Nuclear reflection asymmetry in cluster approach

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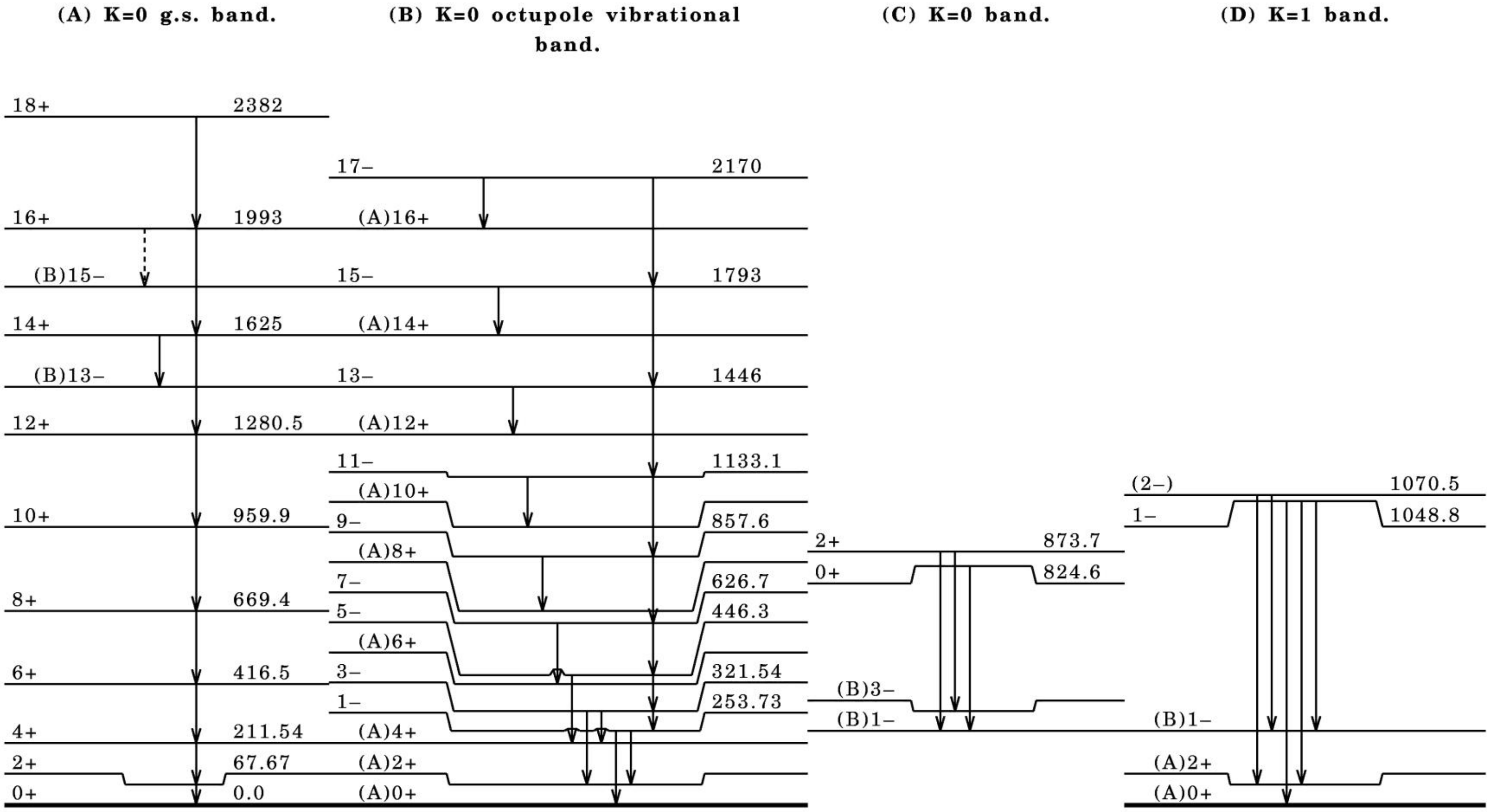
Experimental spectrum : ^{226}Ra

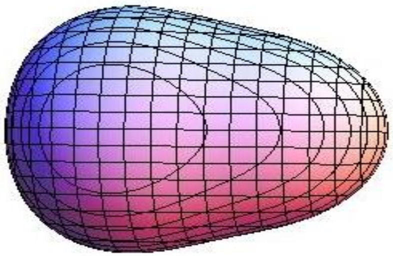


(from www.nndc.bnl.gov/ensdf)

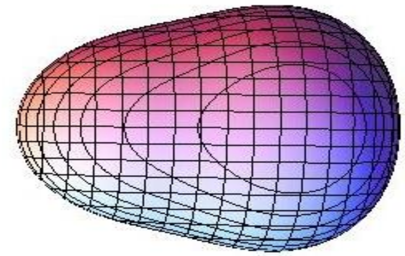
Reflection Asymmetric Deformation

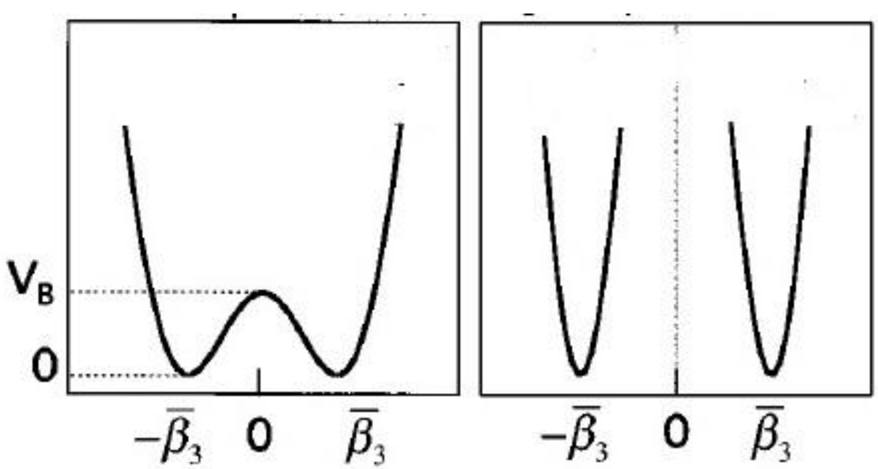
Intrinsic states $\Psi(\beta_{30})$ and $\Psi(-\beta_{30})$ are physically equivalent.

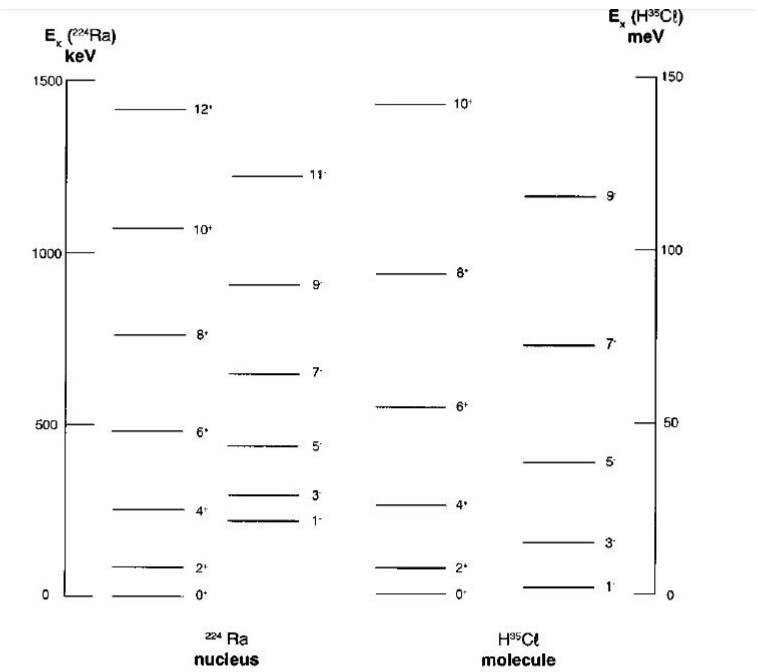
β_{20}=0.6, β_{30}=-0.5



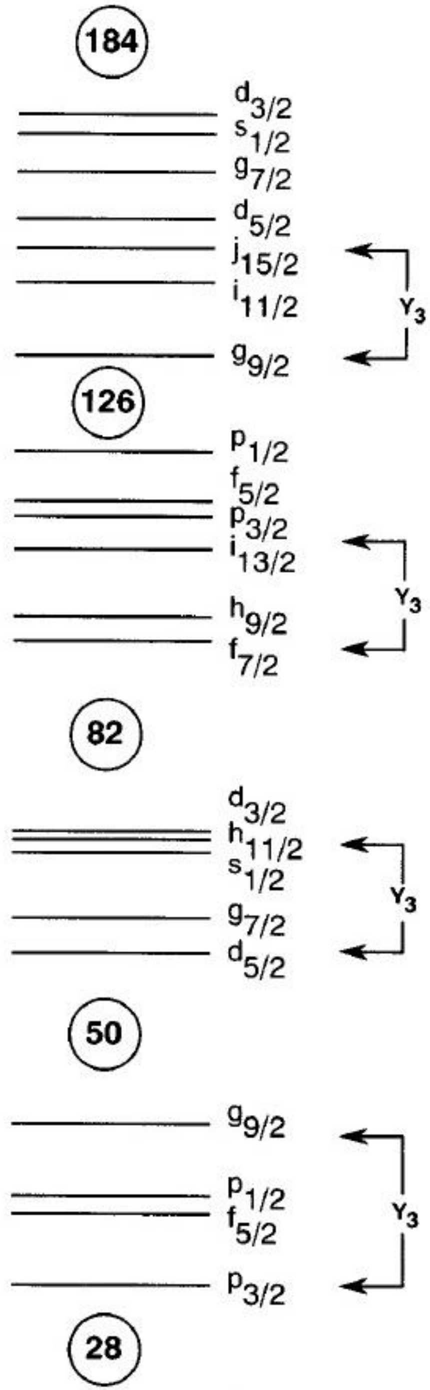
β_{20}=0.6, β_{30}=0.5







ΔL= ΔJ=3 or clustering?



The strong reflection asymmetric correlations near the ground state can be microscopically associated with the appearance of **orbital pairs with Δj = Δ l = 3 near the Fermi surface.**

Besides the actinides, the similar situation occurs in the nuclei with masses near **A ~56 and A ~ 134** that is in agreement with the experimental data.

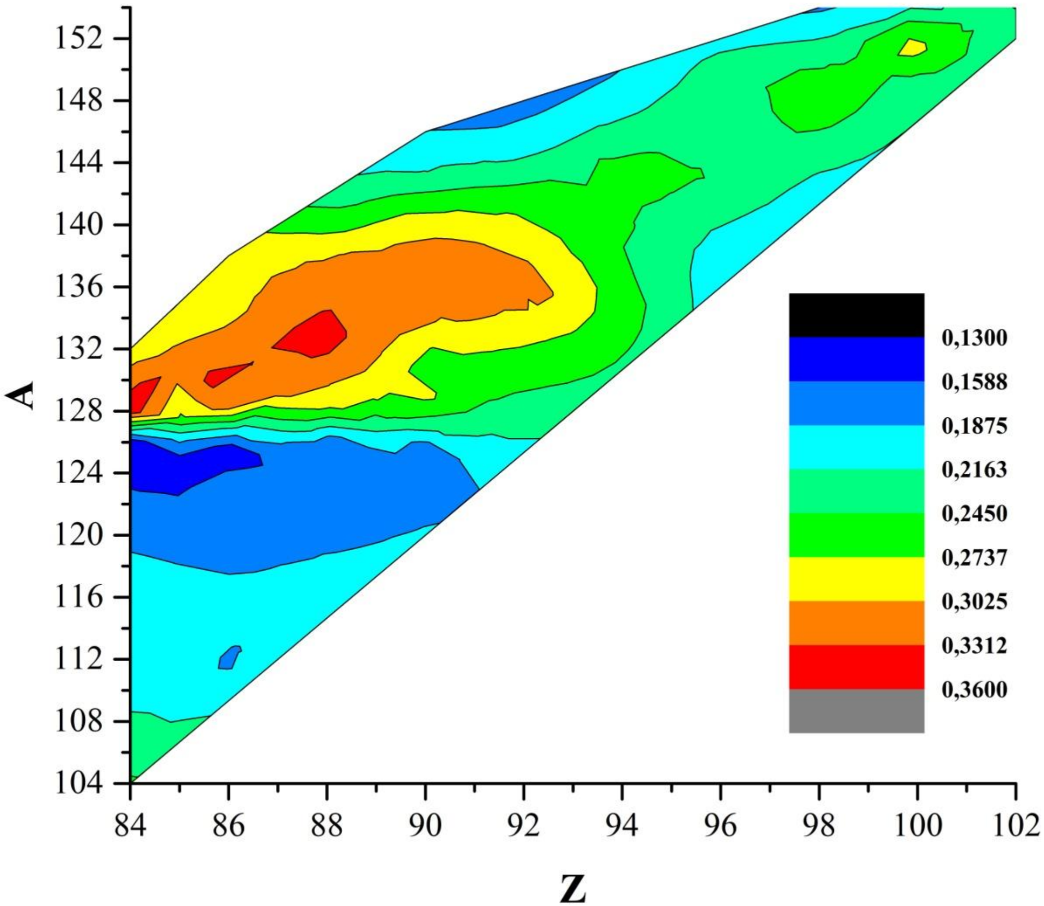
The results of calculations within the shell-corrected liquid drop models and mean-field models show that nuclei in these mass Regions are either soft with respect to the octupole deformation or even octupole-deformed.

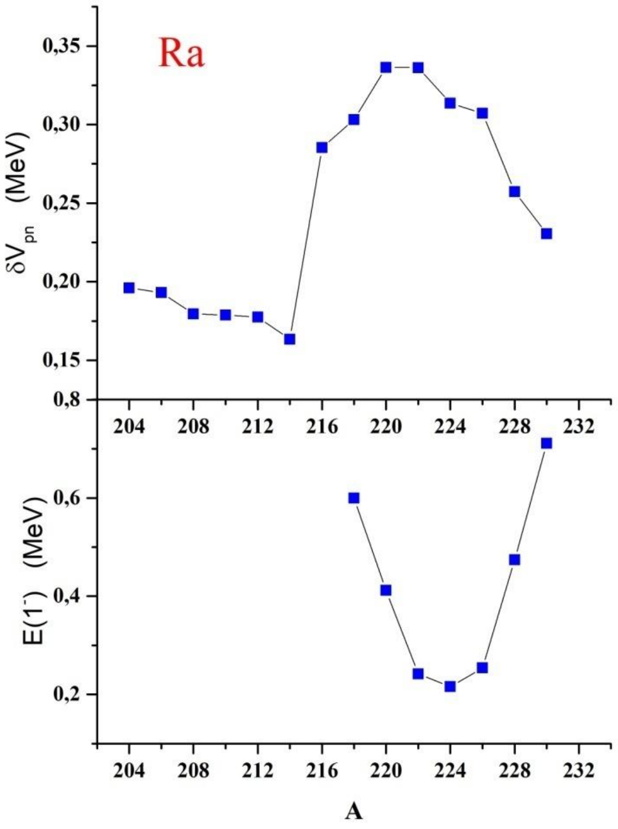
Nuclear spherical single-particle leveles.
Most important octupole couplings are indicated.

P.A. Butler & W. Nazarewicz, Rev. Mod. Phys. 68, 349 (1996)

$$\delta V_{pn} = -0.25\left[B(Z,N) - B(Z,N-2) - B(Z-2,N) + B(Z-2,N-2)\right]$$
$$= -0.25\left[S_{2n} + S_{2p} - S_{\alpha}\right]$$

R.F. Casten & R.B. Cakirli, Phys. Scri 91, 033004 (2016)





Degrees of Freedom of Dinuclear System Model

The dinuclear system (A,Z) consists of a configuration of two touching nuclei (clusters) (A_1,Z_1) and (A_2,Z_2) with $A = A_1 + A_2$ and $Z = Z_1 + Z_2$, which keep their individuality.

DNS has totally 15 collective degrees of freedom which govern its dynamics.

- *Relative motion of the clusters* $\mathbf{R} = (R, \theta_R, \phi_R)$
- *Rotation of the clusters* $\Omega_1 = (\phi_1, \theta_1, \chi_1)$, $\Omega_2 = (\phi_2, \theta_2, \chi_2)$
- *Intrinsic excitations of the clusters* (β_1, γ_1), (β_2, γ_2)
- *Nucleon transfer between the clusters* ξ, ξ_Z

Mass asymmetry $\xi = \frac{2A_2}{A_1+A_2}$. Charge asymmetry $\xi_Z = \frac{2Z_2}{Z_1+Z_2}$

Nuclear wave function is thought as a superposition of DNS configurations with various mass asymmetries, including mononucleus (no clusters) configurations.

Hamiltonian of the DNS model

The kinetic energy operator of the DNS then becomes

$$\hat{T}=-\frac{\hbar^2}{2B(\xi_0)}\frac{1}{\mu^{3/2}(\xi)}\frac{\partial}{\partial\xi}\mu^{3/2}(\xi)\frac{\partial}{\partial\xi}-\frac{\hbar^2}{2\mu(\xi)}\frac{1}{R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R}$$

$$+\frac{\hbar^2}{2\mu(\xi)R^2}\hat{l}_0^2+\frac{\hbar^2}{2}\sum_{n=1}^{2}\sum_{k=1}^{3}\frac{\hat{l}_{(n)k}^2}{I_k^{(n)}(\beta_n,\gamma_n)} \qquad \left(\equiv\hat{T}_{rot}\right)$$

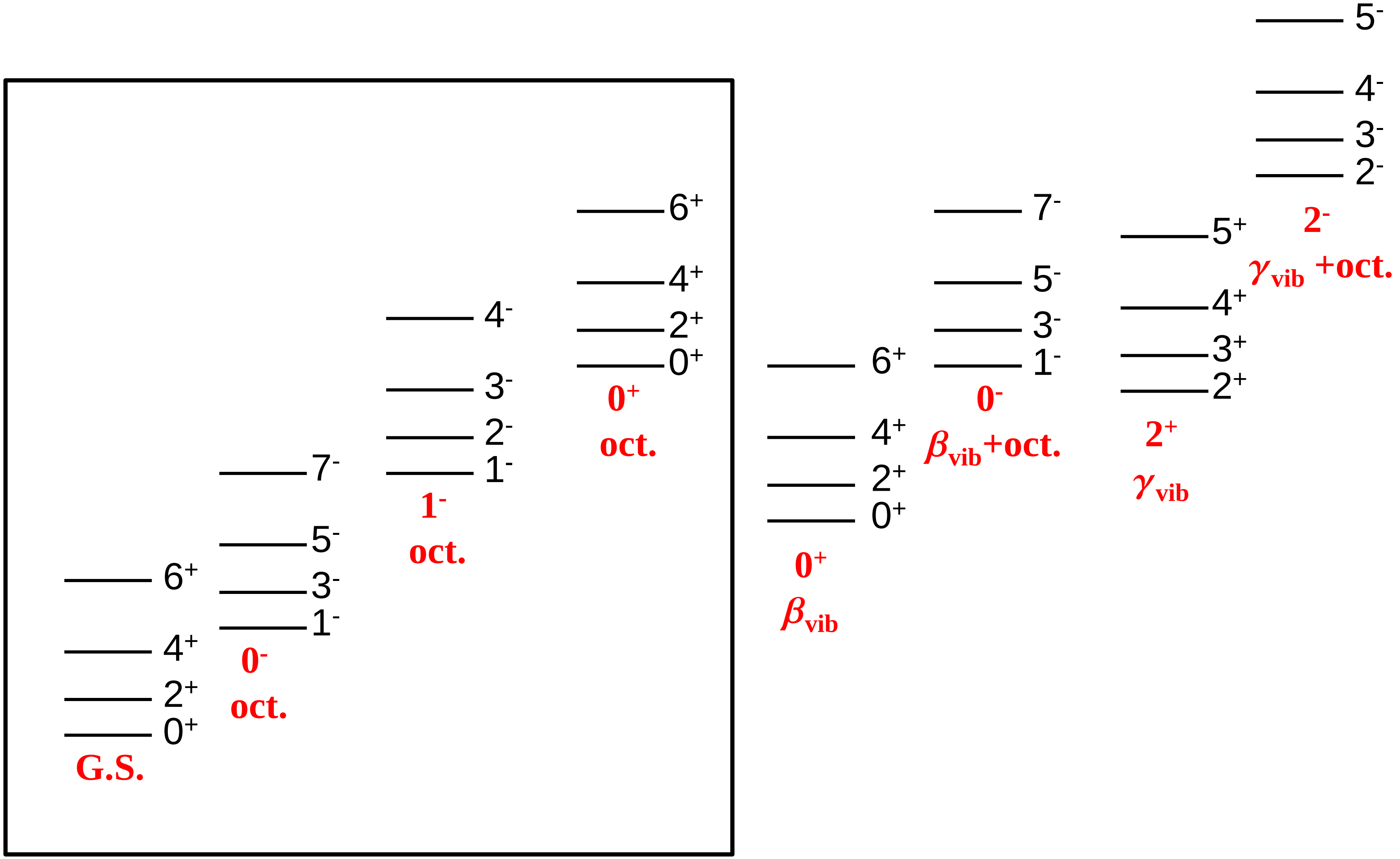
$$-\frac{\hbar^2}{2}\sum_{n=1}^{2}\frac{1}{D_n(\xi_0)}\left(\frac{1}{\beta_n^4}\frac{\partial}{\partial\beta_n}\beta_n^4\frac{\partial}{\partial\beta_n}+\frac{1}{\beta_n^2}\frac{1}{\sin 3\gamma_n}\frac{\partial}{\partial\gamma_n}\sin 3\gamma_n\frac{\partial}{\partial\gamma_n}\right)$$

$$\left(\equiv\hat{T}_{\mathrm{int}\,r}\right)$$

The potential energy of the DNS is

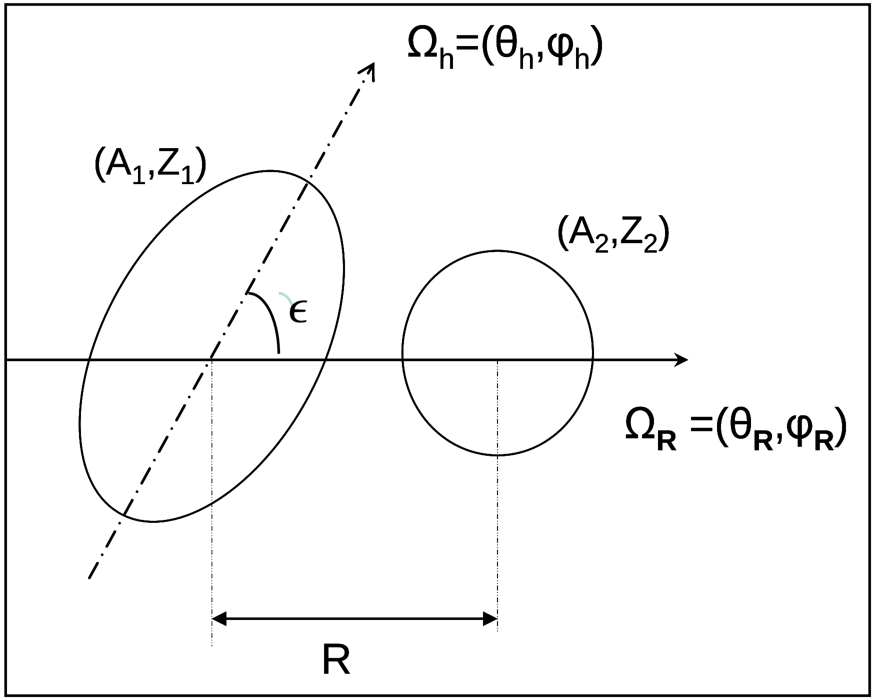
$$V(\xi)=E_1(\xi,\beta_1,\gamma_1)+E_2(\xi,\beta_2,\gamma_2)+V_N(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}})+V_C(R,\xi,\beta_{\{1,2\}},\gamma_{\{1,2\}},\Omega_{\{1,2\}})$$

Schematic Spectrum Produced by DNS Hamiltonian



Degrees of freedom (for Simplified Hamiltonian)

We assume that collective motion of nucleus in mass-asymmetry degree of freedom leads to the admixture of the very asymmetric cluster configurations to the intrinsic nucleus wave function and creates deformations with even- and odd-multipolarities.



-*Mass-asymmetry motion*, $\xi=2\,A_2/A$

-*Rotation of the molecular system*, $\boldsymbol{\Omega_R=(\theta_R,\varphi_R)}$

-*Rotation of the heavy cluster*, $\boldsymbol{\Omega_h=(\theta_h,\varphi_h)}$ *(for deformed heavy fragment)*

or

-*harmonic quadrupole oscillations of the heavy fragment*, $\boldsymbol{\beta_{2\mu}}$ *(for spherical heavy fragment)*

Potential Energy of the Dinuclear System

$$U(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)}) = B_1(\beta^{(1)}) + B_2(\beta^{(2)}) - B_{12} + V(R,\xi,\beta_{2\mu})$$

where, B_1, B_2 and B_{12} are the binding energies of the fragments and the compound nucleus, respectively.

The nucleus-nucleus potential

$$V(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)}) = V_{Coul}(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)}) + V_{nucl}(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)})$$

is the sum of the nuclear interaction potential $V_{nucl}(R,\xi,\beta_{2\mu}^{(1)},\beta_{2\mu}^{(2)})$

and of the Coulomb potential

$$V_{Coul}(R,\xi,\beta_{2\mu}) = \frac{e^2 Z_1 Z_2}{R} + \frac{3}{5}\frac{e^2 Z_1 Z_2}{R^3} R_{01}^2 \sum_{i,\mu} \beta_{2\mu}^{(i)*} Y_{2\mu}(\theta_i,\phi_i) + \ldots$$

Nuclear Interaction in Dinuclear System

$$V_{nucl}(R,\xi,\beta_{2\mu}) = \int \rho_1(\mathbf{r}_1)\rho_2(\mathbf{R}-\mathbf{r}_2)F(\mathbf{r}_1-\mathbf{r}_2)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2$$

$$\rho_i(\mathbf{r}) = \frac{\rho_{00}}{1+\exp\left(\frac{s(\mathbf{r})}{a_{0i}}\right)}, \quad \rho_{00} = 0.17\ \mathrm{fm}^{-3}$$

$$F(\mathbf{r}_1-\mathbf{r}_2) = C_0\left(F_{in}\frac{\rho_0(\mathbf{r}_1)}{\rho_{00}} + F_{ex}\left(1-\frac{\rho_0(\mathbf{r}_1)}{\rho_{00}}\right)\right)\delta(\mathbf{r}_1-\mathbf{r}_2)$$

$$\rho_0(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{r})$$

$$F_{in,ex} = f_{in,ex} + f'_{in,ex}\frac{N_1-Z_1}{A_1}\,\frac{N_2-Z_2}{A_2}$$

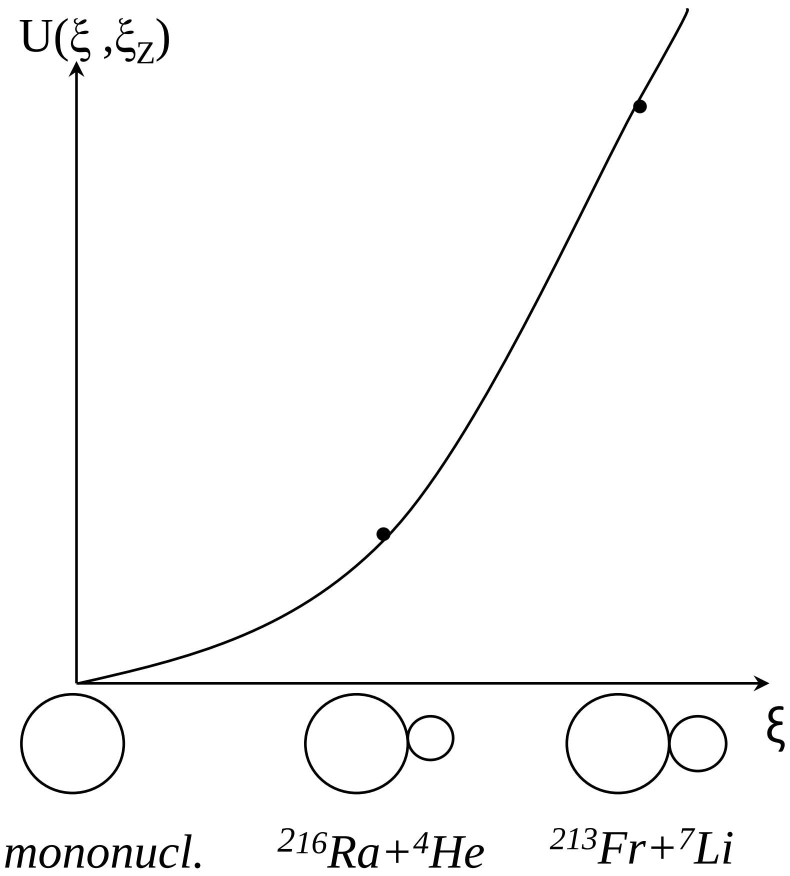
$$C_0 = 300\ \mathrm{MeV\ fm}^3,\ f_{in} = 0.09,\ f_{ex} = -2.59,\ f'_{in} = 0.42,\ f'_{ex} = 0.54$$

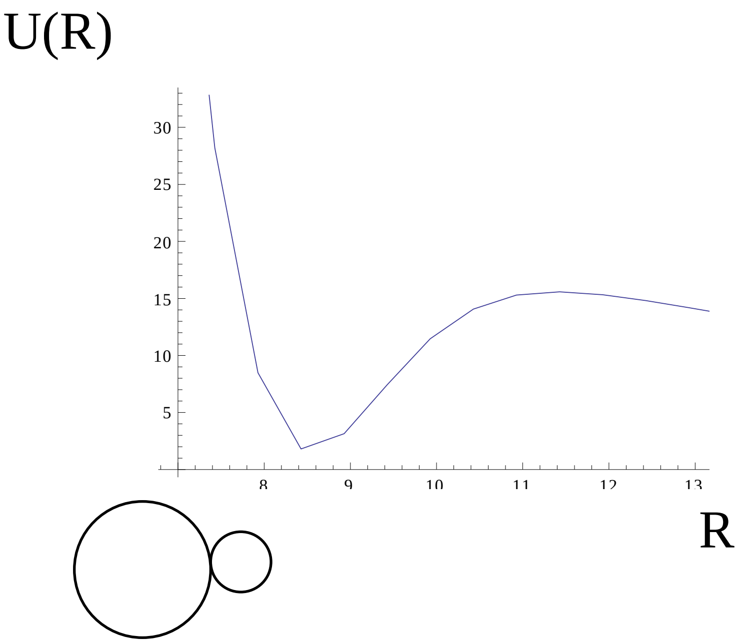
Potential Energy of the Dinuclear System

$$U(\xi,\xi_Z) = B_1(\xi,\xi_Z) + B_2(\xi,\xi_Z) - B + V(\mathbf{R} = \mathbf{R_m},\xi,\xi_\mathbf{Z})$$

$$V(\mathbf{R},\xi,\xi_\mathbf{Z}) = \mathbf{V}_\mathbf{coul}(\mathbf{R},\xi,\xi_\mathbf{Z}) + \mathbf{V}_\mathbf{N}(\mathbf{R},\xi,\xi_\mathbf{Z})$$

$$V_N(\mathbf{R},\xi,\xi_Z) = \int \rho_1(\mathbf{r_1})\rho_2(\mathbf{r_2})F(\mathbf{r_1} - \mathbf{r_2} + \mathbf{R})d\mathbf{r_1}d\mathbf{r_2}$$





Touching configuration
R=R$_m$

Approximation of the Potential Energy

$$U(\xi,\epsilon) = V(\xi) + \frac{C_0\xi}{2}\sin^2(\epsilon)$$

$$V(\xi) = \sum_{n=0,2,4,6} a_n \xi^n$$

$$\sin^2\epsilon = \frac{2}{3}\left(1 - \frac{4\pi}{5}\left[Y_2(\Omega_h)\times Y_2(\Omega_{\mathrm{R}})\right]_{(0,0)}\right)$$

a_{2n} are fitted to reproduce

exp. binding energy of nucleus;

calc. energy of the α*–DNS;*

calc. energy of the Li–DNS;

Nucleus	β	$C\xi_\alpha$
^{230}U	0.173	3.57
^{232}U	0.182	3.96
^{234}U	0.198	4.76
^{236}U	0.207	5.23
^{238}U	0.215	5.57

Hamiltonian of the Model

(Deformed heavy fragment)

Hamiltonian

$$\boxed{\begin{aligned}&\hat{H} = \hat{H}_0 + \hat{V}_{int}\\ &\hat{H}_0 = -\frac{\hbar^2}{2B}\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + \frac{\hbar^2}{2\Im_h}\hat{l}_h^2 + \frac{\hbar^2}{2\mu R_m^2}\hat{l}_{\mathbf{R}}^2 + V_0(\xi)\\ &\hat{V}_{int} = \frac{C_1\xi}{2}\sum_\mu Y_{2\mu}^*(\Omega_h)Y_{2\mu}(\Omega_{\mathbf{R}})\end{aligned}}$$

Wave Functions

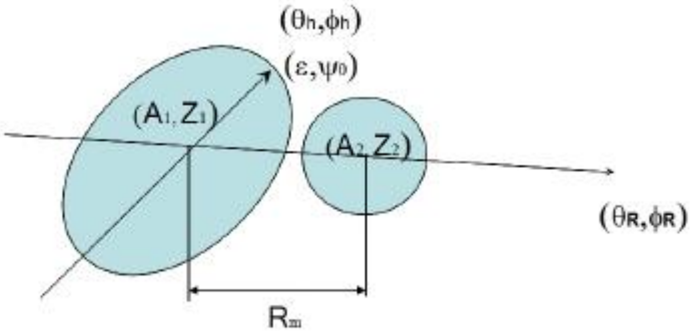
$$\Psi_{LM\pi} = \sum_{l_h l_R n} a_{n l_h l_R}^{(LM)}(\xi)\left[Y_{l_h}(\Omega_h) \times Y_{l_R}(\Omega_R)\right]_{(L,M)}$$

Parity of the states is determined by the angular momentum of the relative motion

$$\pi = (-1)^{l_R}$$

Angular Momentum Dependence of the Parity Splitting

If $\xi_0 \gg 0$, then barrier between mirror shapes is very large $C\xi_0 \gg 0$. Light fragment prefers to stay at the pole of the heavy fragment ($\epsilon \approx 0$). In this case it is more convenient to rewrite the angular Hamiltonian to describe rotation of the DNS as a whole and angular vibration of the light fragment around the pole of the heavy fragment.



(ϵ, ψ_0) — orientation of the deformed fragment with respect to the molecular system.

$$\hat{H} = \hat{H}_{rot} + \hat{H}_{bend} + \hat{V}_{int},$$

$$\hat{H}_{rot} = \frac{\hbar^2}{2\mu R_m^2}(\hat{L}^2 - 2\hat{L}_3')$$

$$\hat{H}_{bend} = \frac{\hbar^2}{2\Im_b}\frac{1}{\epsilon}\frac{\partial}{\partial\epsilon}\epsilon\frac{\partial}{\partial\epsilon} + \frac{\hbar^2}{2\Im_b\epsilon^2}L_3'^2 + \frac{C}{2}\epsilon^2$$

$$\hat{V}_{int} = \frac{\hbar^2}{2\mu R_m^2}\left[\left(\frac{1}{\epsilon}(L_1'L_3' + L_3'L_1'\right) + 2iL_2'\frac{1}{\sqrt{\epsilon}}\frac{\partial}{\partial\epsilon}\sqrt{\epsilon}\right]$$

(L_1', L_2', L_3') — intrinsic components of total angular momentum $\hat{L}$

Angular Momentum Dependence of the Parity Splitting

The angular Hamiltonian for the case of large stiffness (large ξ_0) can be diagonalized analytically

Moment of inertia $\Im_b = \frac{\Im_h \times \mu R_m^2}{\Im_h + \mu R_m^2}$
Frequency of the bending oscillations $\omega_b = \sqrt{\frac{C}{\Im_b}}$

Approximate energies (all matrix elements up to the order ϵ^2 a
taken into account):

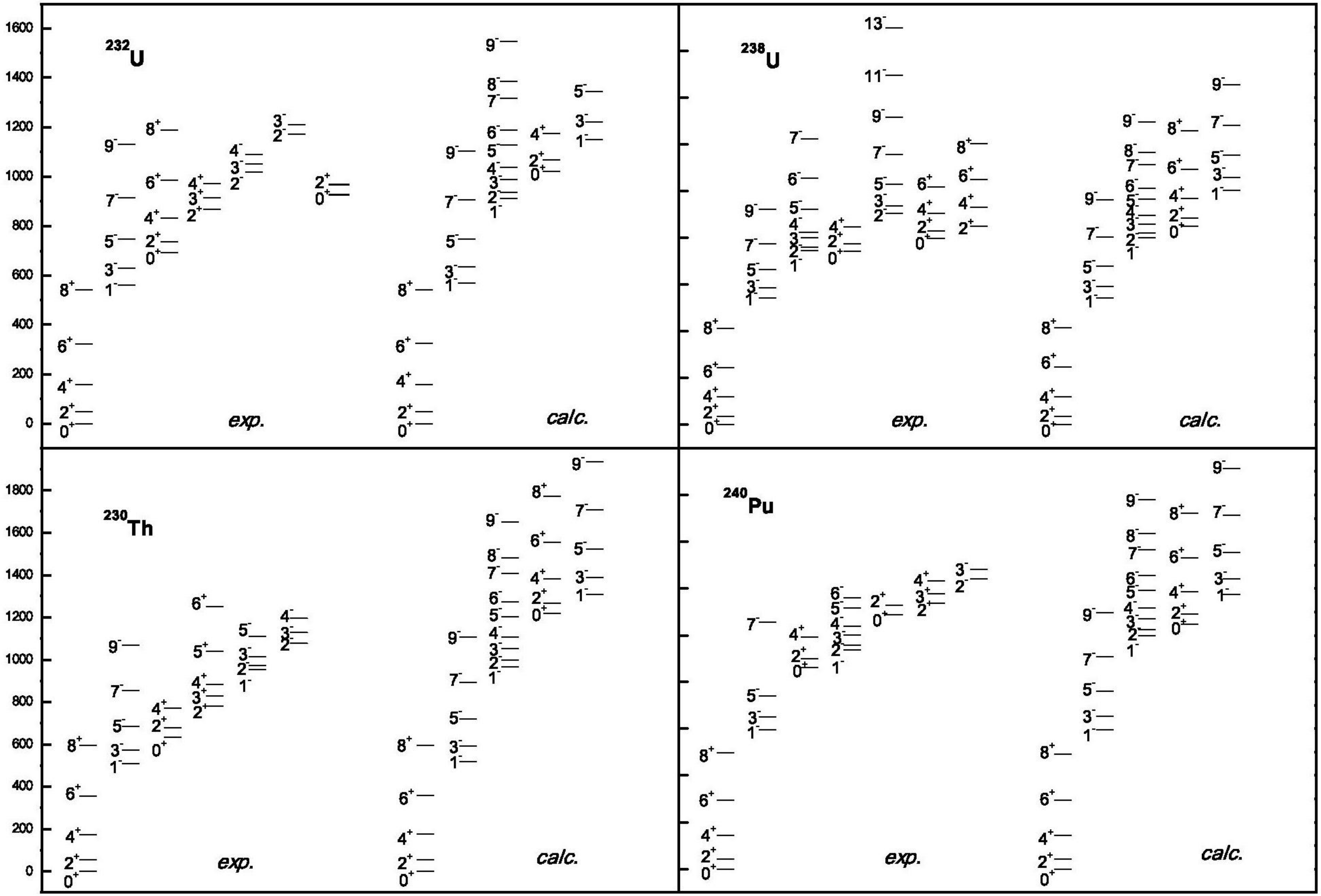
$$E_{nKp} = \hbar\omega_b(2n + K + 1) + \frac{\hbar^2}{\Im_h + \mu R_m^2}\left(I(I+1) - K^2\right)$$

Wave functions:

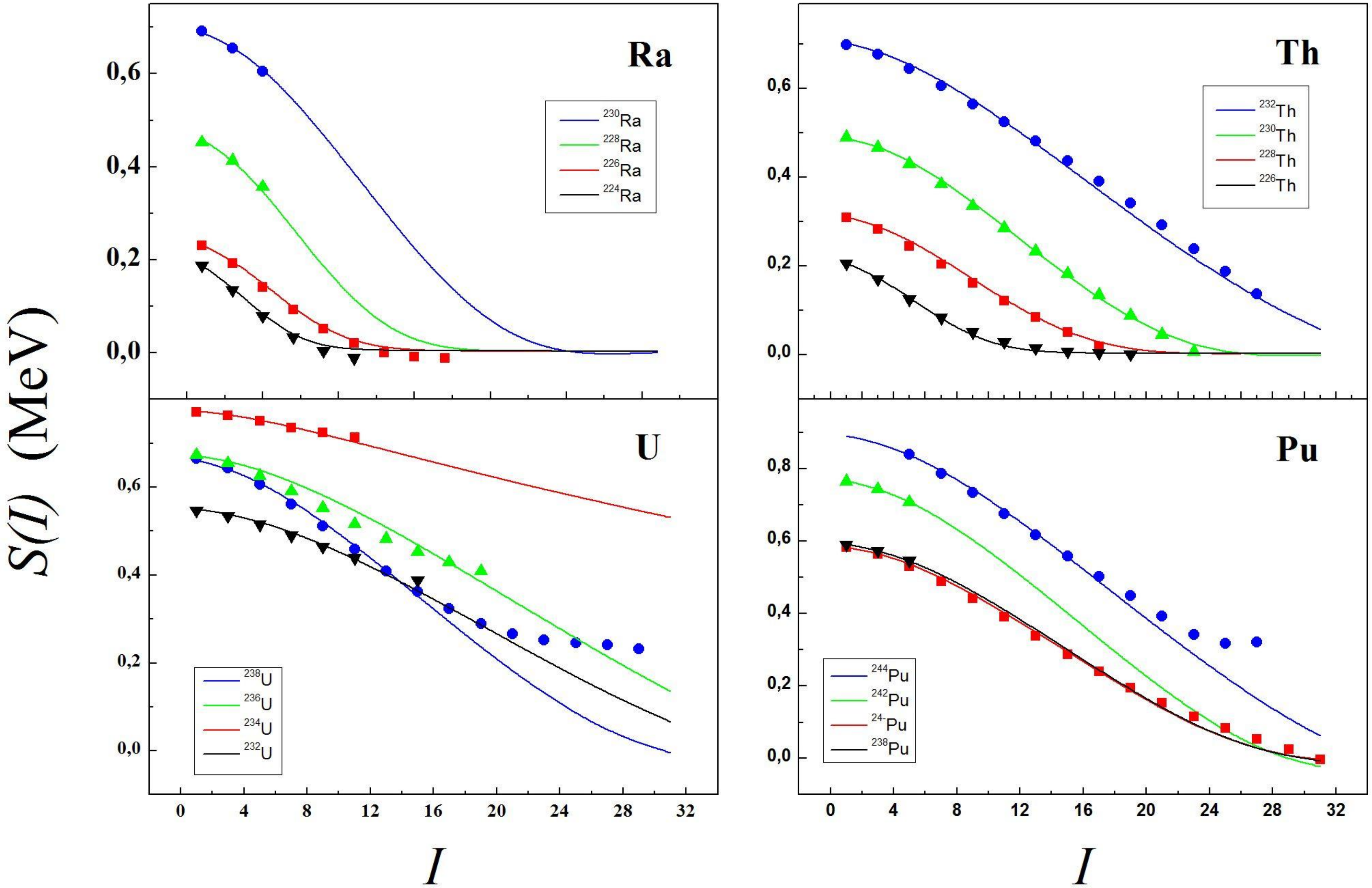
$$\Psi_{np,IKM} = L_{n,|K|}(\epsilon)\left(D^L_{M,K}(\Omega) + p(-1)^{I+K} D^L_{M,-K}(\Omega)\right)$$

$$\boxed{\begin{array}{c} E(L) = \dfrac{\hbar^2 L(L+1)}{2J_{tot}}, L = 0^+, 1^-, 2^+, \ldots \\ S(L^-) \approx 0 \end{array}}$$

Results of Calculation for Heavy Actinides



Parity splitting in alternating parity bands



$$S(I^-) = E(I^-) - \frac{(I+1)E^+_{(I-1)} + IE^+_{(I+1)}}{2I+1}$$

EPJ WC 107, 03009, (2016)

Angular Momentum Dependence of the Parity Splitting

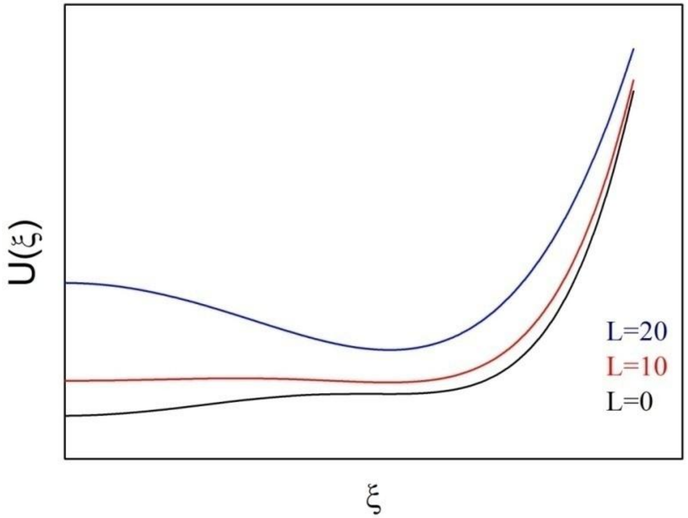
Assumption for wave function

$$\Psi_{LM}(\xi,\Omega_h,\Omega_R)=\Phi(\xi,L)g_{LM}(\xi_0,\Omega_h,\Omega_R)$$

where ξ_0 is the average mass asymmetry.

Hamiltonian in mass asymmetry

$$H(\xi,L)=-\frac{\hbar^2}{2B}\frac{1}{\xi^{3/2}}\frac{\partial}{\partial\xi}\xi^{3/2}\frac{\partial}{\partial\xi}+U_0(\xi)+\frac{\hbar^2 L(L+1)}{2J(\xi)}$$



$\xi=0;$

$$U(\xi,L)=U(\xi,L=0)+\frac{\hbar^2}{2}\frac{L(L+1)}{J_h}$$

$\xi=1;$

$$U(\xi,L)=U(\xi,L=0)+\frac{\hbar^2}{2}\frac{L(L+1)}{J_{tot}}$$

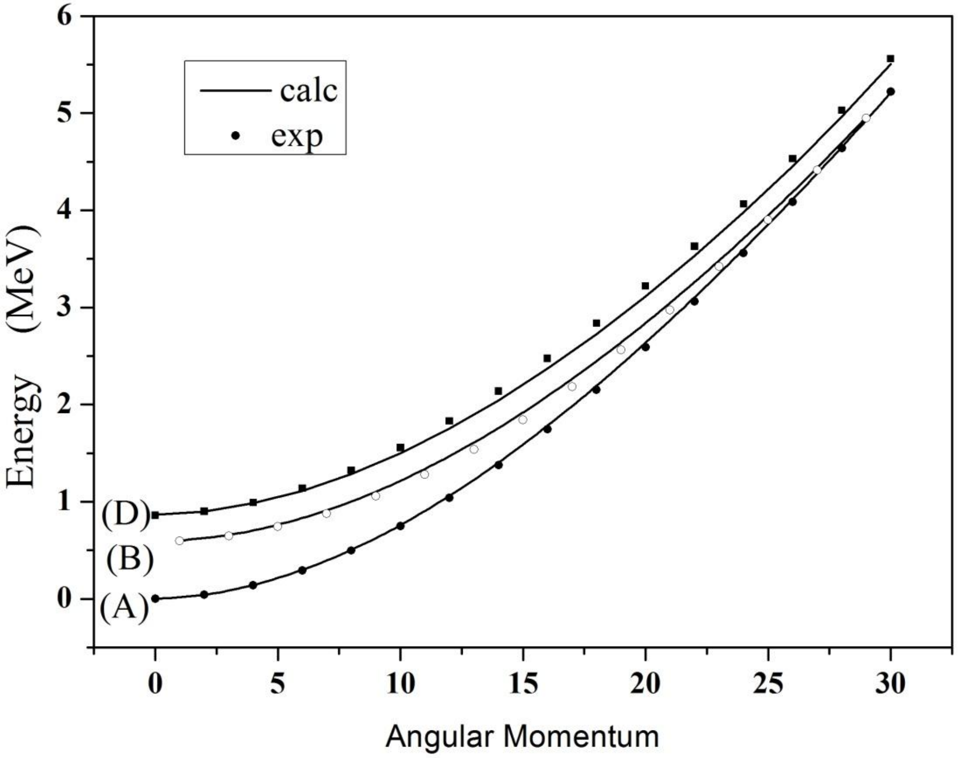
$$\boxed{J_{tot}>J_h}$$

As a result average mass asymmetry ξ_0 increases with angular momentum.

Average mass asymmetry in different rotational bands of ^{240}Pu

Table 1. Calculated average mass asymmetry $\bar{\xi}$ for the members of ground state band (A), second excited positive parity band (D), and lowest excited negative parity band (B).

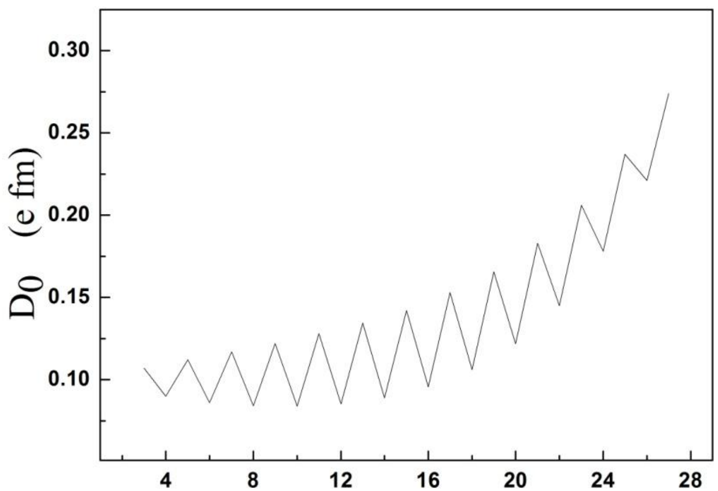
$I^+_{A,D}$	$\bar{\xi}_A$	$\bar{\xi}_D$	I^+_B	$\bar{\xi}_B$
0	0.196	0.396	1	0.334
2	0.197	0.397	3	0.336
4	0.200	0.400	5	0.339
6	0.203	0.404	7	0.344
8	0.209	0.410	9	0.350
10	0.215	0.417	11	0.359
12	0.224	0.426	13	0.369
14	0.235	0.435	15	0.381
16	0.247	0.445	17	0.394
18	0.263	0.455	19	0.409
20	0.282	0.464	21	0.425
22	0.305	0.469	23	0.442
24	0.333	0.474	25	0.460
26	0.367	0.473	27	0.478
28	0.407	0.469	29	0.498
30	0.453	0.464		

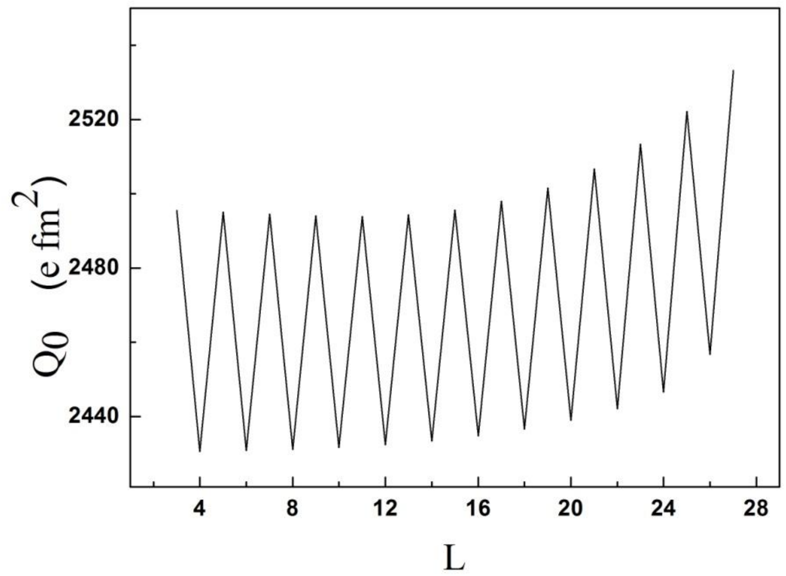


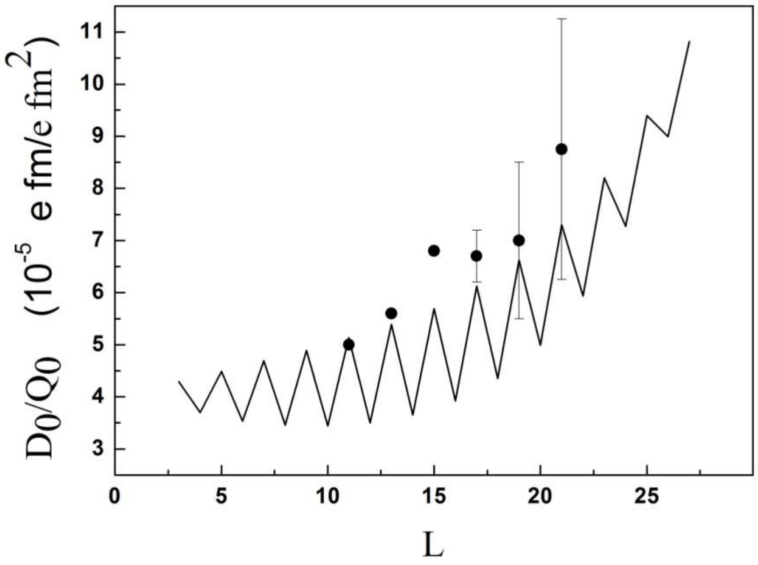
PRC92, 034302 (2015)

Electromagnetic Transition in ^{240}Pu

(*I. Wiedenhöver et al., Phys. Rev. Lett. 83, Number 11, (1999)*)



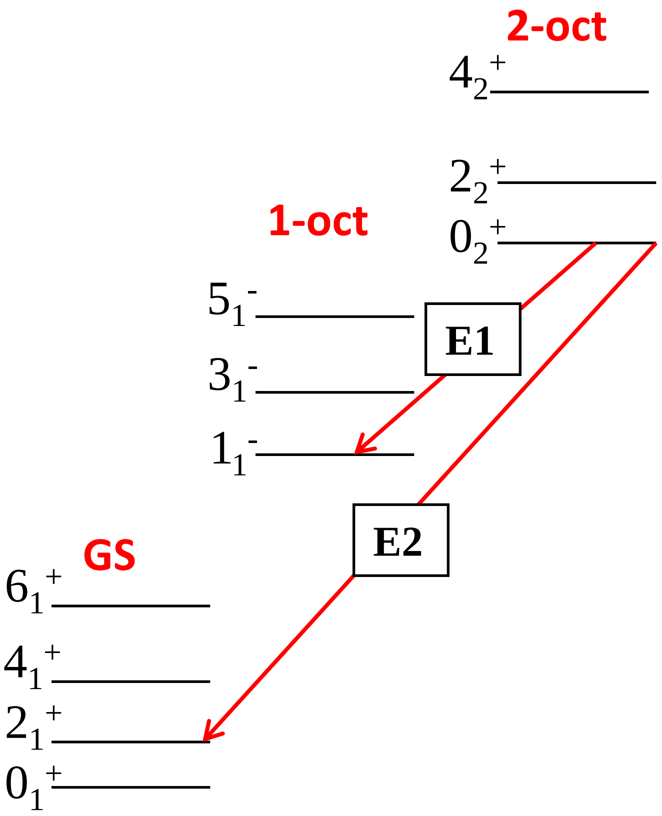




Ratio of transition dipole and quadrupole moments extracted from the *E1* and *E2* branchings $E1(I^- \longrightarrow (I-1)^+)/E2(I^- \longrightarrow (I-2)^-)$ as a function of the initial spin *I*.

PRC92, 034302 (2015)

Electromagnetic Transition in ^{240}Pu

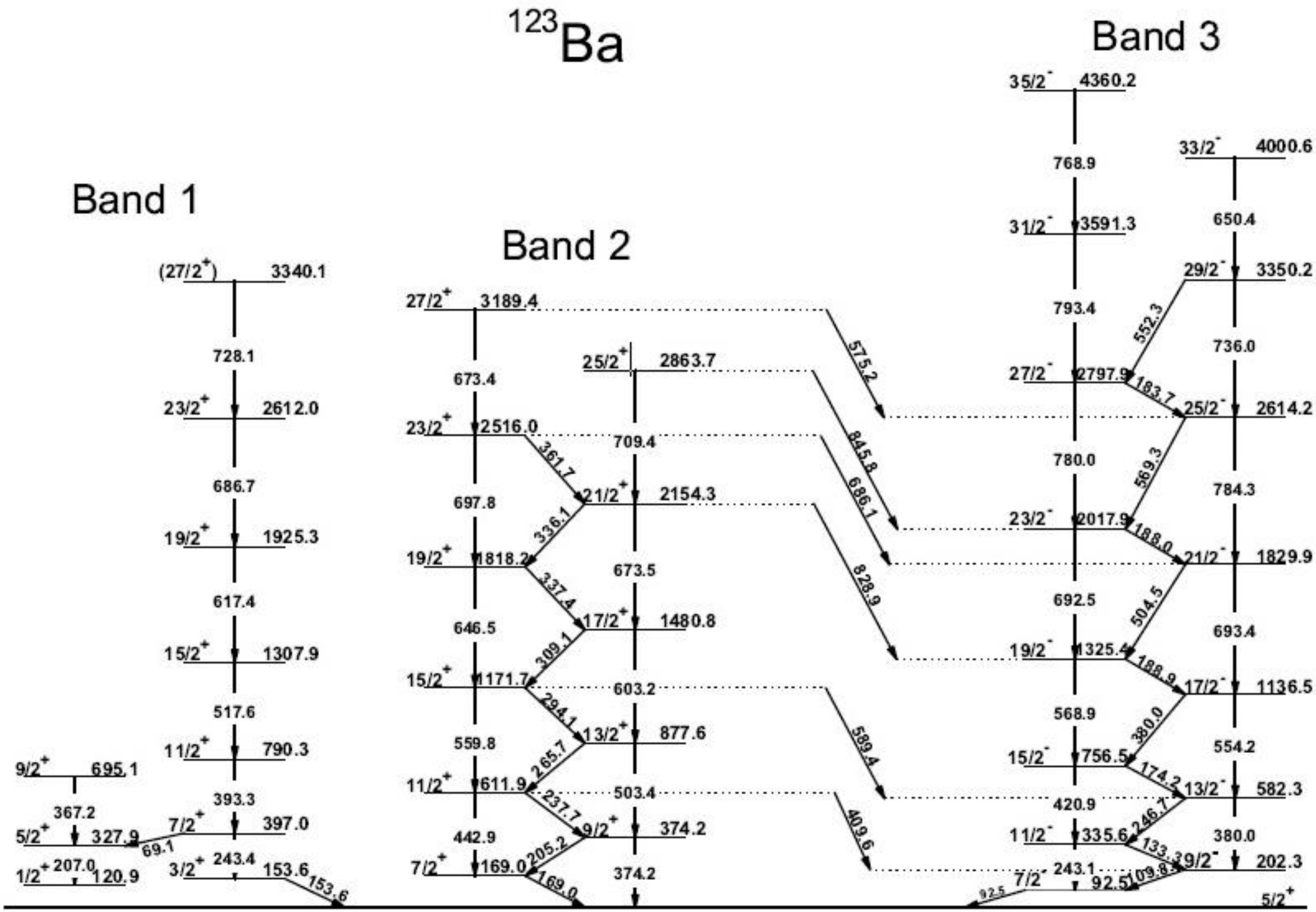


Experimental B(E1)/B(E2) ratios (R_{exp}) are compared to the calculation of our model for the low-spin members of the $K^\pi = 0^+$ 2nd rotational band in ^{240}Pu.

I_i^π	$I_{f,E1}^\pi$	$I_{f,E2}^\pi$	R_{exp} (10^{-6} fm^{-2})	R_{DNS} (10^{-6} fm^{-2})
0_2^+	1_1^-	2_1^+	13.7(3)	19.17
2_2^+	1_1^-	0_1^+	99(15)	99.95
2_2^+	1_1^-	2_1^+	26(2)	39.15
2_2^+	1_1^-	4_1^+	5.9(3)	8.57
2_2^+	3_1^-	0_1^+	149(22)	165.60
2_2^+	3_1^-	2_1^+	39(2)	64.9
2_2^+	3_1^-	4_1^+	8.9(5)	14.2
4_2^+	3_1^-	6_1^+	4.4(11)	6.9
4_2^+	5_1^-	6_1^+	4.7(13)	10.59

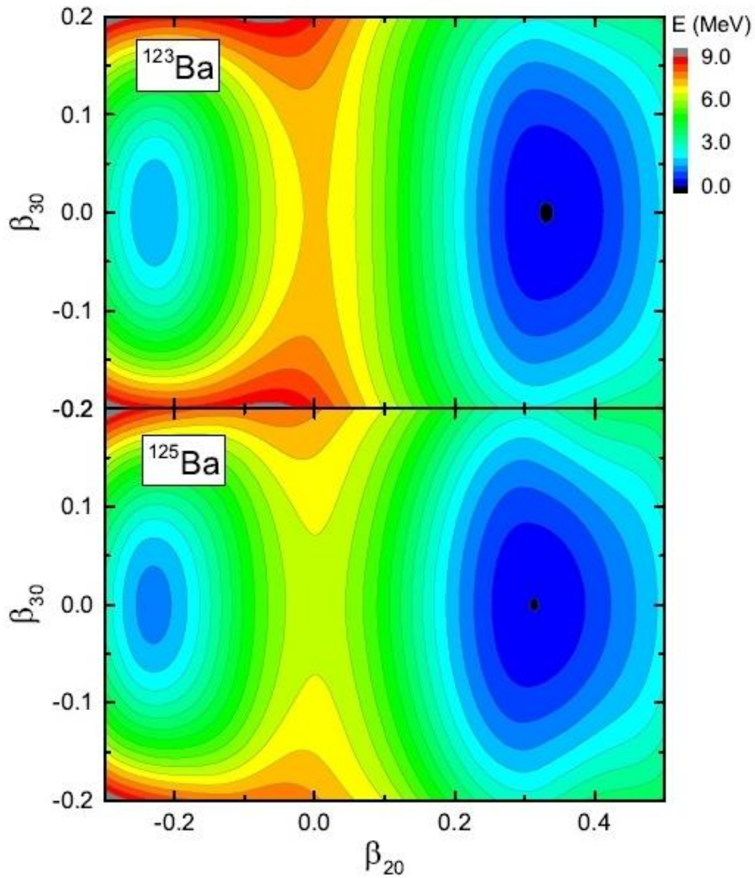
PRC97, 064319(2018)

Reflection-asymmetric correlations in ^{123}Ba



PRC94, 021301 (2016)

PES for ^{123,125}Ba



Calculations have been performed in the frame of MDC-RMF model.

Although the minimum of the nuclear potential energy corresponds to the reflection-symmetric shape, PES for ^{123,135}Ba are very soft with respect to the reflection-asymmetric deformation.

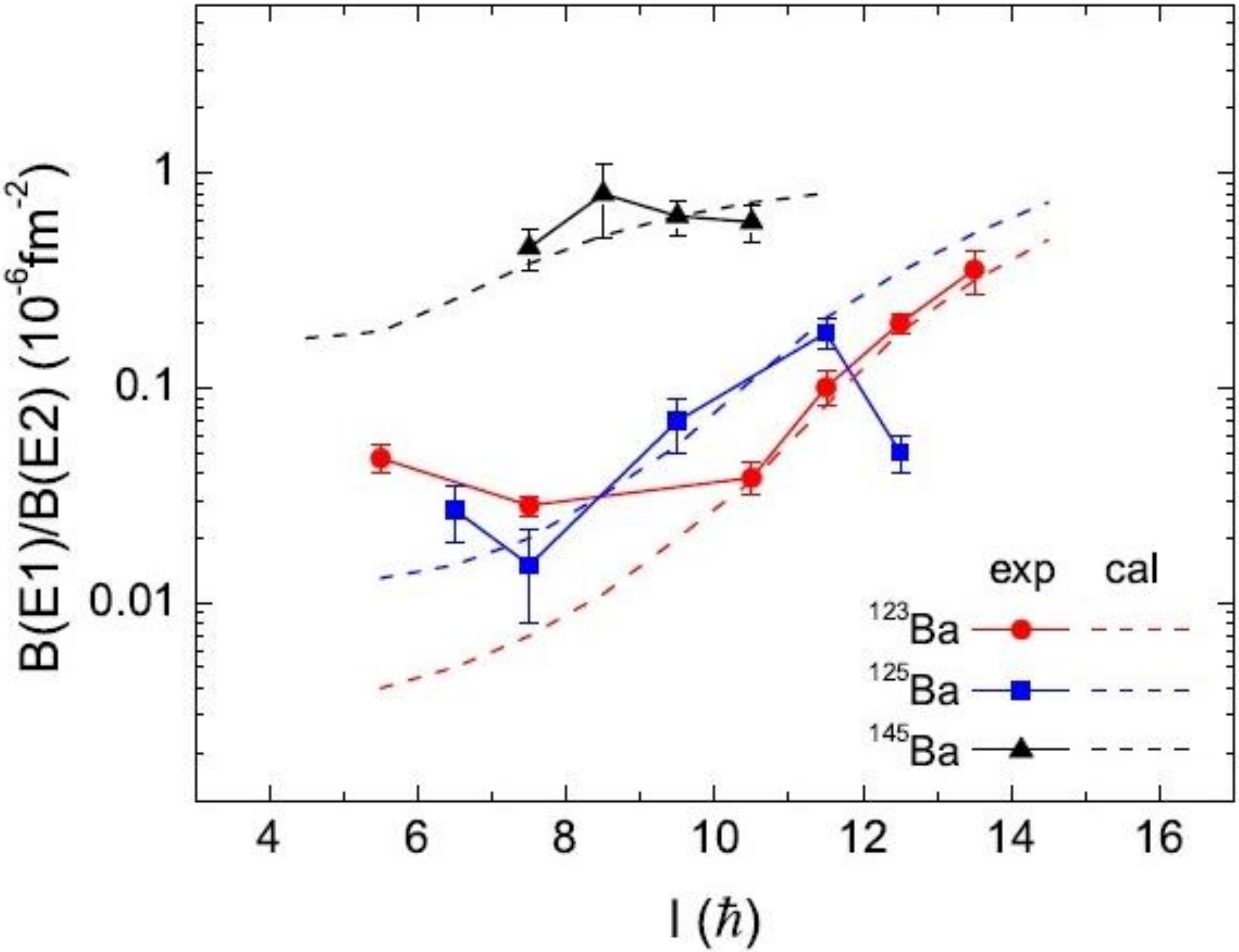
Using the cluster approach one can estimate the critical value of angular momentum at which the stable reflection-asymmetric is developed.

$$I_{crit} \approx 13\hbar \quad \text{- for } {}^{123}\text{Ba},$$

$$I_{crit} \approx 12\hbar \quad \text{- for } {}^{125}\text{Ba}.$$

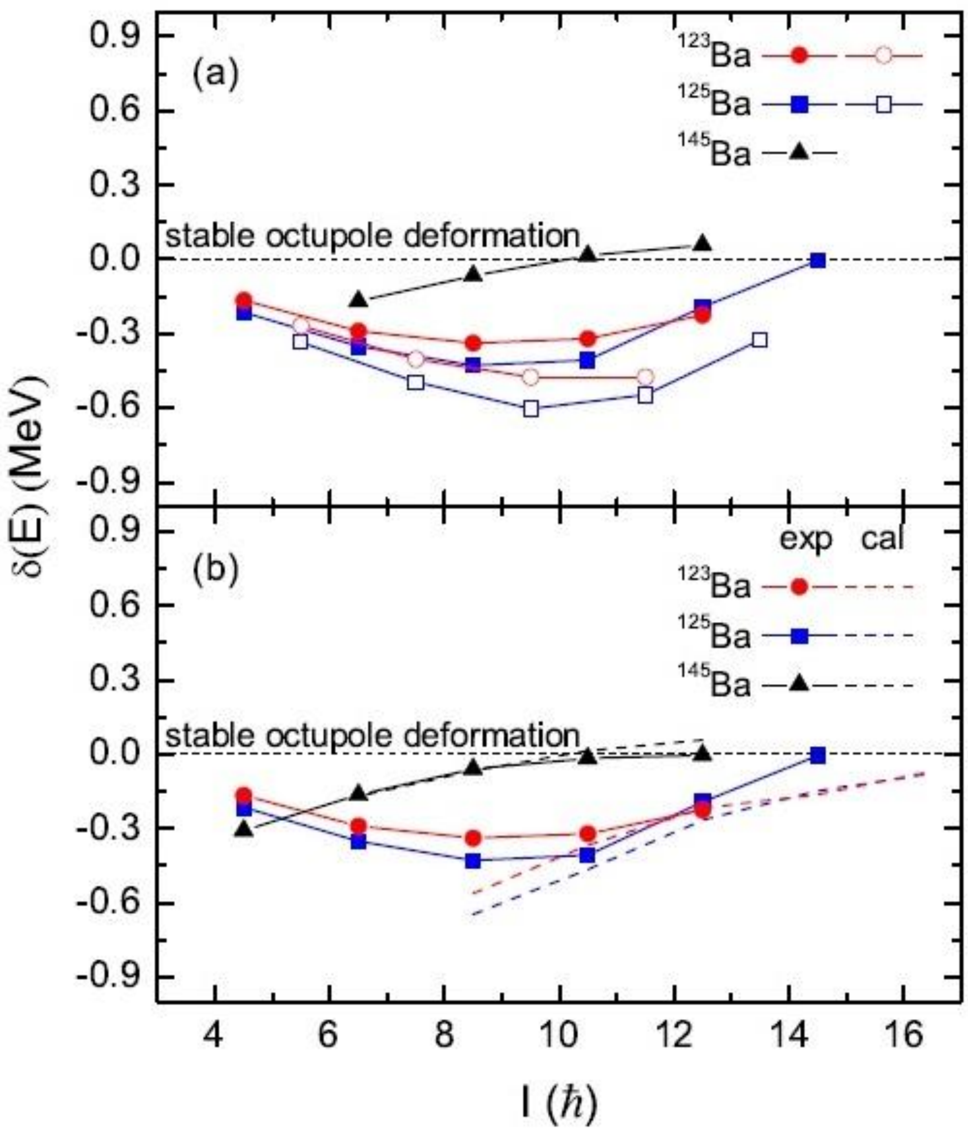
PRC94, 021301(R) (2016)

B(E1)/B(E2)-values for ^{123,125,145}Ba



PRC94, 021301 (2016)

Parity splitting of ^{123,125,145}Ba

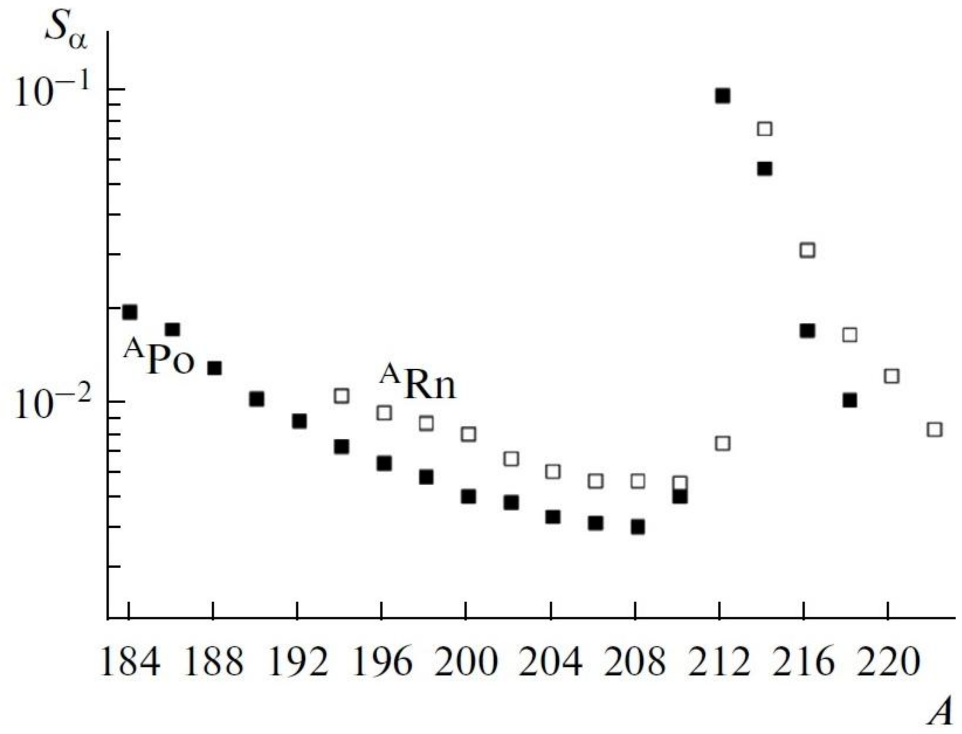


PRC94, 021301 (2016)

Spectroscopic factors for alpha-decay

$$\Psi_{LM}(\xi,\Omega_h,\Omega_R) = \Phi(\xi,L)\, g_{LM}(\xi_0,\Omega_h,\Omega_R)$$

$$S_\alpha = \int (\xi/\xi_\alpha) \left|\Phi(\xi, L=0)\right|^2 d\tau(\xi)$$



S.N. Kuklin et al., Phys. Of Part. And Nuclei 47, 206 (2016)

Conclusion:

- The cluster interpretation of the multiple negative parity bands in actinides and rare-earth nuclei is suggested assuming collective oscillations of nucleus in mass-asymmetry degree of freedom.
- The angular momentum dependence of the parity splitting and electromagnetic transition probabilities *B(E1)* and *B(E2)* are described.
- To take care of non-axially symmetric reflection asymmetric modes, the rotational and vibrational degrees of freedom of the clusters are considered.
- The excited 0^+ bands of reflection-asymmetric nature are explained as a bands built on the first exited state in mass asymmetry degrees of freedom.
- Adding the single-particle degree of freedom of the valence nucleon, the parity doublet rotational bands in odd-mass nuclei are described.