A Nucleation Model Analysis of Neck emission yields in Heavy Ion-Reactions

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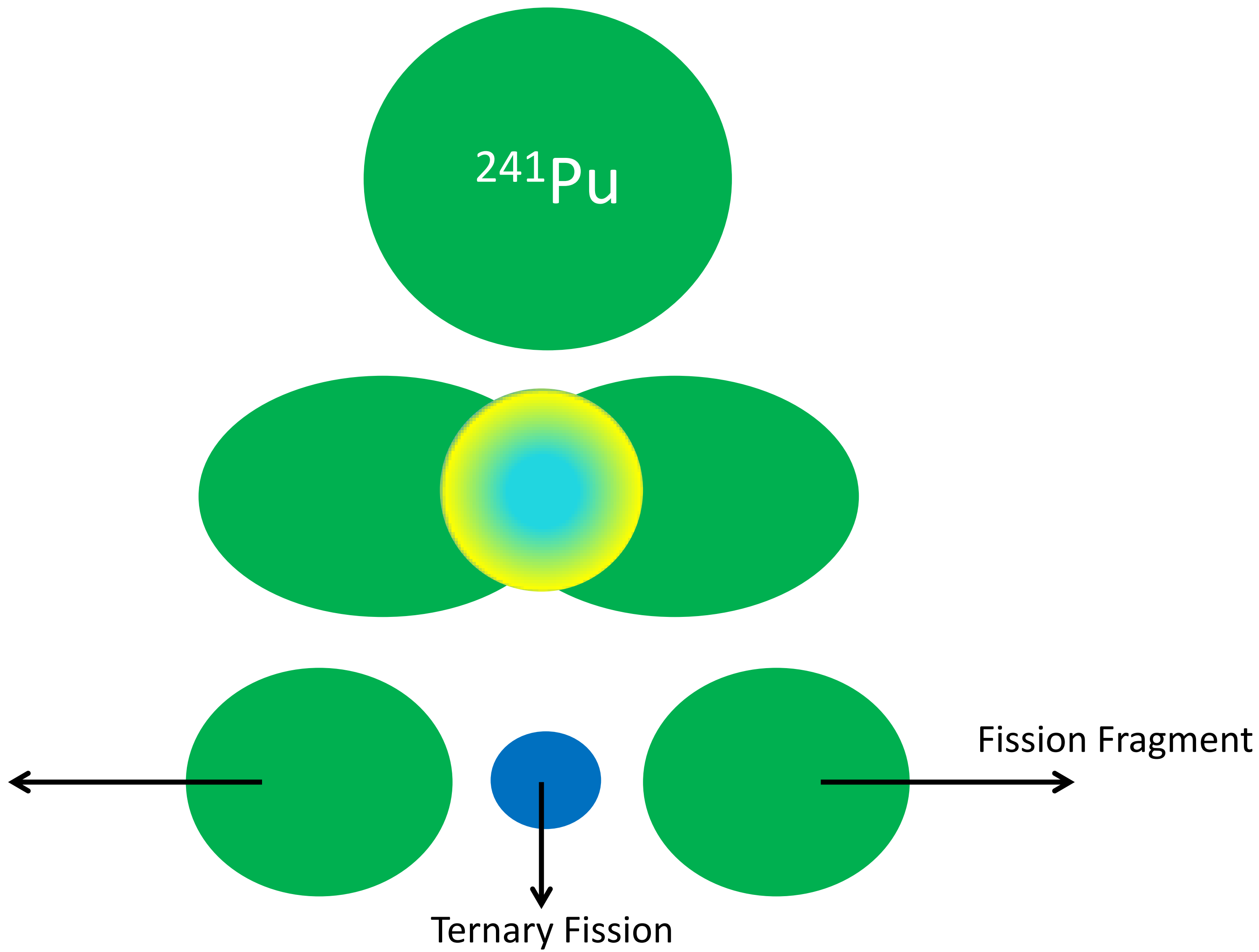
Light clusters in nuclei and nuclear matter: Nuclear structure and decay, heavy ion collisions, and astrophysics

Ternary fission: about 0.3% of heavy nucleus fissions produce a third fragment coming from the low density neck region.



Most of those fragments are α particles.

A high yield of tritons relative to protons is observed and the heavy fragment yields are decreasing with Z.

The use of a nuclear statistical equilibrium (NSE) model based on the chemical potential for low density and temperature is in pretty good agreement with experimental observations for A≤15.

$$\mu(Z,A) = Z\mu_p + (A-Z)\mu_n$$

$$\mu(Z,A) = m(Z,A)c^2 + kT\ln\left\{\frac{\rho N_A Y(Z,A)}{G(Z,A)}\left[\frac{h^2}{2\pi m(Z,A)kT}\right]^{3/2}\right\}$$

To reproduce the heavier fragment yields, one has to take into account the reaction time and the critical cluster size (nucleation).

$$Y(A,\tau) = \frac{1}{2}\rho\exp\left[-\frac{G(A)}{T}\right] \times \operatorname{erfc}\left\{B(T,\sigma)\frac{[(A/A_c)^{1/3}-1]+\left(1-A_c^{-1/3}\right)\exp(-\tau)}{\sqrt{1-\exp(-2\tau)}}\right\},$$

$$B(T,\sigma) = 2R_0\left(\frac{\pi\sigma}{T}\right)^{1/2}A_c^{1/3}, \qquad \tau = \frac{3.967c\rho}{A_c^{2/3}\sqrt{T}}t$$

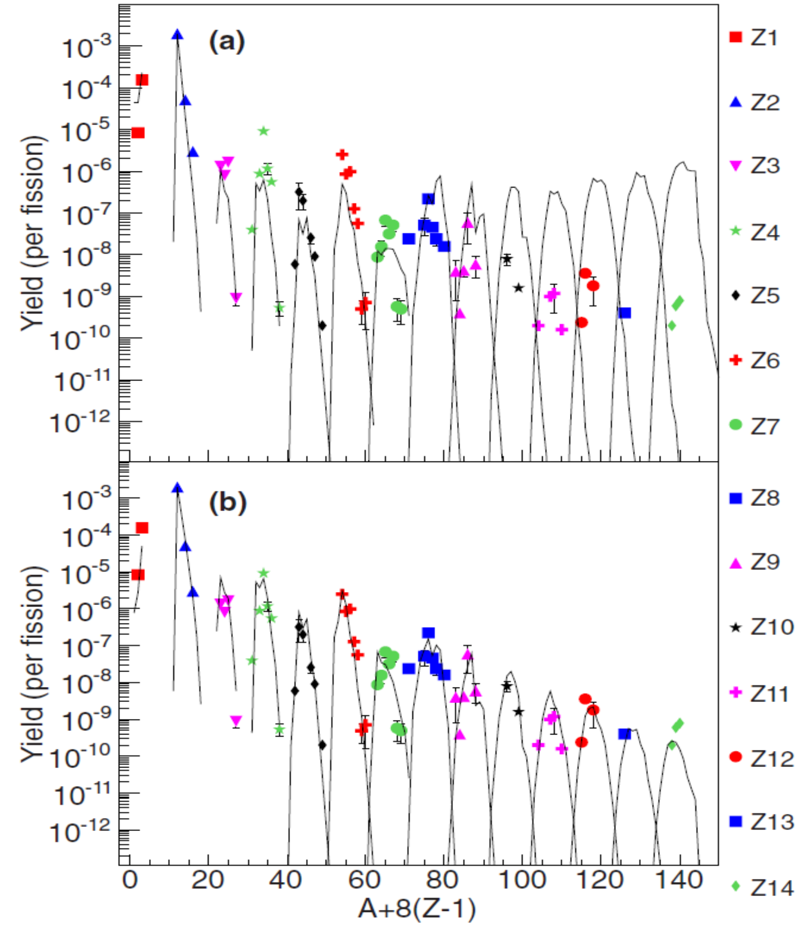


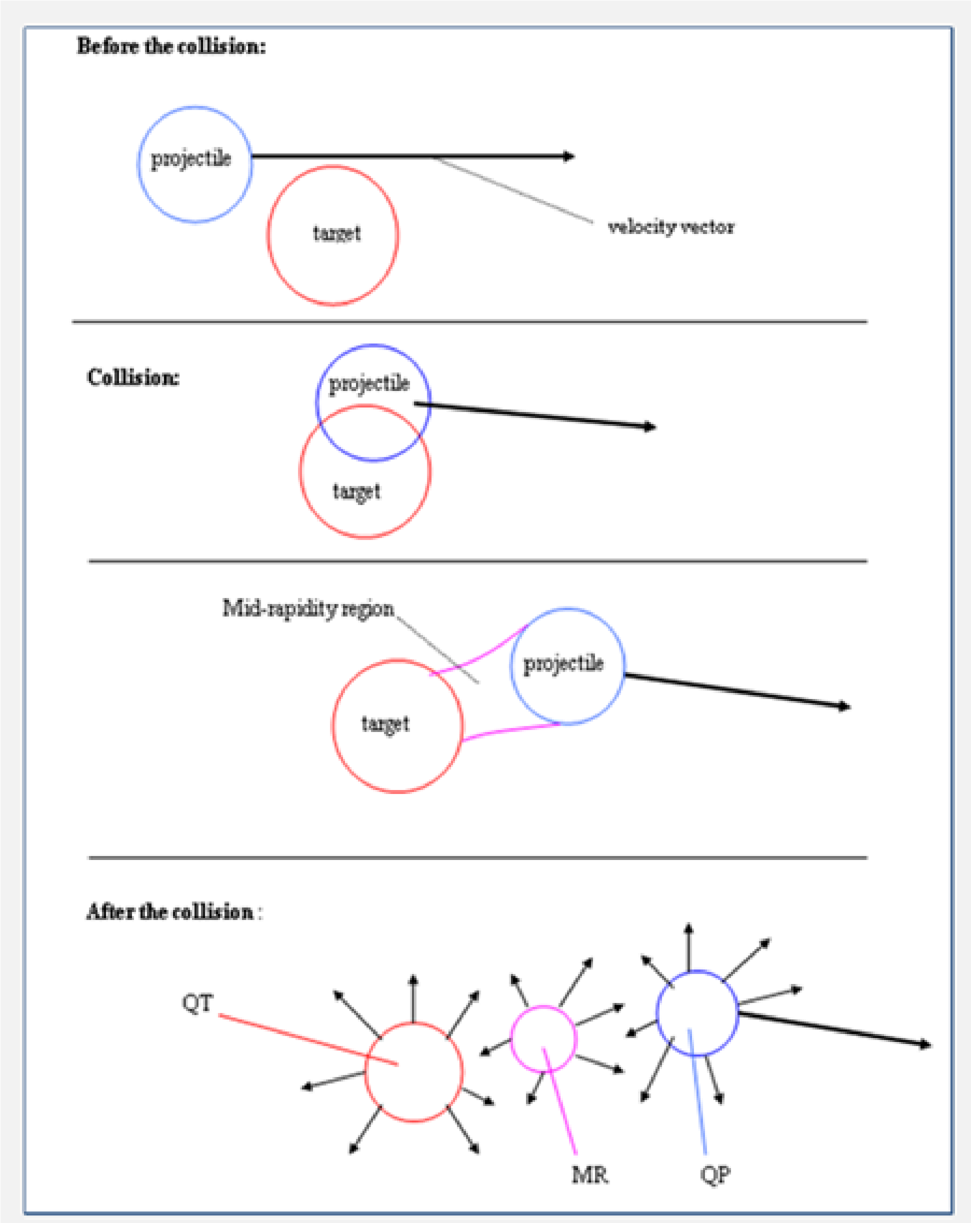
FIG. 2. (Color online) Yield per fission as a function of mass (A) and charge (Z) of products. Solid points represent ^{241}Pu(n_{th}, f) experimental yields from Koester *et al.* [9]. Lines are theoretical predictions from NSE calculation [7]. NSE parameters are $T = 1.4$ MeV, $\rho = 4\times10^{-4}$ fm^{-3}, and $Y_p = 0.34$. (a) NSE calculation only. M^2 fit metric = 4.28. (b) NSE calculation with nucleation. Nucleation parameters are time = 6400 fm/c and $A_c = 5.4$. Fit metric = 1.18.

S. Wuenschel et al., Physical Review C 90, 011601 (2014)

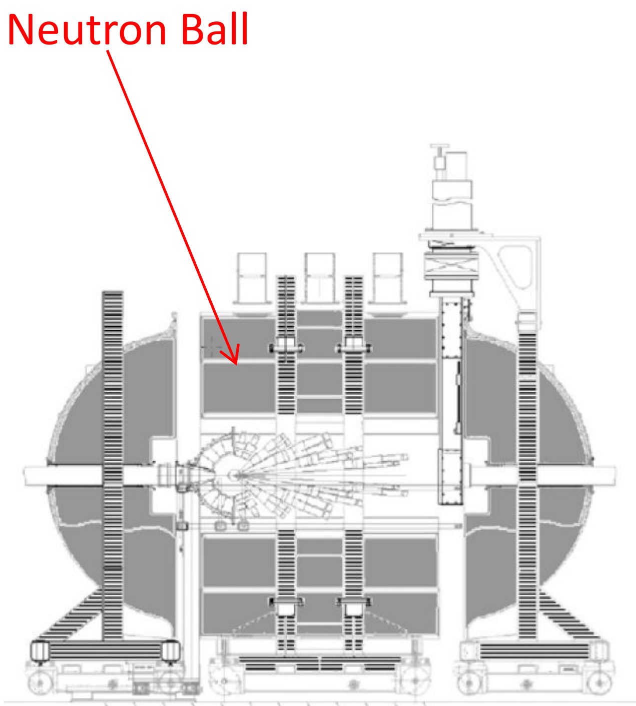
Mid-peripheral collisions with a heavy system are similar to the fission process but at higher temperature and density.

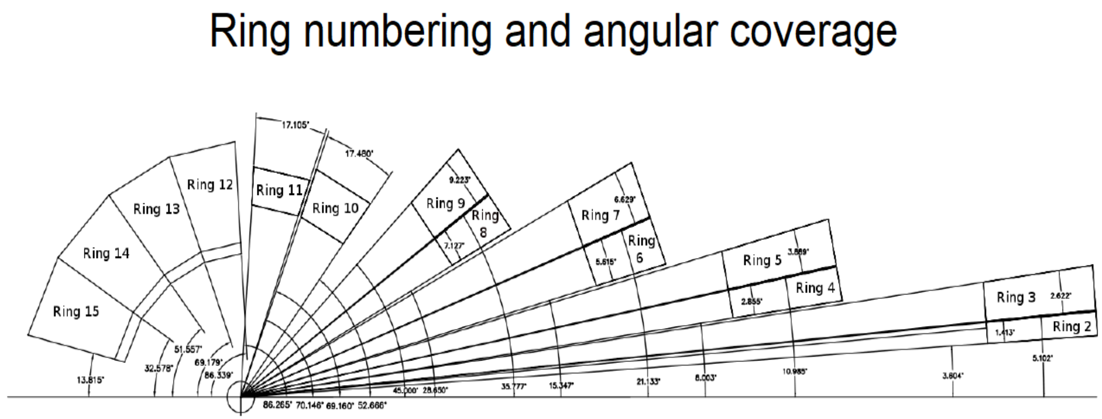
We want to see if NSE with nucleation (NESC) calculation can reproduce the mid-rapidity (ternary fission like) isotopic yields.

We use ^{124}**Sn+**^{124}**Sn** and ^{124}**Sn+**^{112}**Sn** at **26*A* MeV** from the December 2007 **NIMROD** experiment.



NIMROD Detection Array

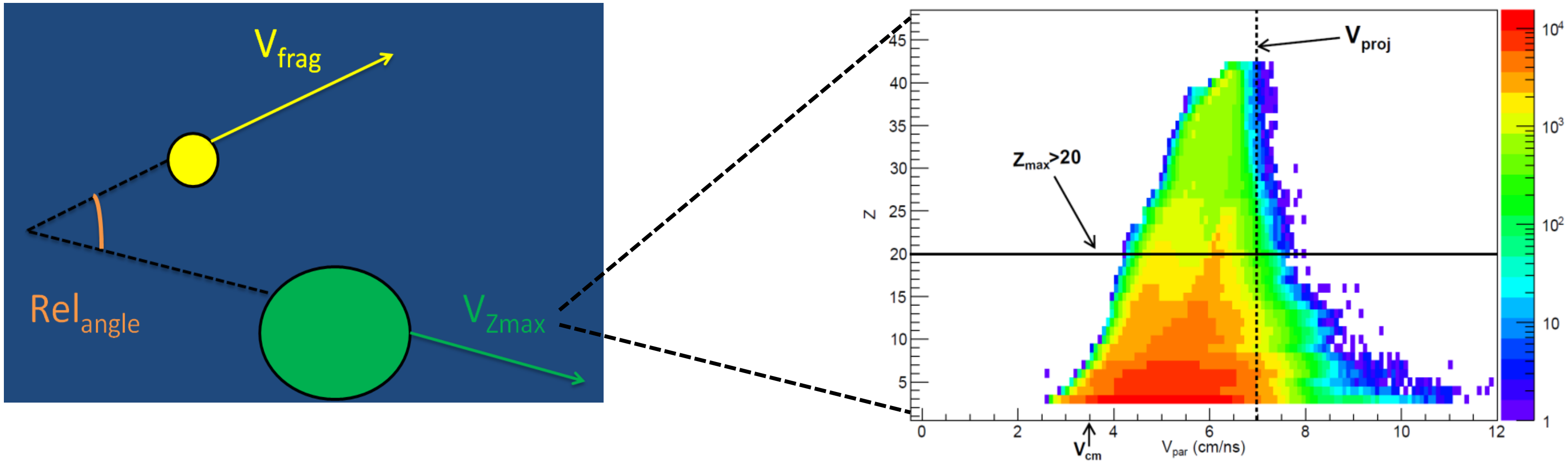




- 156 CsI(Tl) distributed on 11 rings from 3 to 100º.
- 100 Si (300µm)-CsI(Tl) telescopes from ring 2 to 9.
- 30 Si(150µm)-Si(500µ)-CsI(Tl) super telescopes from ring 2 to 9.
- 4π neutron detector array (Neutron Ball).

Relative angle

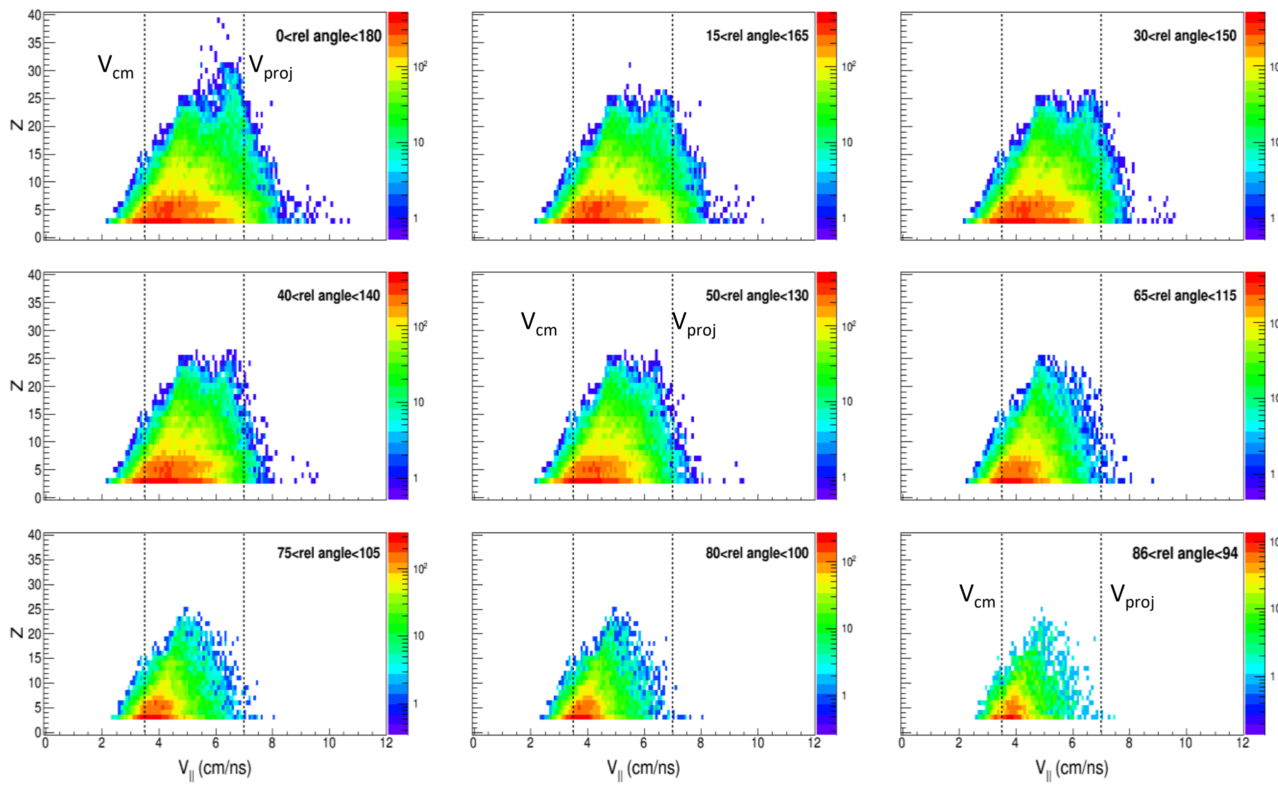
$Z_{max} > 20$



To select the ternary-like-fragments the relative angle (in the center-of-mass) should be close to **90º**.

But the statistic goes down very quickly when shrinking that window.

To maximize the statistic, the **50-130º** selection seems to be a good compromise.



At our first attempt we got reasonable values for:

•Temperature
•Density
•Fit metric

But we got some unexpected results for:

•The time (~6000 fm/c) was in good agreement with quasi-fission and fusion-fission neutron evaporation measurements but is too long for peripheral and semi-peripheral reactions (a few hundreds of fm/c).

•The critical cluster size (~16) was also too high even if we take into account that we are not including hydrogen and helium isotopes in the fit.

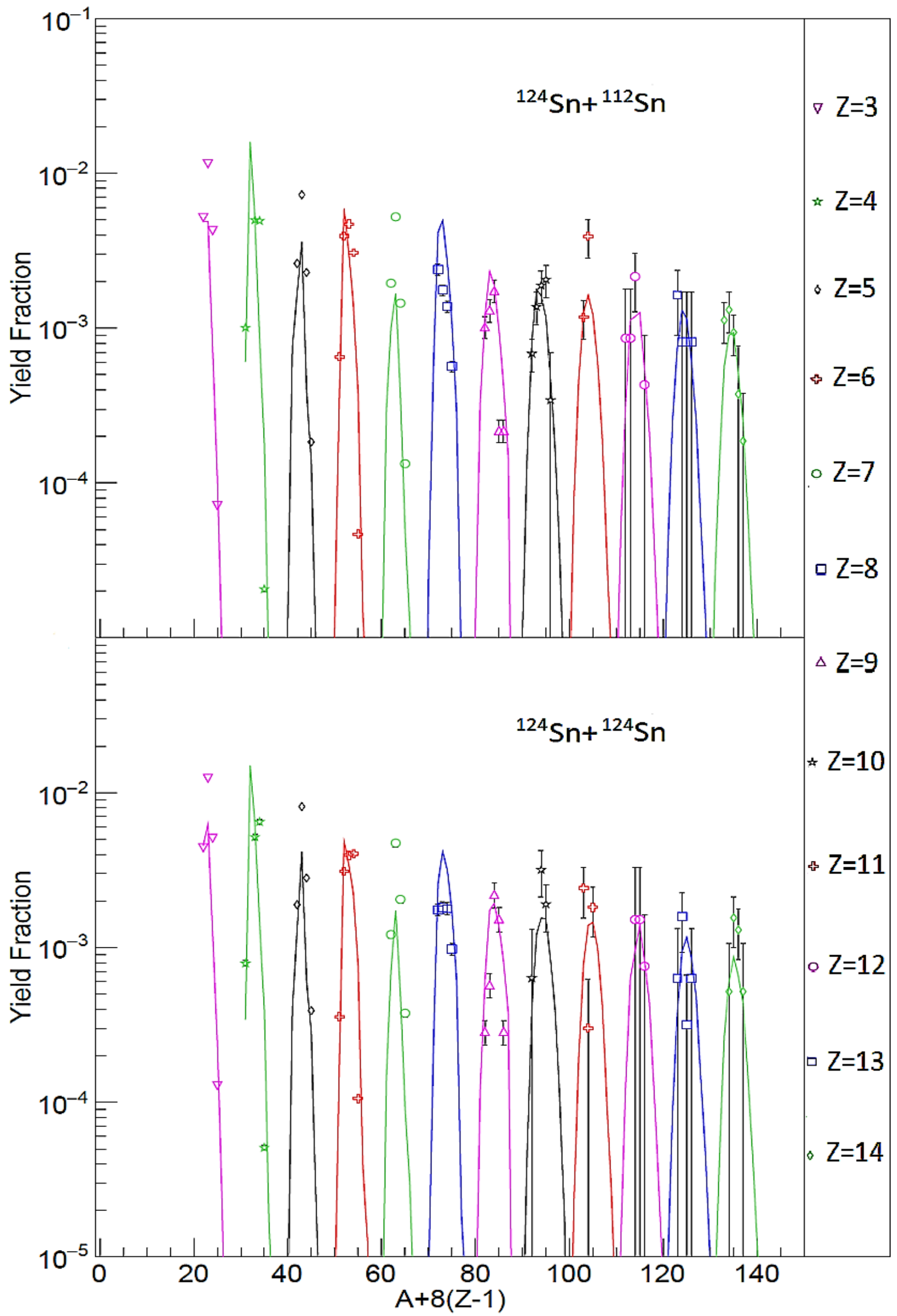
So we kept on working:

•Improved the minimization code
•Improve yield normalization
•Tried different set of initial parameters to find other minima

Fit Parameters

System	^{112}Sn target	^{124}Sn target	^{241}Pu
Temperature (MeV)	2.52	2.43	1.4
Density (10-4 fm-3)	194	150	4
Time (fm/c)	600	950	6400
Ac	6.3	6.2	5.4
Proton ratio (system)	0.47 (0.42)	0.46 (0.40)	0.34 (0.39)
Fit Metric (M^2)	*0.72*	*0.75*	*1.18*

$$M^2 = \sum_j \{ln[Y_{TF}^{exp}(Z_j, A_j)] - ln[Y_{TF}(Z_j, A_j)]\}^2/n$$



Albergo temperature* comparison

$$T_{\text{app}} = \frac{B}{\ln(aR_{\text{app}})} \qquad \frac{1}{T_{\text{app}}} = \frac{1}{T_o} + \frac{\ln k}{B}\ ^{**}$$

$$R = \frac{Y(A_i, Z_i)/Y(A_i + \Delta A, Z_i + \Delta Z)}{Y(A_j, Z_j)/Y(A_j + \Delta A, Z_j + \Delta Z)}$$

$$B = BE(A_i, Z_i) - BE(A_i + \Delta A, Z_i + \Delta Z) - BE(A_j, Z_j) + BE(A_j + \Delta A, Z_j + \Delta Z)$$

$$a = \frac{[2S(A_j, Z_j) + 1]/[2S(A_j, +\Delta A, Z_j + \Delta Z) + 1]}{[2S(A_i, Z_i) + 1]/[2S(A_i, +\Delta A, Z_i + \Delta Z) + 1]} \times \left[\frac{A_j/(A_j + \Delta A_j)}{A_i/(A_i + \Delta A_i)}\right]^{\eta}$$

*S. Albergo et al., Il Nuovo Cimento, vol. 89 A, N. 1 (1985)
**M. B. Tsang, W. G. Lynch, H. Xi, and W. A. Friedman, Phys. Rev. Lett. 78, 3836 (1997)

Albergo temperature comparison

- For $^{2,3}H/^{3,4}He$ within our selection window, we got **T= 2.7 MeV** for $^{124}Sn+^{112}Sn$.

- It's very close to the fit value but the source overlapping is very important for light particles and most are not from the neck.

- So we tried IMF ratios with $^{11,12}C$:

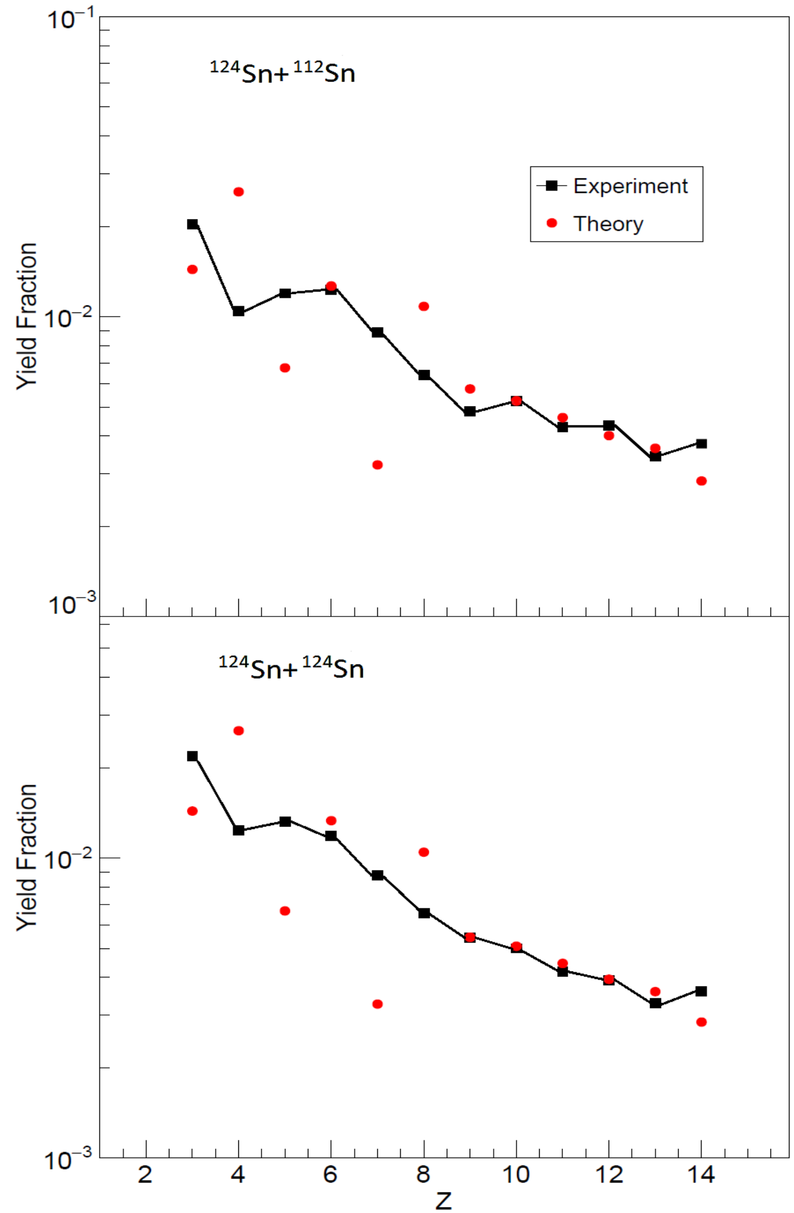
	T_^{112}Sn	T_^{124}Sn
$^{6,7}Li/^{11,12}C$	3.57	3.42
$^{7,8}Li/^{11,12}C$	4.26	3.98
$^{8,9}Li/^{11,12}C$	2.19	2.2
$^{9,10}Be/^{11,12}C$	3.96	3.8
$^{11,12}B/^{11,12}C$	3.94	3.68
$^{12,13}B/^{11,12}C$	2.72	2.82
$^{12,13}C/^{11,12}C$	3.76	3.46
$^{13,14}C/^{11,12}C$	3.94	4.06
$^{15,16}N/^{11,12}C$	3.98	4.05
$^{16,17}O/^{11,12}C$	3.62	3.58
$^{17,18}O/^{11,12}C$	3.55	3.4
Mean (min|max)	**3.59 (2.19|4.26)**	**3.49 (2.2|4.06)**

- The average is higher than the fit values (2.52 and 2.43) but we are still in a realistic range and inside the min-max interval.

- Also the presence of excited states in the case of the experiment can create discrepancies with the model.

What if we move the relative angle window?

- In order to have a sufficient statistic in each window we fit with Z instead of A.
- For each element we assign a A equal to the most probable mass observed in our data for this specific charge.
- These Z distributions show similar trend than the isotopic ones.
- The Z fit gives better fit metric values.
- Temperature, density and time are slightly smaller.



What if we move the relative angle window?

- We do Z fits with twenty degrees relative angle windows from 20^o to 140^o.
- Low relative angle selections must correspond mostly to QP emission while high ones are more related to the neck region.

Fit results for $^{124}Sn+^{124}Sn$

Rel angle (deg)	T (MeV)	density (10-4 fm-3)	Yp	time (fm/c)	Ac	metric
20-40	2.14	187	0.46	140	6.1	0.234831
40-60	2.15	159.6	0.45	320	6.2	0.242627
60-80	2.19	151	0.45	260	6.6	0.231922
80-100	2.22	138.6	0.46	230	6.2	0.248491
100-120	2.43	108.4	0.48	290	6.1	0.290383
120-140	2.87	102.4	0.5	290	6.2	0.28903

- The temperature is increasing and the density is decreasing as a function of the relative angle selection: could be in agreement with a shift from the QP to the mid-rapidity.
- The time parameter doesn’t show any trend but the lower selection has a much smaller value and would be in agreement with QP emission from peripheral collision => short time

Theoretical calculations by Gerd Roepke (preliminary results)

In thermodynamic equilibrium $Y_{A,Z}$ is proportional to the partial density $n_{A,Z}$:

$$n_{A,Z} = \sum_{P,\nu} g_{A,Z,\nu} e^{-(E_{A,Z,\nu}(P)-(A-Z)\mu_n - Z\mu_p)/T} = \left(\frac{2\pi\hbar^2}{AmT}\right)^{-3/2} z_{A,Z}(T) e^{((A-Z)\mu_n + Z\mu_p)/T}$$

where $g_{A,Z,\nu}$ is the degeneration factor of the intrinsic state ν. We assumed $E_{A,Z,\nu}(P) = E_{A,Z,\nu} + \hbar^2 P^2/2Am$, and

$$z_{A,Z}(T) = \sum_{\nu} g_{A,Z,\nu} e^{-E_{A,Z,\nu}/T} = e^{-E_{A,Z,0}/T} g_{A,Z}(T)$$

is the partial partition function for the channel A, Z.

$$\ln[Y_{A,Z}(AT)^{-3/2}/z_{A,Z}(T)] = L_{A,Z}(T)$$

$$L_{A,Z}(T) \propto ((A-Z)\mu_n + Z\mu_p)/T$$

Theoretical calculations by Gerd Roepke (preliminary results)

The isotope distribution (fragments with a given Z):

$$R_{A,Z}(T) \;=\; 2L_{A,Z}(T) - L_{A-1,Z}(T) - L_{A+1,Z}(T)$$

$$\{A,Z\} \;=\; \{11,5\},\{12,5\},\{12,6\},\{13.6\},\{15,7\},\{16,7\},\{17,8\},\{18,8\},\{19,9\},\{20,9\}$$

$$R_{\mathrm{ave}}(T) = (1/N)\sum_{A,Z} R_{A,Z}(T)$$

T is measured at $R_{ave}(T)=0$

$\sigma^2 = (1/N)\sum_{A,Z}[R_{A,Z}(T) - R_{\mathrm{ave}}(T)]^2$

Reaction	T (MeV)	σ^2	$T_{NSE+Nucleation}$
$^{124}Sn+^{112}Sn$ (NSE+Nucleation)	2.32 (2.52)	0.67	2.52
$^{124}Sn+^{124}Sn$ (NSE+Nucleation)	2.91 (2.43)	0.41	2.43

Neglecting excited states:

Reaction	T (MeV)	σ^2
$^{124}Sn+^{112}Sn$	1.92	3.32
$^{124}Sn+^{124}Sn$	2.21	2.994

Theoretical calculations by Gerd Roepke (preliminary results)

The isotope distribution for the neutron chemical potential:

$$R_{A,Z;n}(T) \quad = \quad [L_{A,Z}(T) \; - \; L_{A+1,Z}(T)]T \quad = \quad -\mu_n$$

Sn+Sn:

$$\{A,Z\} = \{10,5\},\{11,5\},\{12,5\},\{11,6\},\{12,6\},\{13.6\},\{14,7\},\{15,7\},$$
$$\{16,7\},\{16,8\},\{17,8\},\{18,8\},\{19,9\},\{20,9\}$$

Pu:

$$\{A,Z\} = \{7,3\},\{8,3\},\{9.4\},\{10,4\},\{11,4\},\{10,5\},\{11,5\},\{14,6\},\{15,6\},\{16.6\},\{17.6\},\{15,7\},\{16,7\}$$

Reaction	T (MeV)	μ_n (MeV)	σ^2
Sn+Sn (NSE+Nucleation)	2.28 (2.43)	-7.62	1.85
^{241}Pu (NSE+nucleation)	1.06 (1.4)	-3.86	0.94

Theoretical calculations by Gerd Roepke (preliminary results)

The isotope distribution for the proton chemical potential:

$$R_{A,Z;p}(T) \quad = \quad [L_{A,Z}(T) \; - \; L_{A+1,Z+1}(T)]T \quad = \quad -\mu_p$$

Sn+Sn:

$$\{A,Z\} \quad = \{10,4\},\{11,5\},\{12,5\},\{13.6\},\{14.6\},\{15,7\},\{16,7\},\{18,8\}$$

Pu:

$$\{A,Z\} = \{8,3\},\{9,3\},\{10,4\},\{11,4\},\{14,6\},\{15,6\},\{16.6\},\{17.6\},\{18,7\},\{19,7\},\{19,8\},\{20,8\},\{21,8\}$$

Reaction	T (MeV)	μ_n (MeV)	σ^2	η_{baryon} ($10^{-4}fm^{-3}$)	Y_p
Sn+Sn (NSE+Nucleation)	2.95 (2.43)	-13.51	2.346	1.27 (150)	0.308 (0.46)
^{241}Pu (NSE+Nucleation)	1.02 (1.4)	-16.94	0.355	? (4)	? (0.34)

Theoretical calculations by Gerd Roepke (preliminary results)

NONEQUILIBRIUM PARAMETERS TO CHARACTERIZE THE DISTRIBUTION OF YIELDS AT FREEZE OUT WITH RESPECT TO THE MASS DISTRIBUTION

$$\bar{Y}_A = \sum_Z Y_{A,Z} \propto (AT)^{3/2} \sum_Z z_{A,Z}(T) e^{\mu_A/T_A A}$$

characterized by the nonequilibrium parameters μ_A, T_A

$$R_A(T) = [\bar{L}_A(T) - \bar{L}_{A+1}(T)]T$$

$$A = 10, 11, 12, 13, 14, 15, 16, 17, 18, 19$$

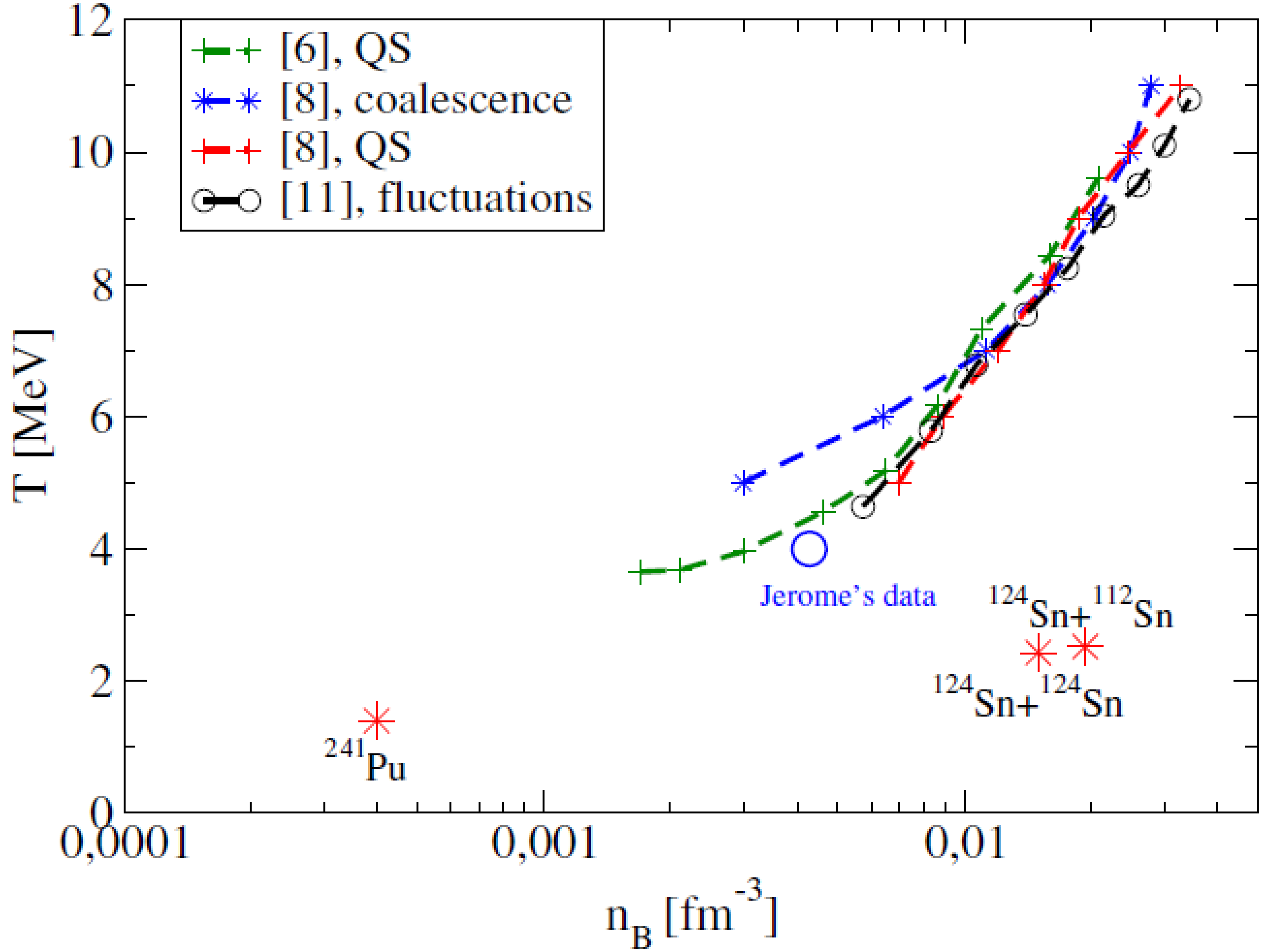
Reaction	T (MeV)	μ_n (MeV)	σ^2
^{124}Sn+^{112}Sn	4.10	-10.577	2.052
^{124}Sn+^{124}Sn	3.945	-10.506	2.189

$\Delta\mu$	η_{baryon} ($10^{-4}fm^{-3}$)	Y_p
0	33.9	0.5
5	42.8	0.37

NSE+nucleation (^{124}Sn+^{124}Sn):
T=1.43 MeV
$\rho=150\times10^{-4}fm^{-3}$
Y_p = 0.46

Theoretical calculations by Gerd Roepke (preliminary results)

NONEQUILIBRIUM PARAMETERS TO CHARACTERIZE THE DISTRIBUTION OF YIELDS AT FREEZE OUT WITH RESPECT TO THE MASS DISTRIBUTION



NSE+nucleation

Nonequilibrium calculation

Summary

- Our NSE+nucleation Sn+Sn fit temperature values are similar to the theoretical equilibration calculations but lower than the non-equilibrium calculations.

- The proton ratios from our NSE+nucleation Sn+Sn fits are higher than the calculation and the expected values for the mid-rapidity as well.

    - A possible explanation could be that our fits only take into account fragments with Z>2. The absence of the stable and neutron rich light isotopes may induce a higher overall proton number in our data.

- Our Sn+Sn fit density values are higher than both equilibrium and non-equilibrium calculations. This still needs to be investigated.

Thank you very much!