Alpha-decay versus alpha-clustering

Doru S. Delion,

Alexandru Dumitrescu and Virgil V. Baran

Horia Hulubei National Institute of Physics and Nuclear Engineering, Bucharest-Magurele

Outline

- I. Basic laws of α -decay
- **II.** α-spectroscopy (fine structure) and clustering
- III. Coupled channels description of the fine structure and clustering
- IV. Gamma and beta decays versus clustering
- V. Microscopic approach for decay and clustering
- VI. Electromagnetic and alpha transitions versus clustering in ²¹²Po
- VII. Proton-neutron correlations and clustering VIII. Conclusions

I. Basic laws of alpha-decay

A. Geiger-Nuttall law for half lives



$$\log_{10} T = a \frac{Z_D}{\sqrt{E}} + b$$

- **H. Geiger and J.M. Nuttall** "The ranges of the α particles from various radioactive substances and a relation between range and period of transformation," *Philosophical Magazine*, Series 6, vol. 22, no. 130, 613-621 (1911).
- **H. Geiger and J.M. Nuttall** "The ranges of α particles from uranium," *Philosophical Magazine*, Series 6, vol. 23, no. 135, 439-445 (1912).

George Gamow in 1909, two years before the discovery of the G-N law ... and in 1930, two years after his explanation





G. Gamow "Zur Quantentheorie des Atomkernes" (On the quantum theory of the atomic nucleus), *Zeitschrift für Physik*, vol. 51, 204-212 (1928).



Decay width can be written



depending exponentially upon the Coulomb parameter

$$\chi = \frac{2Z_D Z_C}{\hbar v} = \frac{2Z_D Z_C}{\hbar \sqrt{2 E/m}}$$

Geiger-Nuttall law relates Log(decay width) to the Coulomb parameter

$$\log_{10}\Gamma = \log_{10}P + \log_{10}2\gamma^2$$

$$\log_{10} P = a\chi + l$$
$$\chi = c \frac{Z_D}{\sqrt{E}}$$

One obtains several parallel lines corresponding to various isotope chains



Geiger-Nuttall law for cluster-decays Magic radioactivity: "Pb decay" Z_D~ 82



Geiger-Nuttall law for proton emission D.S. Delion, R.J. Liotta, R. Wyss, Systematics of proton emission, Physical Review Letters 96, 072501 (2006)



B. Universal law for reduced widths



D.S. Delion **Universal decay rule for reduced widths** Physical Review **C80** (2009) 024310

Evidence of the α-clustering on nuclear surface

(a) α-clustering phase diagram(b) Pairing estimate of the alpha-formation amplitude(c) Potential within the Two Center Shell Model

(a) Phase diagram for deuteron and α-particle



G. Ropke, A. Schnell, P. Schuck, P. Nozieres Four-particle condensate in strongly coupled fermion systems Phys. Rev. Lett. 80, 3177 (1998).

Pairing survives at the equilibrium density ρ_0 and α -quarteting collapses at about 10% ρ_0 , i.e. an α -particle can exist only on the nuclear surface

(b) Microscopic α-particle formation probability within the mean field + pairing approach is peaked on the nuclear surface



(c) Two-center shell model predicts a double humped barrier (similar to fission) with a minimum on the nuclear surface



Conclusion: the cluster-daughter interaction should be a pocket-like potential on the nuclear surface. Thus, α-cluster is hindered inside by the Pauli principle



Conditions for an α-particle moving in a shifted harmonic oscillator potential

1) The first eigenstate energy is the Q-value

2) Its wave function is given by

where the oscillator parameter is

$$Q = E = \frac{1}{2} \hbar \omega$$

$$\Psi(R) = A_0 e^{-\beta(R - R_0)^2/2}$$

$$\beta = \frac{m\omega}{\hbar}$$

One obtains an analytical universal law for the reduced width in terms of the fragmentation potential V_{frag} and cluster amplitude A_o

$$\log_{10} \gamma^{2} = -\frac{\log_{10} e^{2}}{\hbar \omega} V_{frag} + \log_{10} \frac{\hbar^{2} A_{0}^{2}}{2 em R_{B}}$$

It does not depend on the pocket radius and remains valid for any pocket potential,

The fragmentation potential is given by the difference between the Coulomb barrier and Q-value:

$$V_{frag} = \frac{Z_D Z_C}{R_B} - Q$$

THE SLOPE SHOULD BE NEGATIVE!

Experimental universal law for alpha-decay from even-even nuclei has indeed a negative slope and two main regions for spectroscopic factor, divided by ²⁰⁸Pb



One obtains a log(width)-log(penetrability) dependence by using the parameters of the previous plot



This rule is similar to the well known Viola-Seaborg law



One obtains similar dependencies for proton emission. Universal law (b) explains the two lines in the systematics, corresponding to two regions of the fragmentation potential.



Universal law for reduced widths is valid for all emission processes: proton, even-even, odd-mass alpha & cluster decays



One obtains a general log(width)-log(penetrability) dependence for all emission processes by using the corresponding fit parameters



II. α-spectroscopy (fine structure) and clustering



Transitions to the ground band in even-even nuclei $P \to D(J) + \alpha$

Viola-Seaborg graph for α-decays to excited states in even-even nuclei



Observables describing the fine structure

Hindrance factor

$$HF_{J} = \frac{\gamma_{0}^{2}}{\gamma_{J}^{2}} = \frac{\Gamma_{0}}{\Gamma_{J}} \frac{P_{J}}{P_{0}}$$

Intensity

$$I_{J} = \log_{10} \frac{\Gamma_{0}}{\Gamma_{J}} = \log_{10} HF_{J} + \log_{10} \frac{P_{0}}{P_{J}}$$

Ratio of penetrabilities has an almost constant value for considered energies

By using the law for the reduced width one obtains a law for hindrance factors in terms of the excitation energy in the daughter nucleus

$$\log_{10} HF_{J} = \frac{\log_{10} e^{2}}{\hbar \omega} E_{J} + \log_{10} \frac{A_{0}^{2}}{A_{J}^{2}}$$

and intensities

$$I_{J} = \frac{\log_{10} e^{2}}{\hbar \omega} E_{J} + \log_{10} \frac{A_{0}^{2}}{A_{J}^{2}} + \log_{10} \frac{P_{0}}{P_{J}}$$

THE SLOPE SHOULD BE POSITIVE !

Universal law for intensities to excited states in even-even nuclei (a) has a positive slope



III. Coupled channels description of the fine structure and clustering

PHYSICAL REVIEW C 87, 044314 (2013)

Unified description of electromagnetic and α transitions in even-even nuclei

D. S. Delion^{1,2,3} and A. Dumitrescu^{1,4}

Atomic Data and Nuclear Data Tables 101 (2015) 1-40

Contents lists available at ScienceDirect

Atomic Data and Nuclear Data Tables

journal homepage: www.elsevier.com/locate/adt

Systematics of the α -decay fine structure in even-even nuclei



Atomic Data

Nuclear Data Tables

D.S. Delion^{a,b,c,*}, A. Dumitrescu^{a,d}

Gamow states

The decay process is described by resonant solutions (Gamow states) of the stationary Schrodinger equation

$$H\Psi(b_2,\mathbf{R}) = E\Psi(b_2,\mathbf{R})$$

where E=Q-value

The Hamiltonian contains kinetic, daughter and α -daughter terms:

$$H = -\frac{\hbar^2}{2\mu} \nabla_R^2 + H_D(b_2) + V(b_2, \mathbf{R})$$

α-daughter potential contains (a) spherical and (b) QQ interaction

$$V(b_2, \mathbf{R}) = V_0(R) + V_2(b_2, \mathbf{R})$$

(a) Spherical term is given by the external double folding matched to an internal repulsion (simulating Pauli principle)

$$V_0(R) = v_a \overline{V}_0(R) , \quad R > R_m$$

= $c(R - R_{min})^2 - v_0 , \quad R \le R_m$

(b) QQ interaction between daughter nucleus and α -particle is given by the standard expression

$$V_2(b_2, \mathbf{R}) = -C_0(R - R_{min}) \frac{dV_0(R)}{dR}$$
$$\times \hat{2} [Q_2 \otimes Y_2(\Omega)]_0 .$$

Shape of the α-daughter pocket-like potential (solid line) Dotted line plots the original double folding spherical potential. Dashed and dot-dashed lines represent the deformed components.



Repulsive strength is related to the depth of the potential by the matching procedure to the external part. Its value does not affect the fine structure

We fix the repulsive depth by adjusting the resonant energy to the exp. Q-value.`

We use the Coherent State Model (CSM) in terms of quadrupole bosons in order to describe low-lying collective states

Ground state band in even-even nuclei is obtained by projecting out the intrinsic coherent state

$$\varphi_J^{(g)} = \mathcal{N}_J^{(g)} P_{M0}^J \psi_g$$

$$|\psi_g\rangle = e^{d\left(b_{20}^{\dagger} - b_{20}\right)}|0\rangle$$

where the deformation parameter is proportional to the standard quadrupole deformation

$$d = \kappa \beta_2$$

Energy spectrum versus deformation parameter d has a vibrational shape for small d and rotational behavior for large d



Energy ratios versus the CSM deformation parameter are universal functions describing well the experimental values


Channel intensities

$$I_J \equiv \log_{10} \frac{\Gamma_0}{\Gamma_J}$$

define the strength of α -transitions to some excited state with spin J

The CSM deformation parameter d is determined from energy spectra for vibrational, transitional and deformed nuclei.

The only free parameter is the α -daughter coupling strength C which can be determined by the I₂ value for each transition

α-daughter QQ coupling strength C reproducing I₂ versus deformation parameter (a) and mass number (b)



CSM description predicts a linear dependence on the deformation parameter:

$$C(d) = C_0 \left(1 - \sqrt{\frac{2}{7}} a_\alpha d \right)$$

α -daughter coupling strength C reproducing I₂ versus the difference N-N_{magic}



magic

α-daughter QQ coupling strength C is proportional to the clustering probability



IV. Gamma and beta decays versus clustering Electromagnetic B(E:2⁺-->0⁺) value is proportional with respect to the CSM deformation parameter d

As a consequence, the collectivity (given by BE2 values) decreases when clustering (given by reduced width) increases

Gamow-Teller beta versus alpha decays

PHYSICAL REVIEW C 100, 024331 (2019)

Effective axial-vector strength within proton-neutron deformed quasiparticle random-phase approximation

D. S. Delion,^{1,2,3} A. Dumitrescu,^{1,2} and J. Suhonen⁴

Beta matrix element squared is the analog of the alpha decay reduced width (Coulomb effect is extracted from the decay width)

$$D_{1\mu}^{-} = \frac{1}{\sqrt{3}} \sum_{pn} (p \|\sigma\|n) \left[a_{p}^{\dagger} \otimes \tilde{a}_{n}\right]_{1\mu}$$

$$(Z-1,N+1) \beta^{+} \beta^{-1} (Z+1,N-1) \beta^{-1} (Z+1,N-1) \beta^{-1} (Z,N) \beta^{+} (Z,N) \beta$$

$$(Z-1,N+1) \beta^{+} \beta^{-} (Z+1,N-1) \beta^{-} (Z+1,N$$

Exp. beta matrix element in terms of the ft-value h g_A

$$g_{\rm A}\beta_{\rm exp}^{\pm} = \sqrt{\frac{6147 \ (2J_i + 1)}{10^{\log ft}}},$$

Exp. beta matrix elements squared connecting 0⁺ (even-even nuclei) to 1⁺ (odd-odd nuclei) are larger above magic numbers N=50, 82

Comparison to alpha reduced widths

Exp. beta matrix elements squared are proportional to the corresponding alpha reduced widths

Above magic numbers beta transitions within pn-QRPA are mainly given by the closest to the Fermi level p-n pair Therefore the few valence nucleons above closed shells mainly contribute to both alpha clustering and beta transitions. Otherwise Pauli antisymmetrisation hinders clustering/beta transitions

As a consequence, the effective axial-vector strength decreases from $g_A \sim 1$ above magic numbers to $g_A \sim 0.7$ between shells

V. Microscopic approach for decay and clustering How is the emitted cluster formed from protons and neutrons lying in different major shells ?

Microscopic estimate of the formation amplitude

$$\Psi_P \rightarrow \Psi_D + \psi_\alpha$$

H. J. Mang, Phys. Rev. 119, 1069 (1960).

A. Sandulescu, Nucl. Phys. A 37, 332 (1962).

Formation amplitude is the overlap between parent and daughter * alpha wave functions

$$\mathscr{F}(\mathbf{R}_{\alpha}) = \langle \alpha D | P \rangle = \int d\mathbf{x}_{\alpha} d\mathbf{x}_{D} \left[\psi_{\alpha}^{(\beta_{\alpha})}(\mathbf{x}_{\alpha}) \Psi^{(D)}(\mathbf{x}_{D}) \right]^{*} \Psi^{(P)}(\mathbf{x}_{P})$$

By using the cm and relative coordinates it becomes a superposition of ho orbitals depending on alpha-core radius with four times sp ho parameter 4β

$$\mathscr{F}_{\alpha}(\mathbf{R}) = \sum_{L_{\alpha}} \mathscr{F}_{L_{\alpha}}^{(\alpha)}(\mathbf{R}) = \sum_{L_{\alpha}} \sum_{N_{\alpha}} W(N_{\alpha}L_{\alpha}) \phi_{N_{\alpha}L_{\alpha}M_{\alpha}}^{(4\beta)}(\mathbf{R}).$$

where W-coefficients depend on Nilsson expansion coefficients and BCS amplitudes Decay width versus cm radius for N=12 major shells in the diagonalization basis underestimates the exp. value by two orders of magnitude

Ratio exp. / BCS spectroscopic factor (integrated formation probability) >> 1 and it is correlated with the reduced width: alpha-clustering is not described by two-body residual forces

How to correct in a pragmatic way the tail of the α-particle formation amplitude in order to describe the experimental decay width?

A. By using a mixed diagonalization sp basis with two ho parameters

D.S. Delion, A. Insolia, R.J. Liotta, New single particle basis for microscopic description of decay processes, Physical Review **C54**, 292 (1996).

B. By including surface alpha-like correlations in the sp mean field one obtains a larger tail

D.S. Delion and R.J. Liotta, Shell-model representation to describe alpha emission Physical Review **C87**, 041302(R) (2013).

A. Woods-Saxon mean field diagonalized within a two harmonic oscillator (ho) basis

The effect of the second (cluster) part of the sp basis

B. Woods-Saxon mean field plus a Gaussian surface cluster component enhances the tail of sp orbitals

This picture is confirmed by microscopic calculations G. Roepke, et.al., Phys. Rev. C 90, 034304 (2014)

Proton and neutron formation probabilities with cluster component (a) and without cluster component (b)

Cluster component increases the p-n overlap by creating p & n orbitals with the same principal quantum number.

Thus, the effective p-n correlation increases.

Decay width with the cluster component reproduces the exp. value and weakly depends on the cm radius (a). Decay width without the cluster component is much smaller than the exp. value (b).

Cluster strength (a), Log(cluster probability) (b) and Fragmentation potential (c) versus neutron number are very similar

Spectroscopic factor versus reduced width

 $S = rac{\Gamma_{\mathrm{expt}}}{\Gamma_{\mathrm{theor}}} = rac{T_{\mathrm{theor}}}{T_{\mathrm{expt}}},$

PHYSICAL REVIEW C 92, 021303(R) (2015)

Systematics of *α*-decay transitions to excited states

D. S. Delion^{1,2,3} and A. Dumitrescu^{1,4}

FIG. 1. Spectroscopic factor versus the α -formation probability x 100 for even-even, even-odd, odd-even, and odd-odd emitters.

Spectroscopic factor versus neutron number

Theoretical investigation of α-like quasimolecules in heavy nuclei

D. S. Delion, 1,2,3 A. Dumitrescu, 1,2 and V. V. Baran1,4

FIG. 3. Spectroscopic factor vs neutron number of the daughter nucleus for even-even emitters. The symbols denote different regions of the nuclear chart divided by magic numbers.

VI. Electromagnetic and alpha transitions versus clustering in ²¹²Po

D.S. Delion, R.J. Liotta, P. Schuck, A. Astier, and M.-G. Porquet Shell model plus cluster description of negative parity states in ²¹²Po Phys. Rev. C85, 064306 (2012)

> Positive parity states 2⁺, 4⁺, 6⁺, 8⁺ are given by neutron broken pairs

> $|^{212}\mathrm{Po}(J^+)\rangle = |^{210}\mathrm{Pb}(J^+) \otimes {}^{210}\mathrm{Po}(\mathrm{g.s.})\rangle$

Negative parity states 4⁻, 6⁻, 8⁻ are given by neutron broken pairs coupled to an octupole state

 $|^{212}\text{Po}(I^{-})\rangle = |[^{210}\text{Pb}(J^{+}) \otimes {}^{210}\text{Pb}(3^{-})]_{I^{-}} \otimes {}^{210}\text{Po}(g.s.)\rangle$

Viewpoint: Do alpha particles cluster inside heavy nuclei?

Michael P. Carpenter, Argonne National Laboratory, Argonne, IL 60439, USA

January 25, 2010 • Physics 3, 8

New excited states have been observed in ²¹²Po that are associated with a configuration in which an alpha particle is combined with a doubly-magic ²⁰⁸Pb core.

PRL 104, 042701 (2010) PHYSICAL REVIEW LETTERS

week ending 29 JANUARY 2010

S

Novel Manifestation of α -Clustering Structures: New " $\alpha + {}^{208}$ Pb" States in 212 Po Revealed by Their Enhanced *E1* Decays

A. Astier,¹ P. Petkov,^{1,2} M.-G. Porquet,¹ D. S. Delion,^{3,4} and P. Schuck⁵

Mean field with surface α-clustering in ²¹²Po explains decay width between ground states

Surface α -clustering term with the amplitude ≈ 0.3 explains large electromagnetic E1 transitions in ²¹²Po

B(E2:J+2→J)-values										
$\overline{J' o J}$	210 Po $B(E2)_{exp}$	$B(E2)_{th}$	210 Pb $B(E2)_{exp}$	$B(E2)_{th}$	212 Po $B(E2)_{exp}$	$B(E2)_{th}$				
$2 \rightarrow 0$	0.56(12)	6.7	1.4(4)	3.9		9.2				
$4 \rightarrow 2$	4.6(2)	12.9	3.2(7)	3.5		20.8				
$6 \rightarrow 4$	3.0(1)	8.9	2.2(3)	2.4	13.5(36)	14.4				
$8 \rightarrow 6$	1.18(3)	3.9	0.62(5)	1.0	4.60(9)	5.8				

$B(E1:I \rightarrow J)$ -values

<i>I</i> ⁻	J^+	E_{MSM} (MeV)	$\frac{E(^{212}\text{Po}(I^{-}))}{(\text{MeV})}$	$\frac{E_{\exp}(^{212}\text{Po}(I^{-}))}{(\text{MeV})}$	$B(E1)_{th}^{(1)}$ (10 ⁴ W.u.)	$B(E1)_{th}^{(2)}$ (10 ⁴ W.u.)	$B(E1)_{exp}$ (10 ⁴ W.u.)
2-	2+	-0.407	1.236		5	1	
	4+	-0.204	1.907		15	63	
4-	4+	-0.303	1.808	1.744	9	11	25
	6+	-0.107	2.201	1.946	2	4	11
6-	6+	-0.213	1.886	1.787	37	122	66
	8+	-0.490	2.197	2.016	3	8	19
8-	6+	-0.489	1.816	1.751	43	148	200
	8+	-0.215	2.240	1.986	8	24	
10-	8+	-0.360	2.135	2.465	2	1	18

VII. Proton-neutron correlations are larger in N~Z nuclei

Above N=Z=50 formation amplitude is similar for various theoretical approaches Influence of the proton-neutron pairing on the alpha-formation probability is small

Universal behavior of the surface Gaussian potential strength, which is proportional to the reduced width, for Z>50 and Z>82 regions

VIII. Conclusions

- **1)** Surface α-daughter interaction leads to
- the universal law for reduced width versus the fragmentation potential and for hindrance factors versus the excitation energy
- 2) α-daughter QQ strength describing fine structure is proportional to the clustering probability and has the maximal value above magic numbers
- 3) Nuclear collectivity linearly decreases when alpha-clustering increases
- 4) Reduced widths are proportional to beta matrix elements squared: clustering and p-n transitions are given by few valence nucleons above magic numbers and are hindered by exchange effects between shells
- 5) As a clear evidence of the alpha-clustering the ratio between exp. and BCS spectroscopic factors >> 1 and linearly depends on the clustering probability
- 6) Absolute decay widths can be described microscopically by using a mixed sp basis, containing additional clustering components
- 7) Proton-neutron corrrelations have a small influence on the alpha-clustering.
- 8) Alpha-clustering induced by the surface interaction has an universal behavior for both Z>50 and Z>82 regions

THANK YOU !

Doru S. Delion

LECTURE NOTES IN PHYSICS 819

Theory of Particle and Cluster Emission

