

Localization and cluster formation in light nuclei

Dario Vretenar



Localization and clustering in nucleonic matter

B. Mottelson - quantality parameter: $\Lambda_{\text{Mot}} \hat{=} \frac{\hbar^2}{m\bar{r}^2|V_0|}$. ratio of the zero-point kinetic energy of the confined particle to its potential energy.

The transition between a solid phase (small kinetic energy compared to the potential at equilibrium) and a liquid (large kinetic energy in comparison to the depth of the potential) occurs for $\Lambda_{\text{Mot}} \cong 0.1$.

Nuclear matter: $\bar{r} \sim 1$ fm, $|V_0| \sim 100$ MeV $\rightarrow \Lambda_{\text{Mot}} \sim 0.4$ quantum liquid phase.

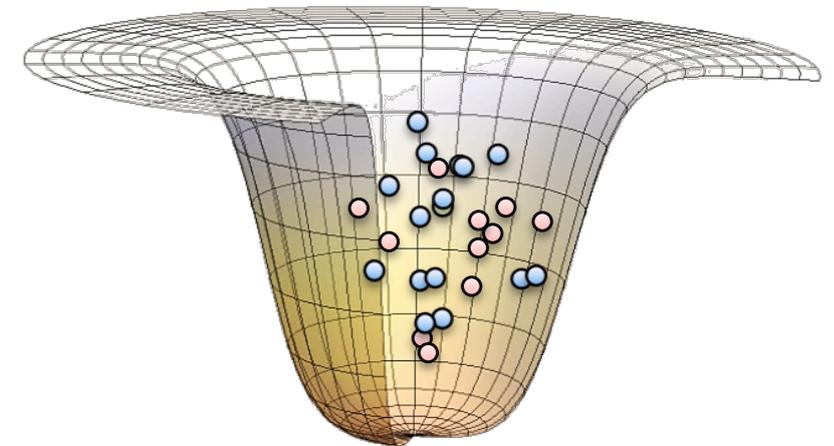
Liquid-cluster transition in finite nuclei

...dimensionless localisation parameter:

$$\alpha_{\text{loc}} \hat{=} \frac{\Delta r}{\bar{r}},$$

\bar{r} is the average inter-nucleon distance, and Δr the spatial dispersion of the wave function:

$$\Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2}.$$

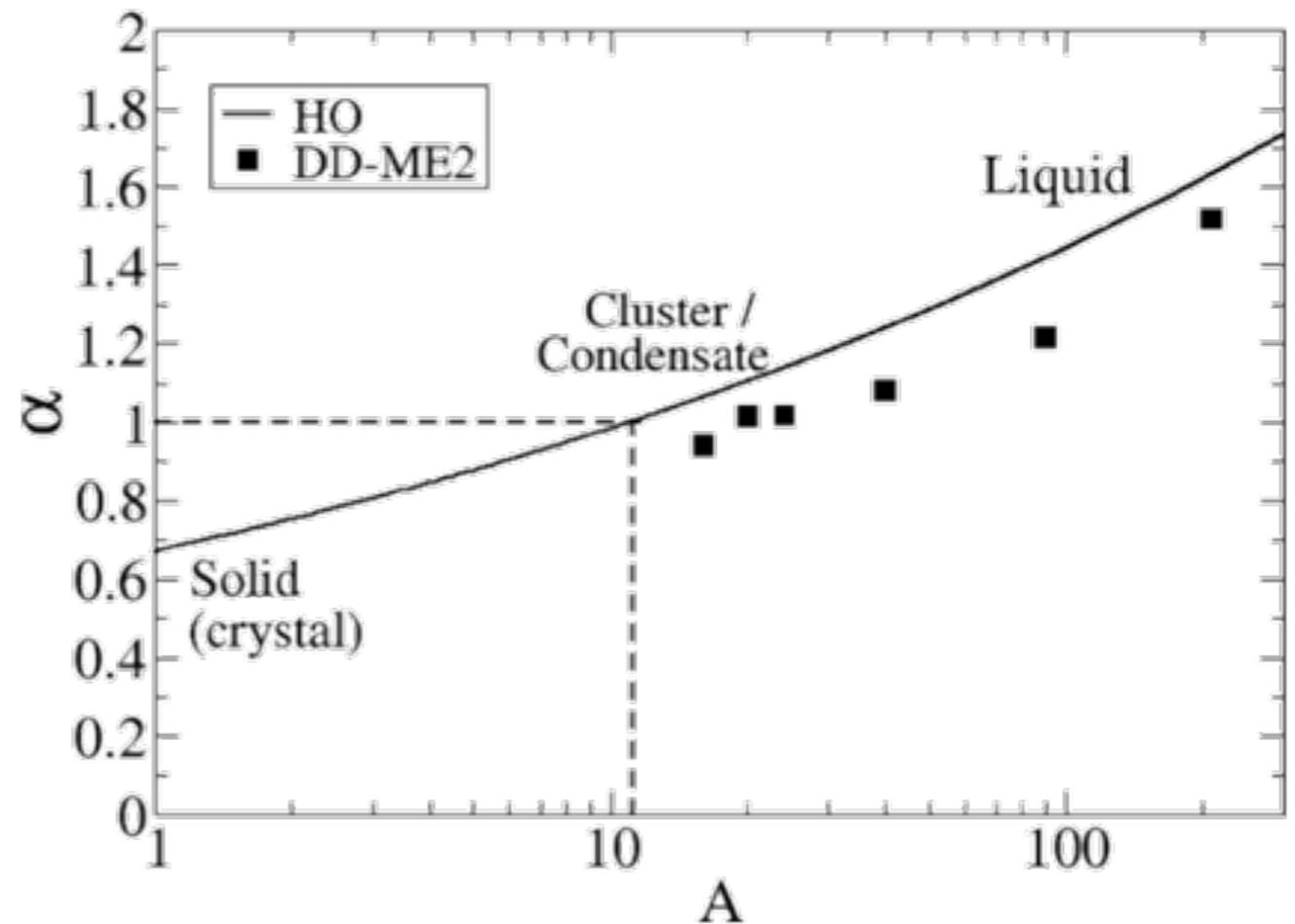


When the confining nuclear potential is approximated by a three-dimensional isotropic harmonic oscillator:

$$\alpha_{\text{loc}} \simeq \frac{b}{r_0} = \frac{\sqrt{\hbar R}}{r_0(2mV_0)^{1/4}}$$

b is the oscillator length and $r_0 = 1.25$ fm. Using the liquid-drop parameterization for the radius $R = r_0 A^{1/3}$

$$\alpha_{\text{loc}} = \frac{\sqrt{\hbar} A^{1/6}}{(2mV_0 r_0^2)^{1/4}} \simeq 0.67 A^{1/6}$$



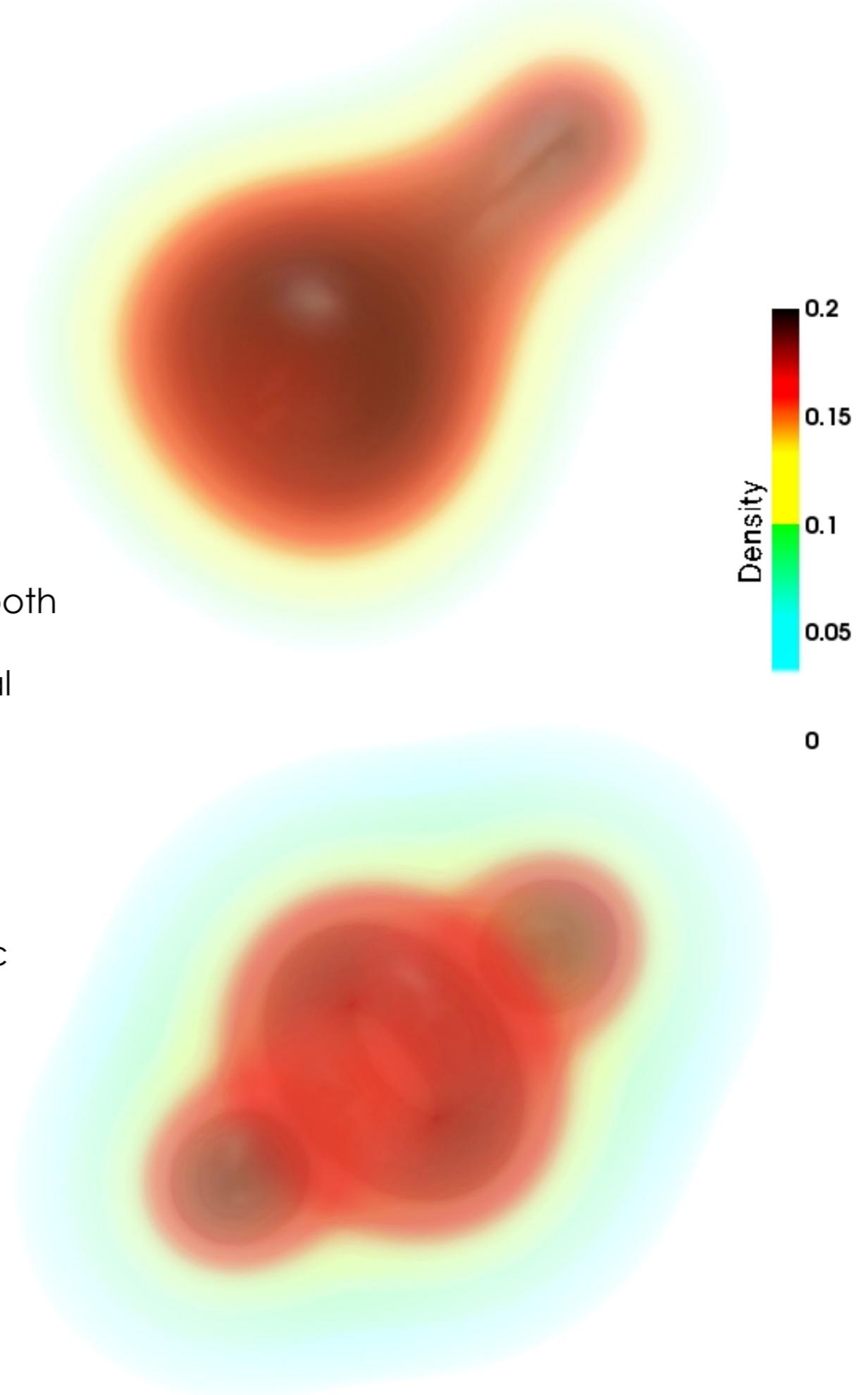
The transition from coexisting cluster and mean-field states to a Fermi liquid state occurs for $A \sim 20 - 30$. In heavier systems $\alpha_{\text{loc}} > 1 \rightarrow$ heavy nuclei consist of largely delocalized nucleons characterized by a large mean free path.

Clusters in light α -conjugate nuclei

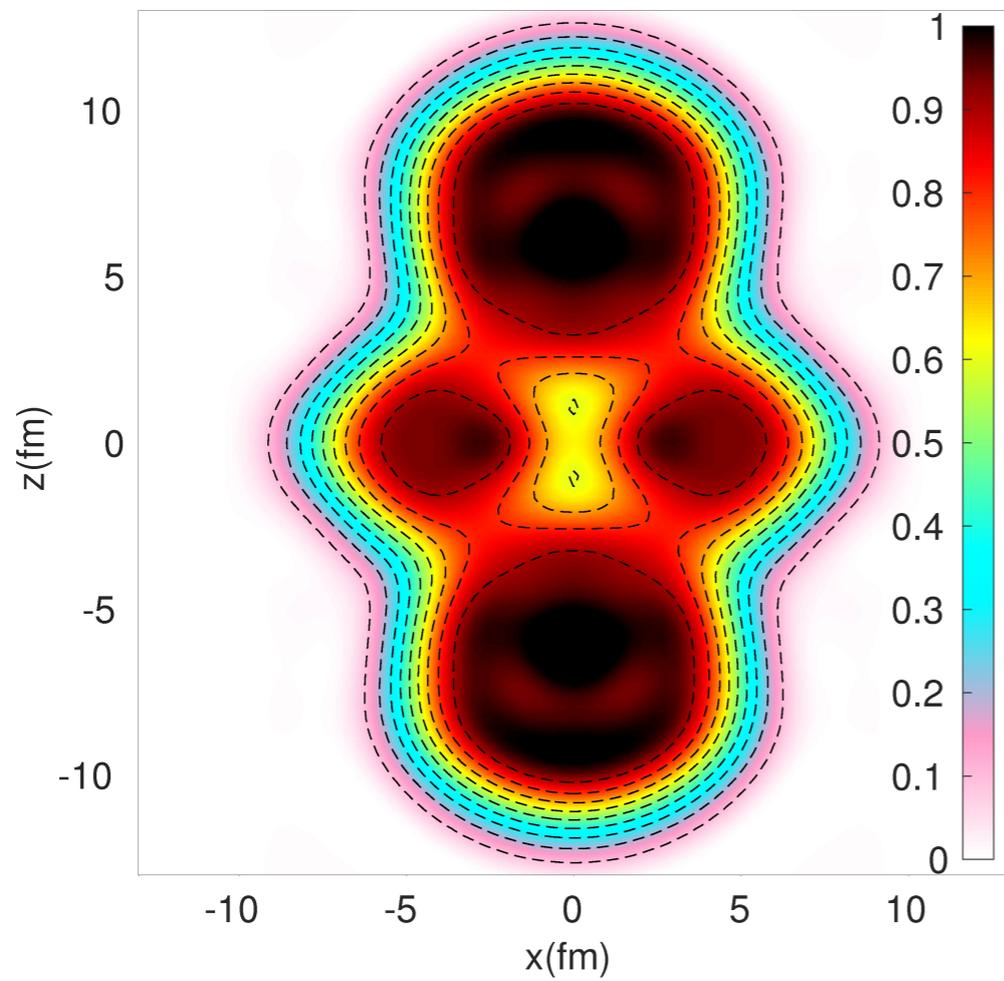
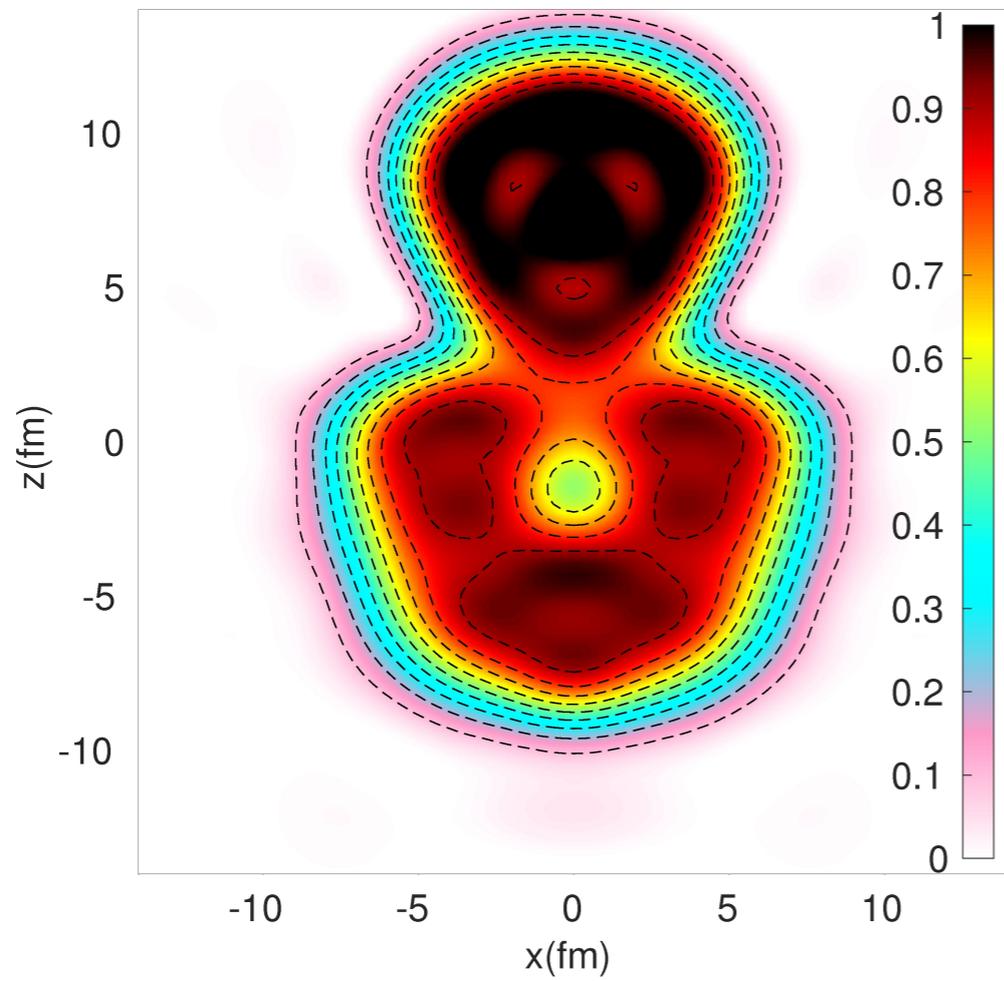
Important role of nuclear shape deformation: removes the degeneracy of single-nucleon levels associated with spherical symmetry.

The saturation of inter-nucleon forces, effective when both spin and isospin are coupled to zero, produces a particularly strong binding of the α -cluster and a central density that is by almost a third larger than central densities in most nuclei.

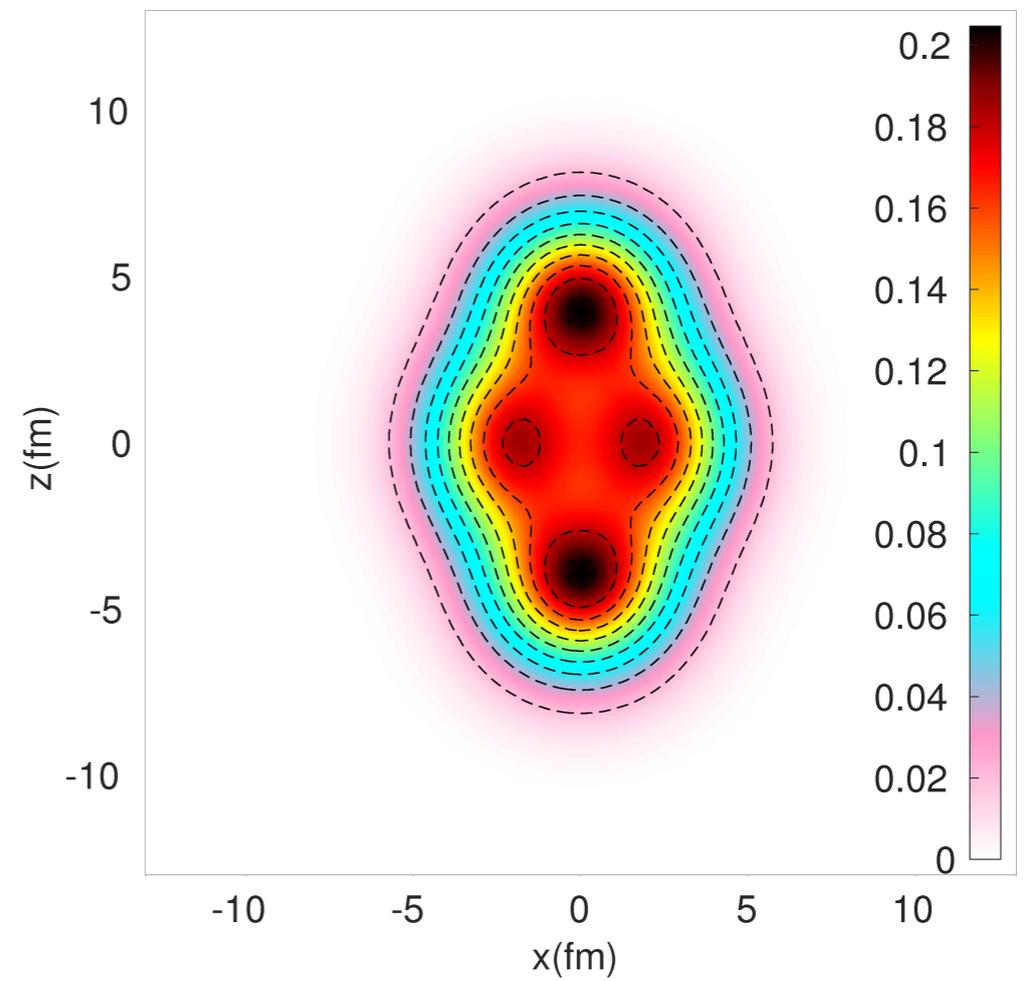
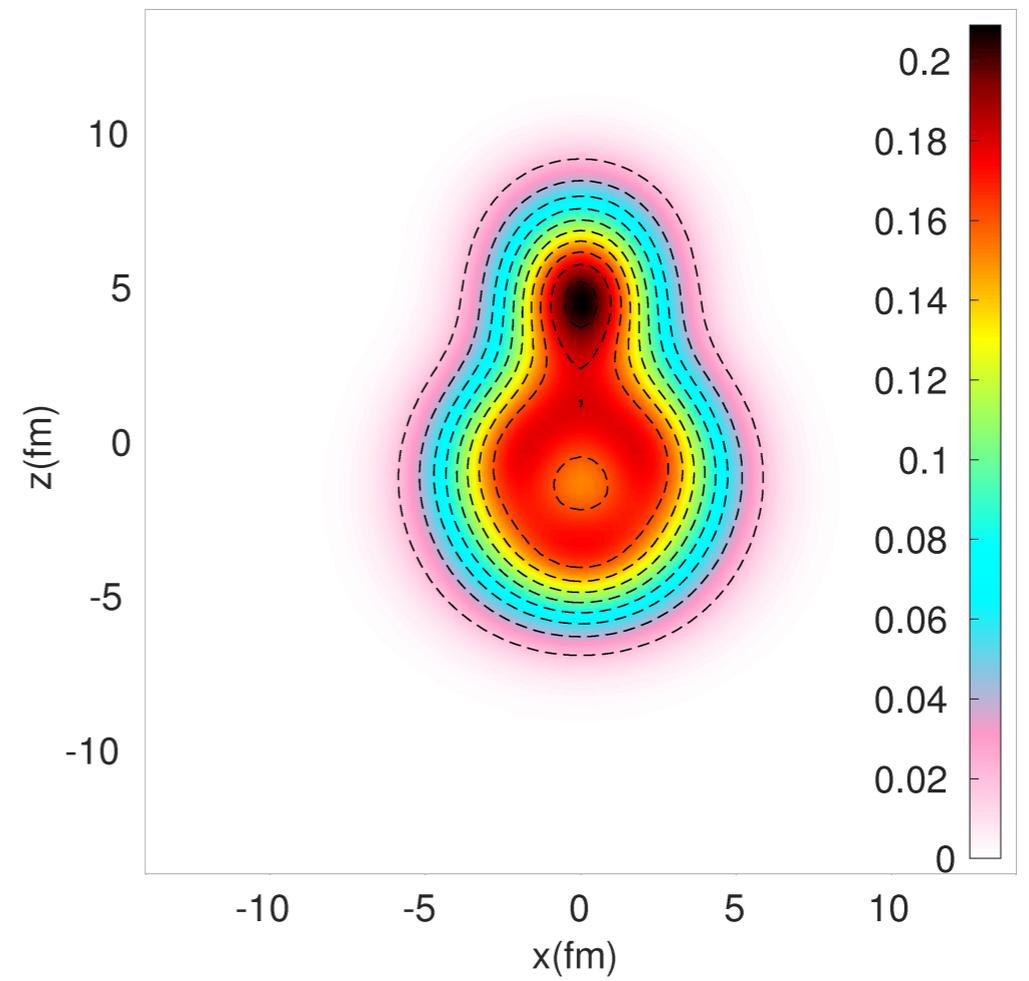
Self-consistent mean-field intrinsic reflection-asymmetric and reflection-symmetric axial densities of ^{20}Ne .



Localization function

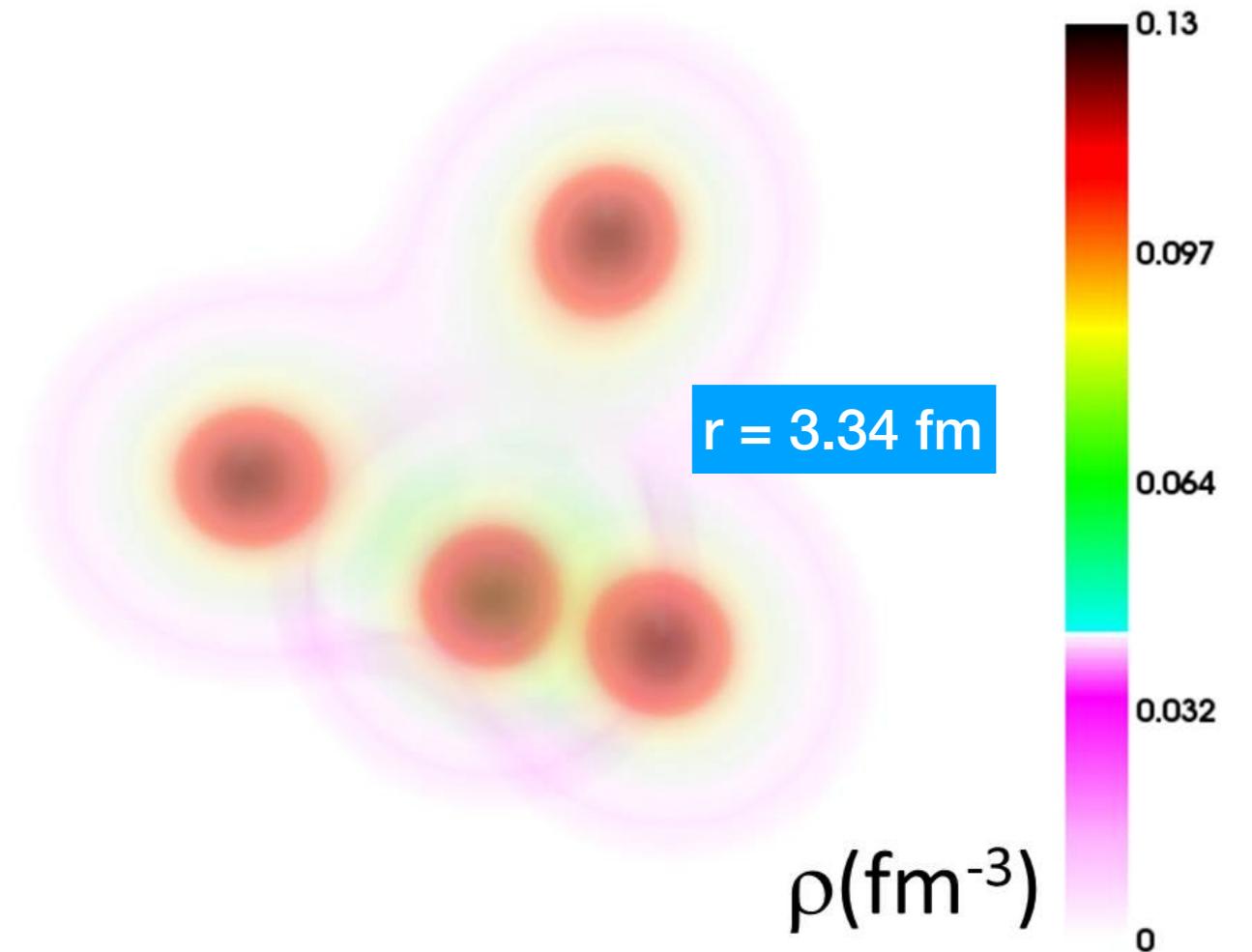
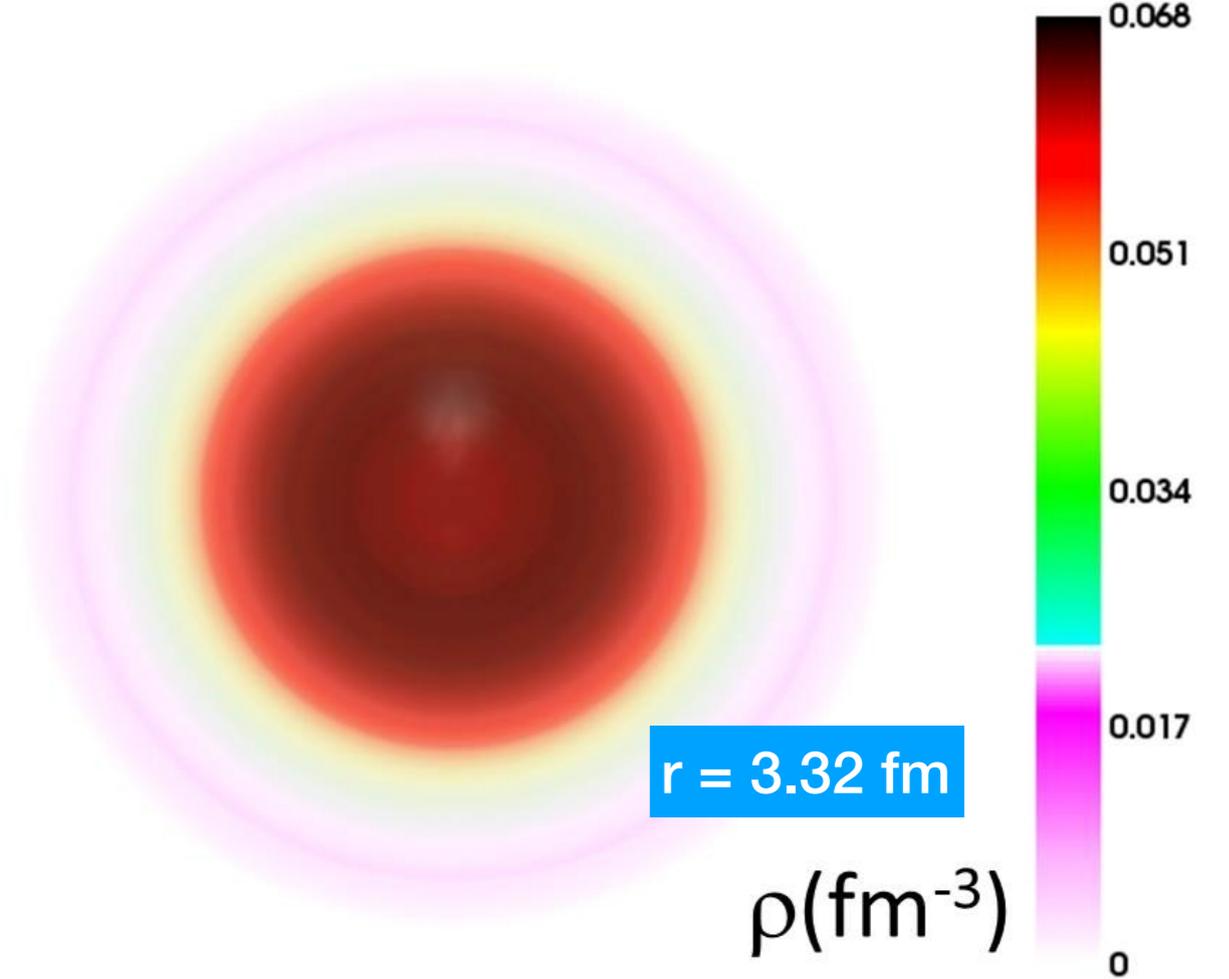


Single-nucleon density

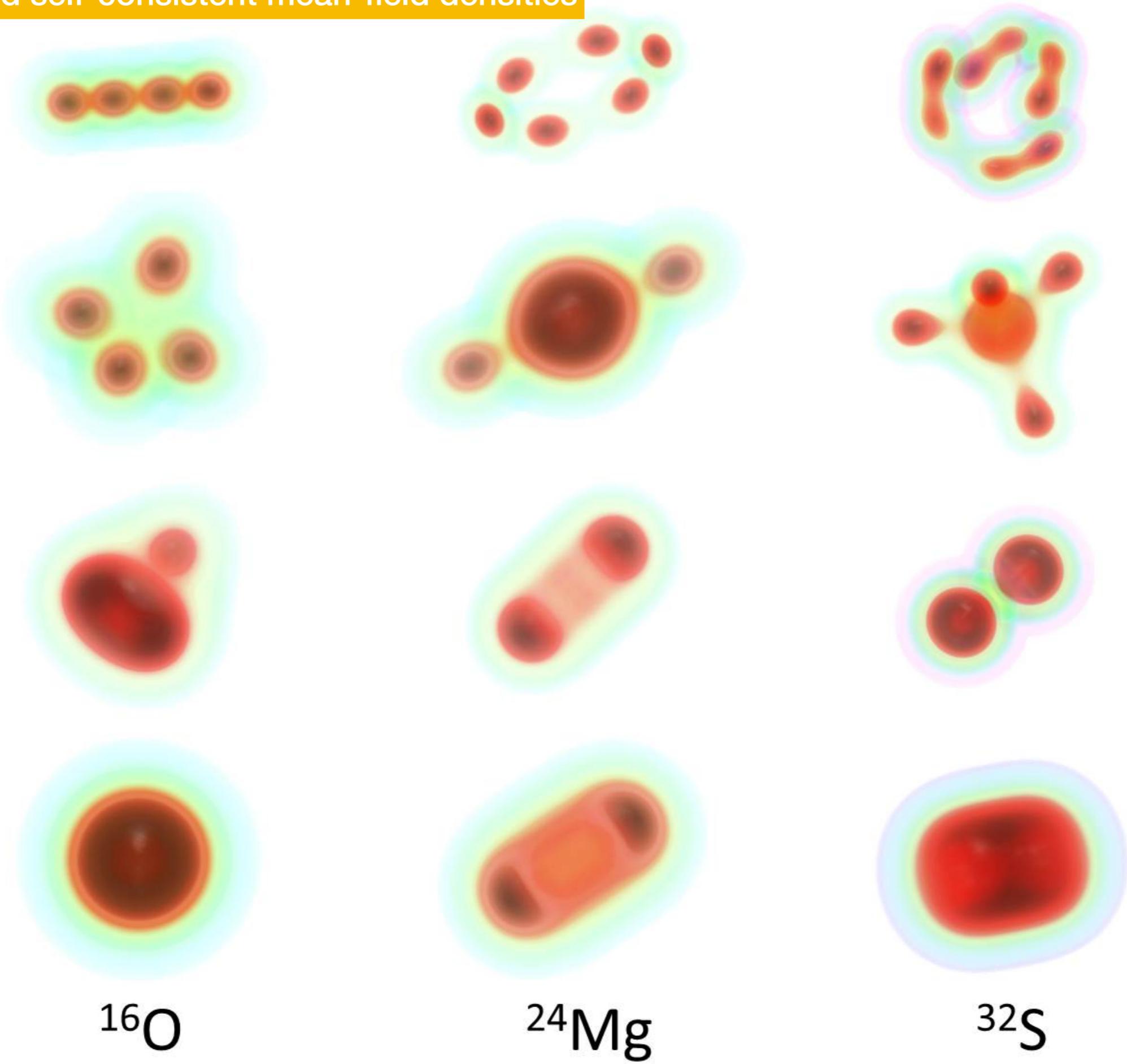


Dilute nuclear matter in excited configurations => the formation of clusters locally enhances the nucleonic density toward its saturation value, increasing the binding of the system.

Constrained self-consistent mean-field densities of ^{16}O .

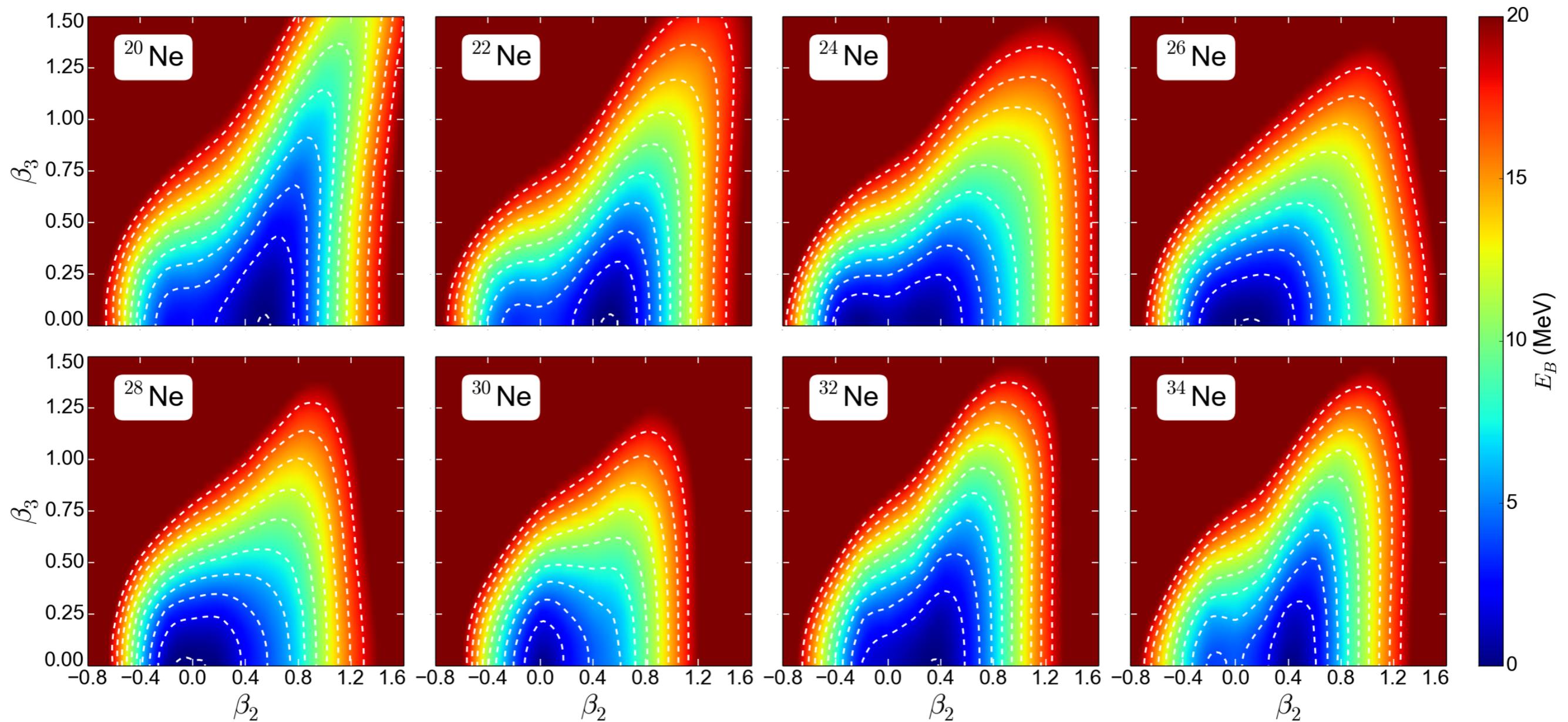


Constrained self-consistent mean-field densities

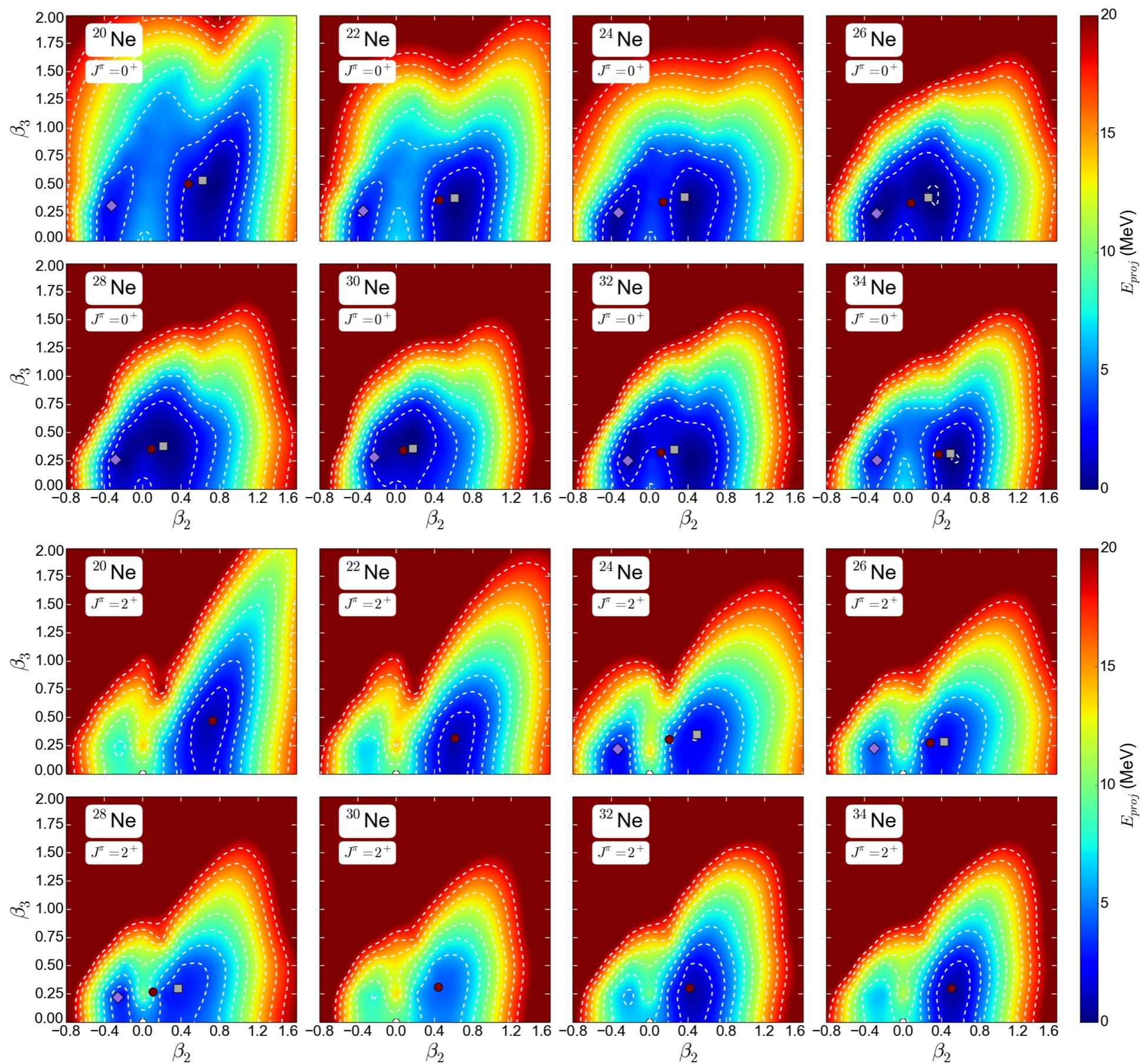


Beyond self-consistent mean field: collective correlations related to symmetry restoration and nuclear shape fluctuations

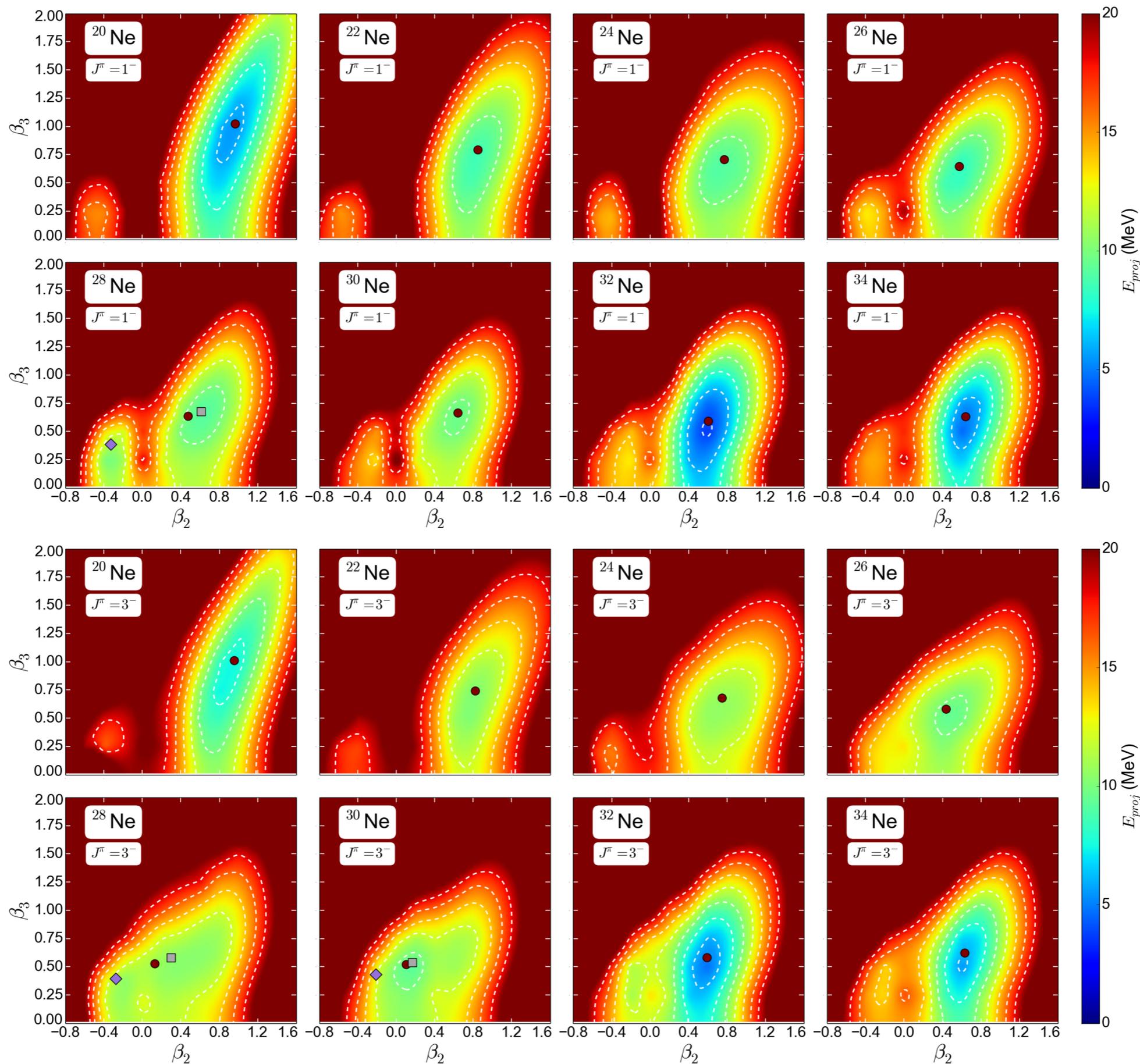
Quadrupole and octupole collectivity and cluster structures in neon isotopes



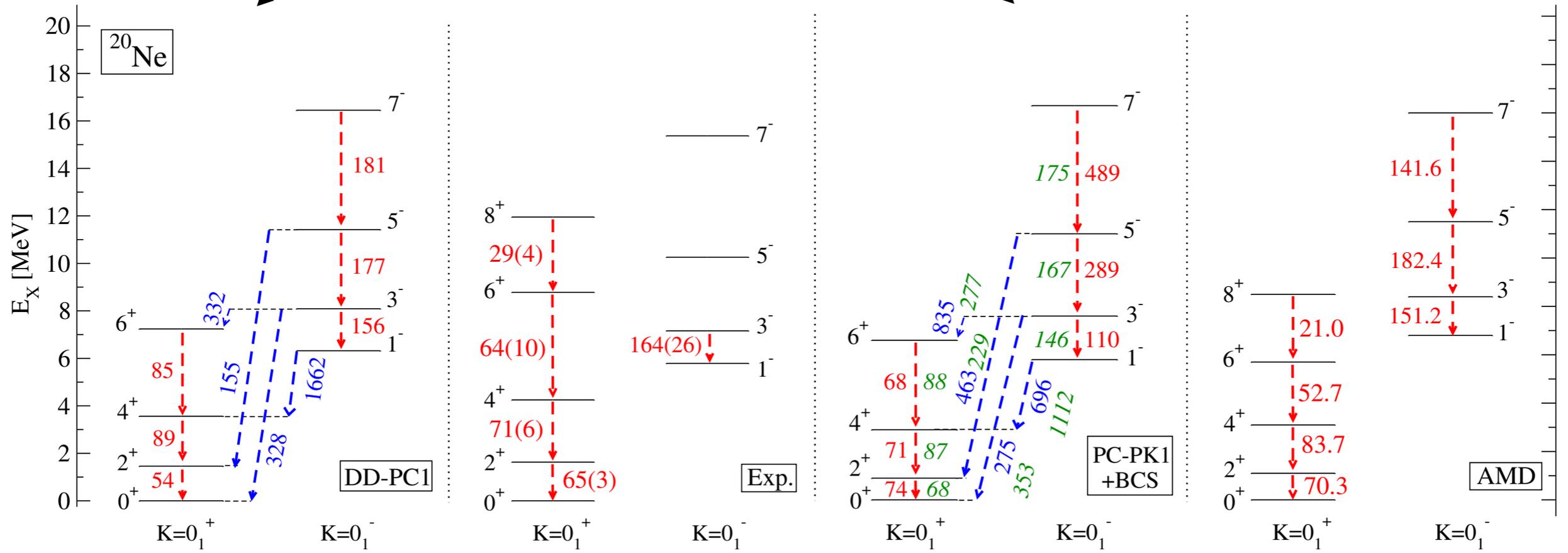
GCM configuration mixing of angular-momentum and parity projected SCMF states



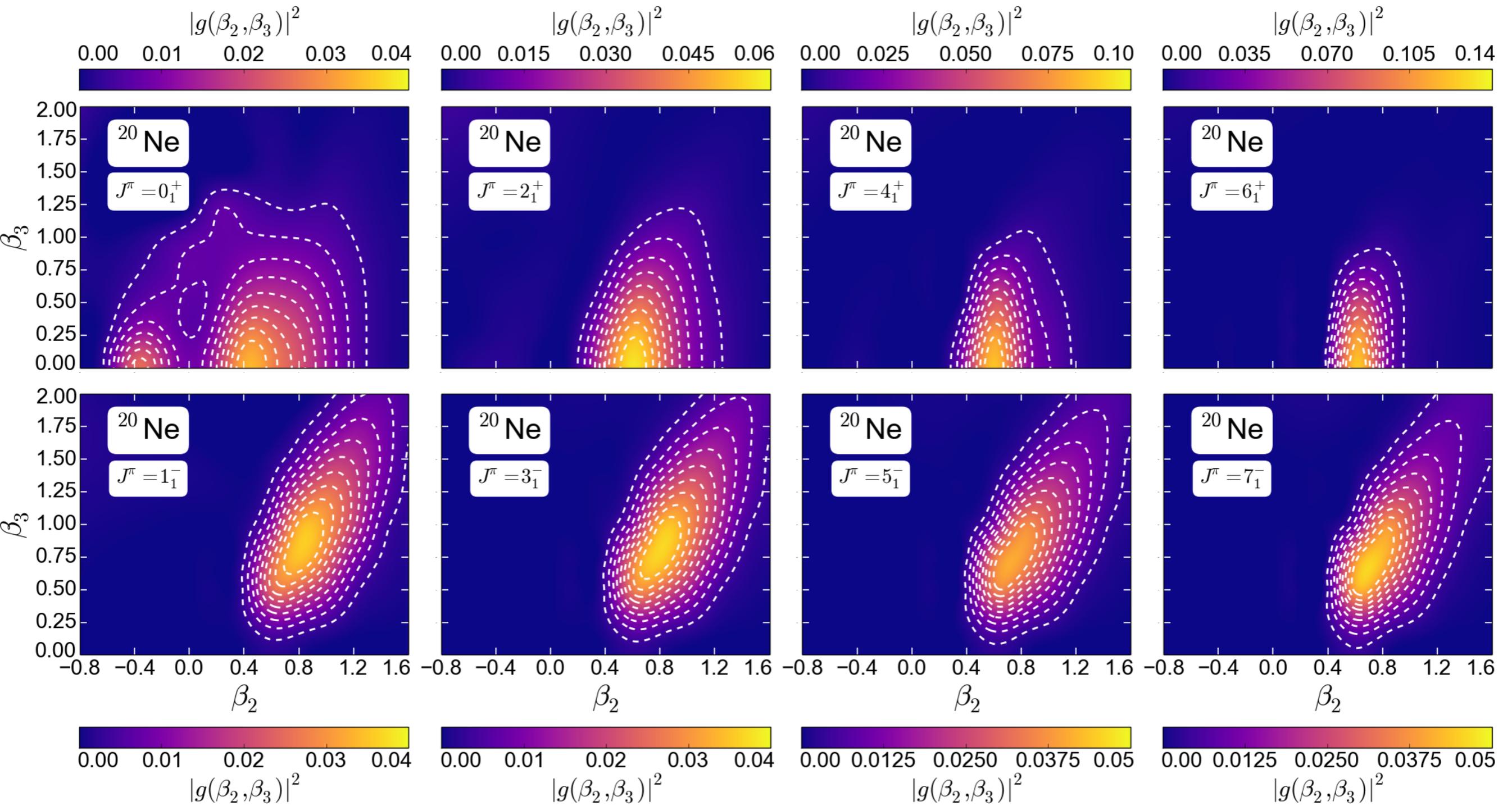
GCM configuration mixing of angular-momentum and parity projected SCMF states



GCM configuration mixing of angular-momentum and parity projected SCMF states

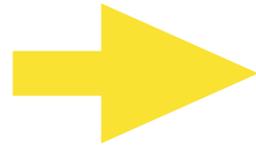


Amplitudes of GCM collective wave functions squared.

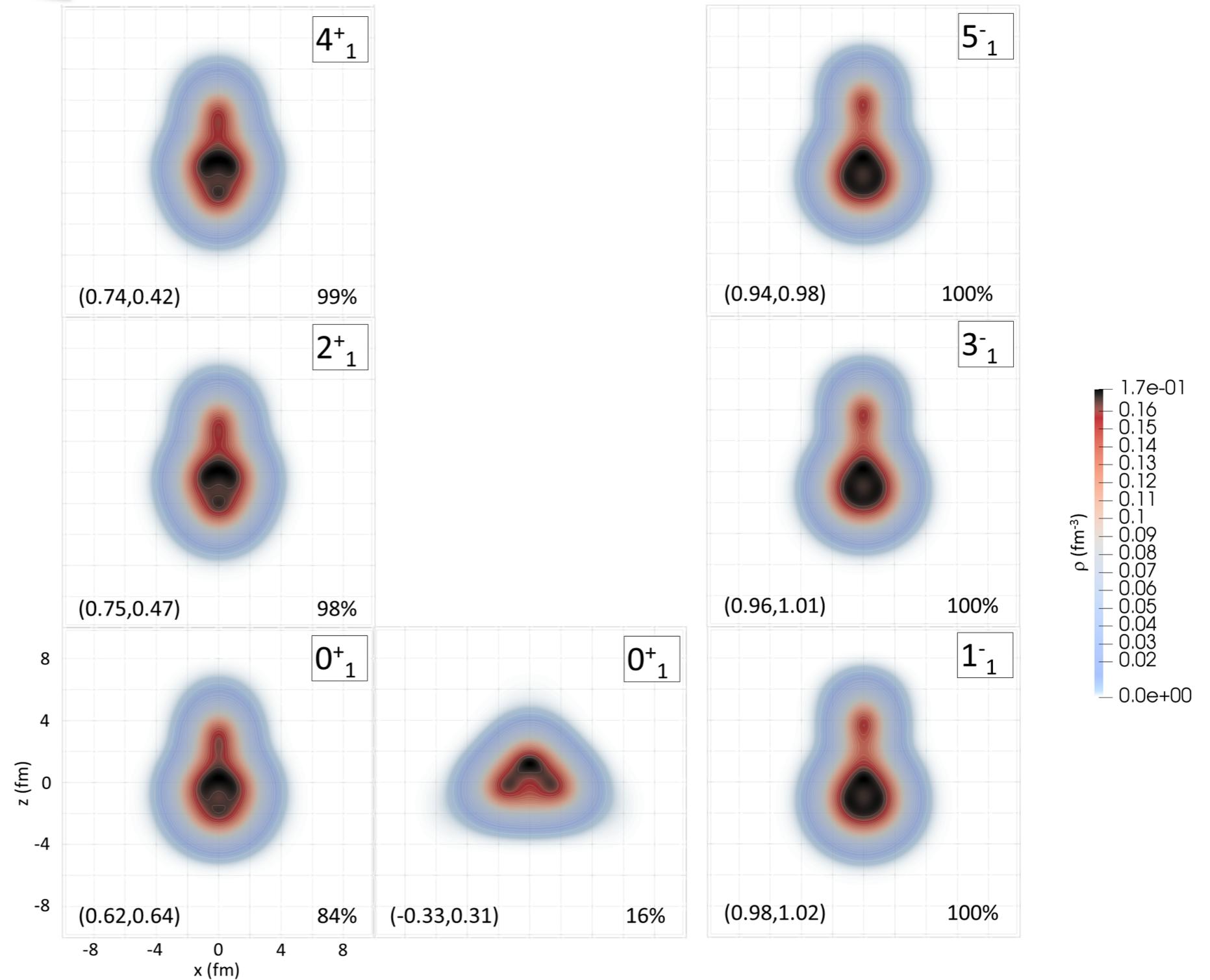


Expectation values of of the quadrupole and octupole deformation parameters in the correlated GCM states:

$$\langle \beta_\lambda \rangle_\alpha^{J\pi} = \sum_i |g_\alpha^{J\pi}(q_i)|^2 \beta_{\lambda i}$$



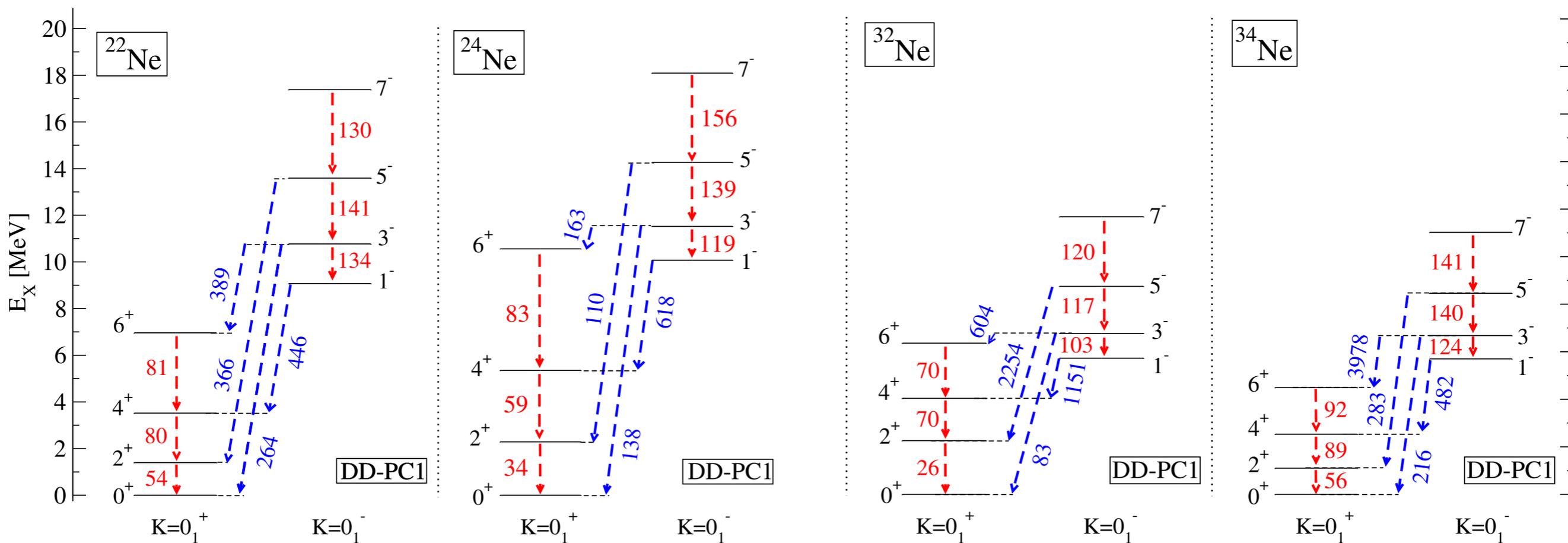
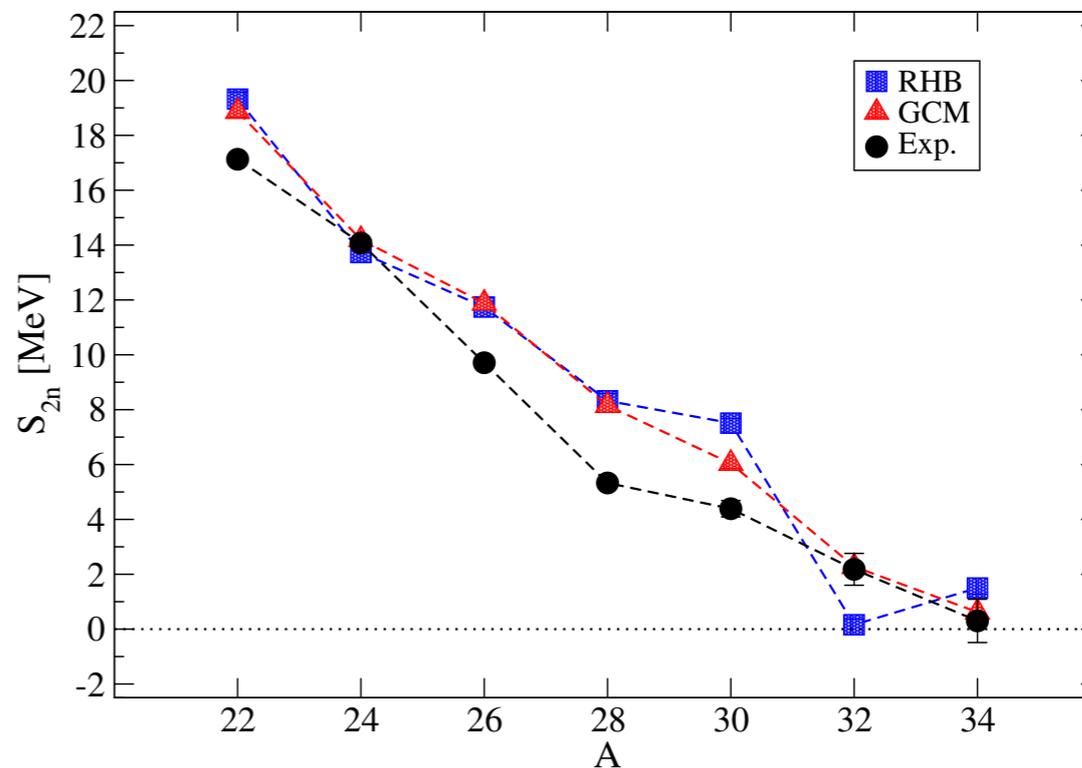
characteristic intrinsic densities at these deformations:



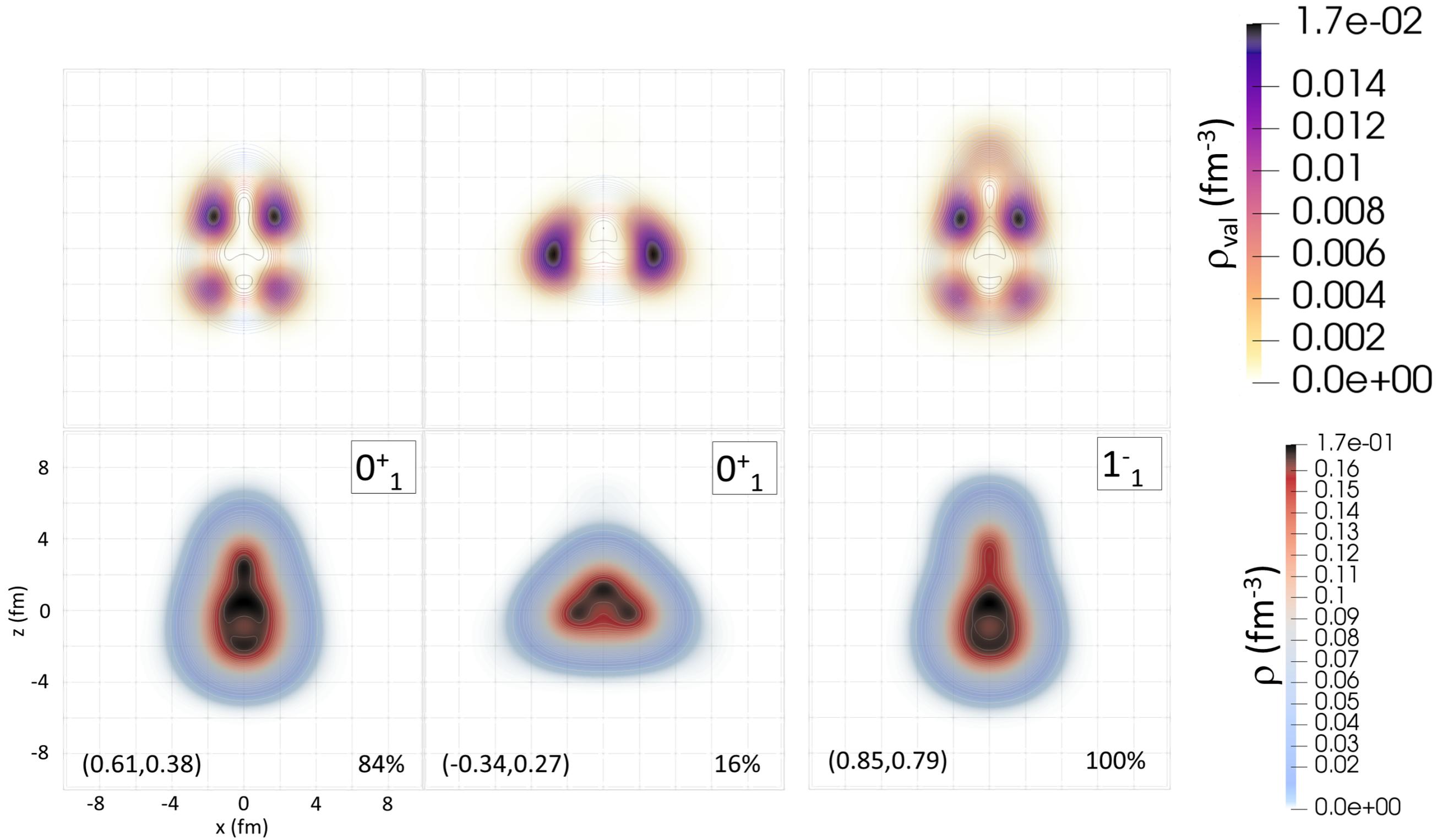
Neutron-rich Ne isotopes

PHYSICAL REVIEW C **97**, 024334 (2018)

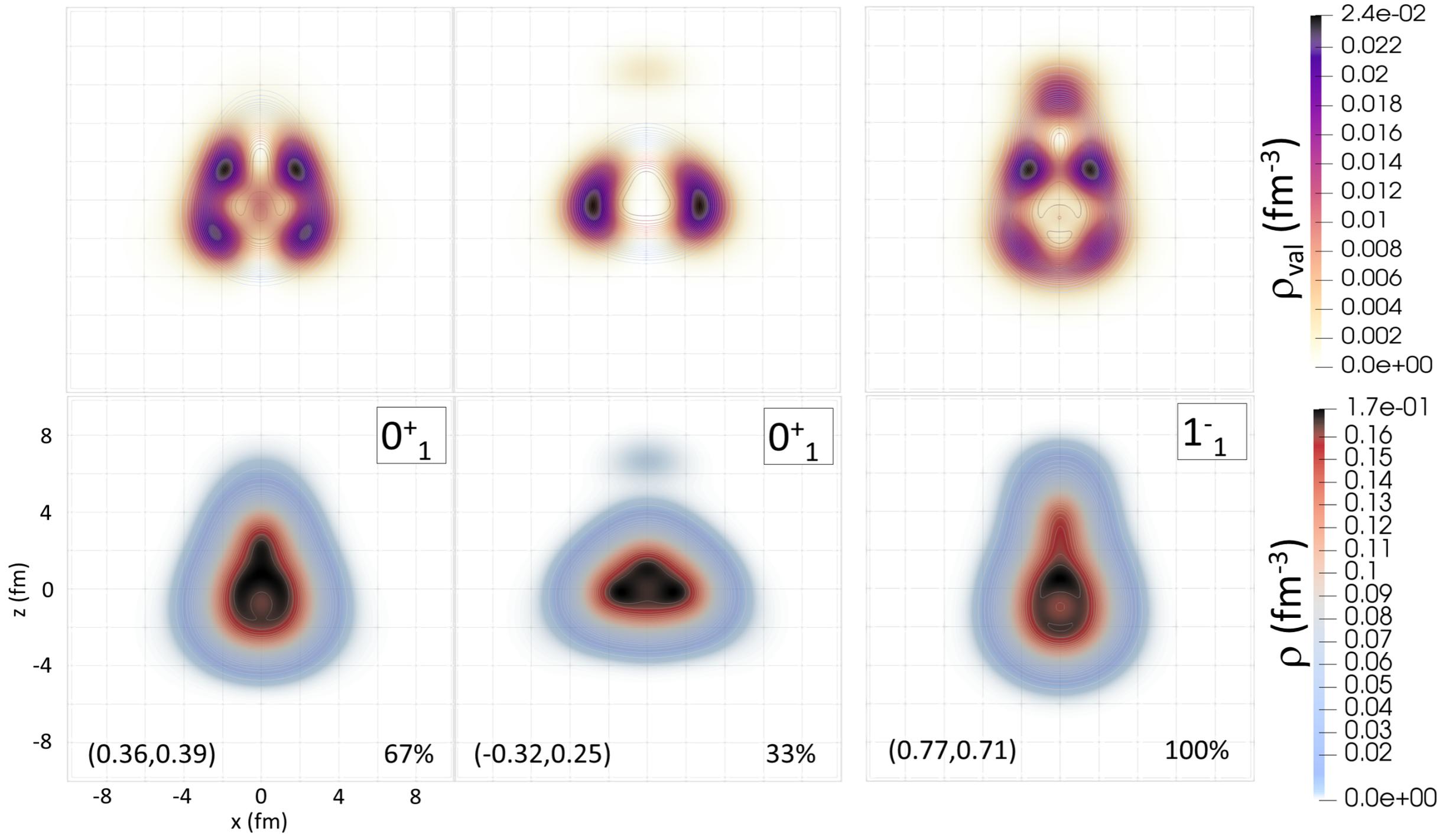
GCM spectra: DD-PC1 + separable pairing



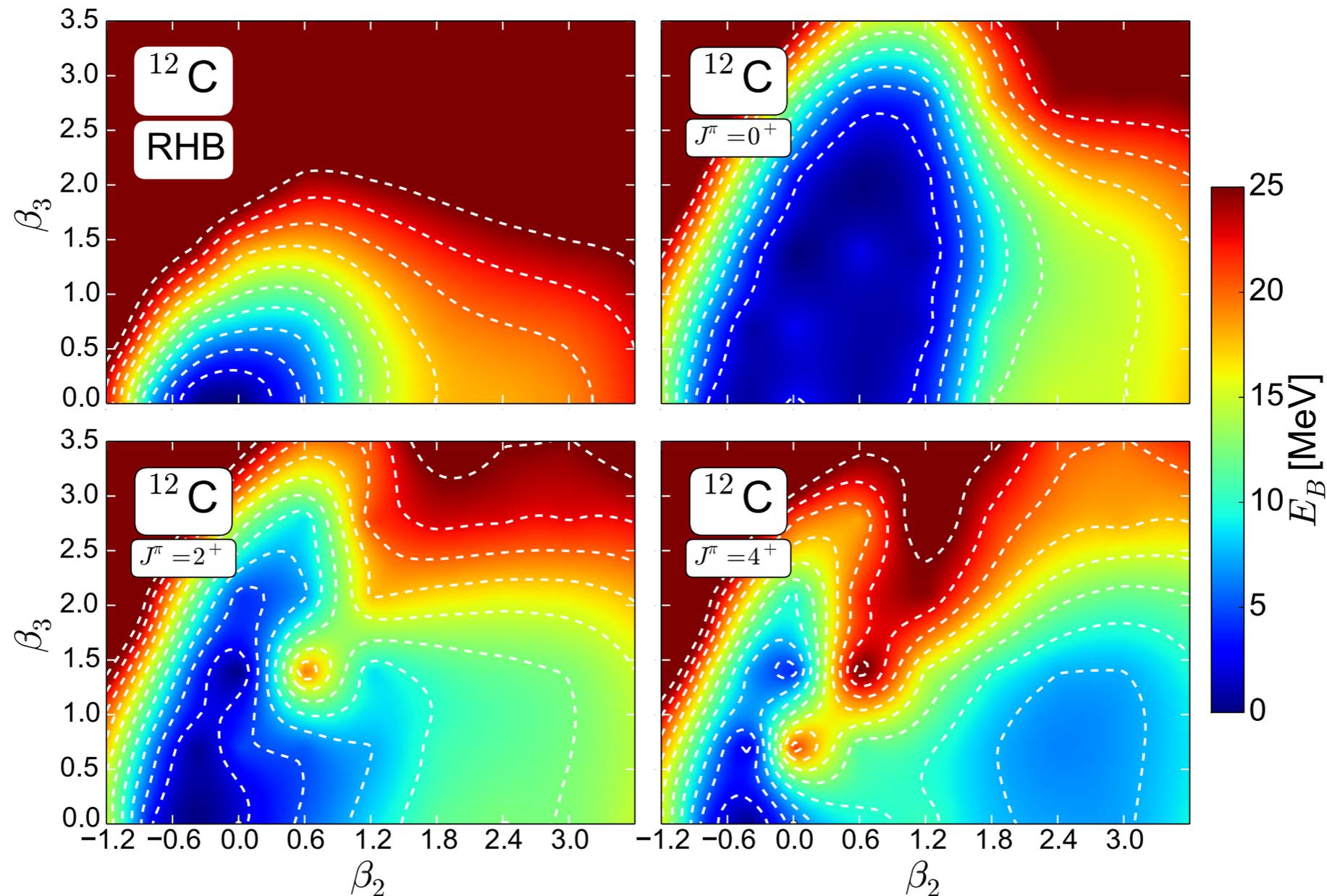
^{22}Ne - intrinsic densities at characteristic prolate (oblate) deformations:



^{24}Ne - intrinsic densities at characteristic prolate (oblate) deformations:

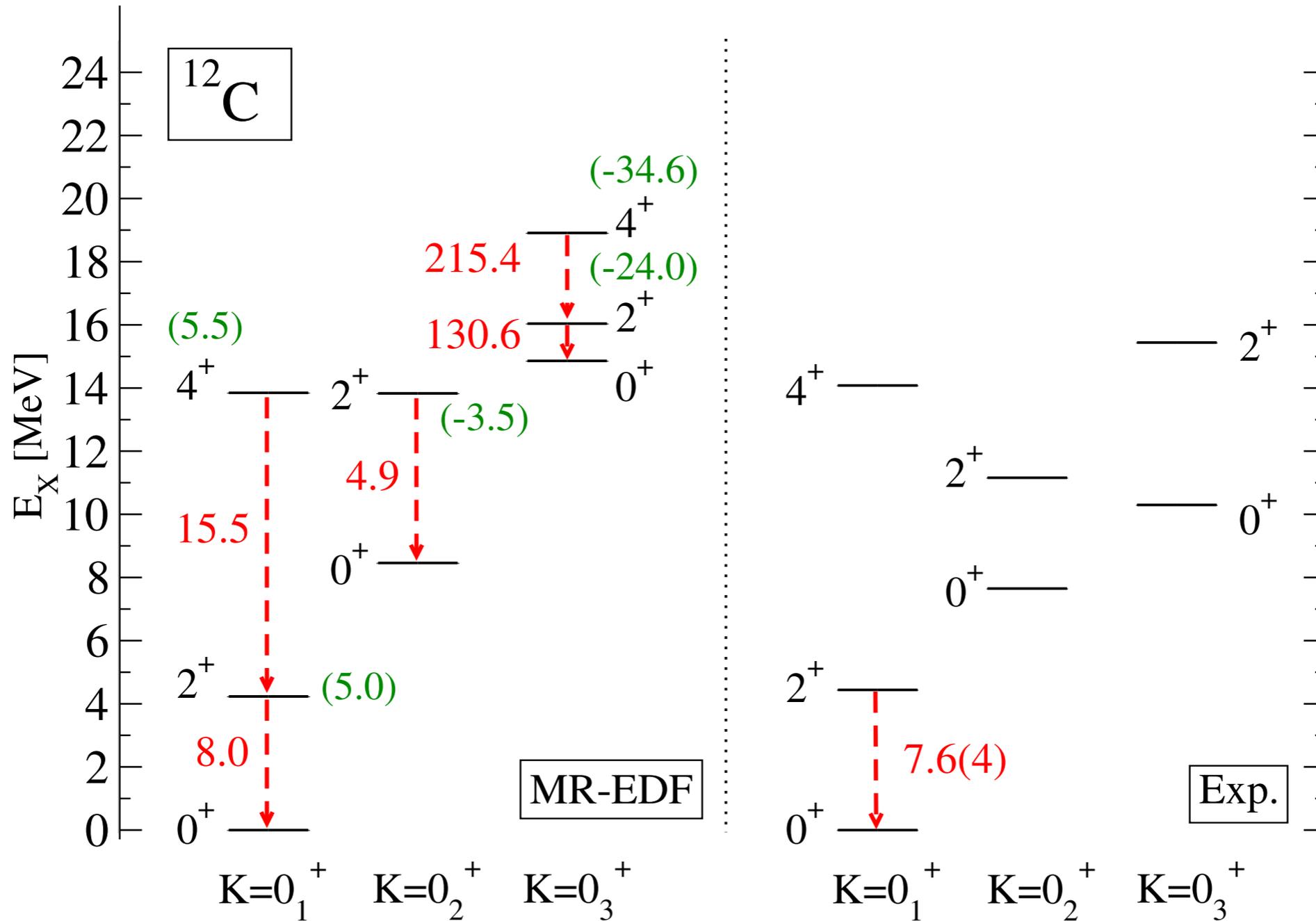


Energy density functional DD-PC1 + separable pairing interaction

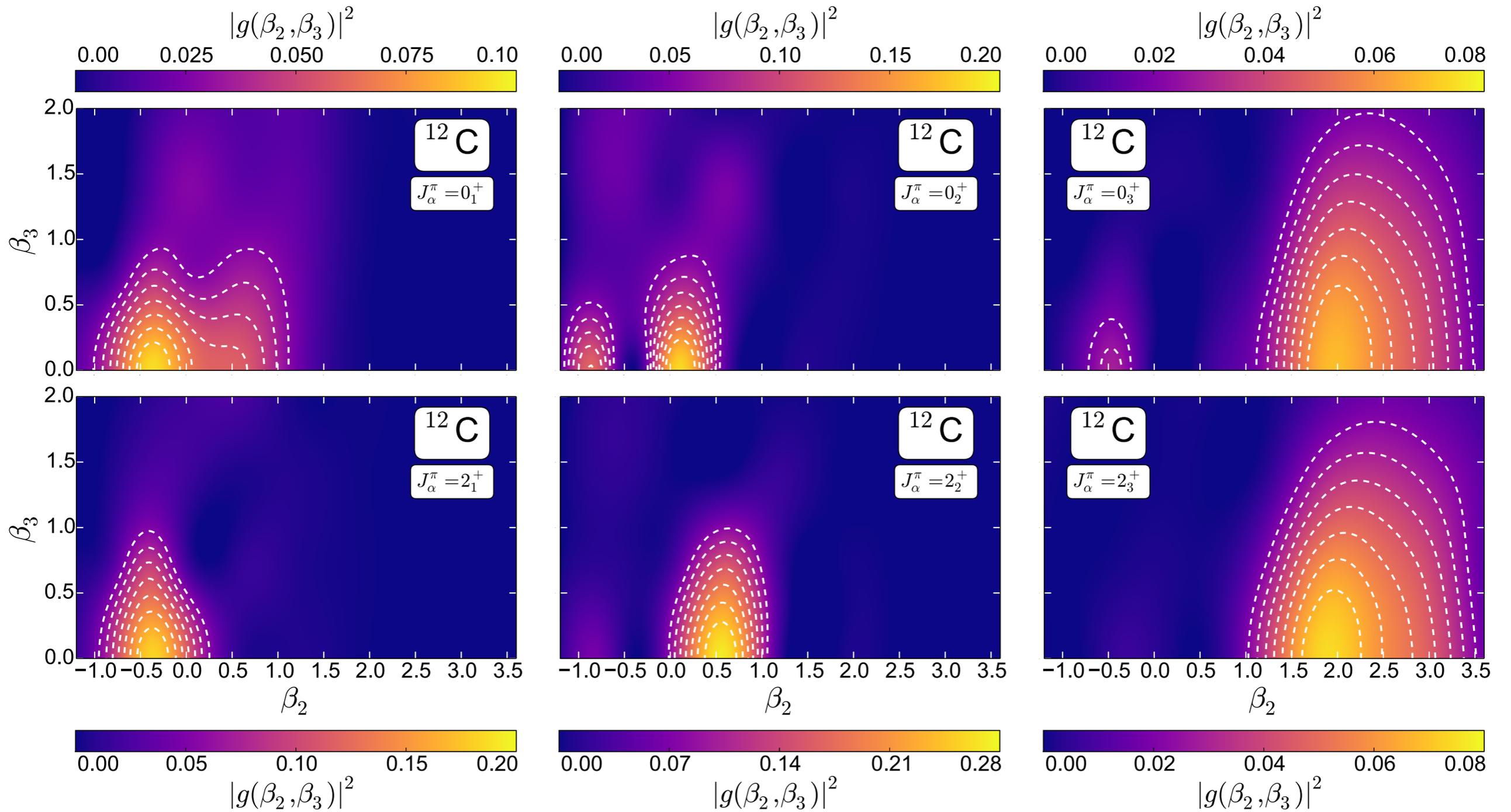


Deformation energy surfaces of ^{12}C in the β_2 - β_3 plane: self-consistent mean-field RHB, angular momentum-, particle number-, and parity-projected for spin-parity values $J^\pi = 0^+, 2^+, 4^+$.

GCM configuration mixing of angular momentum-, particle number-, and parity-projected SCMF states:



Amplitudes of collective wave functions squared $|g(\beta_2, \beta_3)|^2$ for the low-energy levels of ^{12}C .

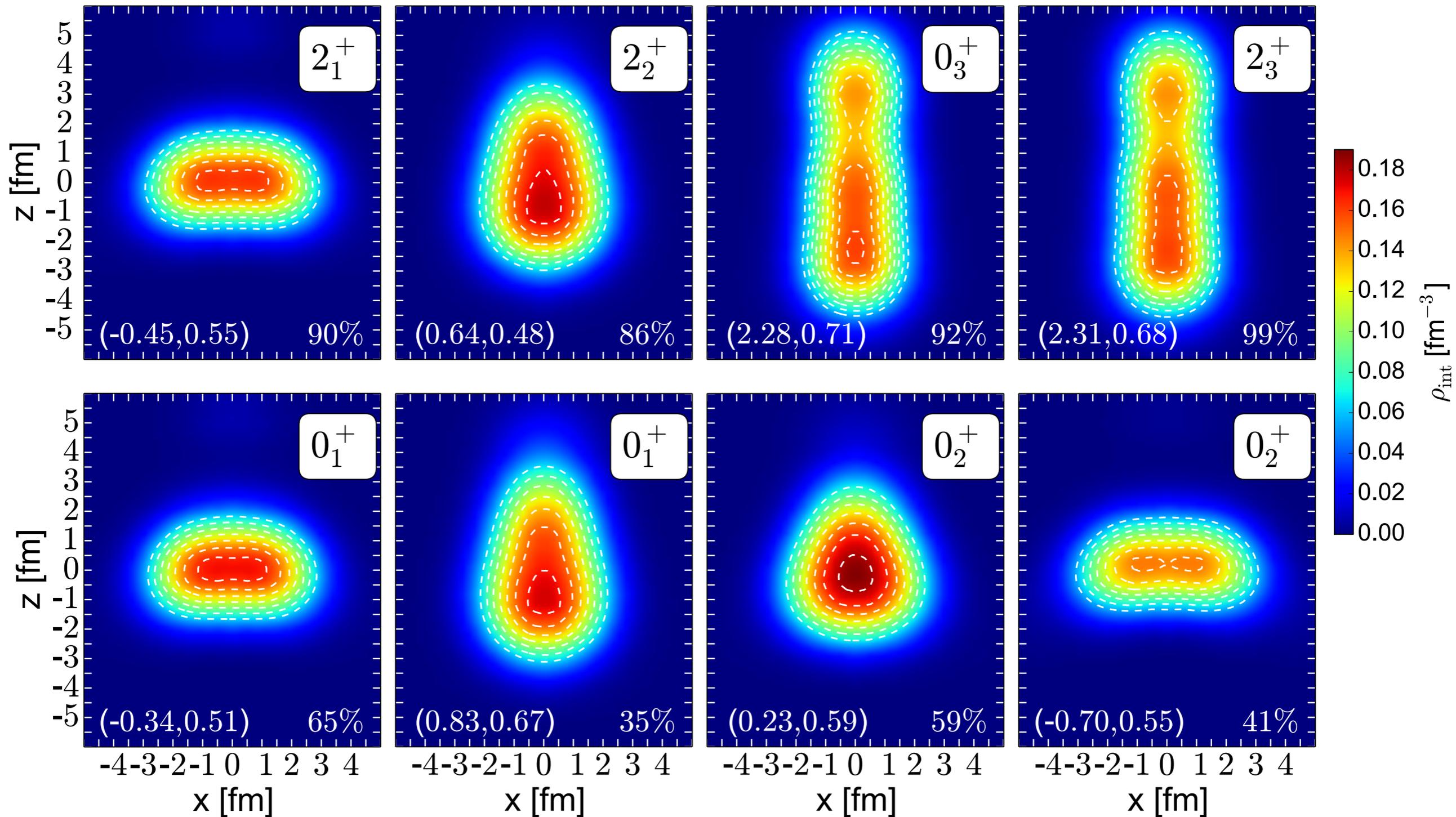


Collective states in the intrinsic frame

...calculate the expectation values of deformation parameters β_2 and β_3 for each collective state:

$$\langle \beta_2 \rangle_{\alpha}^{J;NZ;\pi} = \sum_i |g_{\alpha}^{J;NZ;\pi}(q_i)|^2 \beta_{2i},$$

$$\langle |\beta_3| \rangle_{\alpha}^{J;NZ;\pi} = \sum_i |g_{\alpha}^{J;NZ;\pi}(q_i)|^2 |\beta_{3i}|,$$



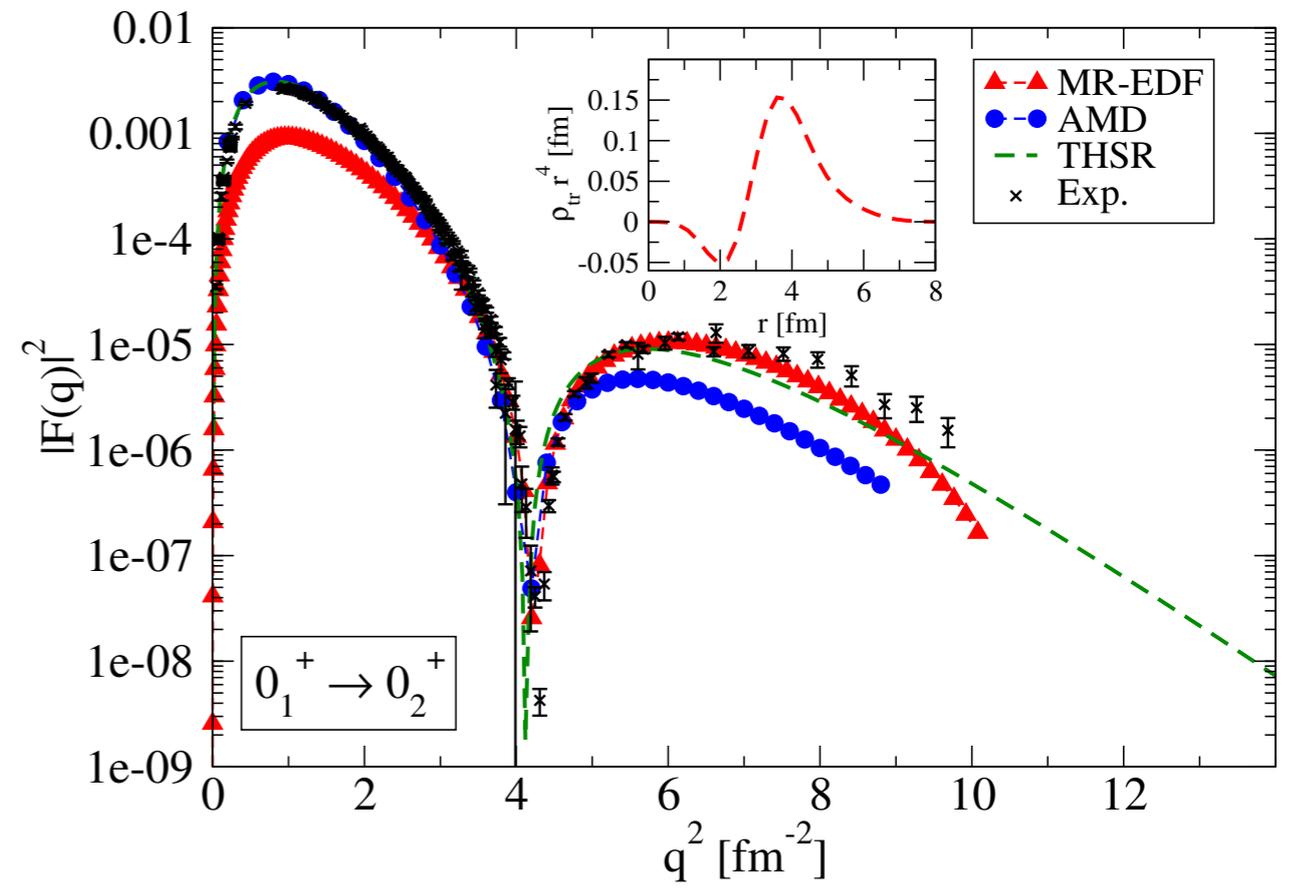
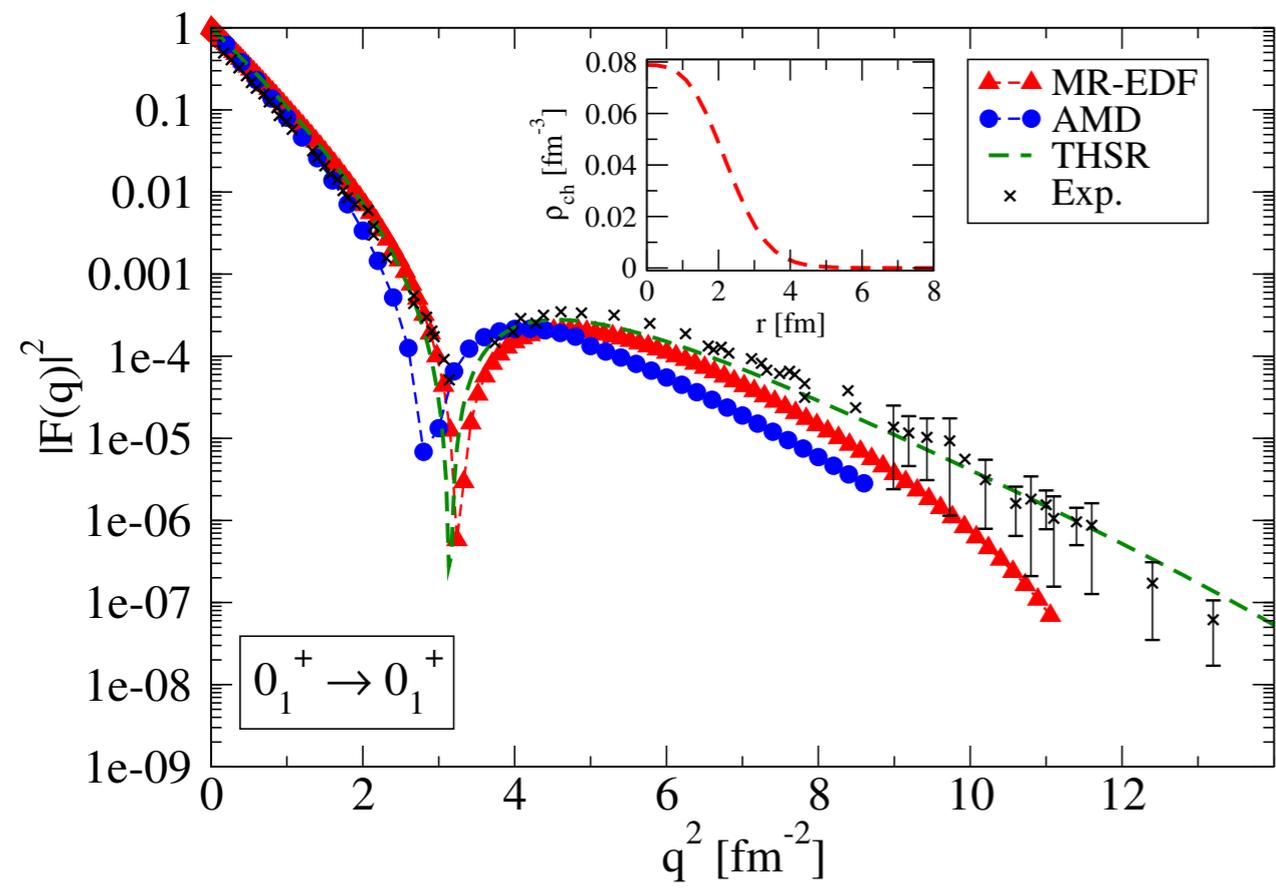
Electron-nucleus scattering form factors

...longitudinal Coulomb form factor:

$$F_L(q) = \frac{\sqrt{4\pi}}{Z} \int_0^\infty dr r^2 \rho_{J_i \alpha_i, L}^{J_f \alpha_f}(r) j_L(qr),$$

reduced proton transition density:

$$\rho_{J_i \alpha_i, L}^{J_f \alpha_f}(r) = (-1)^{J_f - J_i} \frac{2J_f + 1}{2J_i + 1} \sum_K \langle J_f 0 L K | J_i K \rangle \times \int d\hat{r} \rho_{\alpha_f \alpha_i}^{J_f J_i K 0}(\mathbf{r}) Y_{LK}^*(\hat{r}),$$





- *P. Marević*
- *R.-D. Lasserri*
- *J.-P. Ebran*
- *E. Khan*
- *T. Nikšić*
- *D. Vretenar*