Alpha condensates and dynamics of cluster formation

Yasuro Funaki

(Kanto Gakuin U.)

ECT Workshop on Light clusters in nuclei and nuclear matter @Trento, Italy. September 2-6, 2019.*

Excited states above the Hoyle state

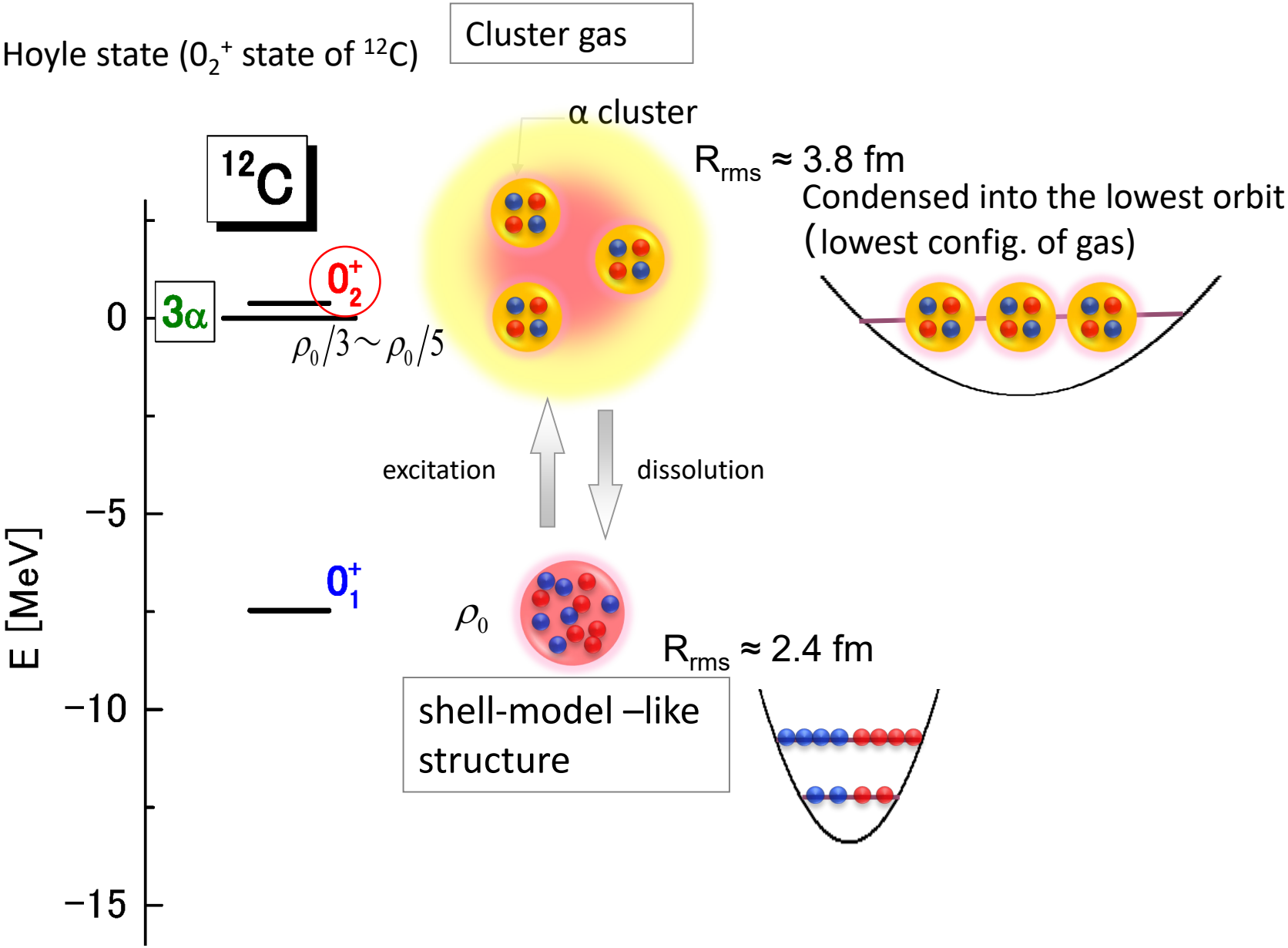
Alpha condensate and clusters in ^{16}O

Excited states above the Hoyle state

Alpha condensate and clusters in ^{16}O

Alpha condensate state

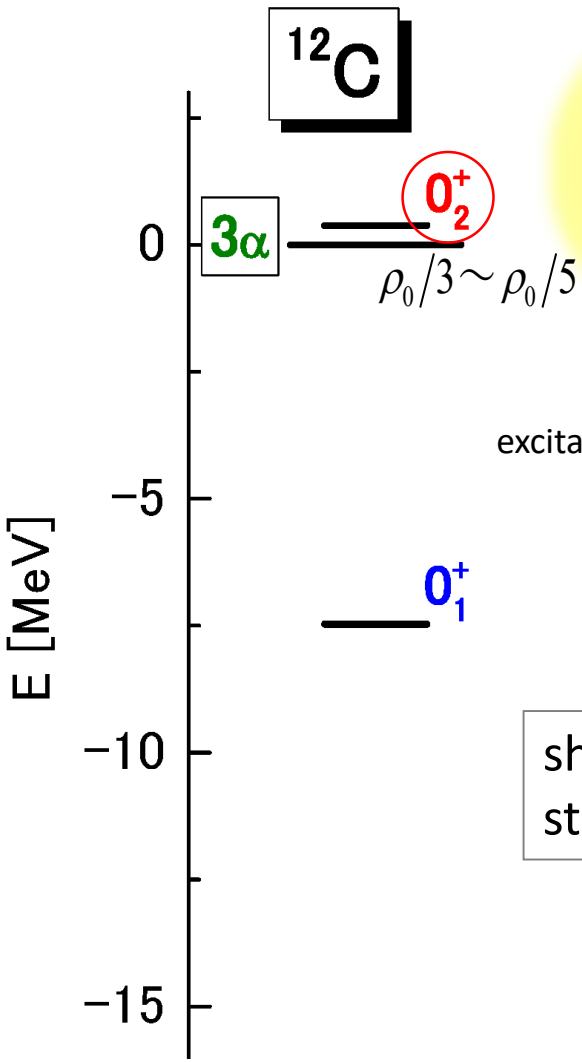
The Hoyle state (0_2^+ state of ^{12}C)

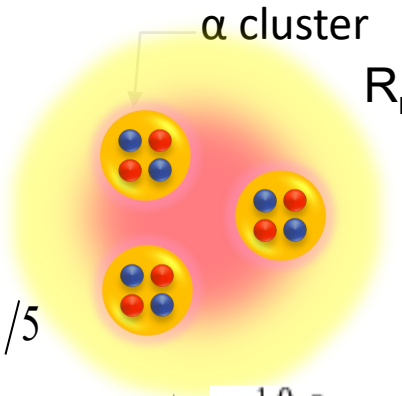


Alpha condensate state

The Hoyle state (0_2^+ state of ^{12}C)

Cluster gas

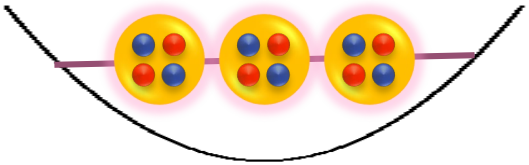




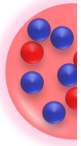
$R_{rms} \approx 3.8$ fm

Condensed into the lowest orbit

(lowest config. of gas)

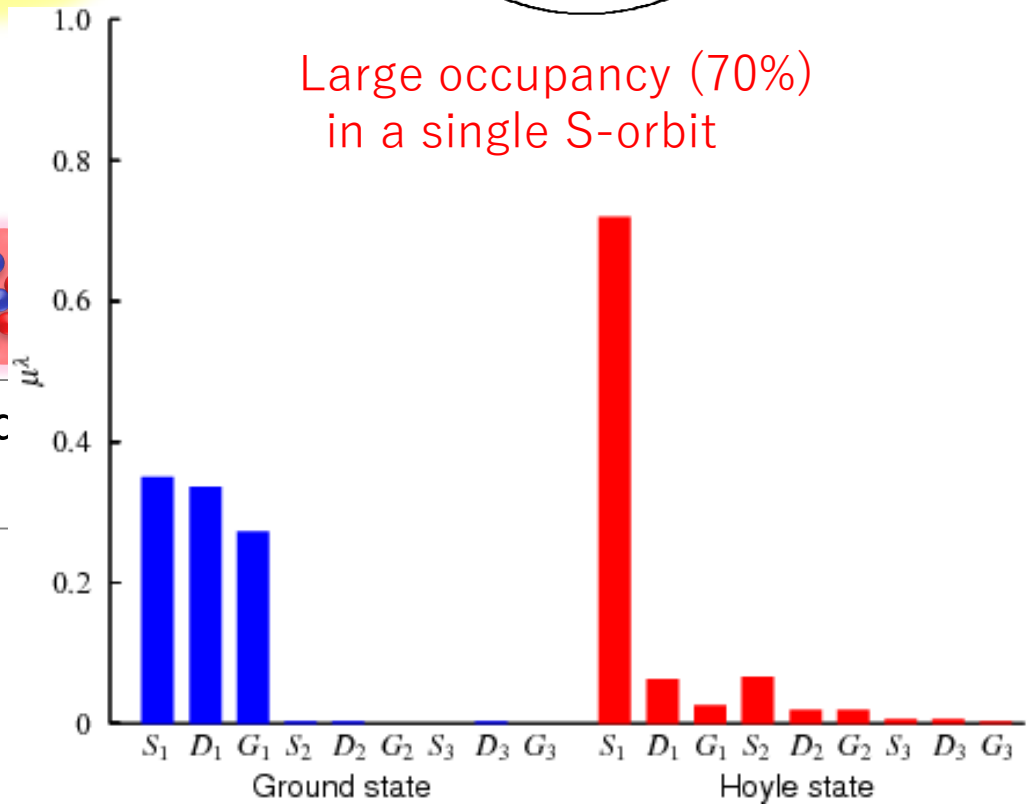


excitation



shell-moc
structure

Large occupancy (70%)
in a single S-orbit

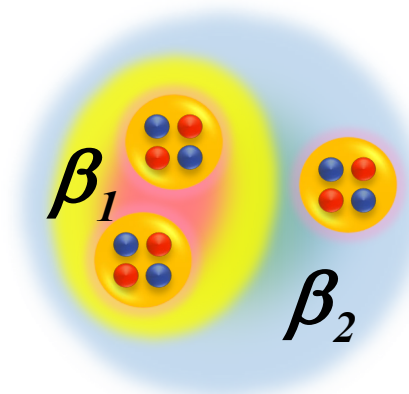


3α extended THSR wave function ($\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2$: original THSR)

$$\Phi_\alpha(\boldsymbol{\beta}, b) = \exp\left(-2\sum_k^{x,y,z} \frac{r_k^2}{b^2 + 2\beta_k^2}\right)\phi_\alpha(b)$$

$$\Phi_{^{12}\mathrm{C}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) = \Psi_G^{-1}\mathcal{A}\{\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\ \Phi_\alpha(\boldsymbol{\beta}_2, b)\}$$

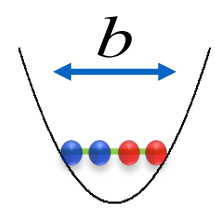
Ψ_G: Total center-of-mass w.f. to be eliminated



Internal w.f. of α particle

b=1.35 fm: fixed

$\phi_\alpha(b) :=$



3α extended THSR wave function ($\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2$: original THSR)

$$\Phi_\alpha(\boldsymbol{\beta}, b) = \exp\left(-2\sum_k^{x,y,z} \frac{r_k^2}{b^2 + 2\beta_k^2}\right)\phi_\alpha(b)$$

$$\Phi_{^{12}\mathrm{C}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) = \Psi_G^{-1}\mathcal{A}\{\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\,\Phi_\alpha(\boldsymbol{\beta}_2, b)\}$$

Ψ_G: Total center-of-mass w.f. to be eliminated

Hill-Wheeler eq. or GCM (generator coordinate method)

$$\sum_{\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2} \left\langle \hat{P}_{MK}^J \Phi_{^{12}\mathrm{C}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) \middle| \hat{H} - E \middle| \hat{P}_{MK}^J \Phi_{^{12}\mathrm{C}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2, b) \right\rangle f(\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2) = 0$$

$\hat{P}_{MK}^J$: Angular momentum projection operator

Hamiltonian (NN force: Volkov No.2 force)

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i^{12} \nabla_i^2 - T_G + \sum_{i<j}^{12}(V_{ij}^{(N)} + V_{ij}^{(C)})$$

$\boldsymbol{\beta}_i = (\beta_{ix} = \beta_{iy}, \beta_{iz})$

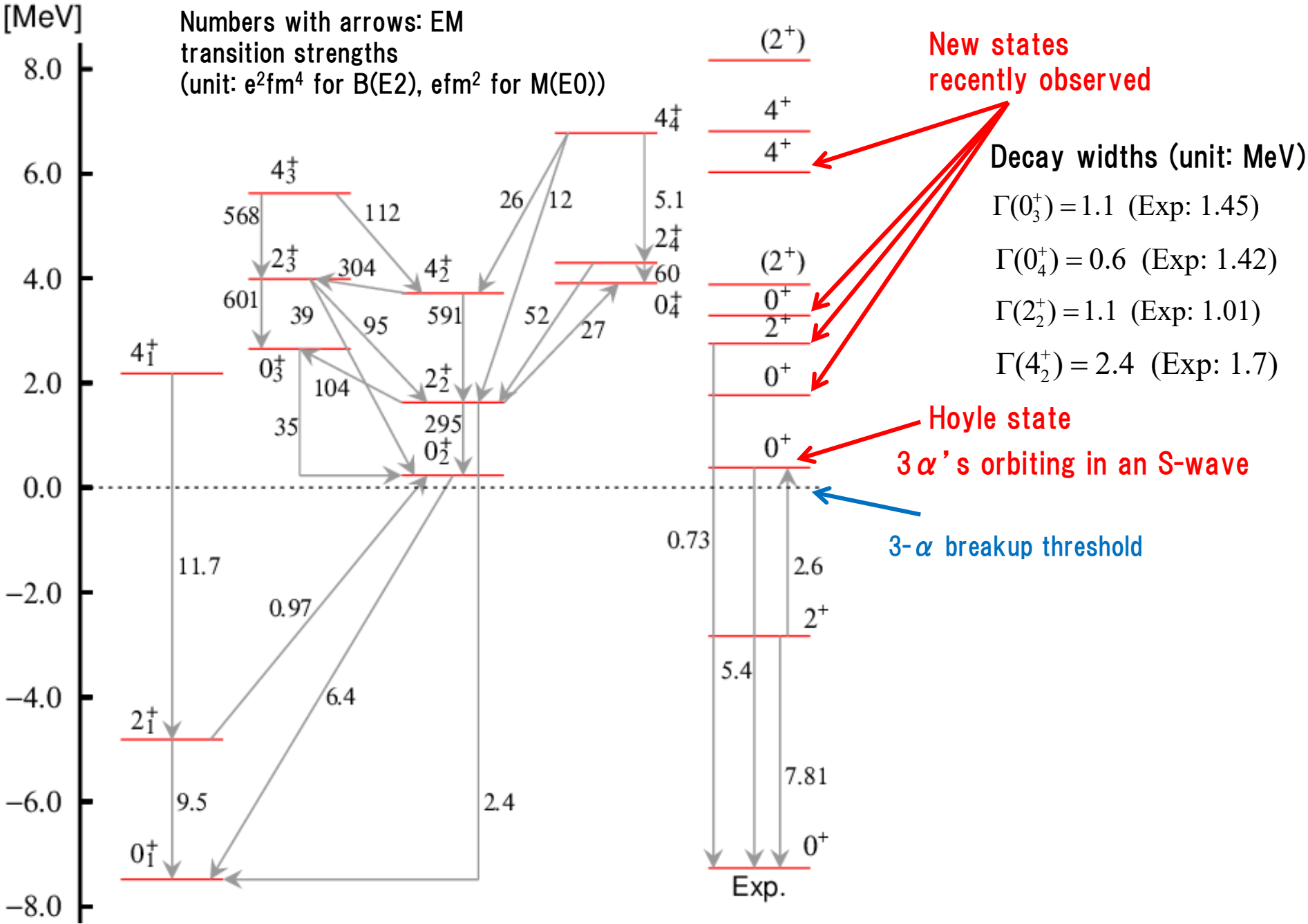
With (axially symmetric) deformation

Spurious continuum components are effectively eliminated by r^2 constraint method.

See Y. F. et al., PTP **115,** 115 (2006).

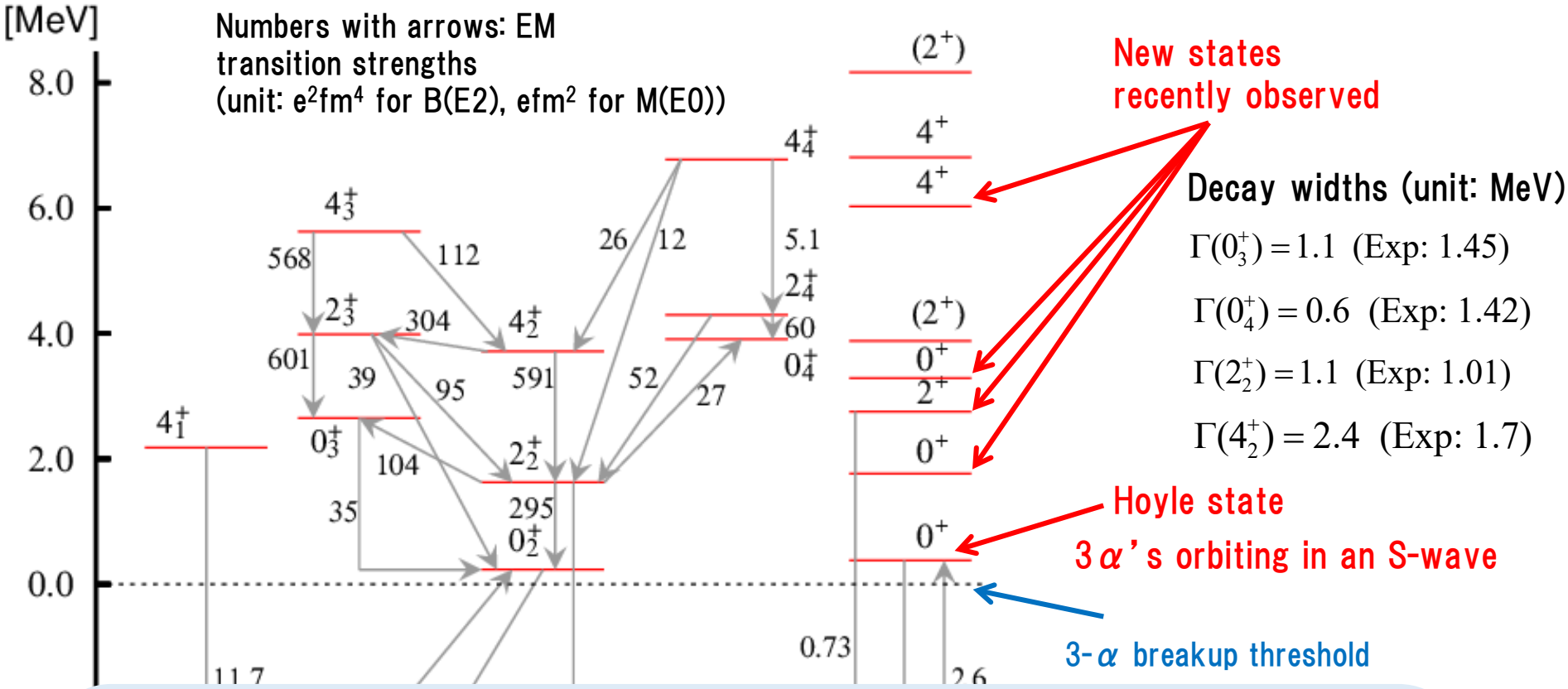
Results (of ^{12}C)

Y. F. et al., PPNP **82**, 78-132 (2015).



Results (of ^{12}C)

Y. F. et al., PPNP **82**, 78-132 (2015).



New observed states are consistently reproduced.

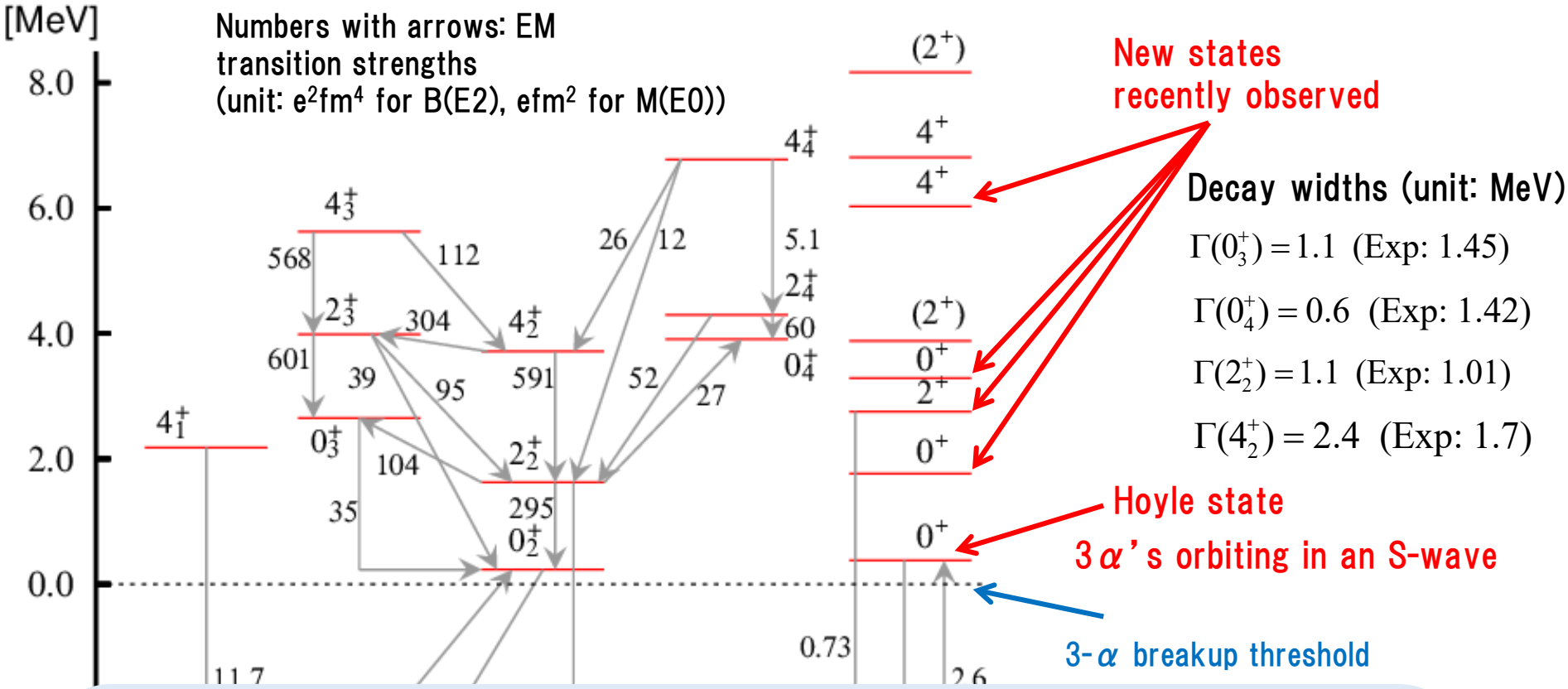
Large spatial size
3.7 fm ~ 4.7 fm

except for the shell-model-like states
($0_1^+, 2_1^+, 4_1^+$: ~2.4 fm)

All excited states above the threshold are governed by cluster dynamics

Results (of ^{12}C)

Y. F. et al., PPNP **82**, 78-132 (2015).



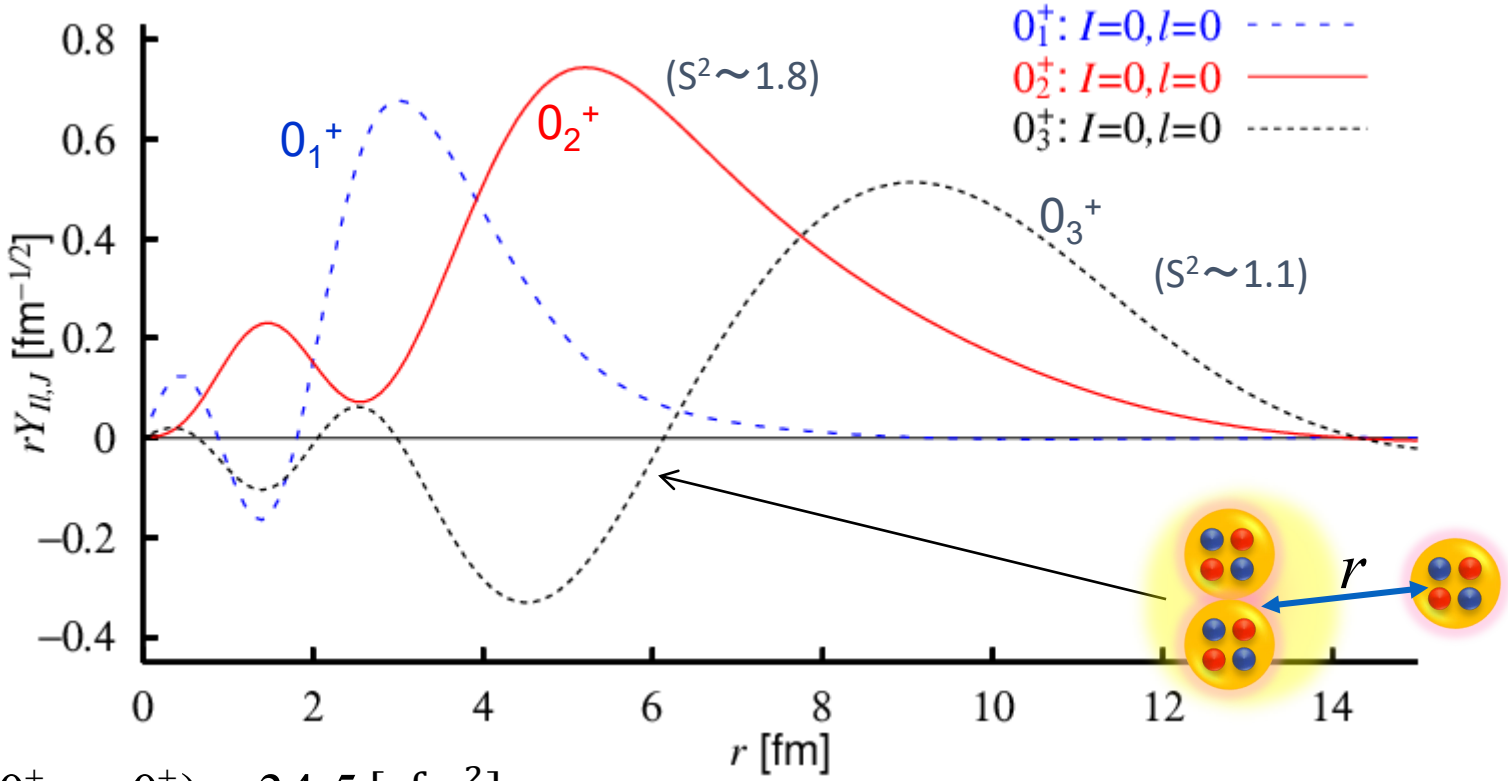
New observed states are consistently reproduced.

Rich alpha cluster dynamics built on the Hoyle state, as if the Hoyle state were the g.s. of cluster excitations

All excited states above the threshold are governed by cluster dynamics

0_3^+ state: higher nodal excitation of the Hoyle state

Overlap functions of the 0_1^+ , 0_2^+ , 0_3^+ states for ^{8}Be(0+)+α(S) channel



$$M(E0;\ 0_3^+ \to 0_2^+) = 34.5\ [e\mathrm{fm}^2]$$

Very large monopole transition strength between the 0_2^+ and 0_3^+ states

c.f. $M(E0;\ 0_2^+ \to 0_1^+) = 6.4\ [e\mathrm{fm}^2]$

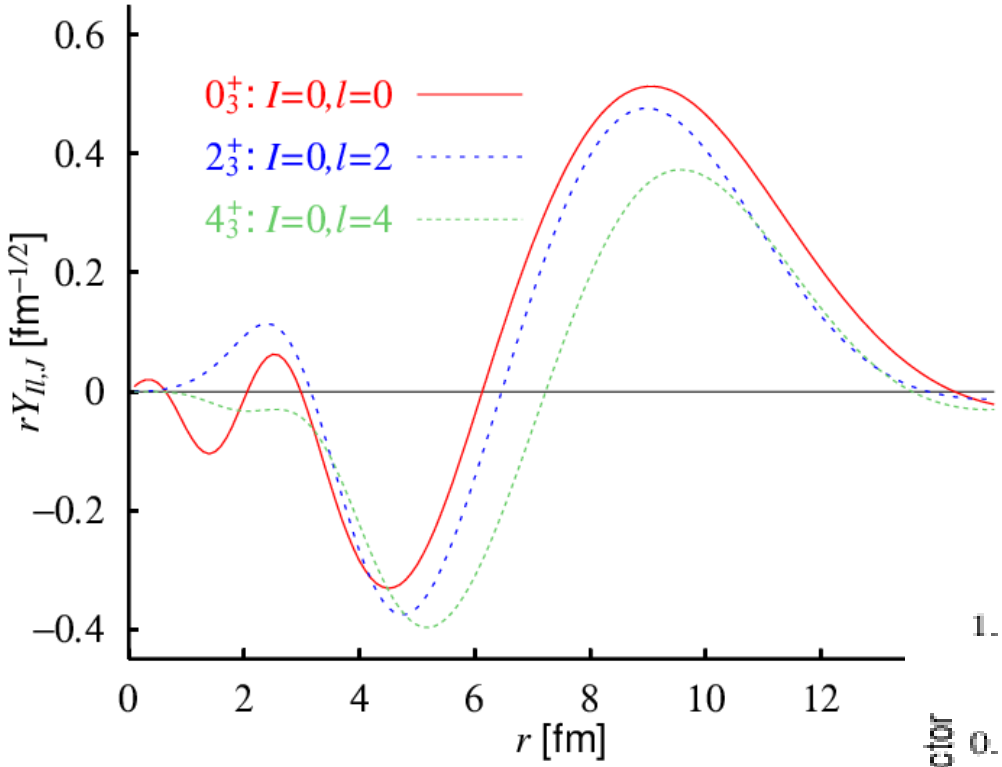
0_1^+ state: 2 nodes

0_2^+ state: 3 nodal oscillation (nodes disappear due to the dissolution of ^{8}Be core)

0_3^+ state: 4 nodes (higher nodal structure)

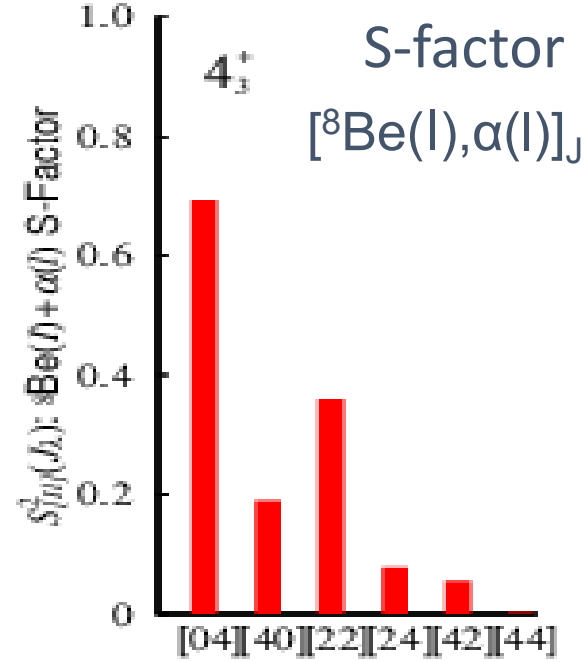
Y. F., PRC **92**, 021302(R) (2015).

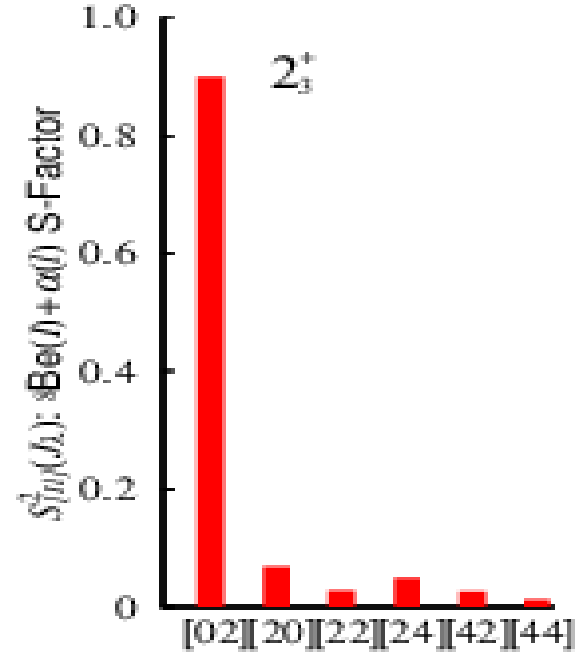
RWA of 0_3^+ , 2_3^+ , 4_3^+ states: ^{8}Be(0^+) + alpha(l) rotational band

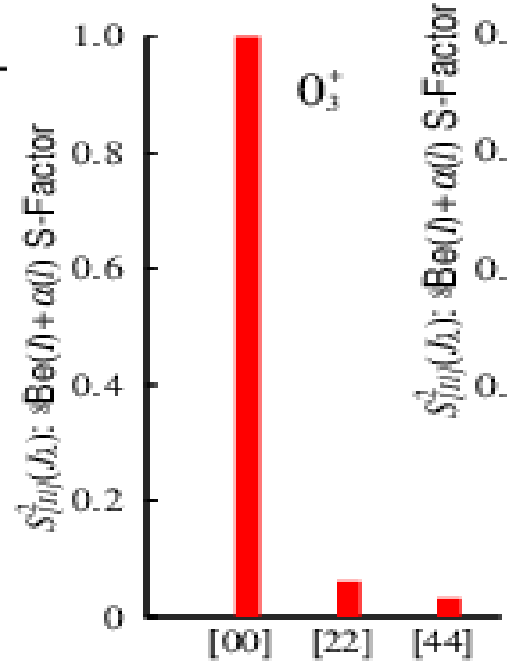


S-factor

[^{8}Be(l),α(l)]$_J$







^{8}Be + alpha : well developed, constructed on the Hoyle state

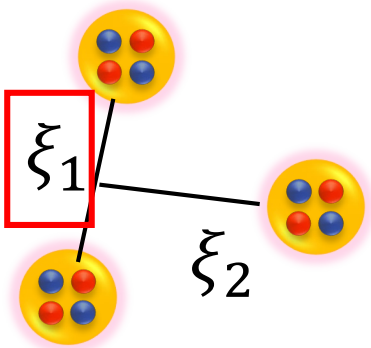
0_3^+ : higher nodal behavior (4S)

Alpha-alpha correlation in the 0_3^+ state

$$M(E0;\ 0_3^+ \to 0_2^+) = 34.5\ [e\mathrm{fm}^2]$$

Very large monopole transition strength between the 0_2^+ and 0_3^+ states

$$O_B = \sum_{i=1}^{12} (\boldsymbol{r}_i - \boldsymbol{r}_{\mathrm{cm}})^2 = \sum_{k=1}^{3} \sum_{i \in \alpha_k} (\boldsymbol{r}_i - \boldsymbol{X}_k)^2 + \boxed{2\xi_1^2} + \frac{8}{3}\xi_2^2$$



Natural to consider a large portion from alpha-alpha relative motion (ξ_1), too.

B. Zhou et al., PRC **94**, 044319 (2016) .

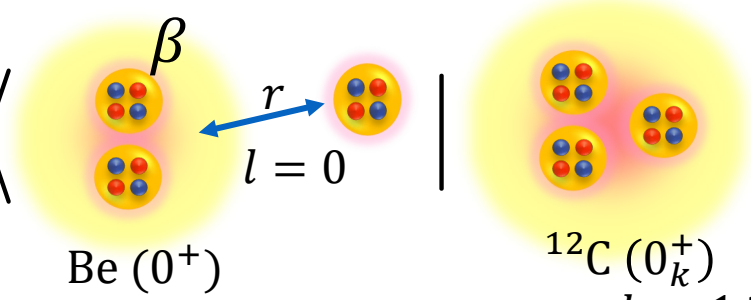
How S-factor changes as virtually $^{8}\mathrm{Be}(0^{+}:\beta)$ is varied

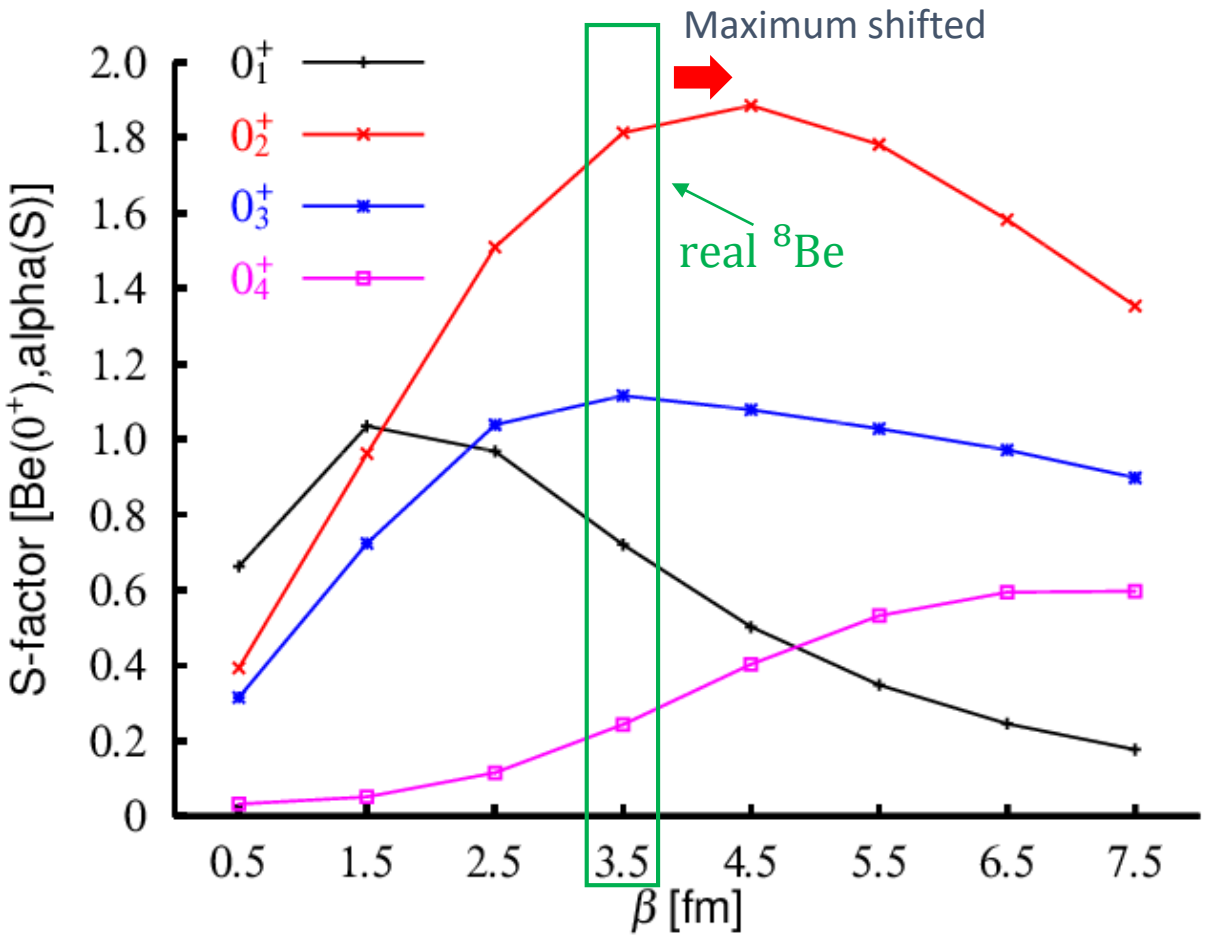
Overlap amplitude between ^{8}Be+alpha and ^{12}C

Definition of S-factor

$$S^2 = \int dr \left(r \times Y_{[0,0]}(r)\right)^2$$

$$Y_{[0,0]}(r) = \left\langle \; \text{Be}\,(0^+) \; \middle| \; {}^{12}\text{C}\,(0_k^+) \; \right\rangle$$





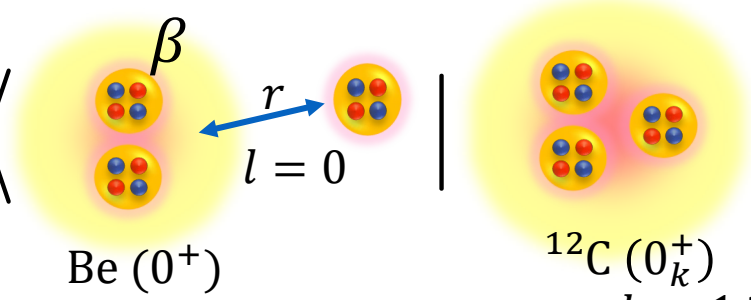
How S-factor changes as virtually $^{8}\mathrm{Be}(0^{+}:\beta)$ is varied

Overlap amplitude between ^{8}Be+alpha and ^{12}C

Definition of S-factor

$$S^2 = \int dr \left(r \times Y_{[0,0]}(r)\right)^2$$

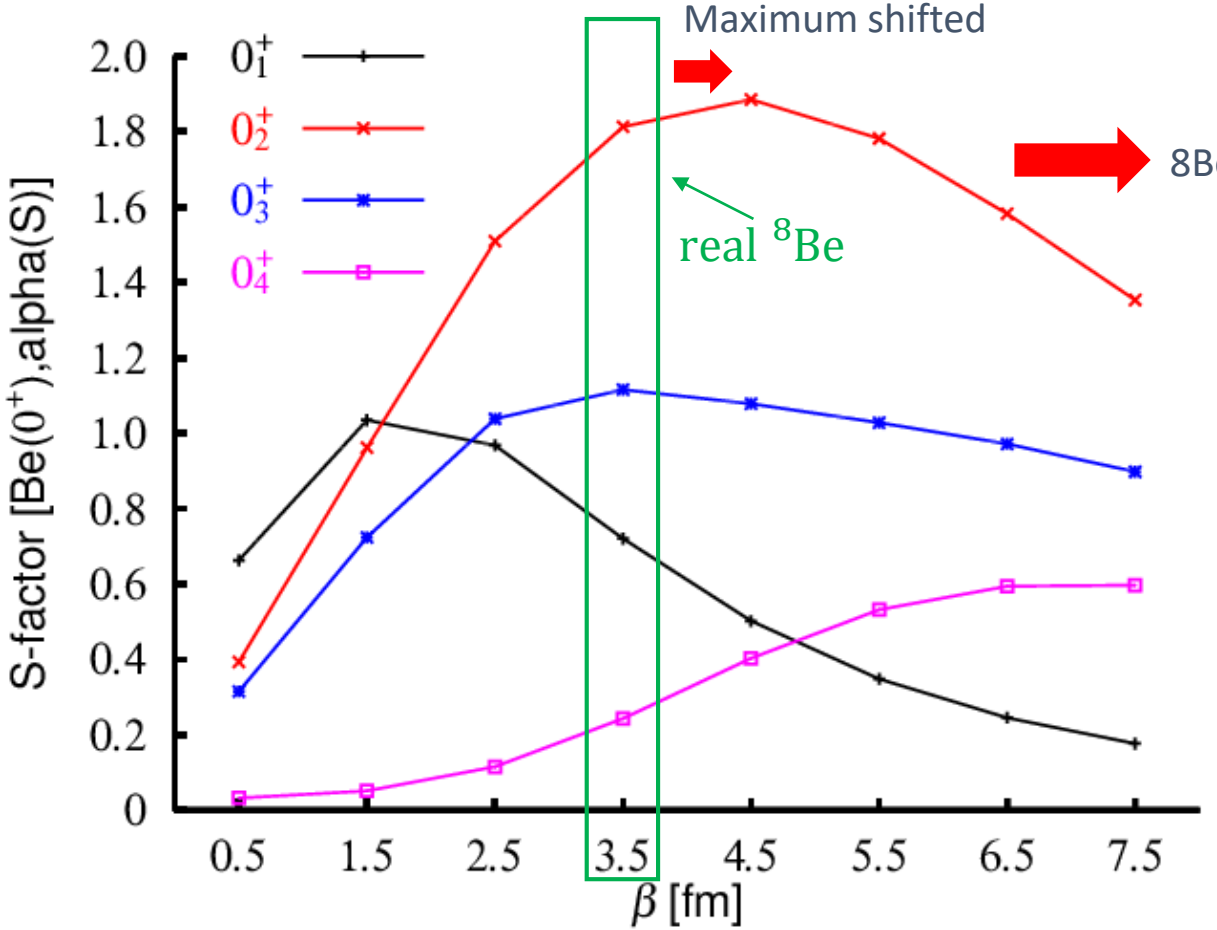
$$Y_{[0,0]}(r) = \langle$$



$$\rangle$$

Maximum shifted

8Be becomes larger in the 02+ state



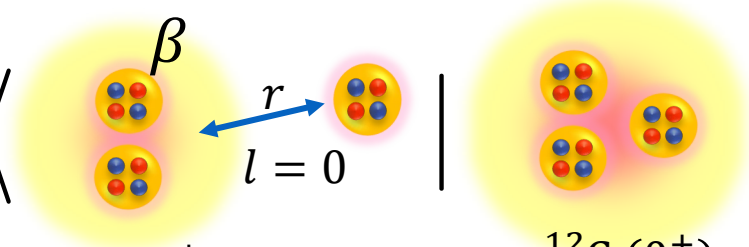
How S-factor changes as virtually $^{8}\mathrm{Be}(0^{+}:\beta)$ is varied

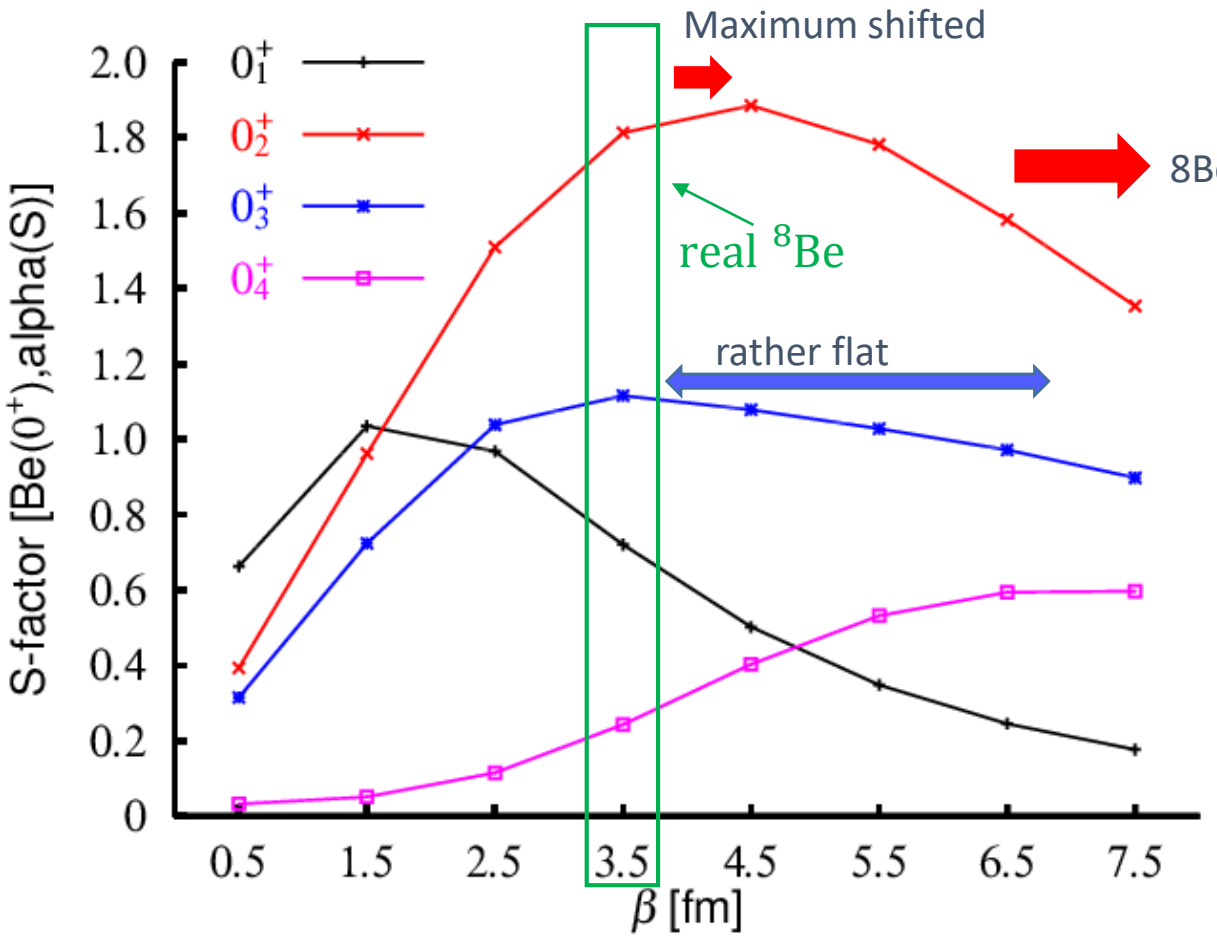
Overlap amplitude between ^{8}Be+alpha and ^{12}C

Definition of S-factor

$$S^2 = \int dr \left(r \times Y_{[0,0]}(r)\right)^2$$

$$Y_{[0,0]}(r) = \left\langle \; \beta \; r \; l = 0 \; \middle| \; \right\rangle$$





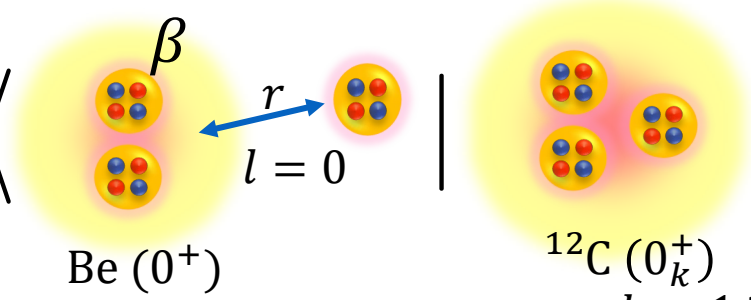
How S-factor changes as virtually $^8\mathrm{Be}(0^+:\beta)$ is varied

Overlap amplitude between ^{8}Be+alpha and ^{12}C

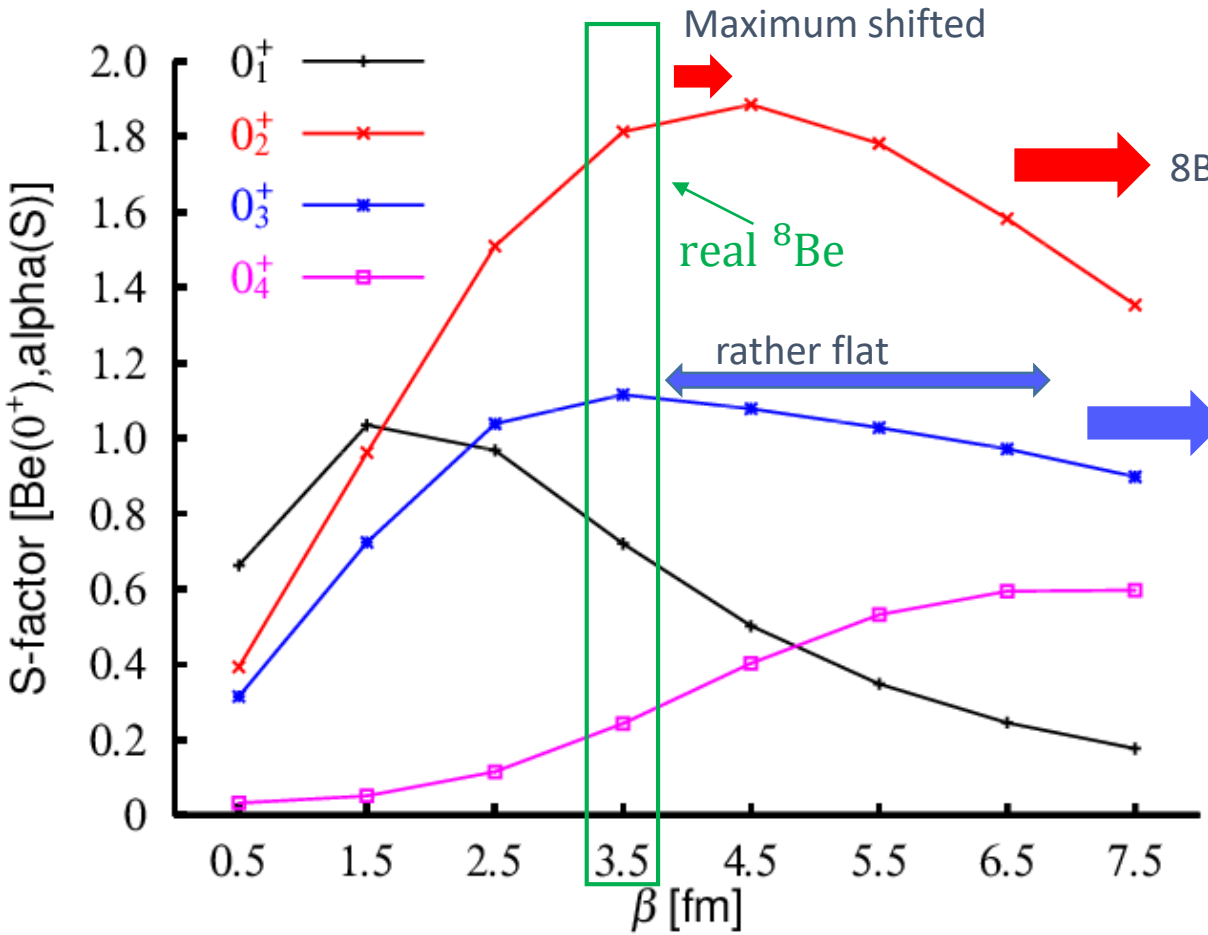
Definition of S-factor

$$S^2 = \int dr\,(r \times Y_{[0,0]}(r))^2$$

$$Y_{[0,0]}(r) = \langle$$



$$\rangle$$



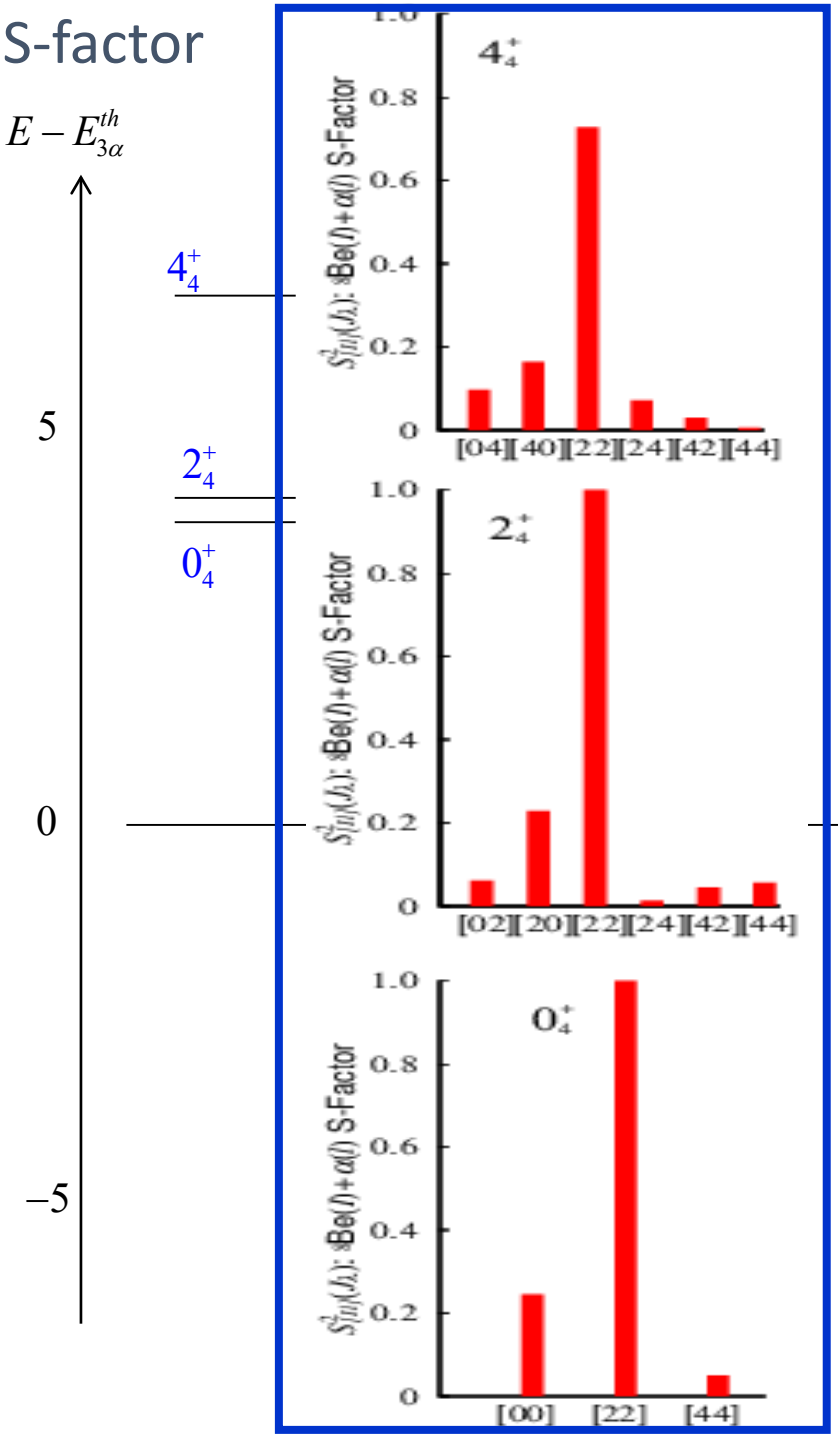
8Be becomes larger in the 02+ state

8Be: very fragile

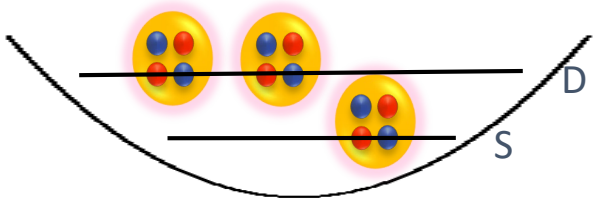
Large B(E0) strength also coming from 8Be excitation

$$M(E0;\ 0_3^+ \to 0_2^+) = 34.5\ [e\mathrm{fm}^2]$$

S-factor



Large components from
$[2,2]_J$ channel
-> corresponding to vibrational mode ?

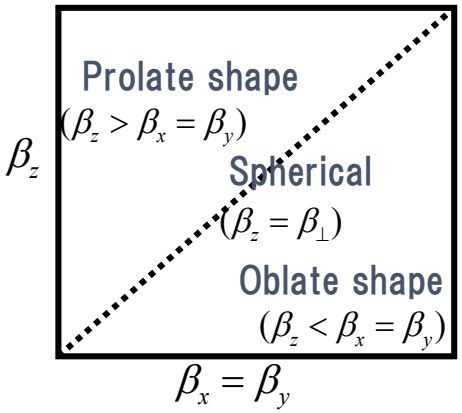


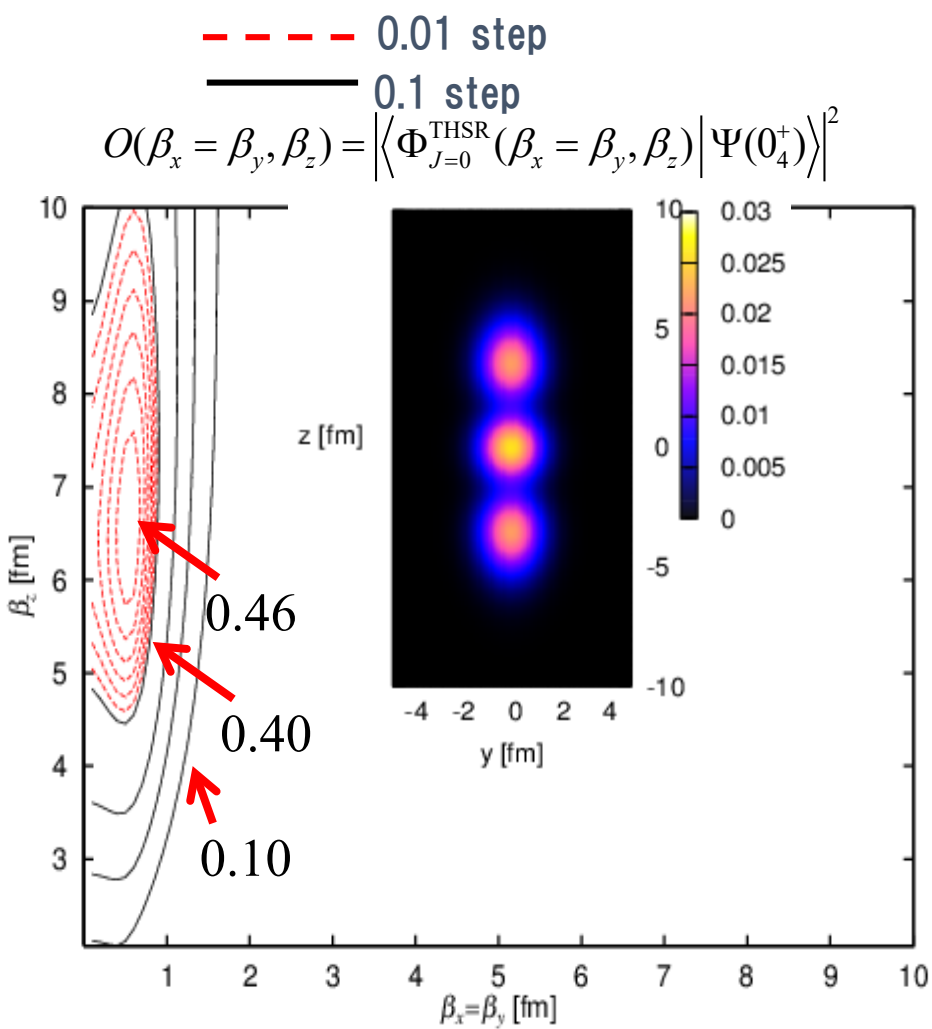
Two α's jump into D-orbit, coupling to J=0,2,4 ?

Squared overlap with single THSR config. for the 0_4^+ state of ^{12}C

For the 0_4^+ state

Clear linear-chain structure

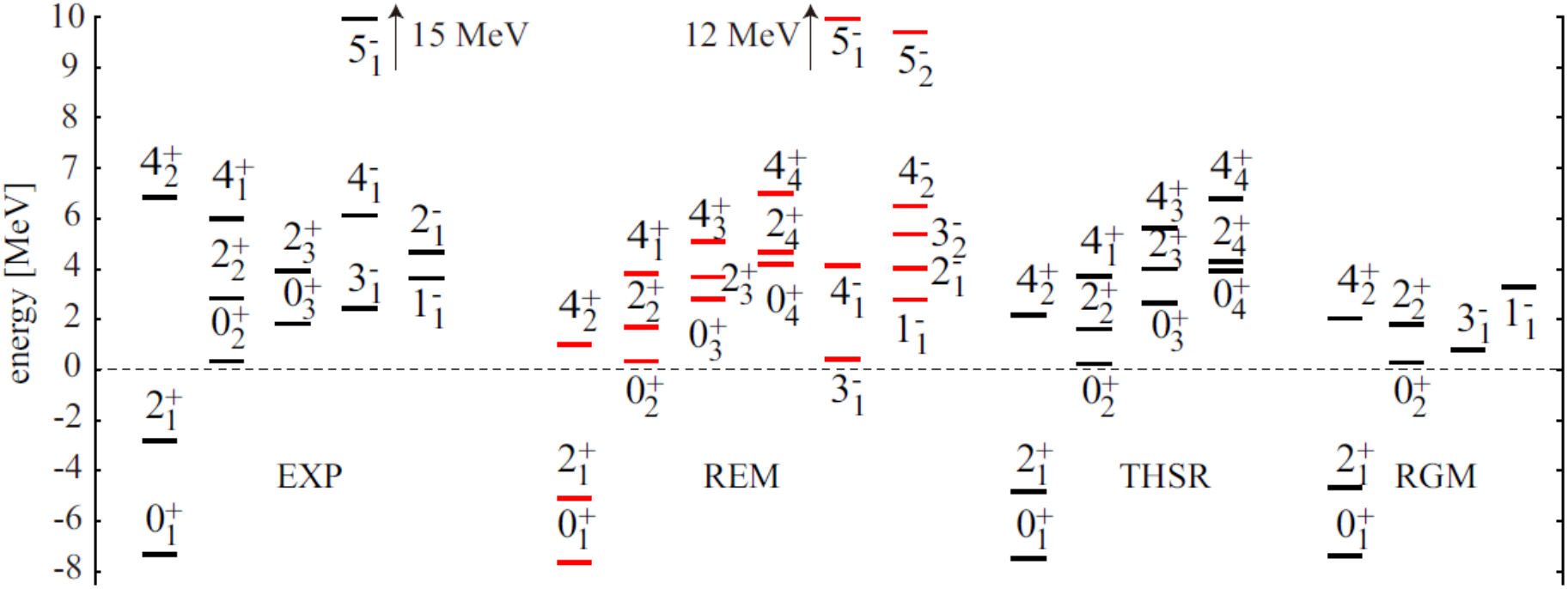




Y. F., PRC **94**, 024344 (2016) .

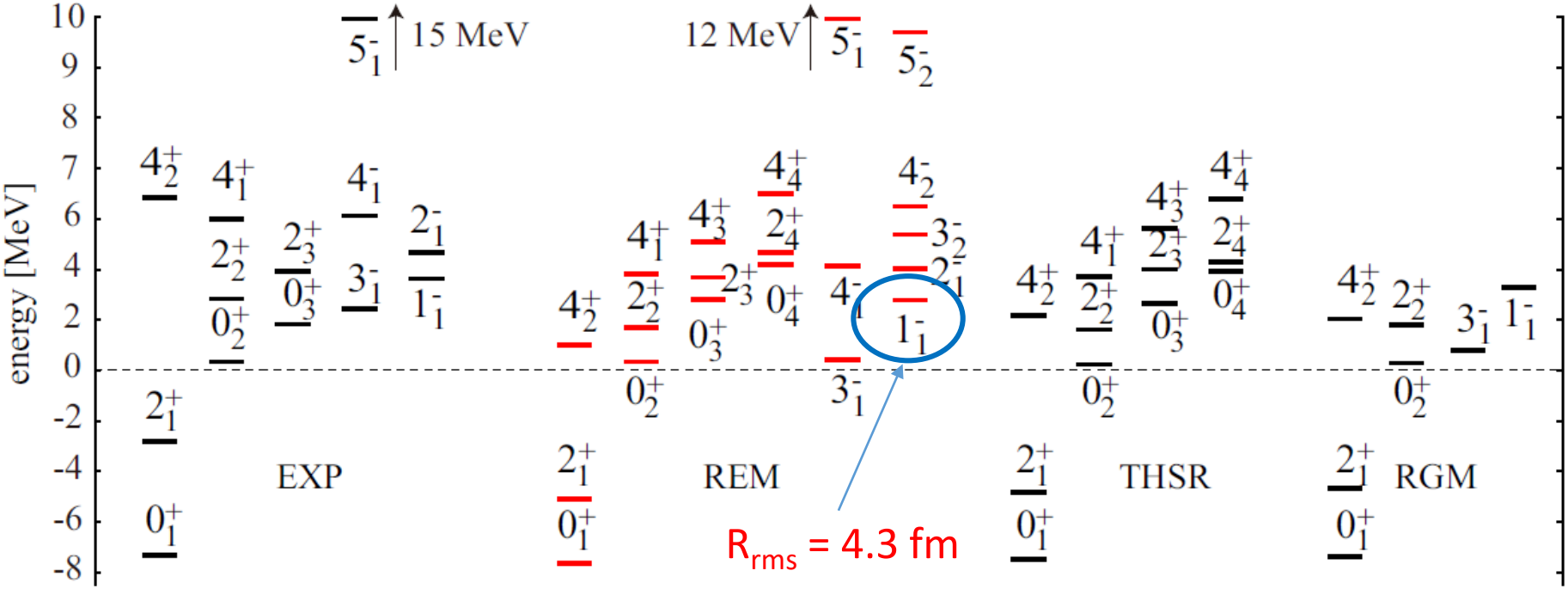
Well reproduced by REM.

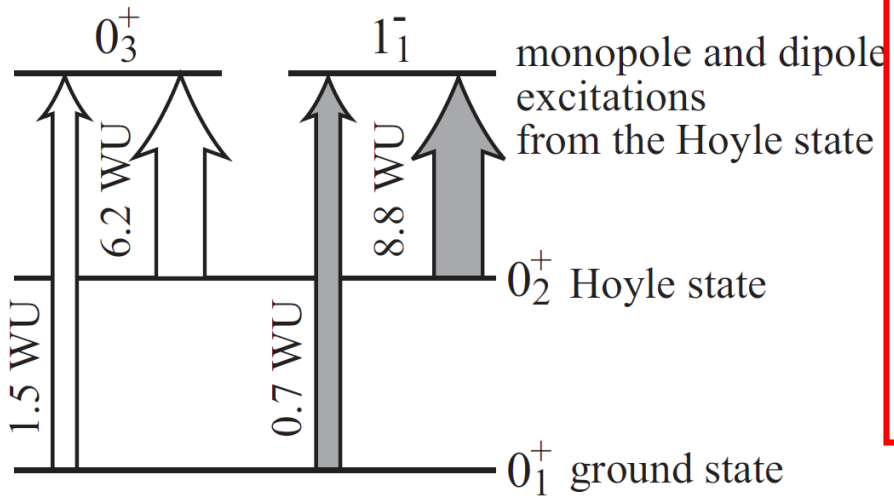
R. Imai et al., arXiv: 1802.03523.

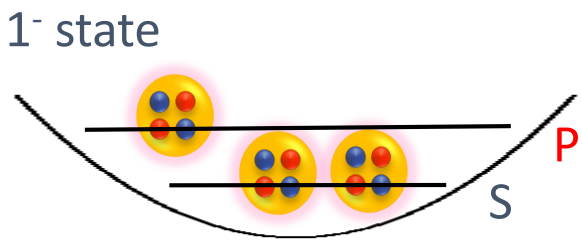


Well reproduced by REM.

R. Imai et al., arXiv: 1802.03523.







P-state occupancy: 35.3 %

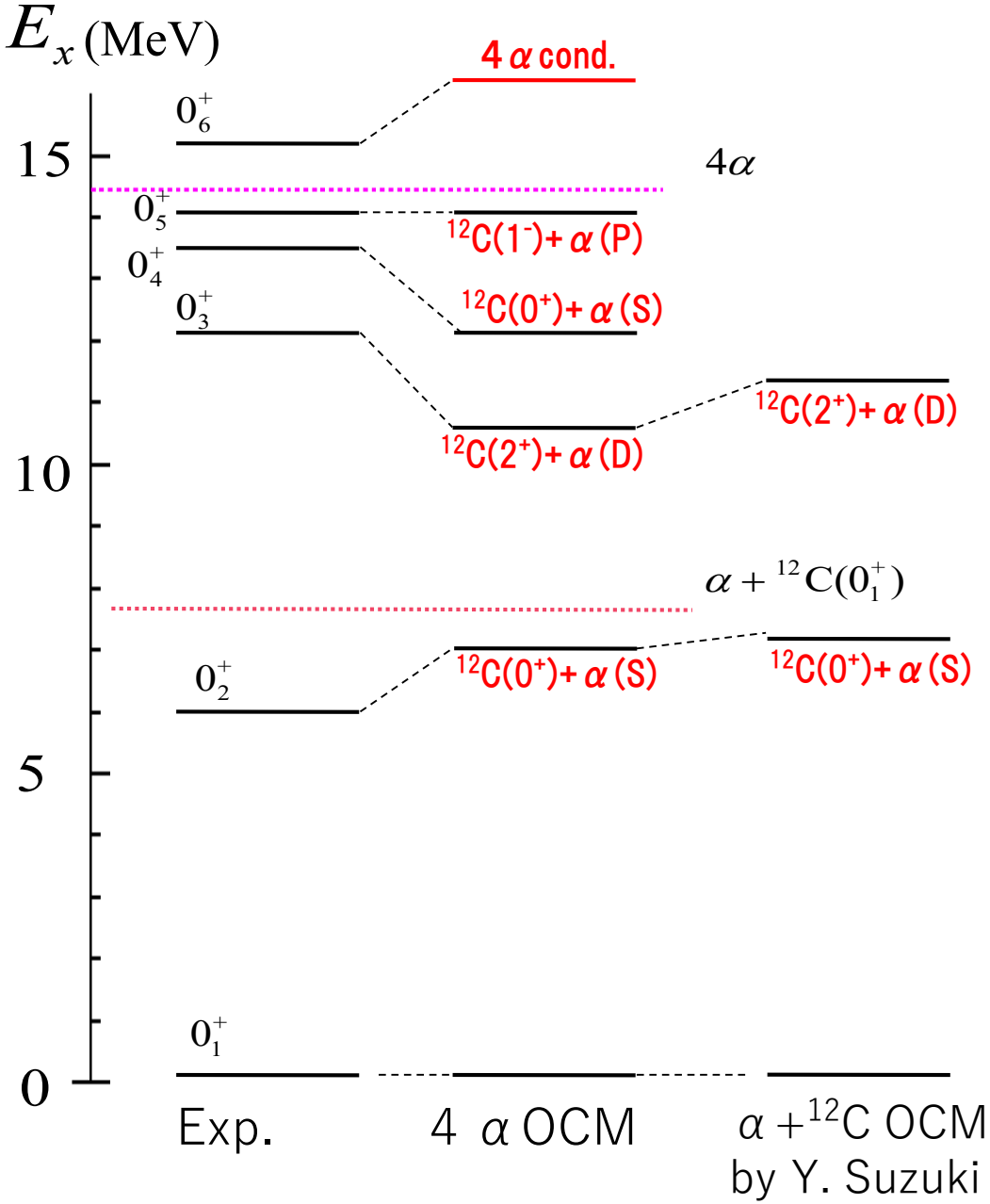
R_{rms} = 3.3 fm

Yamada & Schuck,.EPJA **26**, 185 (2005) .

3-alpha RGM: R_{rms} = 3.4 fm

Alpha condensate and clusters in ^{16}O

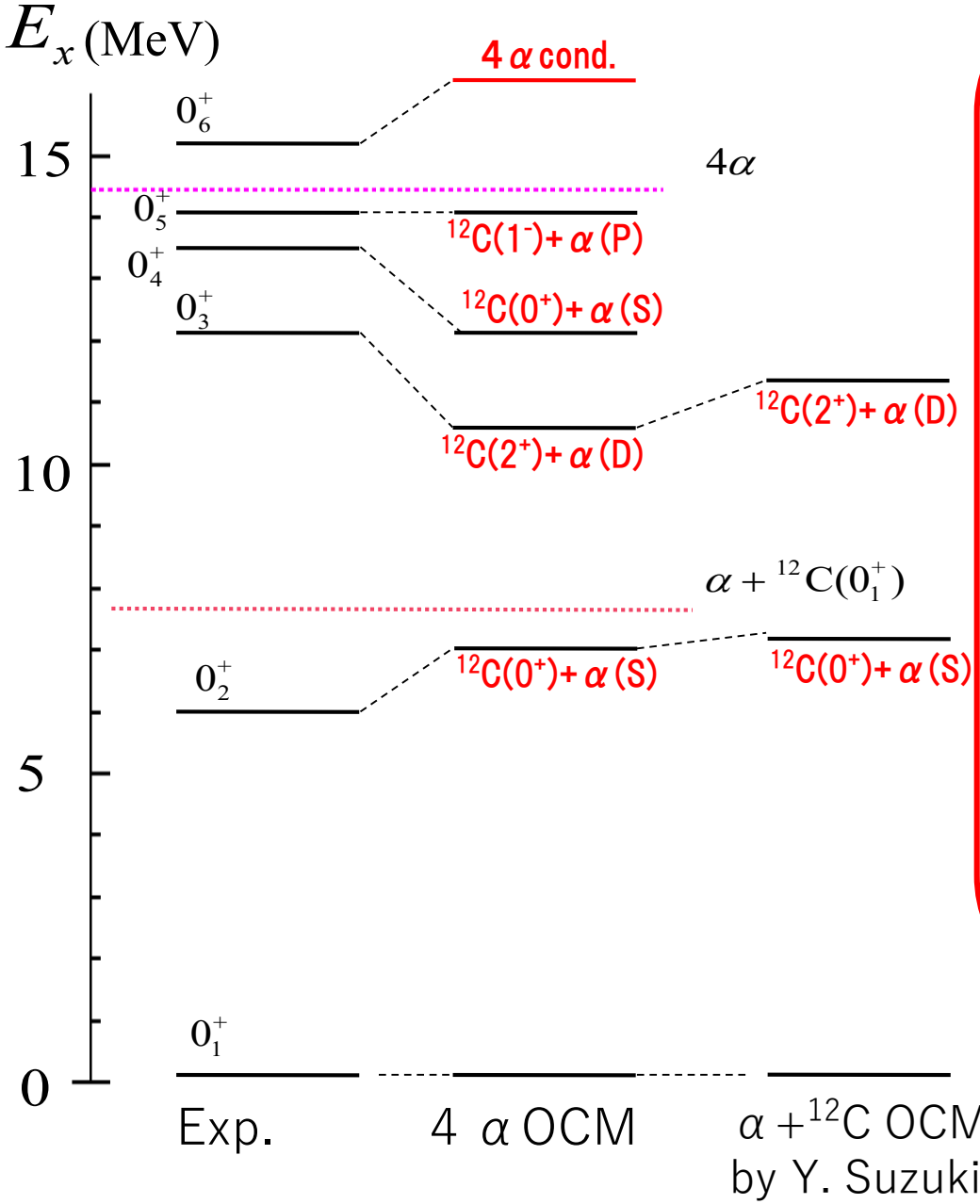
4-alpha condensate state in ^{16}O



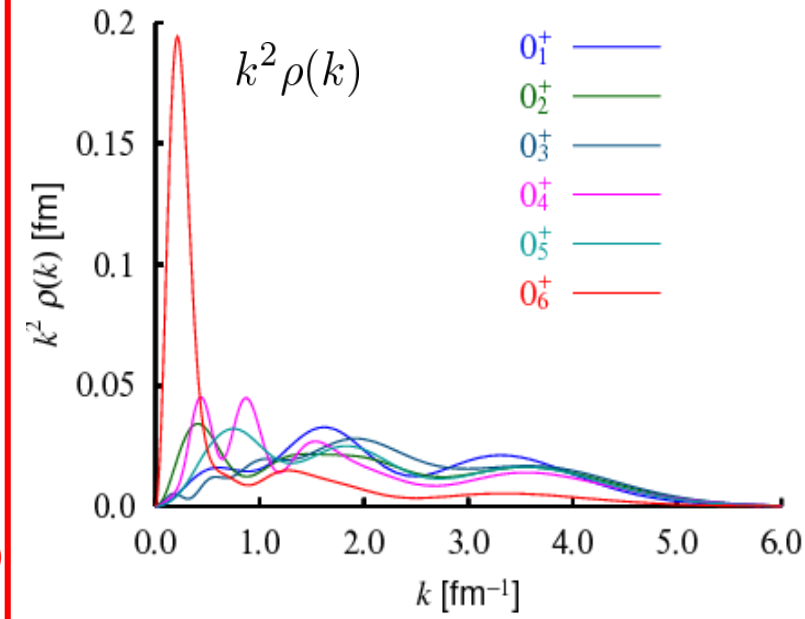
0_4^+ state: T. Wakasa, Y. F. et al., PLB 653, 173 (2007).

Y. F. et al., PRL 101, 081502 (2008).

4-alpha condensate state in ^{16}O



Momentum distributions of α particle



0_6^+: 4 α condensate

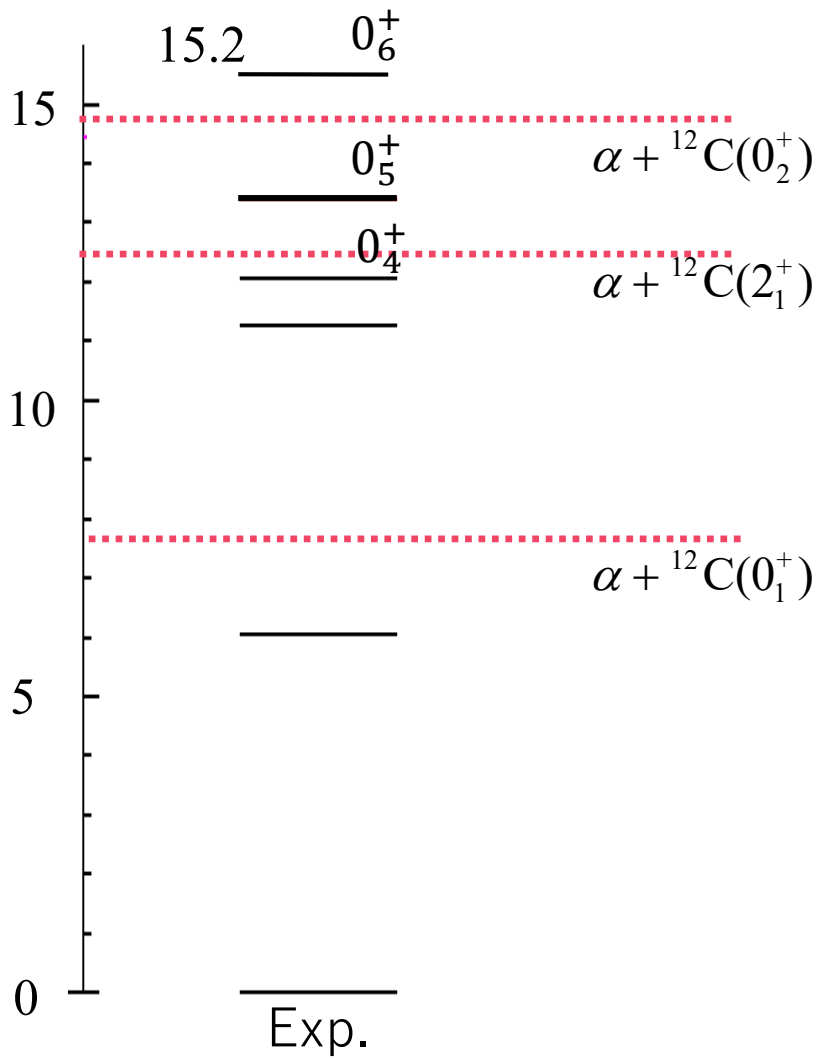
Corresponding de Broglie length

$$\lambda = \frac{2\pi}{\sqrt{\langle k^2 \rangle}} \geq 20 \text{ fm}$$

0_4^+ state: T. Wakasa, Y. F. et al., PLB 653, 173 (2007).

Y. F. et al., PRL 101, 081502 (2008).

Decay widths are well reproduced.



$\Gamma\ (0_4^+)_{\mathrm{OCM}} \sim 0.8$ MeV

$\Gamma\ (0_5^+)_{\mathrm{OCM}} < 0.2$ MeV

$\Gamma\ (0_6^+)_{\mathrm{OCM}} = 0.13$ MeV

(calculated based on R-matrix theory)

$\Gamma = P \cdot \gamma^2$: (partial) decay width

γ^2: reduced width
(alpha+12C components)

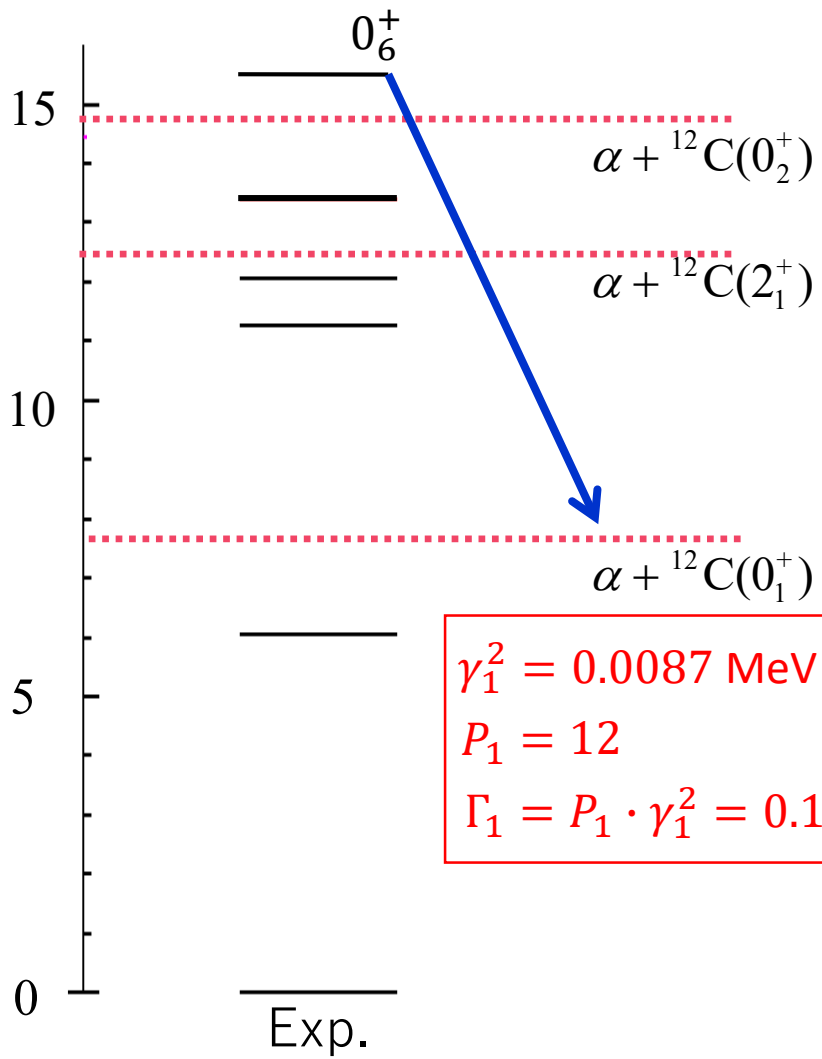
P: penetration factor

$\Gamma\ (0_4^+$ at 13.6 MeV$) = 0.6$ MeV

$\Gamma\ (0_5^+$ at14.0 MeV$) = 0.19$ MeV

$\Gamma\ (0_6^+$ at15.2 MeV$) = 0.17$ MeV

The reason why 0_6^+ is narrow in spite of such high excitation



$\gamma_1^2 = 0.0087$ MeV

$P_1 = 12$

$\Gamma_1 = P_1 \cdot \gamma_1^2 = 0.104$ MeV

$\Gamma\ (0_4^+)_{\mathrm{OCM}} \sim 0.8$ MeV

$\Gamma\ (0_5^+)_{\mathrm{OCM}} < 0.2$ MeV

$\Gamma\ (0_6^+)_{\mathrm{OCM}} = 0.13$ MeV

(calculated based on R-matrix theory)

$\Gamma = P \cdot \gamma^2$: (partial) decay width

γ^2: reduced width
(alpha+12C components)

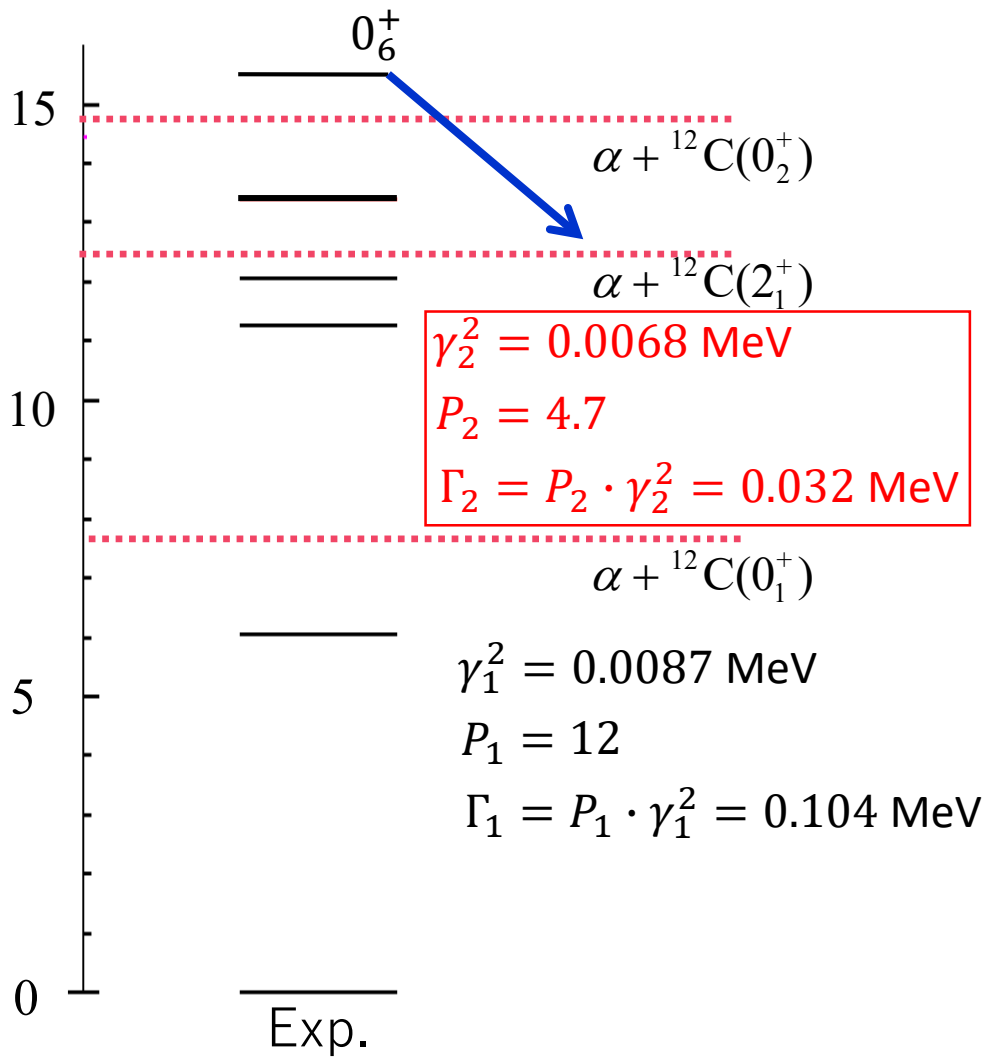
P: penetration factor

$\Gamma\ (0_4^+$ at 13.6 MeV$) = 0.6$ MeV

$\Gamma\ (0_5^+$ at14.0 MeV$) = 0.19$ MeV

$\Gamma\ (0_6^+$ at15.2 MeV$) = 0.17$ MeV

The reason why 0_6^+ is narrow in spite of such high excitation



$\Gamma\ (0_4^+)_{\mathrm{OCM}} \sim 0.8$ MeV

$\Gamma\ (0_5^+)_{\mathrm{OCM}} < 0.2$ MeV

$\Gamma\ (0_6^+)_{\mathrm{OCM}} = 0.13$ MeV

(calculated based on R-matrix theory)

$\Gamma = P \cdot \gamma^2$: (partial) decay width

γ^2: reduced width
(alpha+12C components)

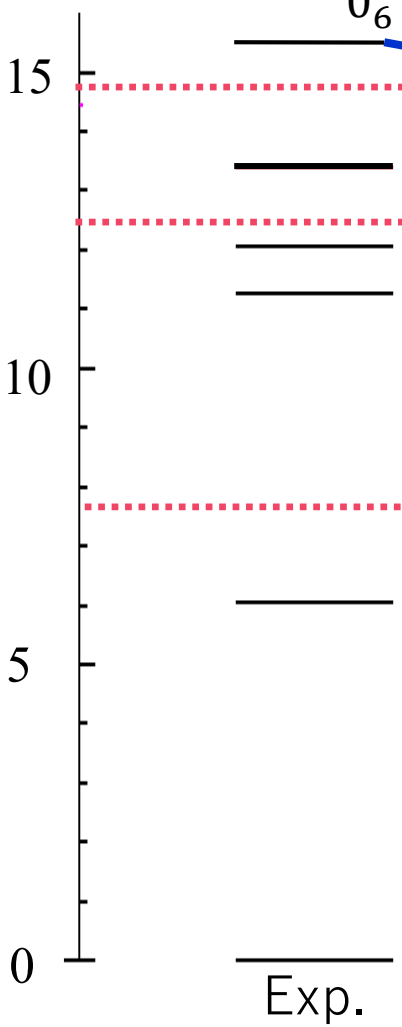
P: penetration factor

$\Gamma\ (0_4^+$ at 13.6 MeV$) = 0.6$ MeV

$\Gamma\ (0_5^+$ at14.0 MeV$) = 0.19$ MeV

$\Gamma\ (0_6^+$ at15.2 MeV$) = 0.17$ MeV

The reason why 0_6^+ is narrow in spite of such high excitation



0_6^+

$\gamma_3^2 = 0.218$ MeV

$P_3 = 3.7 \times 10^{-6}$

$\Gamma_3 = P_3 \cdot \gamma_3^2 = 8 \times 10^{-8}$ MeV

$\alpha + {}^{12}\mathrm{C}(0_2^+)$

$\alpha + {}^{12}\mathrm{C}(2_1^+)$

$\gamma_2^2 = 0.0068$ MeV

$P_2 = 4.7$

$\Gamma_2 = P_1 \cdot \gamma_1^2 = 0.032$ MeV

$\alpha + {}^{12}\mathrm{C}(0_1^+)$

$\gamma_1^2 = 0.0087$ MeV

$P_1 = 12$

$\Gamma_1 = P_1 \cdot \gamma_1^2 = 0.104$ MeV

$\Gamma\ (0_4^+)_{\mathrm{OCM}} \sim 0.8$ MeV

$\Gamma\ (0_5^+)_{\mathrm{OCM}} < 0.2$ MeV

$\Gamma\ (0_6^+)_{\mathrm{OCM}} = 0.13$ MeV

(calculated based on R-matrix theory)

$\Gamma = P \cdot \gamma^2$: (partial) decay width

γ^2: reduced width
(alpha+12C components)

P: penetration factor

$\Gamma\ (0_4^+$ at 13.6 MeV$) = 0.6$ MeV

$\Gamma\ (0_5^+$ at14.0 MeV$) = 0.19$ MeV

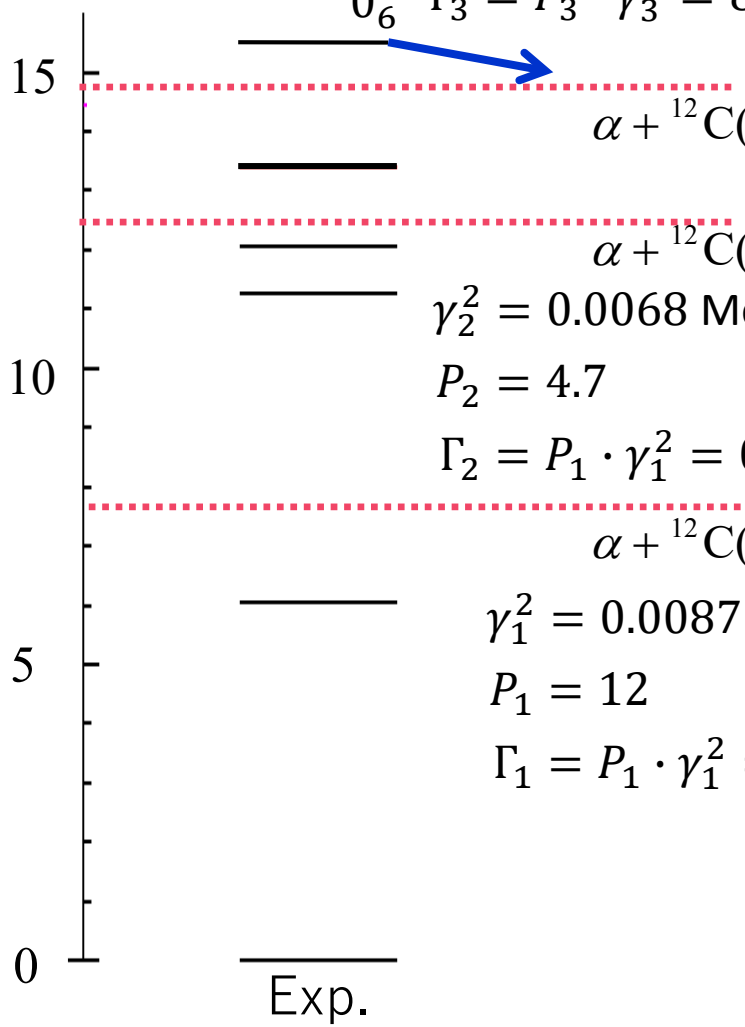
$\Gamma\ (0_6^+$ at15.2 MeV$) = 0.17$ MeV

The reason why 0_6^+ is narrow in spite of such high excitation

$\gamma_3^2 = 0.218$ MeV

$P_3 = 3.7 \times 10^{-6}$

0_6^+ $\Gamma_3 = P_3 \cdot \gamma_3^2 = 8 \times 10^{-7}$ MeV



$\gamma_2^2 = 0.0068$ MeV

$P_2 = 4.7$

$\Gamma_2 = P_1 \cdot \gamma_1^2 = 0.032$ MeV

$\gamma_1^2 = 0.0087$ MeV

$P_1 = 12$

$\Gamma_1 = P_1 \cdot \gamma_1^2 = 0.104$ MeV

$\Gamma\ (0_4^+)_{\mathrm{OCM}} \sim 0.8$ MeV

$\Gamma\ (0_5^+)_{\mathrm{OCM}} < 0.2$ MeV

$\Gamma\ (0_6^+)_{\mathrm{OCM}} = 0.13$ MeV$=\Gamma_1 + \Gamma_2 + \Gamma_3$

(calculated based on R-matrix theory)

$\Gamma = P \cdot \gamma^2$: (partial) decay width

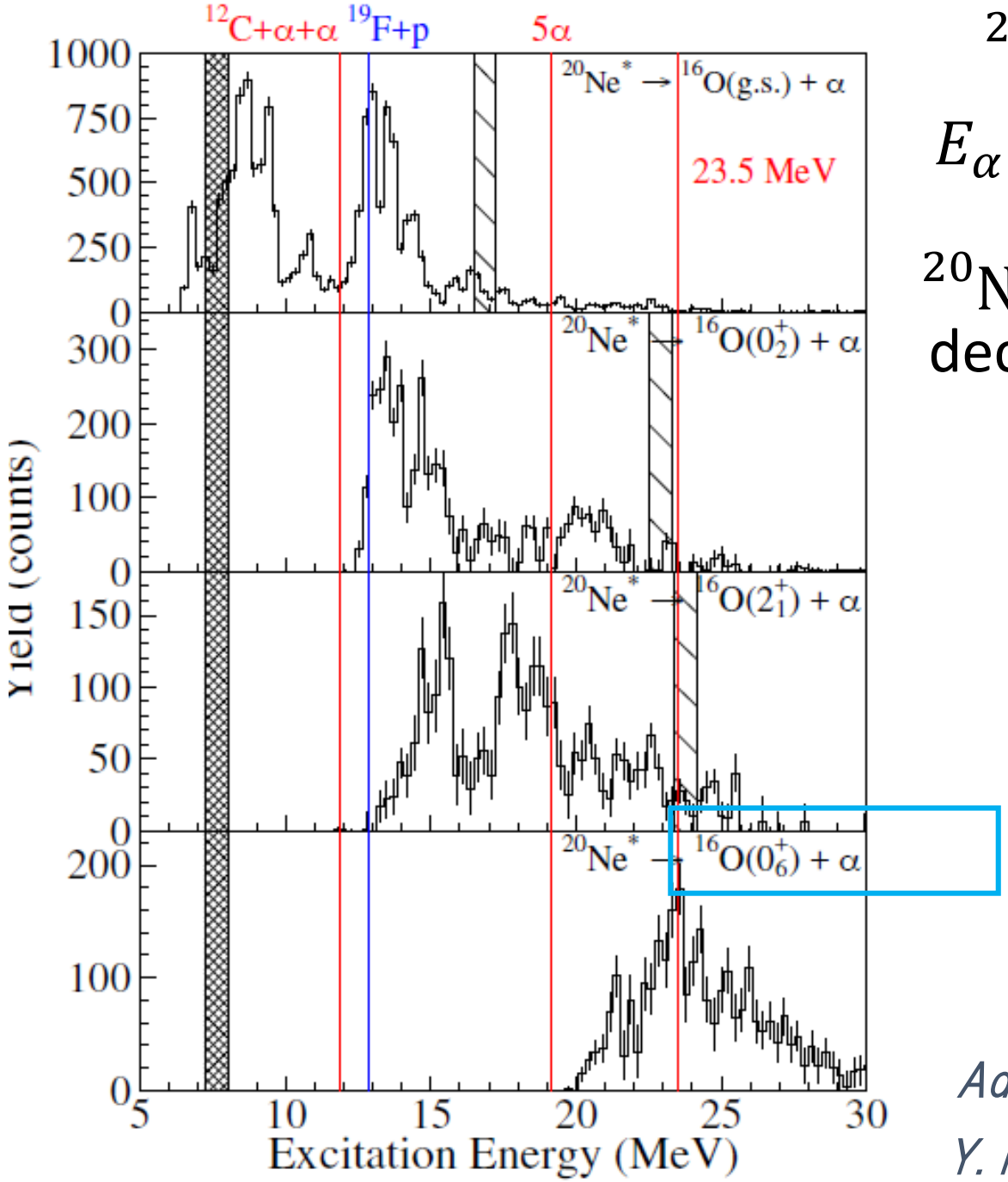
γ^2: reduced width
(alpha+12C components)

P: penetration factor

$\Gamma\ (0_4^+$ at 13.6 MeV$) = 0.6$ MeV

$\Gamma\ (0_5^+$ at14.0 MeV$) = 0.19$ MeV

$\Gamma\ (0_6^+$ at15.2 MeV$) = 0.17$ MeV



$^{20}\mathrm{Ne}(\alpha, \alpha')$

$E_\alpha = 389\ \mathrm{MeV}$

$^{20}\mathrm{Ne}$
decay into several channels

$E_x = 23.5\ \mathrm{MeV}$

$^{16}\mathrm{O}(0_6^+)$ analog state

5α condensate

Adachi, Fujikawa, Kawabata et al.

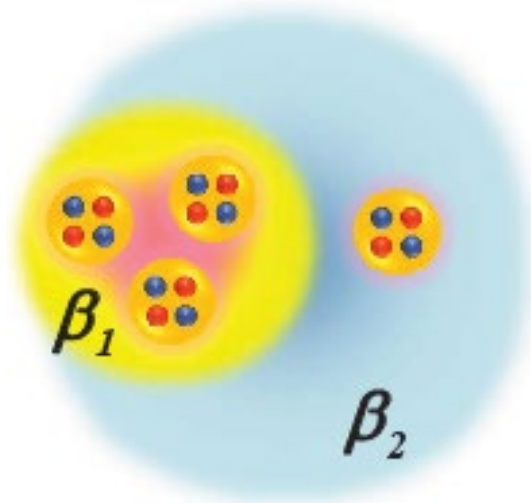
Y. Fujikawa, Master thesis(2019).

4α extended THSR wave function ($\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2$: original THSR)

$$\Phi_\alpha(\boldsymbol{\beta}, b) = \exp\left(-2\sum_k^{x,y,z} \frac{r_k^2}{b^2 + 2\beta_k^2}\right)\phi_\alpha(b)$$

$$\Phi_{^{16}\mathrm{O}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) = \Psi_G^{-1}\mathcal{A}\{\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_2, b)\}$$

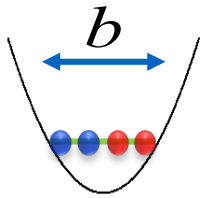
Ψ_G: Total center-of-mass w.f. to be eliminated



Internal w.f. of α particle

b=1.44 fm: fixed

$\phi_\alpha(b) :=$



4α extended THSR wave function ($\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2$: original THSR)

$$\Phi_\alpha(\boldsymbol{\beta}, b) = \exp\left(-2\sum_k^{x,y,z} \frac{r_k^2}{b^2 + 2\beta_k^2}\right)\phi_\alpha(b)$$

$$\Phi_{^{16}\mathrm{O}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) = \Psi_G^{-1}\mathcal{A}\{\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_2, b)\}$$

Ψ_G: Total center-of-mass w.f. to be eliminated

Hill-Wheeler eq. or GCM (generator coordinate method)

$$\sum_{\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2} \left\langle \hat{P}_{MK}^J \Phi_{^{16}\mathrm{O}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) \middle| \hat{H} - E \middle| \hat{P}_{MK}^J \Phi_{^{16}\mathrm{O}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2, b) \right\rangle f(\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2) = 0$$

$\hat{P}_{MK}^J$: Angular momentum projection operator

Hamiltonian (NN force: F1 force)

A. Tohsaki, PRC **49**, 1814 (1994).

$\boldsymbol{\beta}_i = (\beta_{ix} = \beta_{iy}, \beta_{iz})$

With (axially symmetric) deformation

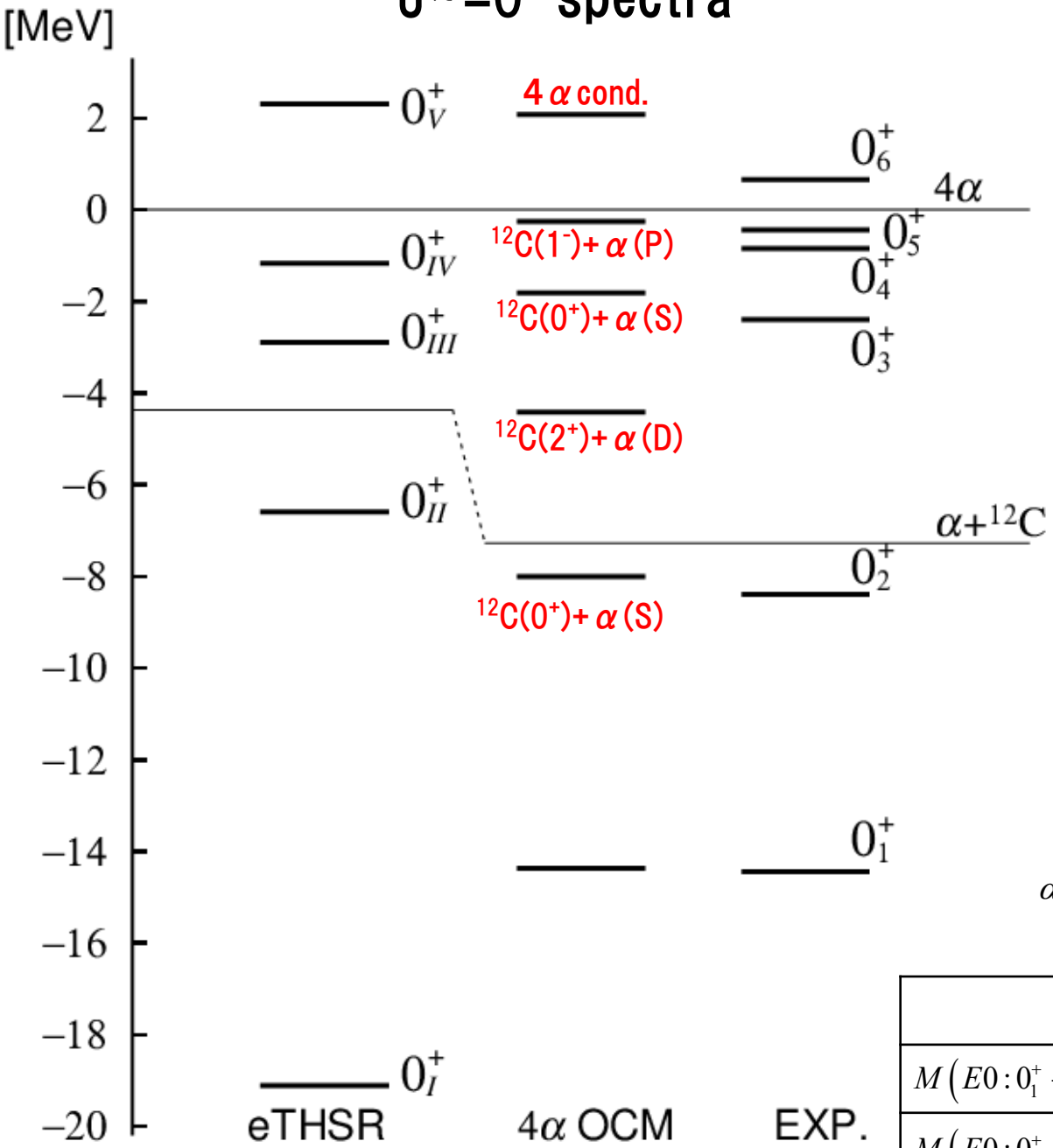
$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i^{16} \nabla_i^2 - T_G + \sum_{i<j}^{16}(V_{ij}^{(N)} + V_{ij}^{(C)}) + \sum_{i<j<k}^{16} V_{ijk}^{(N)}$$

Spurious continuum components are effectively eliminated by r^2 constraint method.

See Y. F. et al., PTP **115,** 115 (2006).

$J^{\pi}=0^{+}$ spectra

Y. F, PRC 97, 021304(R)(2018)

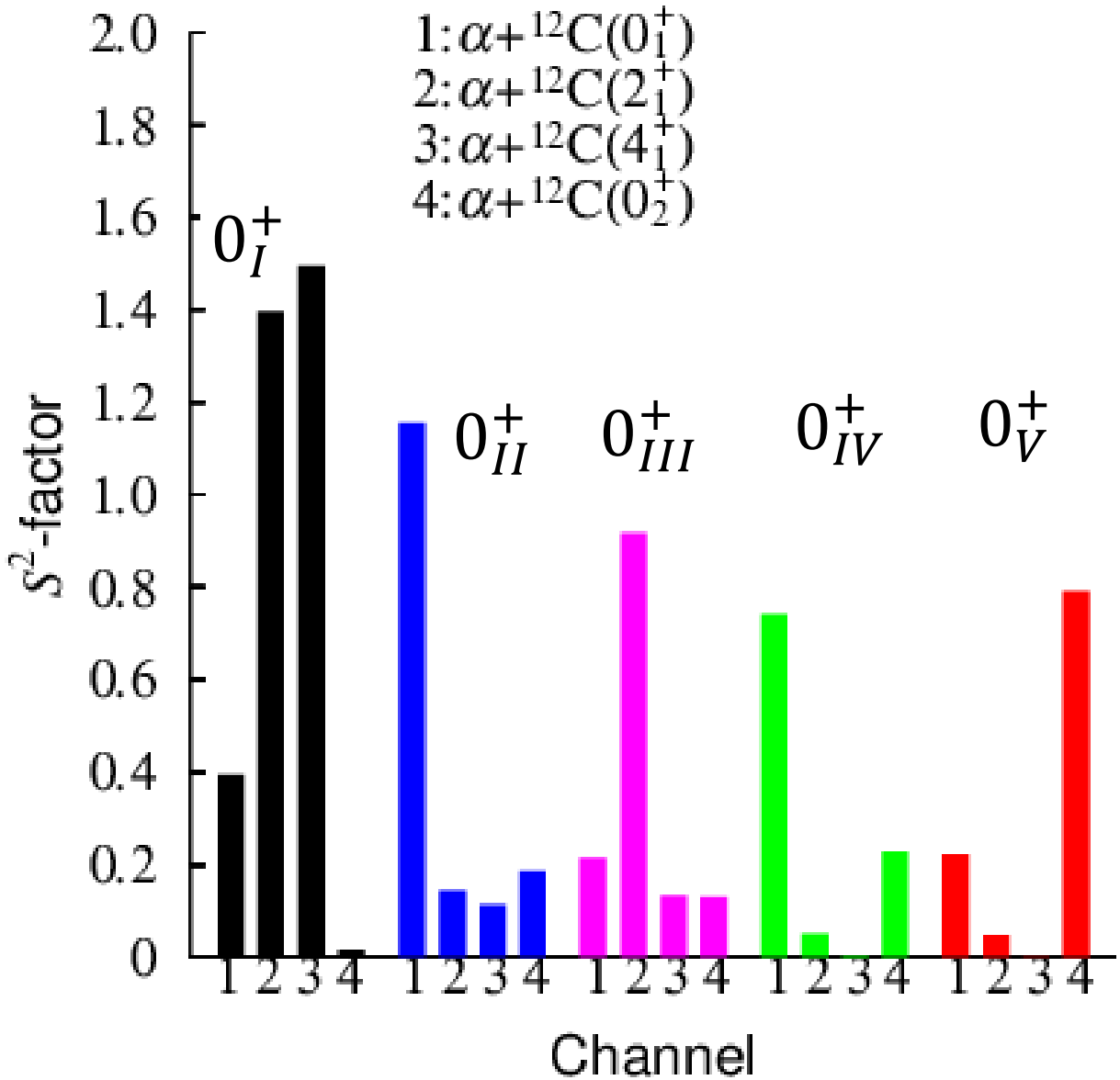


$\alpha+^{12}$C OCM

Y. Suzuki, PTP **55**, 1751 (1976); **56**, 111 (1976).

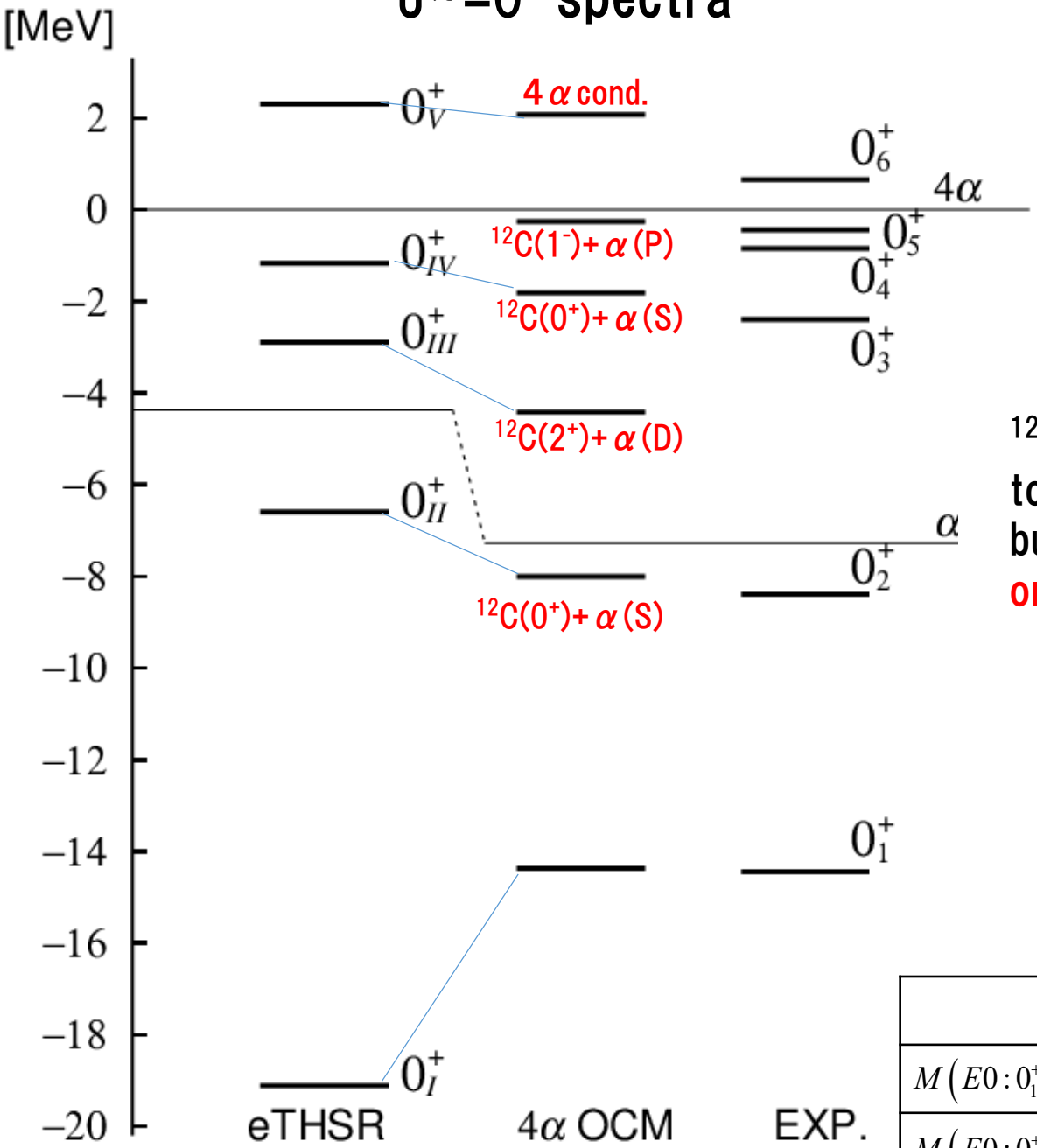
	eTHSR	$\alpha+^{12}$C OCM	EXP.
$M\left(E0:0_1^+\rightarrow 0_2^+\right)$	5.9	3.88	3.66 ± 0.55
$M\left(E0:0_1^+\rightarrow 0_3^+\right)$	5.7	3.50	4.40 ± 0.44

S^2-factor


1:α+12C(0+1)
2:α+12C(2+1)
3:α+12C(4+1)
4:α+12C(0+2)
0+I
0+II
0+III
0+IV
0+V
S2-factor
Channel
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4

$J^{\pi}=0^{+}$ spectra

Y. F, PRC 97, 021304(R)(2018)



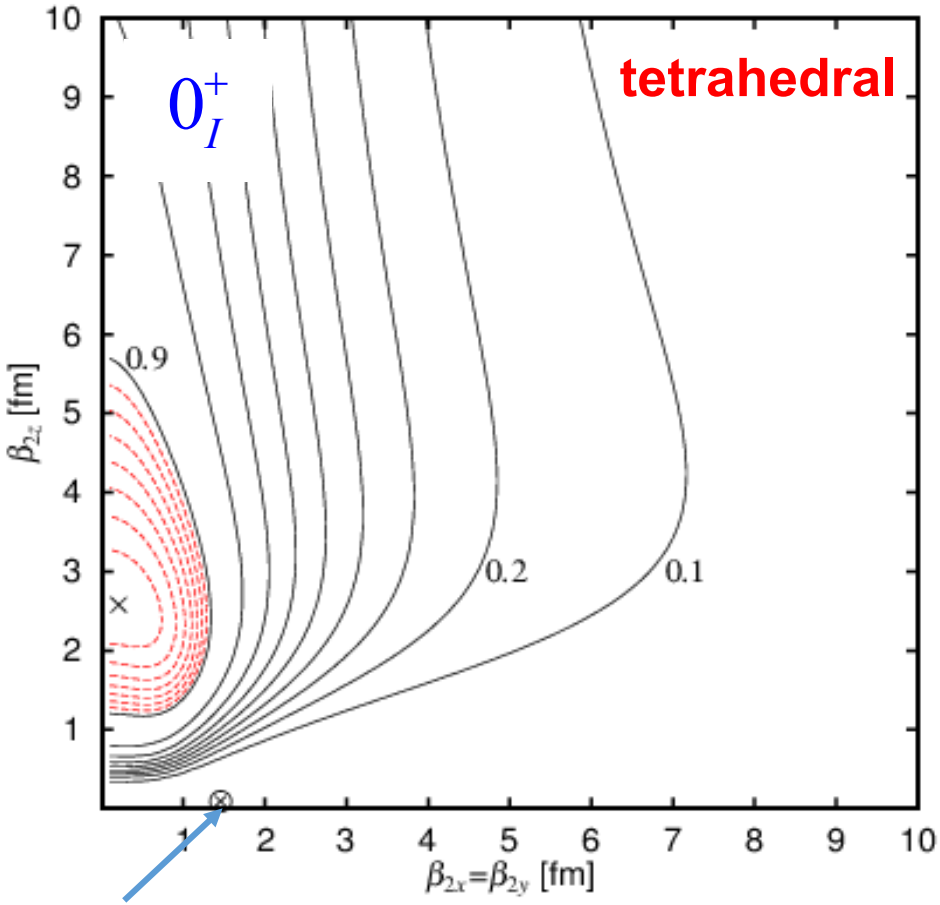
^{12}C(1$^-$)+α(P) structure is difficult to describe by the present eTHSR but extension is possible and now on going.

$\alpha+^{12}$C OCM

Y. Suzuki, PTP **55**, 1751 (1976); **56**, 111 (1976).

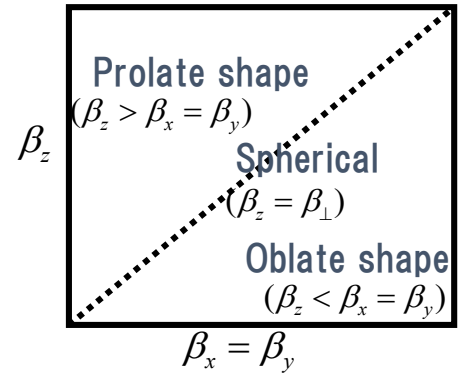
	eTHSR	$\alpha+^{12}$C OCM	EXP.
$M\left(E0:0_1^+\rightarrow 0_2^+\right)$	5.9	3.88	3.66 ± 0.55
$M\left(E0:0_1^+\rightarrow 0_3^+\right)$	5.7	3.50	4.40 ± 0.44

Squared overlap surface with single config. of eTHSR



0.01 step

0.1 step

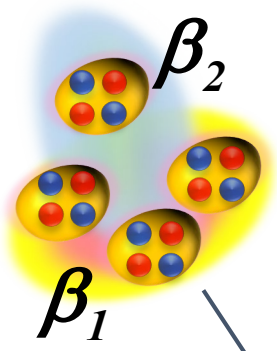


$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at ⊗

Container for 3 α

× : maximum

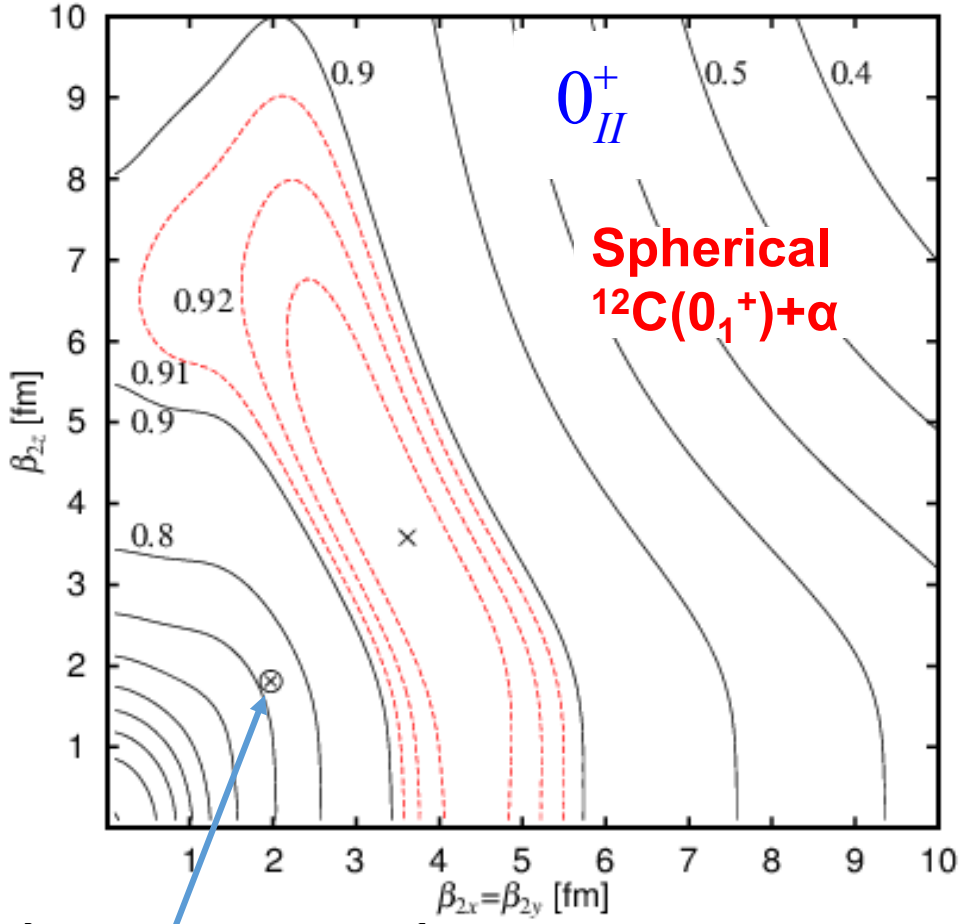
For the fourth α



THSR+GCM

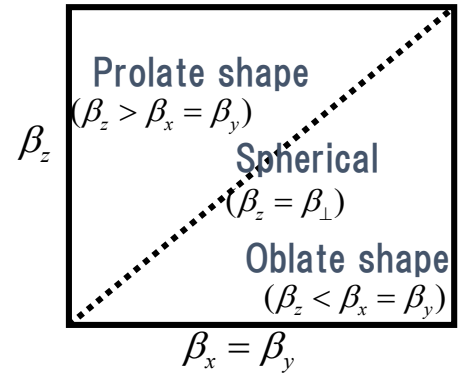
$$\left|\langle \Phi(\beta_1, \beta_2) | 0_\lambda^+ \rangle\right|^2$$

Squared overlap surface with single config. of eTHSR



0.01 step

0.1 step



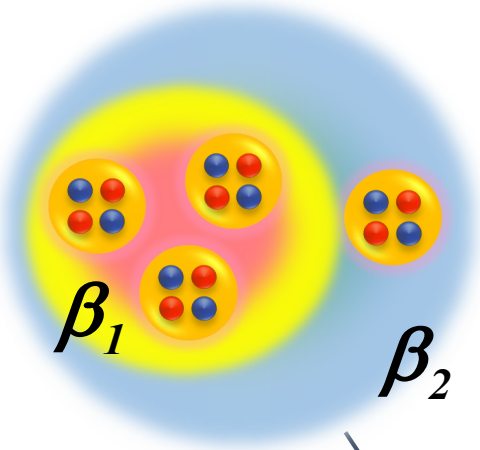
	Sq. overlap	$(\beta_{1x} = \beta_{1y}, \beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9,0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at ⊗

Container for 3 α

× : maximum

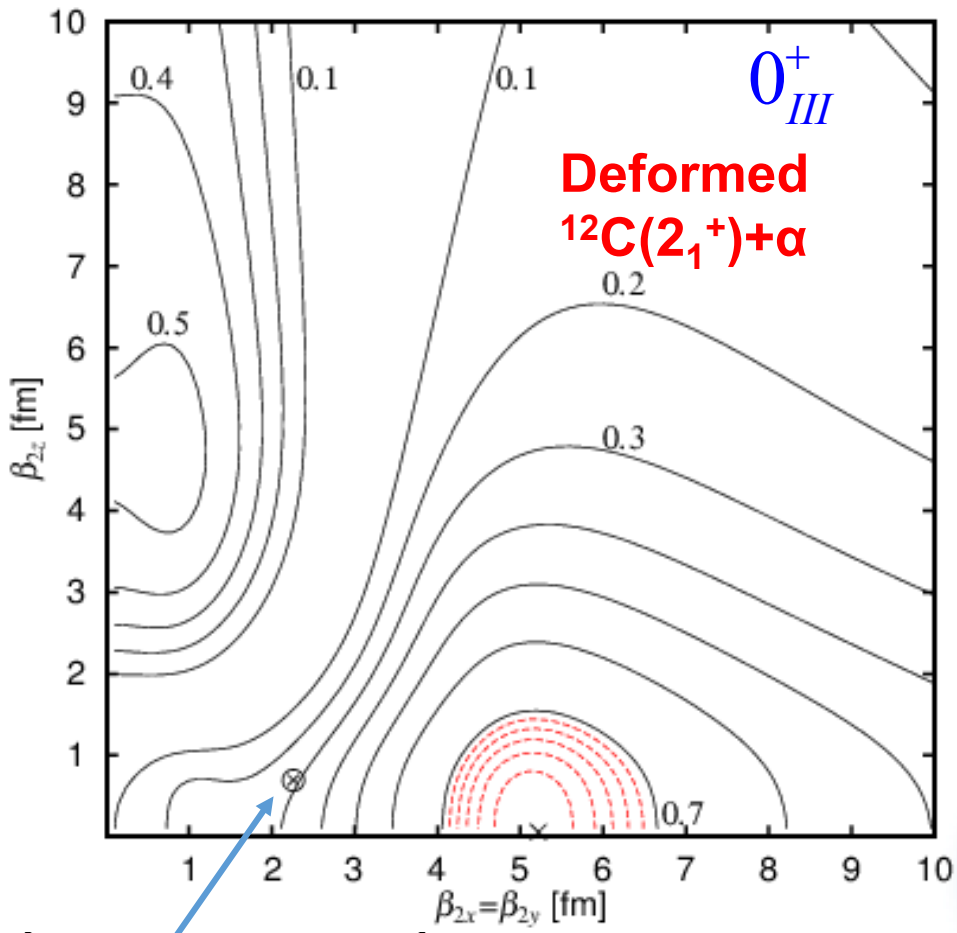
For the fourth α



THSR+GCM

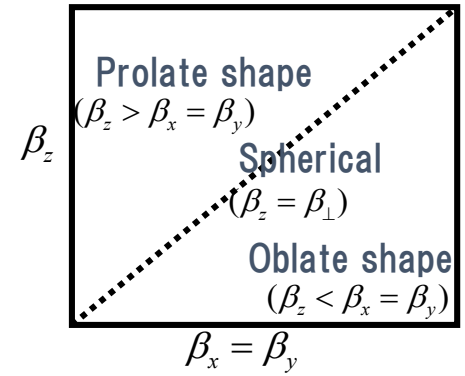
$$\left|\langle \Phi(\beta_1, \beta_2) | 0_\lambda^+ \rangle\right|^2$$

Squared overlap surface with single config. of eTHSR



0.01 step

0.1 step



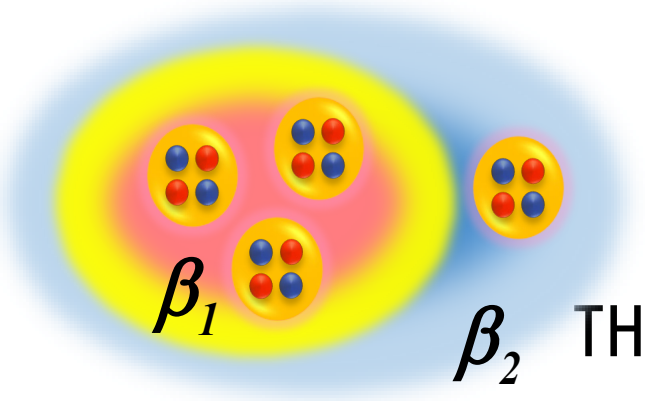
	Sq. overlap	$(\beta_{1x} = \beta_{1y}, \beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9, 0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at ⊗

Container for 3 α

× : maximum

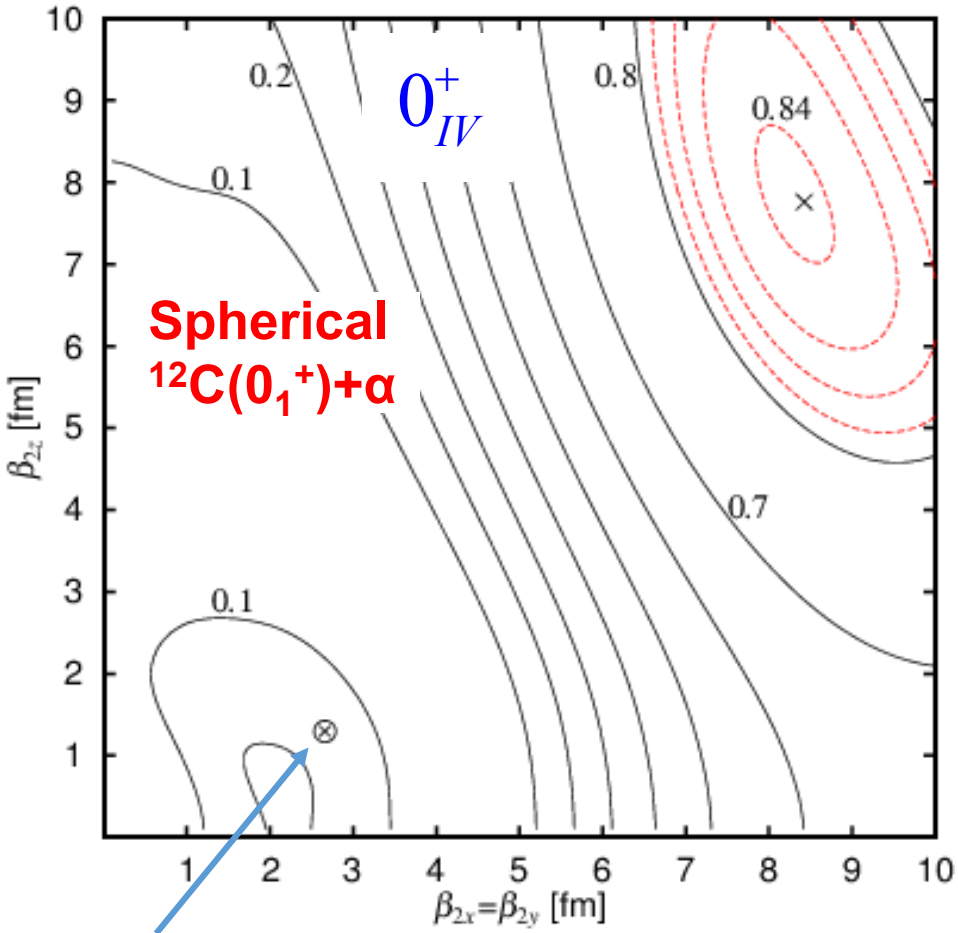
For the fourth α



THSR+GCM

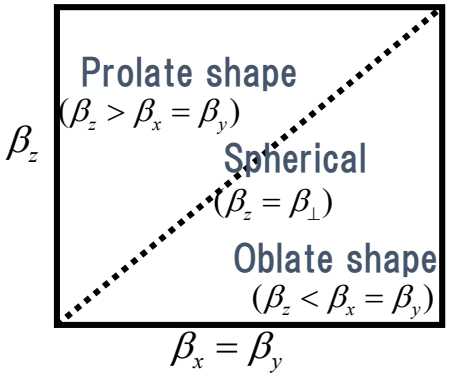
$$\left|\langle \Phi(\beta_1, \beta_2) | 0_\lambda^+ \rangle\right|^2$$

Squared overlap surface with single config. of eTHSR



0.01 step

0.1 step



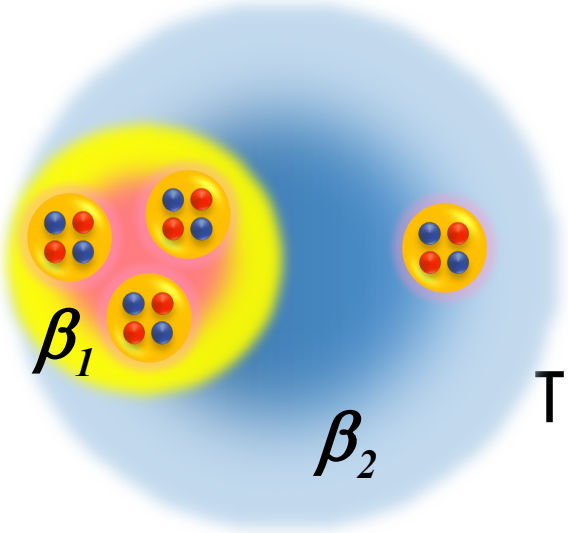
	Sq. overlap	$(\beta_{1x} = \beta_{1y}, \beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9,0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at ⊗

Container for 3 α

× : maximum

For the fourth α

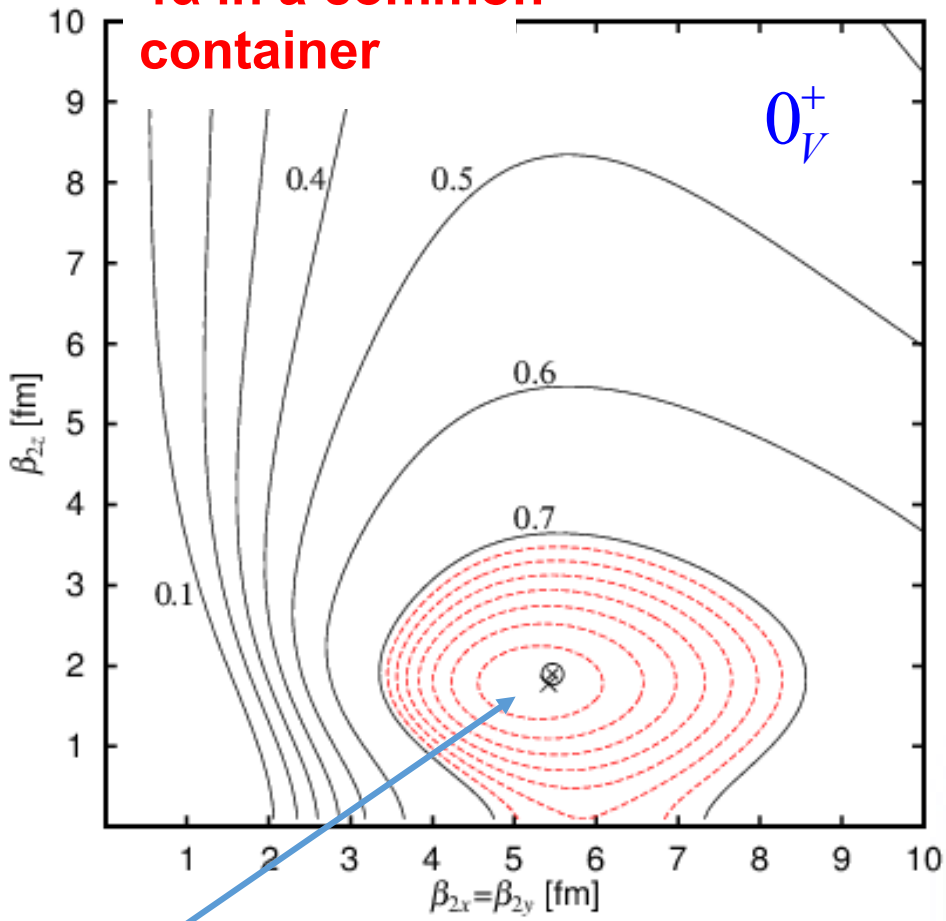


THSR+GCM

$$\left|\langle \Phi(\beta_1, \beta_2) | 0_\lambda^+ \rangle\right|^2$$

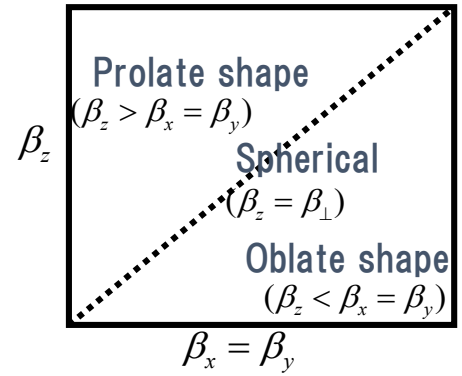
Squared overlap surface with single config. of eTHSR

4α in a common container



- - - - 0.01 step

——— 0.1 step



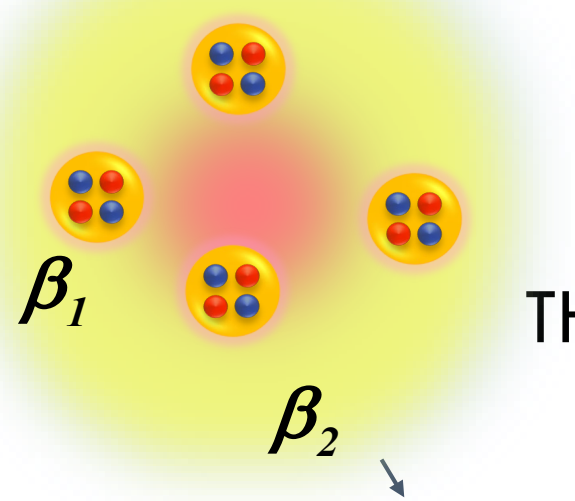
	Sq. overlap	$(\beta_{1x} = \beta_{1y}, \beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9,0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at ⊗

Container for 3 α

× : maximum

For the fourth α



THSR+GCM

$$\left|\langle \Phi(\beta_1, \beta_2) | 0_\lambda^+ \rangle\right|^2$$

Summary and future work

Rich spectra above the Hoyle state

03+: higher nodal, vibration··· ?

04+: linear chain, triplet state,··· ?

1−: gas of alphas!?, $1P(0S)^2$?
as well as 3−, by Bo−san tomorrow

16O: cluster states −> describable by `container' evolution
0_V^+ (0_6^+ in OCM) 4 alphas in an identical orbit

A strong peak around 23.5 MeV in 20Ne, which strongly decays into the 06+ state in 16O, by Kawabata group.

Strong candidate of 5−alpha condensate!
−> 12C+2alpha OCM, 5−alpha OCM, 5−alpha THSR
Now on going

Thanks

to my Collaborators

Bo Zhou (Hokkaido U.)

Zhongzhou Ren (Tongji U.)

Chang Xu (Nanjing U.)

Lyu Meng Jiao (RCNP)

Zhao Qing (Nanjing U.)

Taiichi Yamada (Kanto Gakuin U.)

Tadahiro Suhara (Matsue)

Hisashi Horiuchi (RCNP)

Akihiro Tohsaki (RCNP)

Peter Schuck (IPN, Orsay)

Gerd Röpke (Rostock U.)

and for your attention.