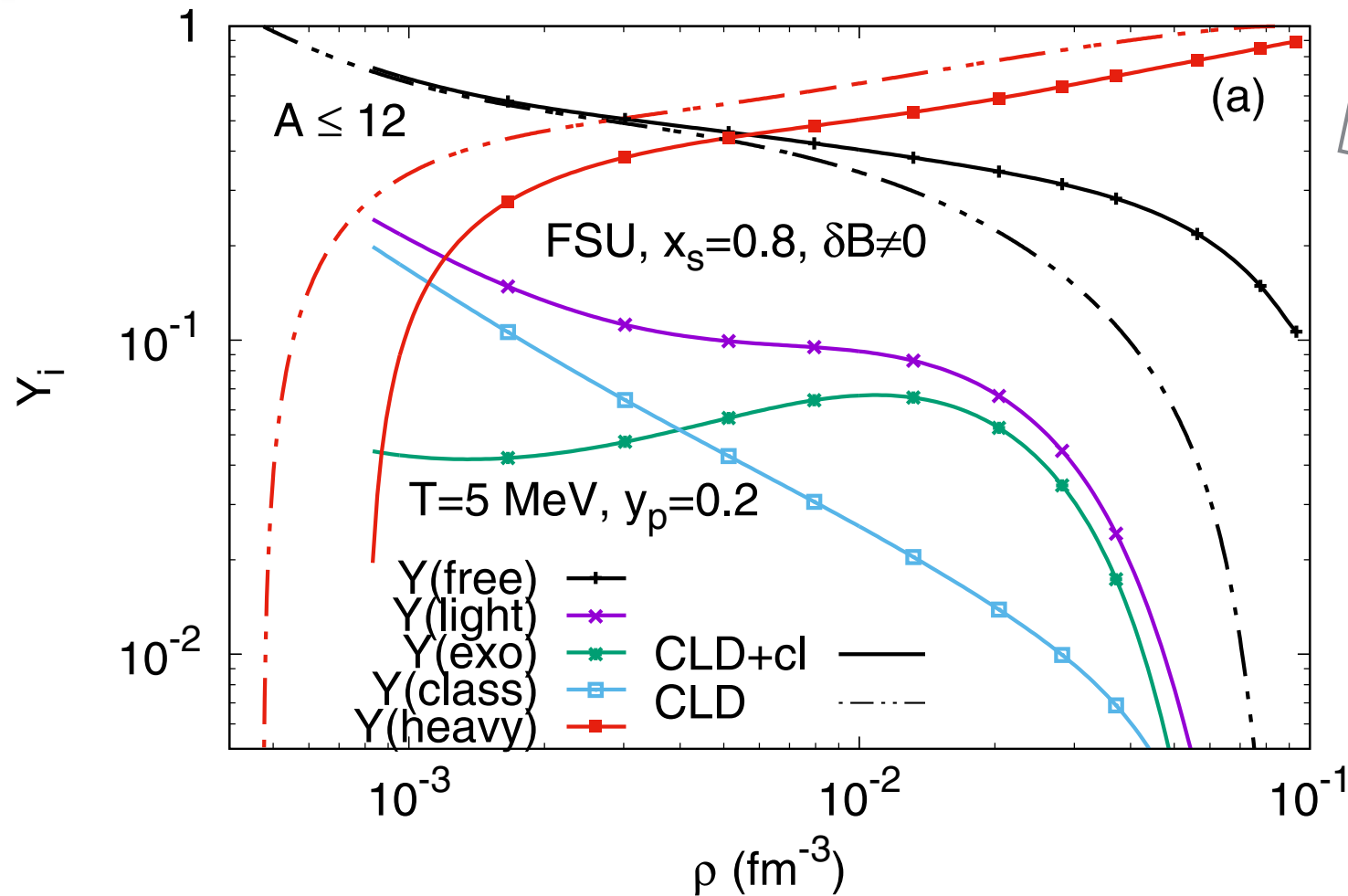
Cluster fractions - CLD vs HM

- Heavy cluster with light clusters (CLD+cl) **VS.** homogeneous matter with light clusters (HM+cl).
- Light clusters with $A \leq 12$.



- The heavy cluster (CLD+cl) calculation: light clusters less abundant but increase their melting density.
- Increasing T : the onset of both heavy and light clusters moves to larger densities.

Pasta versus Cluster fractions with pastas



PRC 99, 055806 2019

The inclusion of light clusters

- moves the onset of the heavy cluster to larger densities
- reduces the mass fraction of the heavy cluster
- increases the fraction of free nucleons in the background

Summary

Thank you!

- A simple parametrisation of in-medium effects acting on light clusters is proposed in a RMF framework.
- Interactions of clusters with medium described by modification of sigma-meson coupling constant.
- Clusters dissolution obtained by the density-dependent extra term on the binding energy.
- $x_{sj} = 0.85 \pm 0.05$ reproduces both virial limit and Kc from Texas HIC data.
- Exotic clusters ($4 < A < 12$) have effects on the clusters abundances and equilibrium constants.
- Light clusters and pasta structures are relevant and should be explicitly included in EoS for CCSN simulations and NS mergers.
- The two data sets, Texas and INDRA, are compatible, but one prefers a lower x_s than the other. More work is needed to understand this behaviour.