

# Cluster states probed by alpha (and proton) inelastic scattering

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# 1. Introduction

low-energy (LE)  
monopole/dipole excitations  
&  
cluster states

# LE-ISM, LE-ISD for cluster states

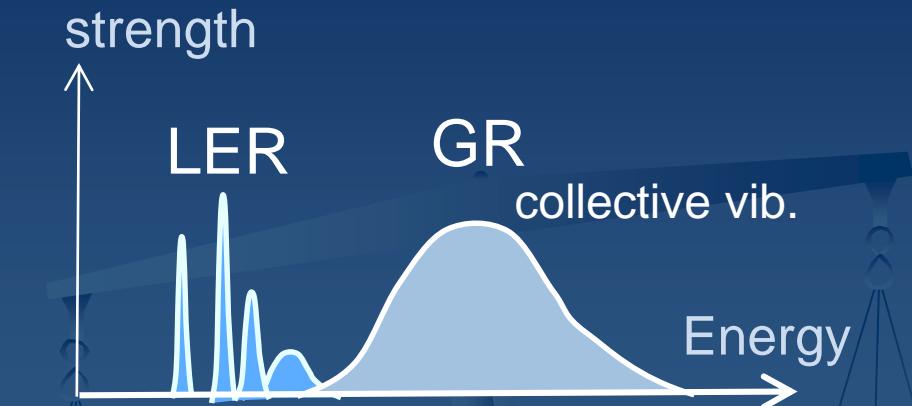
LE strengths (<15 MeV):  
observed by hadron probes  
alpha/proton inelastic scattering  
What is the origin?  
maybe, various LE modes

Yamada et al. PRC**85**, 034315 (2012)  
Chiba et al. PRC**93** (2016) no.3, 034319

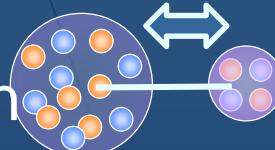
ISM, ISD: compressional  
operators excite inter-cluster motion

$$M(ISO) = \sum_i [r_i^2] Y_{00}(\hat{\mathbf{r}}_i) \sqrt{4\pi}$$

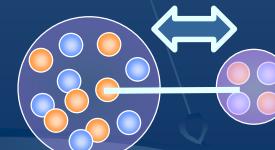
$$M(ISD; \mu) = \sum_i [r_i^3] Y_{1\mu}(\hat{\mathbf{r}}_i)$$



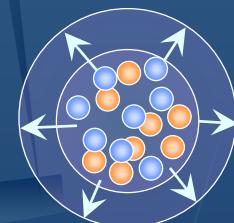
LE  
Inter-cluster motion



Inter-cluster motion



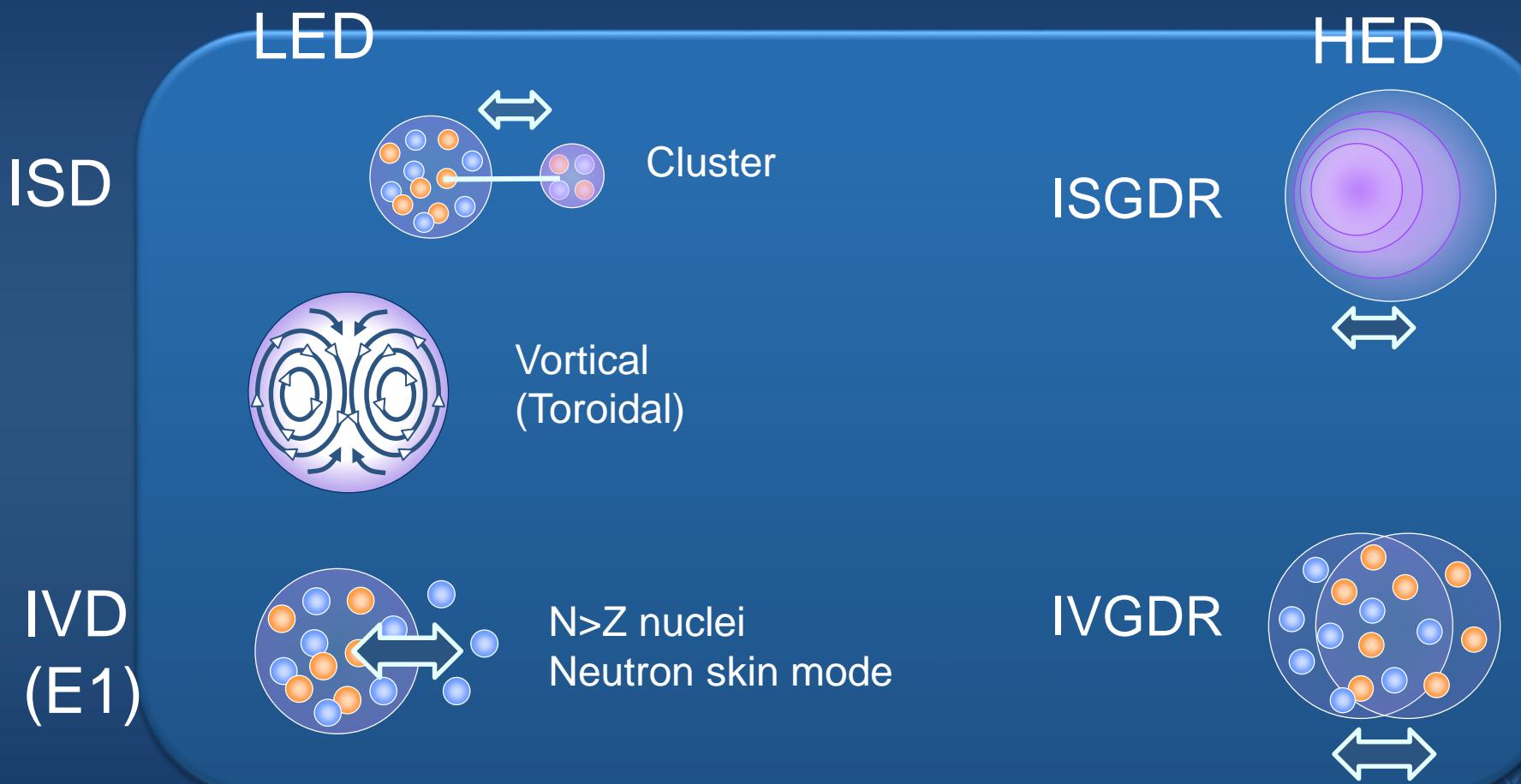
IS-GR  
compressive



compressive



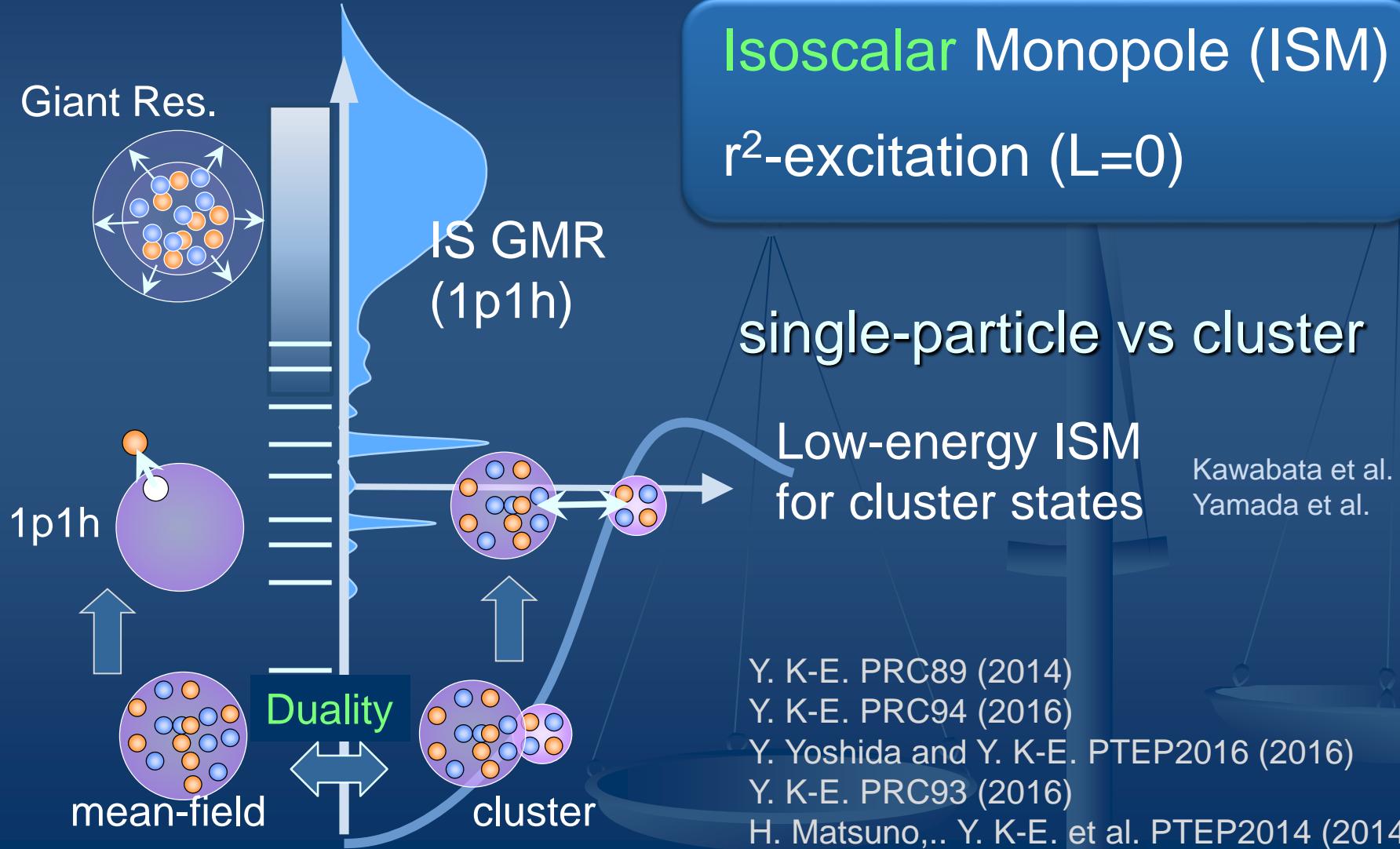
# Various modes contribute to LE strengths ex) case of Dipole



Many modes may arise from various Dof:  
1p1h ex, cluster, nuclear currents etc.

# Isoscalar monopole (dipole) excitations

Various modes arise from various Dof: 1p1h ex, cluster, nuclear currents etc.



# Aims

- To clarify natures and origins of LE modes:
  - How do they decouple from HE GRs.
  - Why do they come down to low energy

Structure calculation with AMD+cl-GCM  
cluster, deformation, 1p1h, vib/rot, GR

- How to experimentally probe them

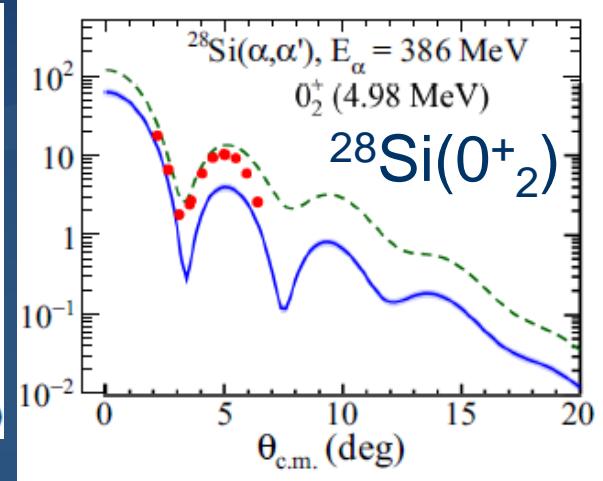
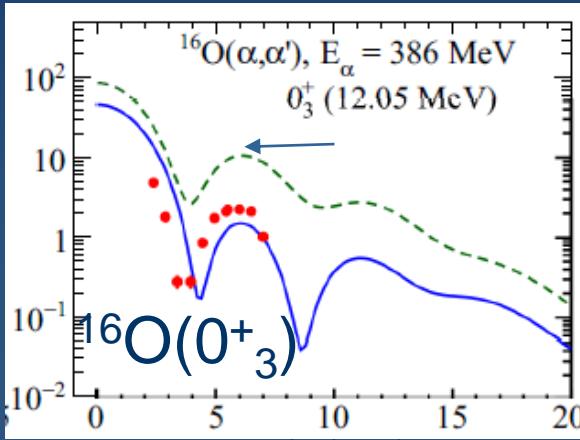
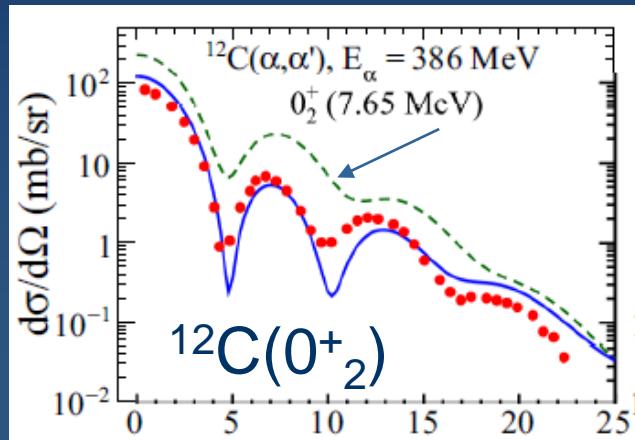
Reaction analysis of alpha inelastic scattering

# alpha (and/or proton) inelastic scattering can be good probe

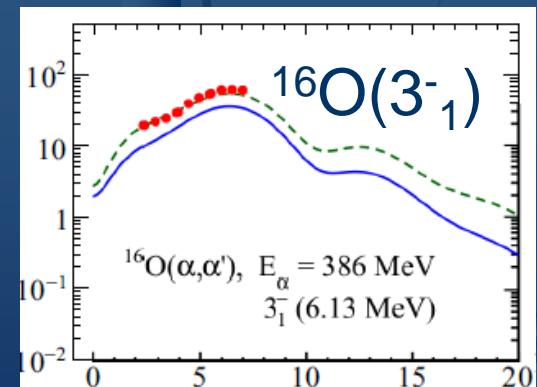
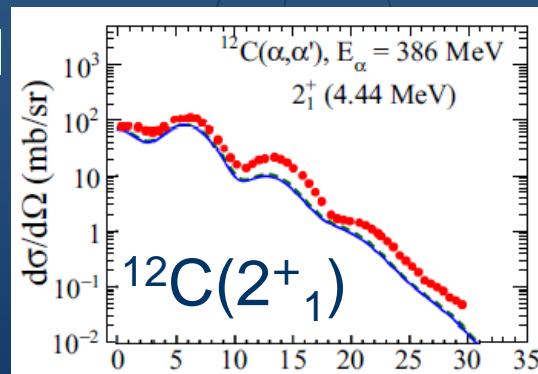
- Isoscaler(and isovector) monopole, dipole excitations
- search for new cluster states
- Determination of  $B(\text{IS0})$ ,  $B(\text{ISD})$ ?

But, ... a problem in reaction calculation:  
It largely overestimate cross sections of  
only  $0^+$  excitations by a factor of 3-5.

# A puzzle: overestimation of monopole cross sections by factors of 3-5

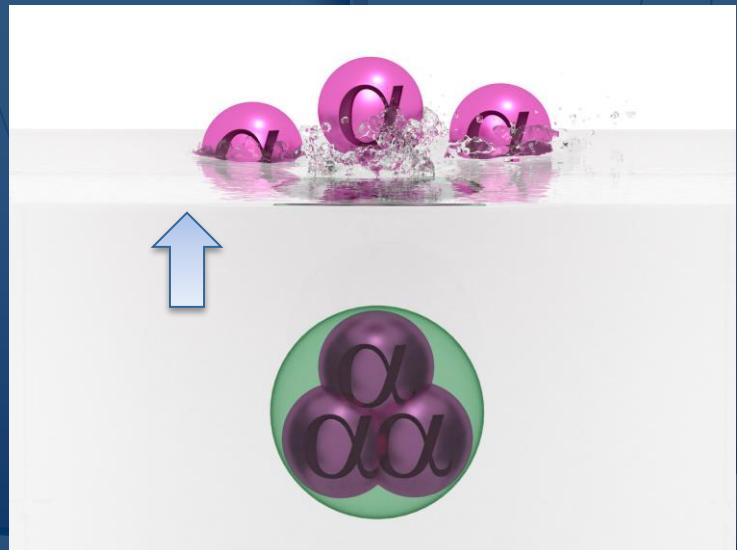
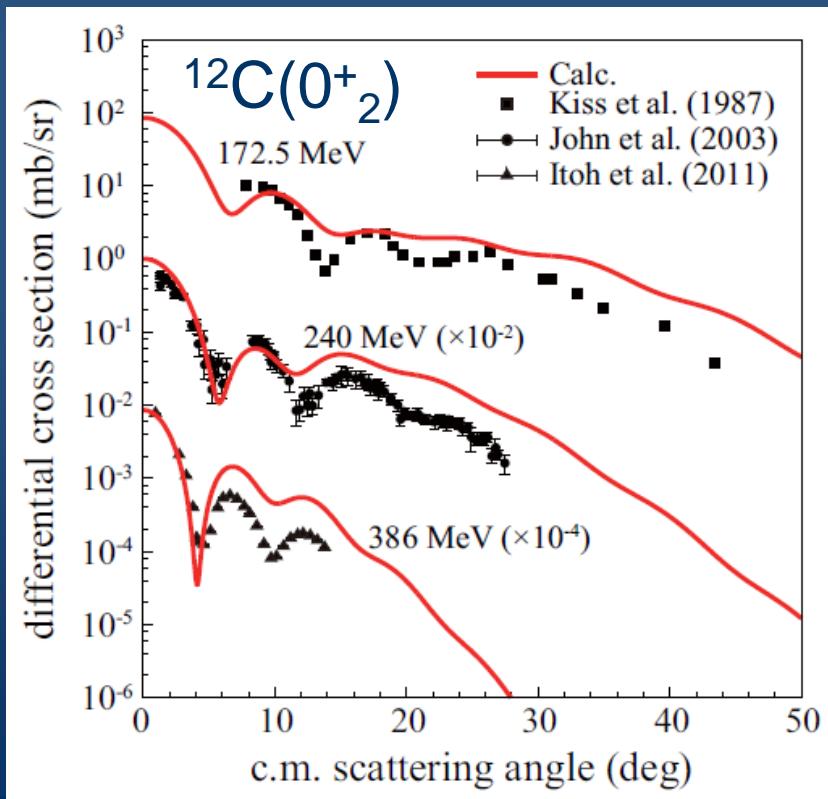


traditional reaction model  
no problem in 2+, 3-,  
but overshoot  
only 0+ cross section  
by a factor of 3-5



# Success of MCC calc. with g-matrix folding model

- microscopic coupled-channel (MCC)  
successful reproduction of  $^{12}\text{C}(0^+_2)$  cross sections with  $3\alpha$ -RGM density      *Minomo et al. C 93, 051601(R) (2016)*



# MCC: strategy

## Reaction part

bare NN forces  
(Bonn-B)

g-matrix theory

Effective NN int.  
(Melbourne g-matrix)

double folding

$\alpha$ -nucleus potentials

coupled-channels calc.

$(\alpha, \alpha')$  cross sections

## Structure part

micro. structure model  
(AMD, Cluster-GCM, RGM)



Matter and transition  
densities



great merits:  
no phenomenological adjustable  
parameters for real and imaginary  
potentials.

# 2. Formulation of structure model: sAMD+GCM

Shifted basis AMD

# AMD method for structure study

AMD wave fn.

$$\Phi = c\Phi_{\text{AMD}} + c'\Phi'_{\text{AMD}} + c''\Phi''_{\text{AMD}} + \dots$$

$$\Phi_{\text{AMD}} = \det \{\varphi_1, \varphi_2, \dots, \varphi_A\}$$

Slater det.

$$\varphi_i = \phi_{Z_i} \chi_i$$

**Gaussian**

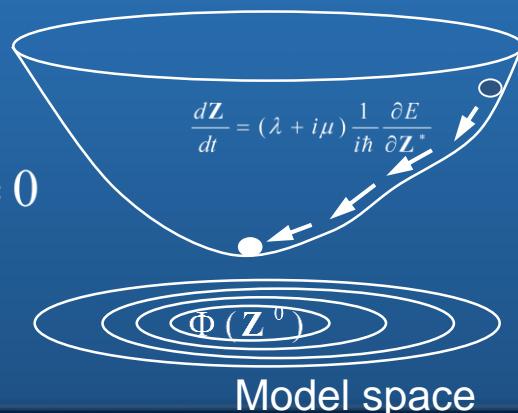
$$\phi_{Z_i}(r_j) \propto \exp \left[ -\nu \left( \mathbf{r} - \frac{\mathbf{Z}_i}{\sqrt{\nu}} \right)^2 \right]$$

$$\chi_i = \begin{cases} \frac{1}{2} + \xi_i & p \text{ or } n \\ \frac{1}{2} - \xi_i & \text{isospin} \end{cases}$$

Intrinsic spins

Energy Variation

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0$$

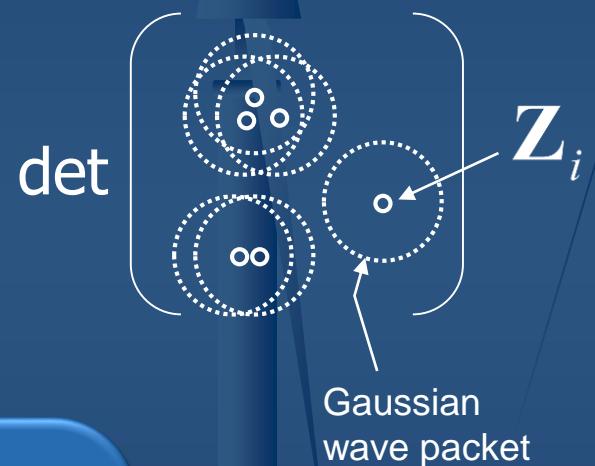


$$\Phi_{\text{AMD}}(\mathbf{Z})$$

$$\mathbf{Z} = \{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A, \xi_1, \dots, \xi_A\}$$

Variational parameters:

Gauss centers, spin orientations



Initial states

**J<sup>π</sup>-VAP**  
to get the ground state wave function.

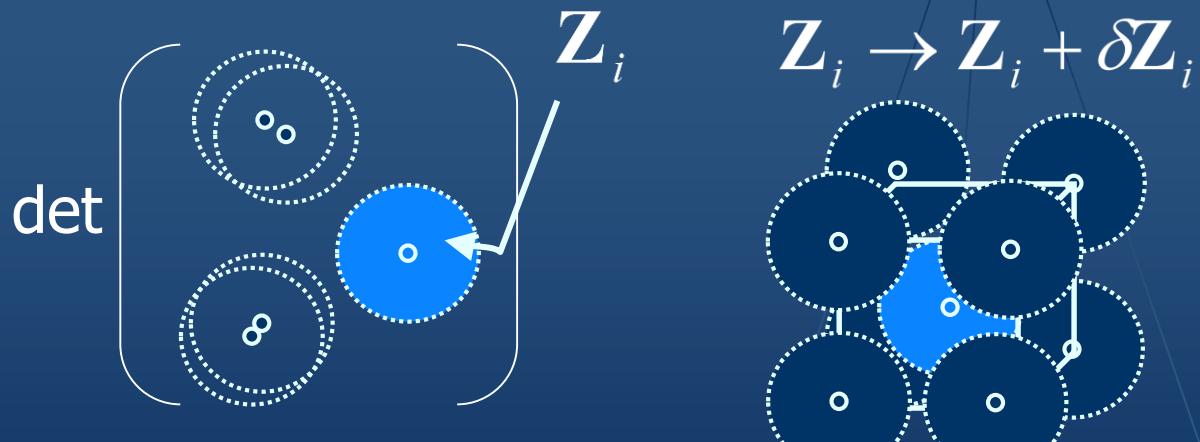
Energy minimum

# Shifted basis AMD (sAMD)

$$\Phi = \det \{\varphi_1, \varphi_2, \dots, \varphi_A\} \quad \text{Ground st. wave functions}$$

$$\downarrow \quad \varphi_i + \delta\varphi_i = \phi_{Z_i + \delta Z_i} \chi_i \quad \text{small shift of spatial part}$$

$$\det \{\varphi_1, \dots, \varphi_i + \delta\varphi_i, \dots, \varphi_A\} \quad \text{A shifted basis}$$



Small shift for  
8 orientations  
(8A basis)

8A basis is enough for IS0,E1,IS1 in 12C and Be

# sAMD+GCM

AMD(VAP)

Ground state wave function

→ g.s. cluster correlation

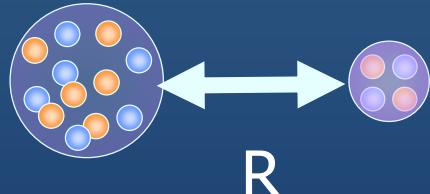
sAMD

1p1h excitations on g.s.

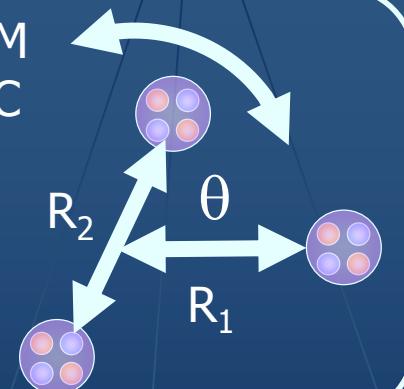
→ GRs

GCM various cluster configurations

$\alpha$ -GCM  
for 10Be and 16O



$3\alpha$ -GCM  
For 12C



Large amp. cluster motion in LE

sAMD+GCM: all bases are superposed.

$J\pi$ -projection, cm motion are treated microscopically

# Today's topics

1. cluster states in  $^{12}\text{C}$
2. alpha scattering off  $^{12}\text{C}, ^{16}\text{O}$
3. cluster states in  $^{14}\text{C}$

# Topic 1

## Cluster states in $^{12}\text{C}$

### with sAMD+3 $\alpha$ GCM

# multi-cluster: gas, triangle, linear chain

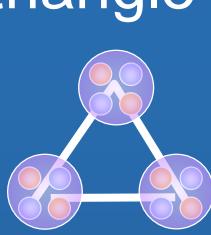
$^{12}\text{C}$

triangle ?

$0_3^+$



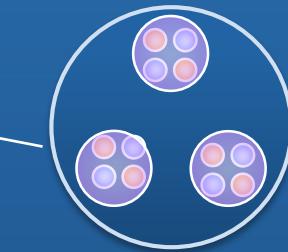
$2_2^+ \ 3_1^-$



$0_2^+$

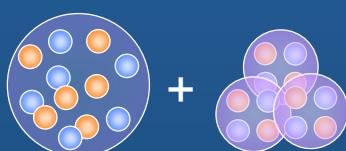
$7.65 \text{ MeV}$

$^{8}\text{Be} + \alpha$

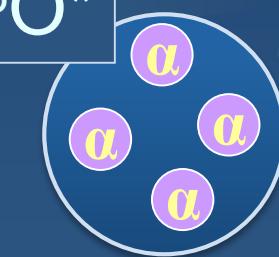


Cluster (boson)  
gas

$0_1^+$



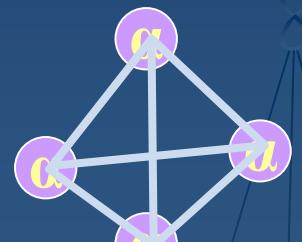
$^{16}\text{O}^*$



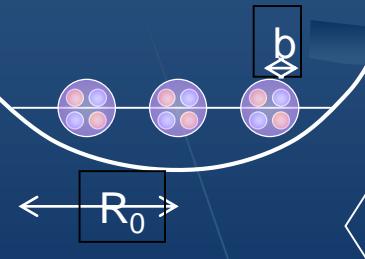
4 $\alpha$  gas

Tohsaki et al.,  
Funaki et al

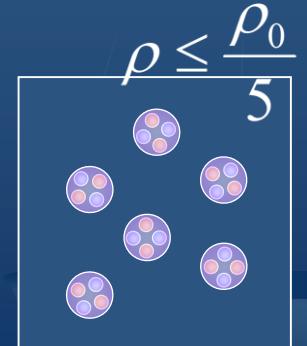
Yamada et al.,  
Y. K-E PRC96 (2017)  
Y. K-E. PRC89 (2014)



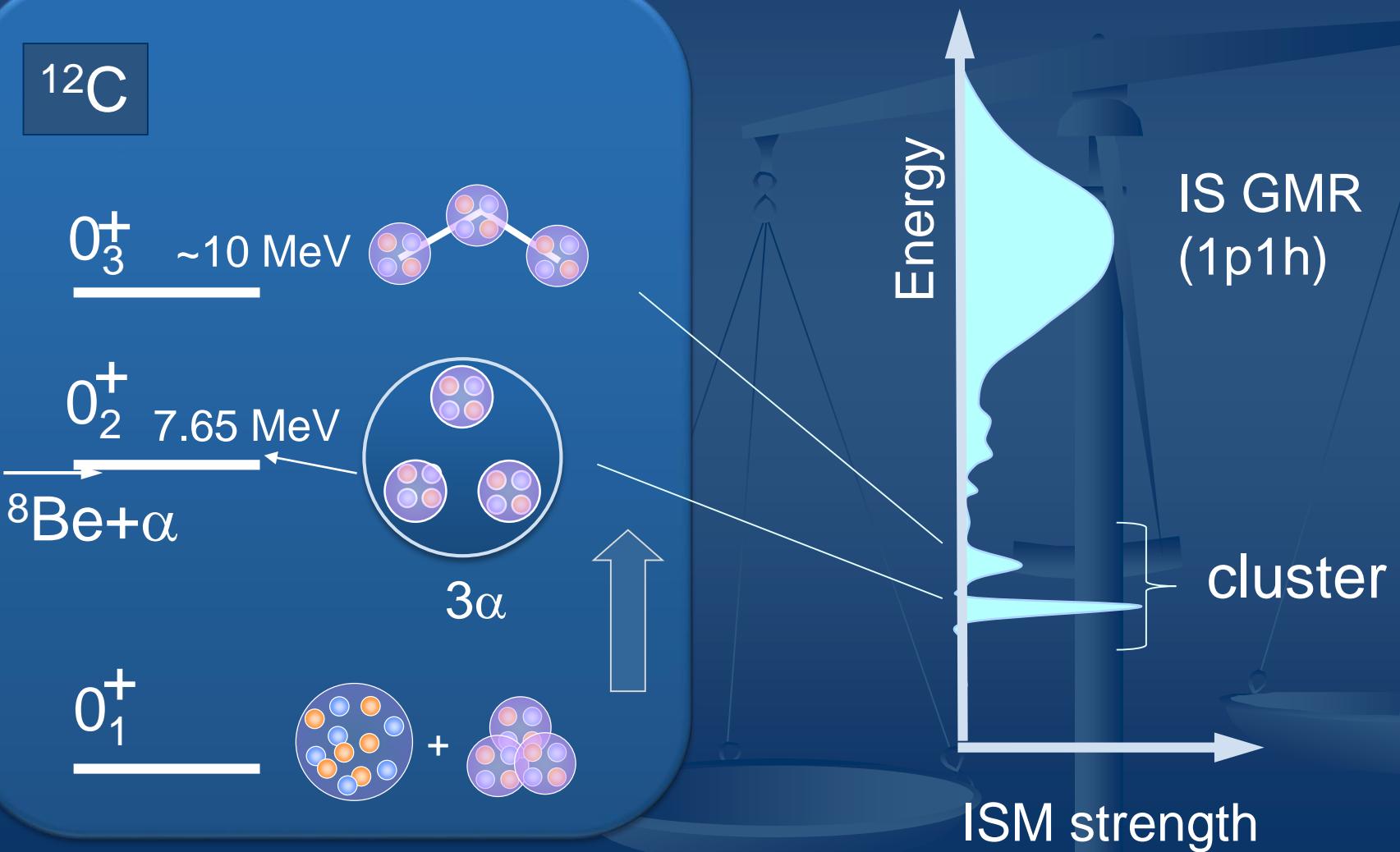
tetrahedral



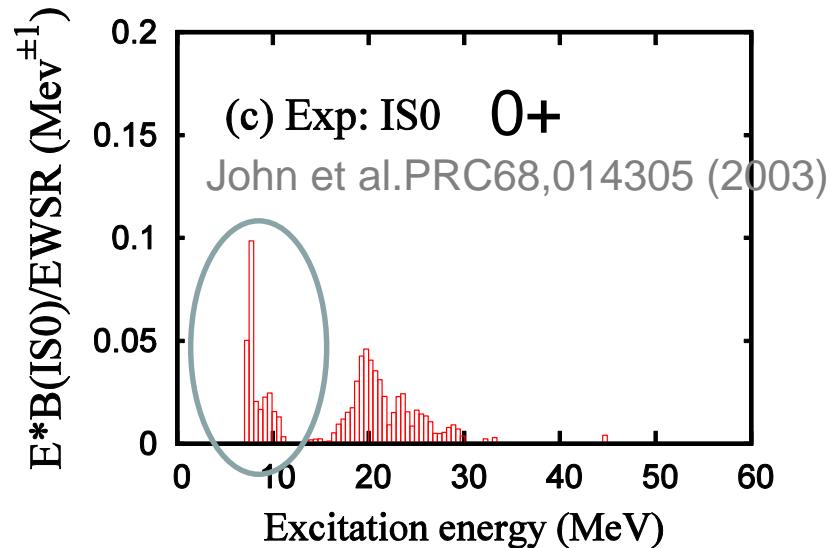
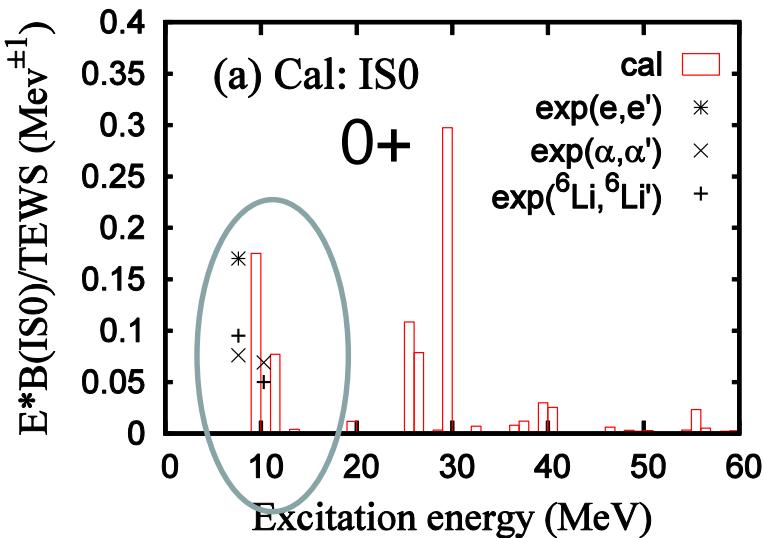
Analogy to  
BEC in dilute nuclear matter



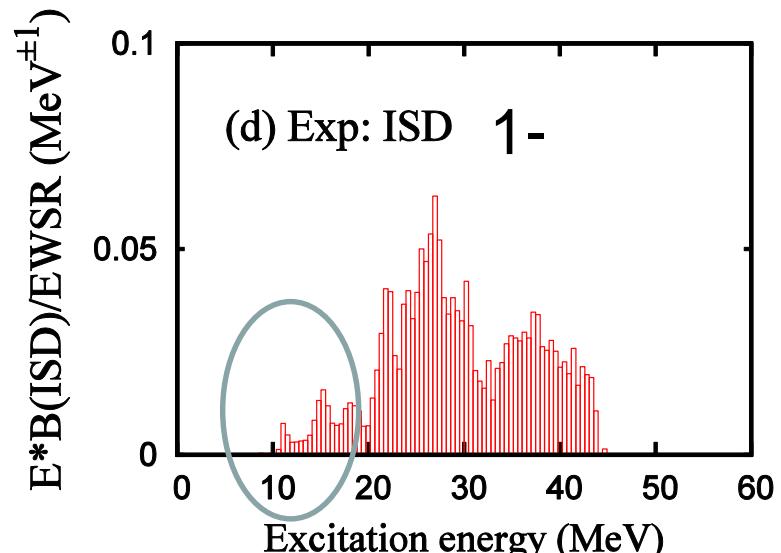
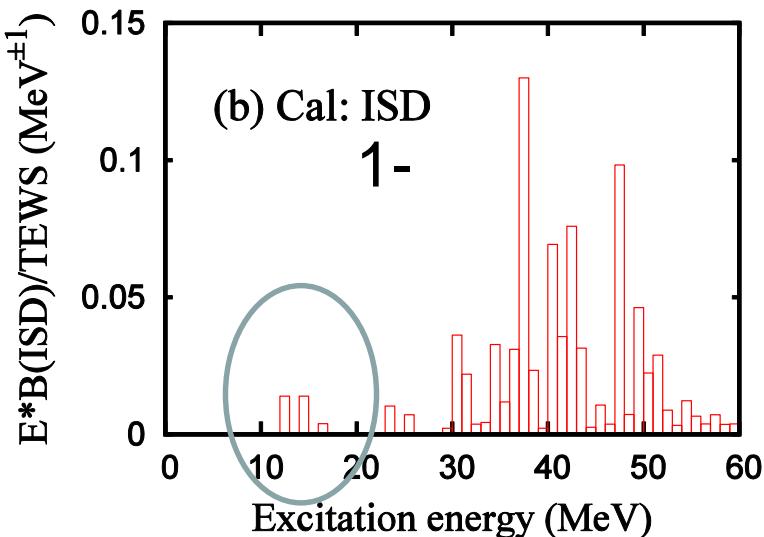
# Results of $^{12}\text{C}$ : sAMD+3 $\alpha$ GCM for ISM and ISD excitations



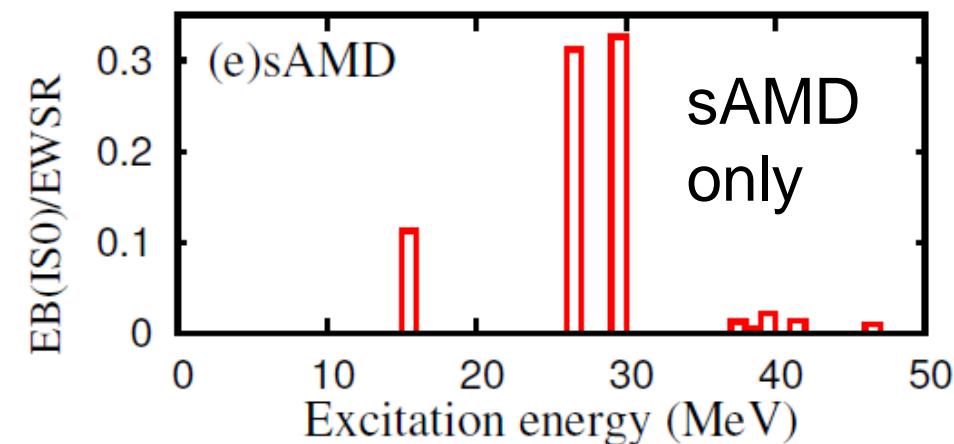
# IS Monopole, Dipole strengths in $^{12}\text{C}$ : sAMD+GCM( $3\alpha$ )



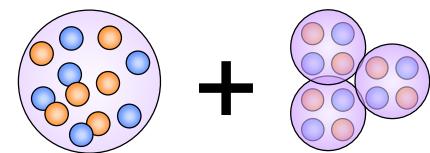
Low-energy strengths for cluster states separately from GRs.  
Why the LE strengths are fragmented into a few cluster states?



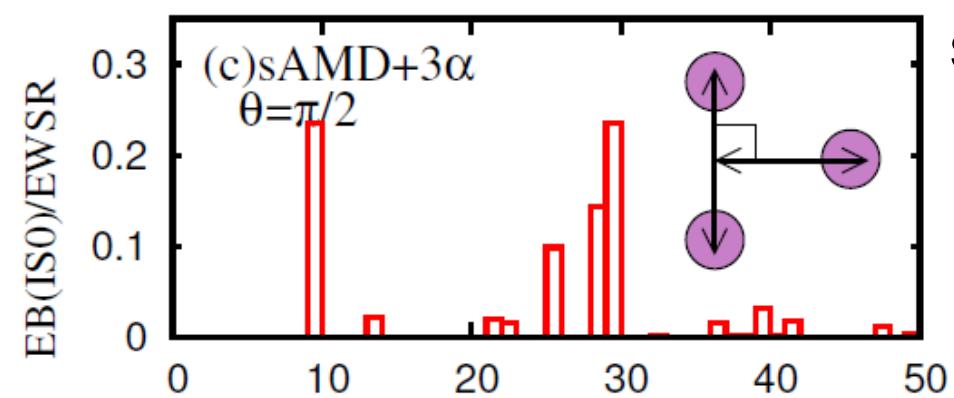
# B(ISM) of $^{12}\text{C}$



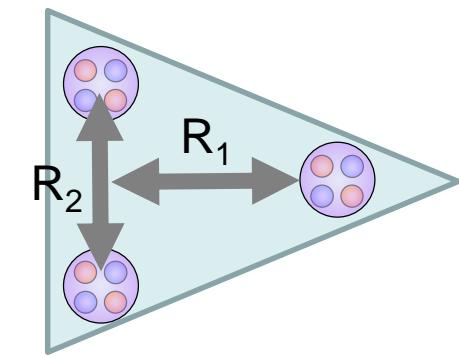
sAMD small amp: 1p-1h



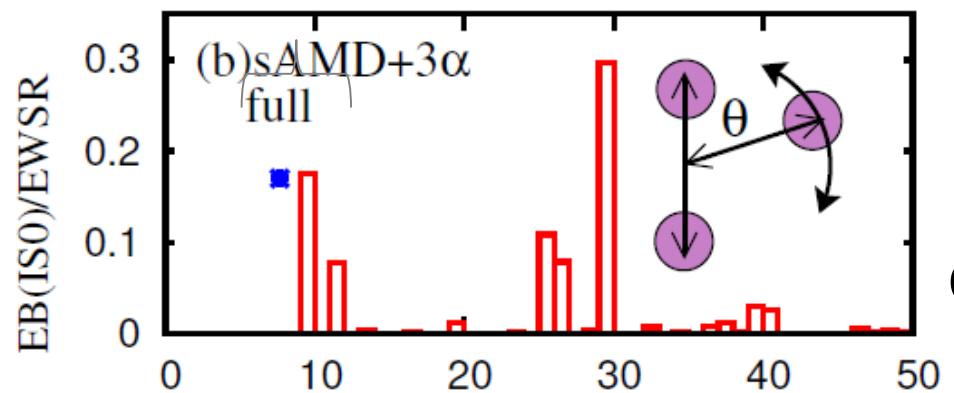
GMR strengths



sAMD+ $3\alpha(\pi/2)$   
distance mode

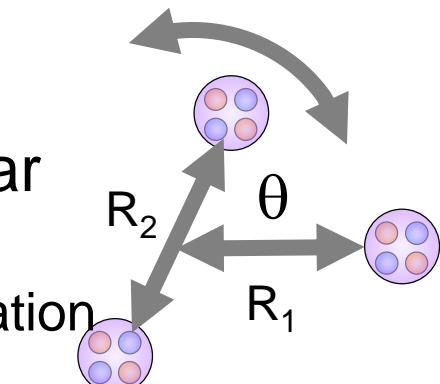


Large amplitude  
cluster motion

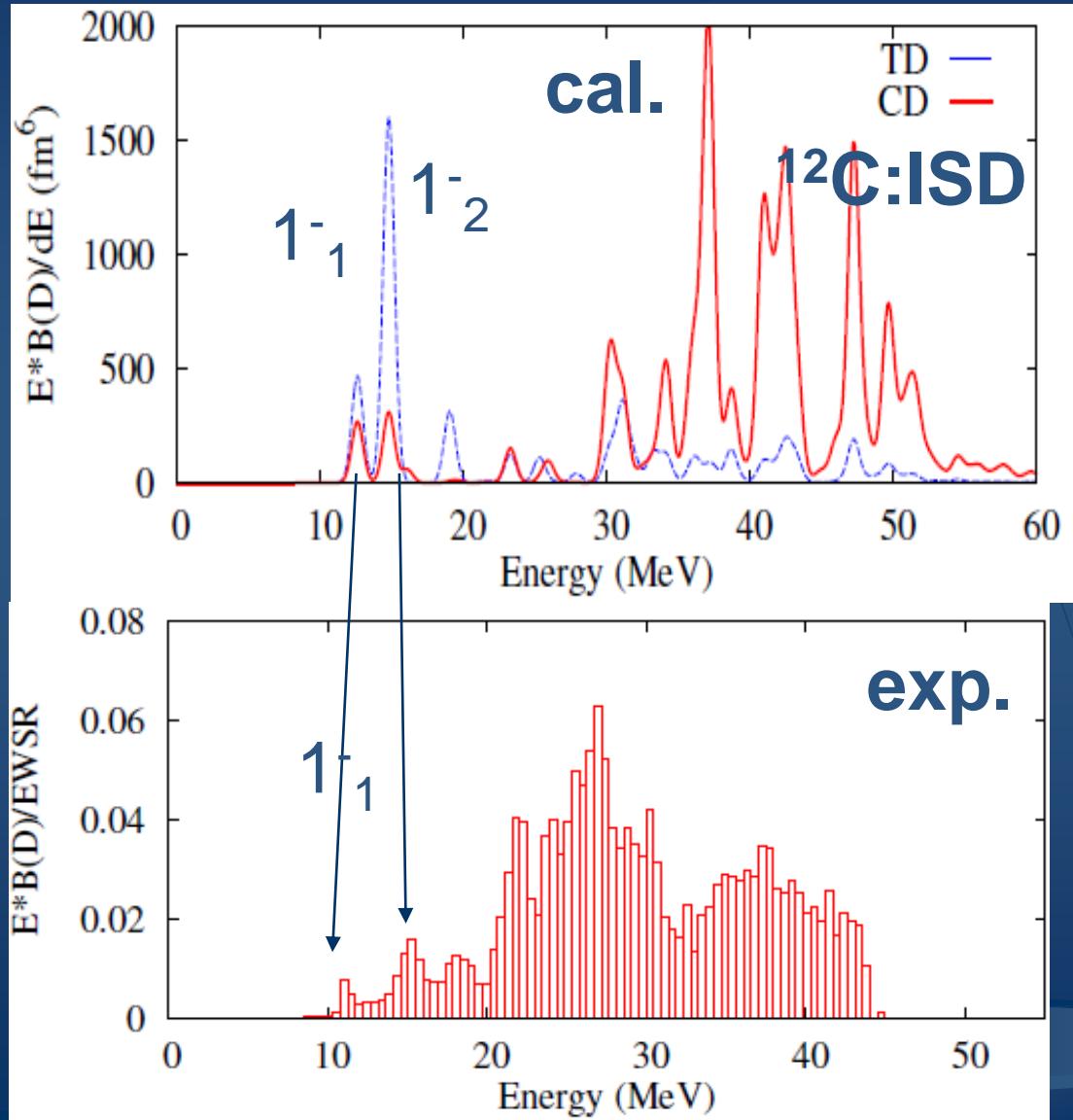


sAMD+ $3\alpha(\text{full})$   
distance& angular

Coupling of 8Be-rotation  
splits LE-ISM

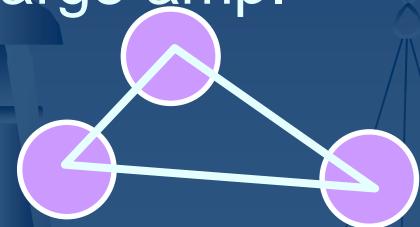


# Dipole in $^{12}\text{C}$



$1^-_1$

Cluster mode  
Large amp.

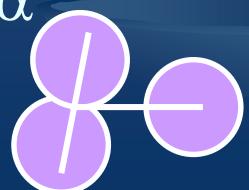


Distorted  $3\alpha$ -triangle  
= Uegaki's  $3\alpha$ -GCM

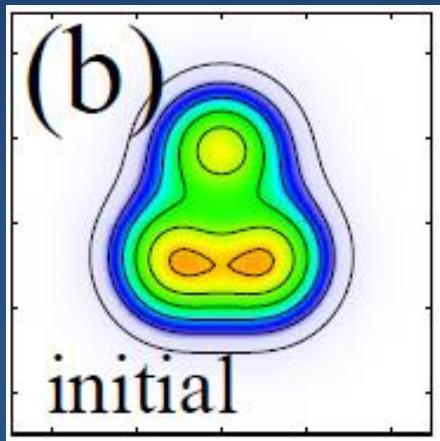
$1^-_2$

vibration

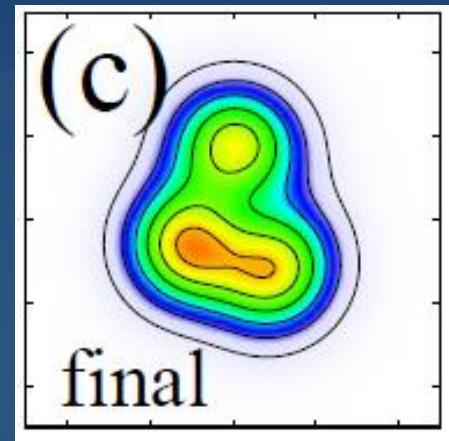
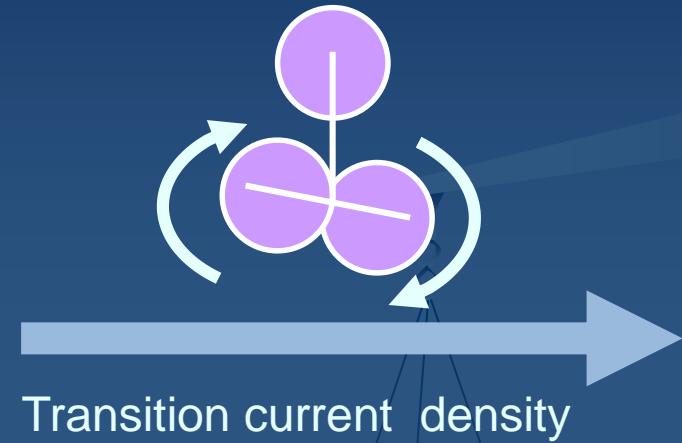
Small amp. 1p1h mode  
Compact  $3\alpha$



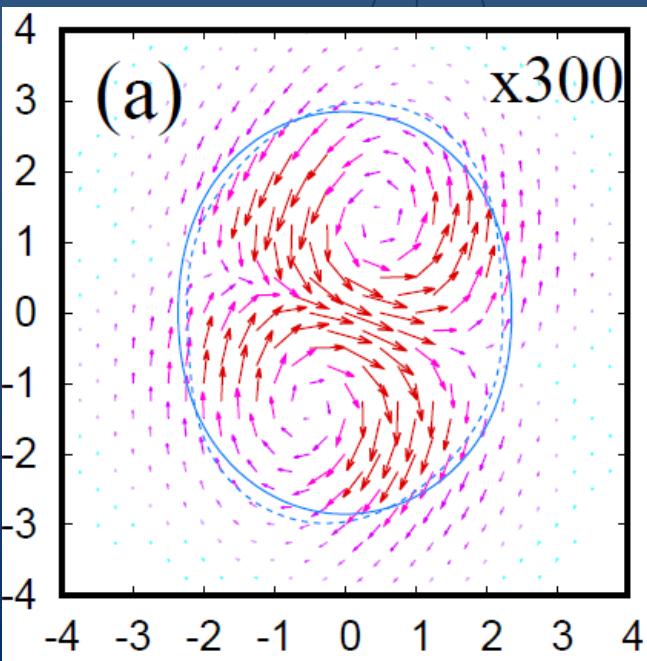
# Vortical nature in $0^+_1$ to $1^-_2$



$0^+$



$1^-_2$



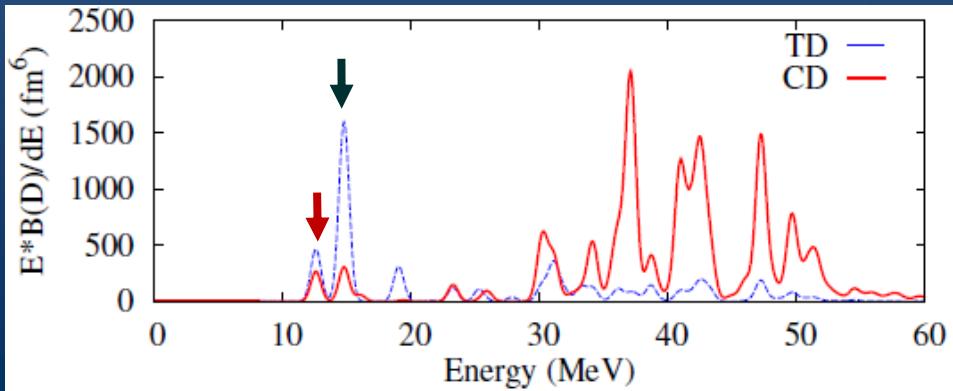
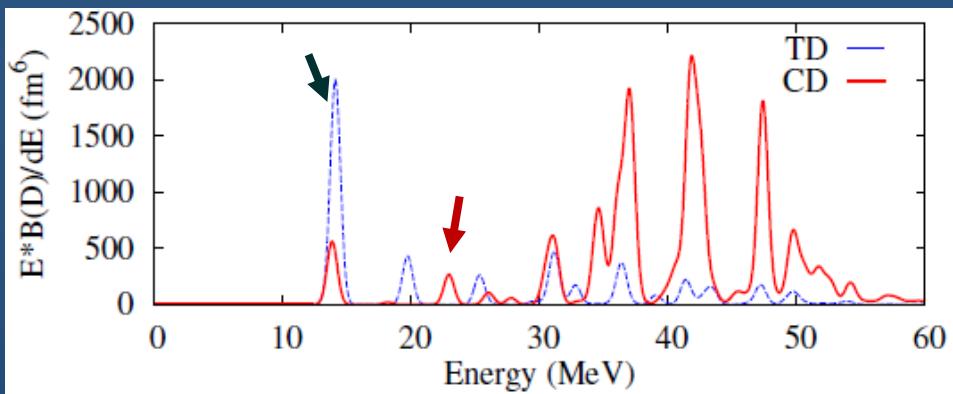
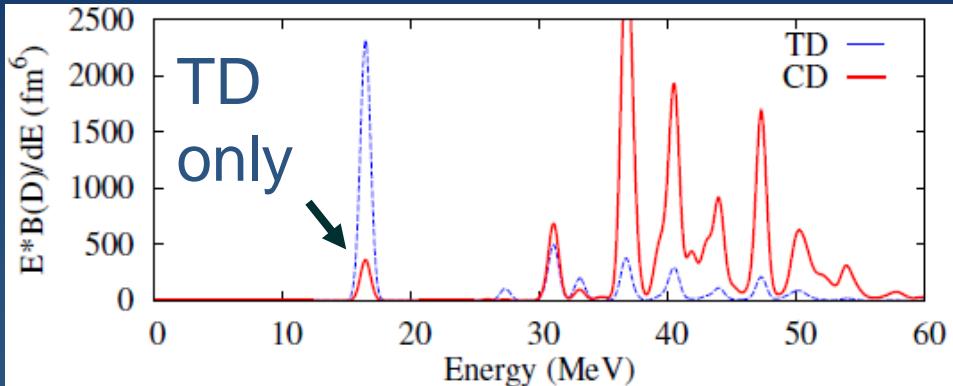
2 $\alpha$ -cluster rotation  
induces vortical  
(toroidal) current.

# Compressive and toroidal dipole strengths

CD:compressive    TD:toroidal

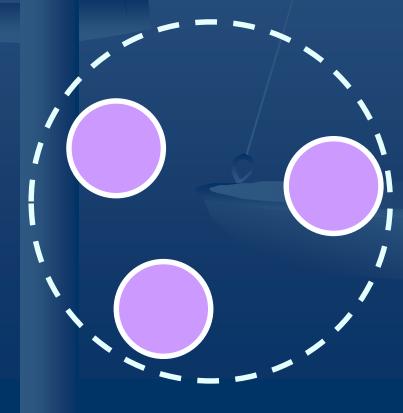
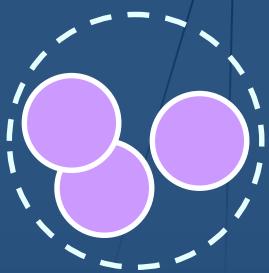
$$(\nabla \cdot \mathbf{j}) r^3 Y_{1\mu} \quad (\nabla \times \mathbf{j}) \cdot r^3 \mathbf{Y}_{11\mu}$$

sAMD: small amp.



sAMD  
+ $3\alpha(R < 3 \text{ fm})$

sAMD  
+ $3\alpha(\text{full})$



# Topic 2

Alpha inelastic scattering  
with MCC

# microscopic reaction calculations with g-matrix folding model

- successful reproduction of  $^{12}\text{C}(0^+_2)$  cross sections with RGM density by Minomo et al.

*Minomo et al. C 93, 051601(R) (2016)*

No overshooting problem of 0+ cross sections

Reliable reaction approach:

There is no phenomenological adjustable parameters in the reaction part

# MCC

Minomo et al. PRC93 (2016)  
K-E, Ogata PRC99 (2019)  
K-E. Ogata ORC99 (2019)

## Reaction part

bare NN forces  
(Bonn-B)

g-matrix theory

Effective NN int.  
(Melbourne g-matrix)

double folding

$\alpha$ -nucleus potentials

coupled-channels calc.

$(\alpha, \alpha')$  cross sections

## Structure part

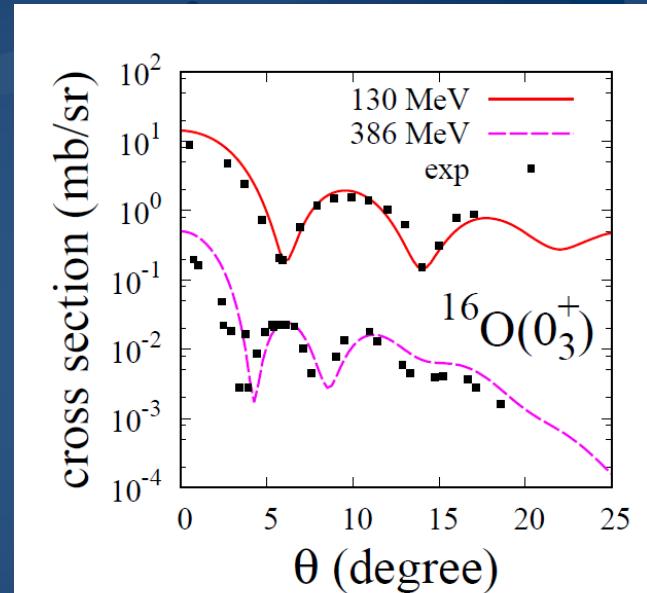
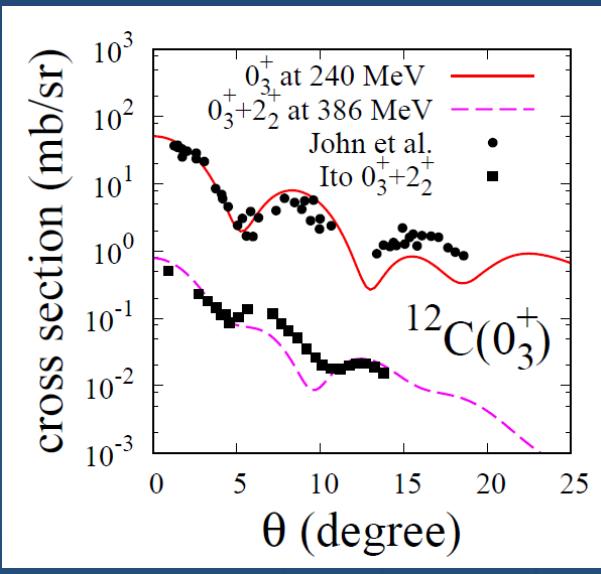
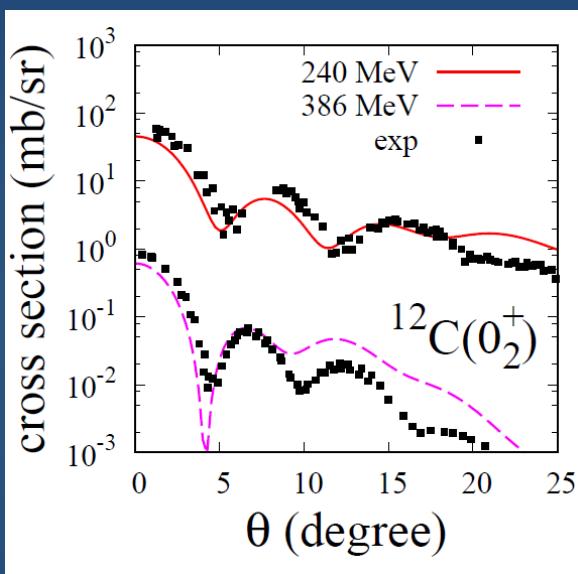
micro. structure model  
(AMD, Cluster-GCM, RGM)  
1p1h, cluster, vib/rot,  
vortical, GR



Matter and transition  
densities

great merits:  
no phenomenological adjustable  
parameters for real and imaginary  
potentials.

# Results of $^{12}\text{C}(\alpha, \alpha')$ and $^{16}\text{O}(\alpha, \alpha')$ MCC with AMD densities

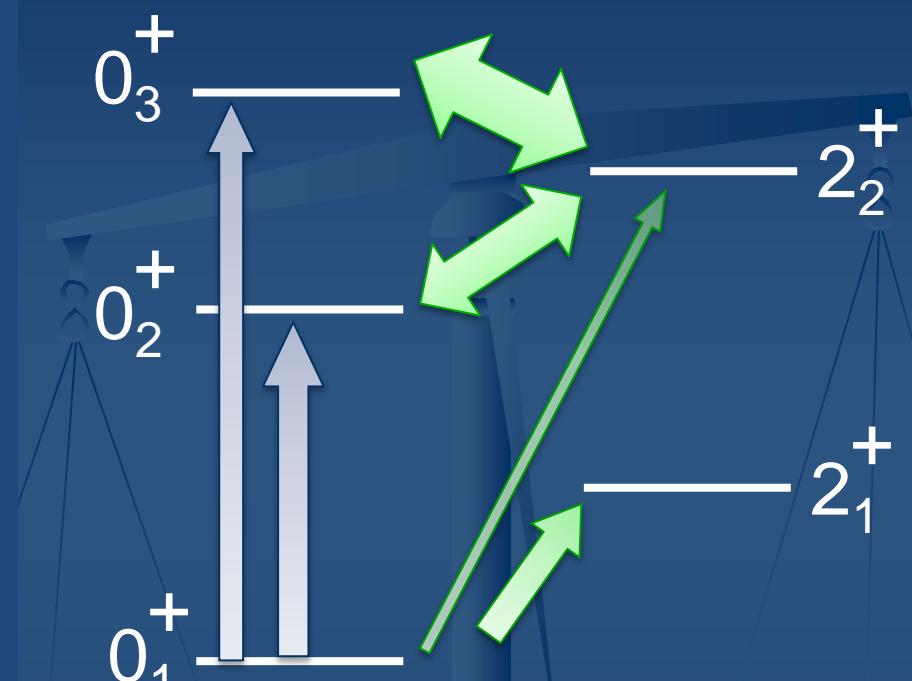
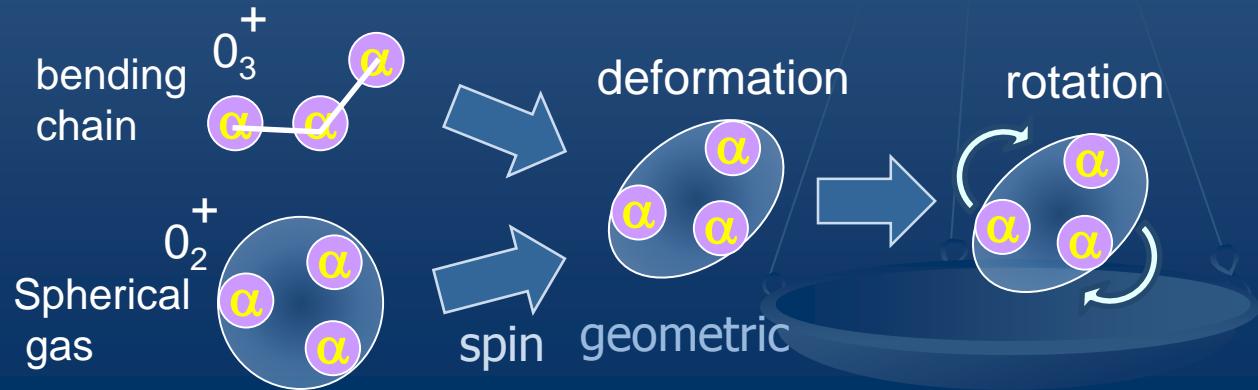
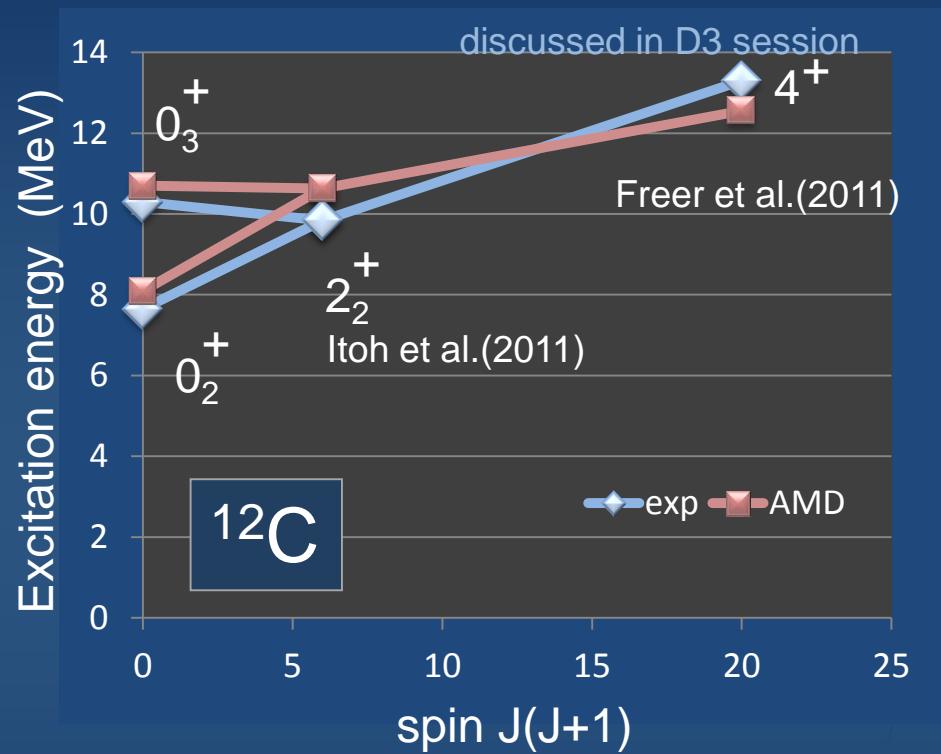


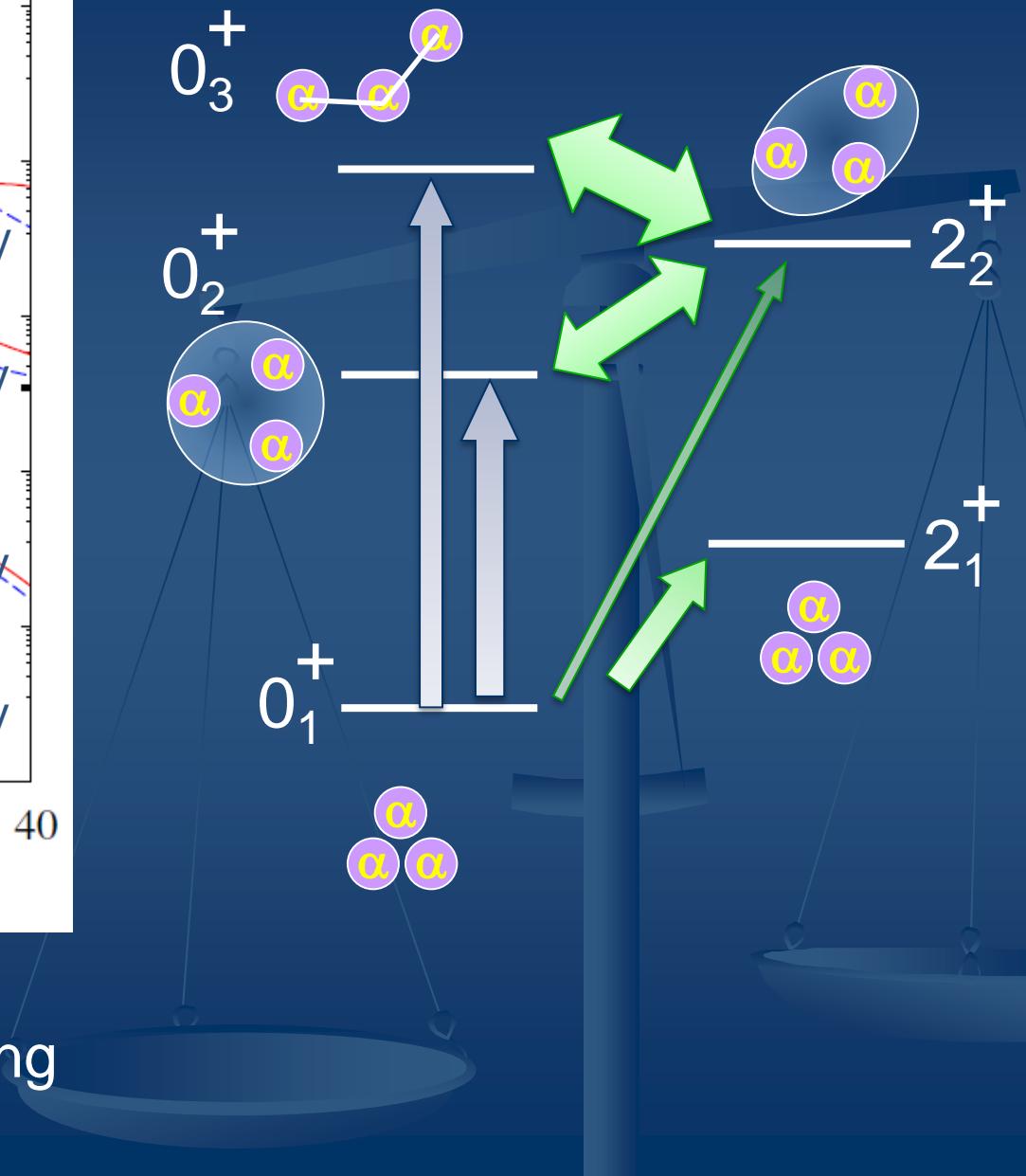
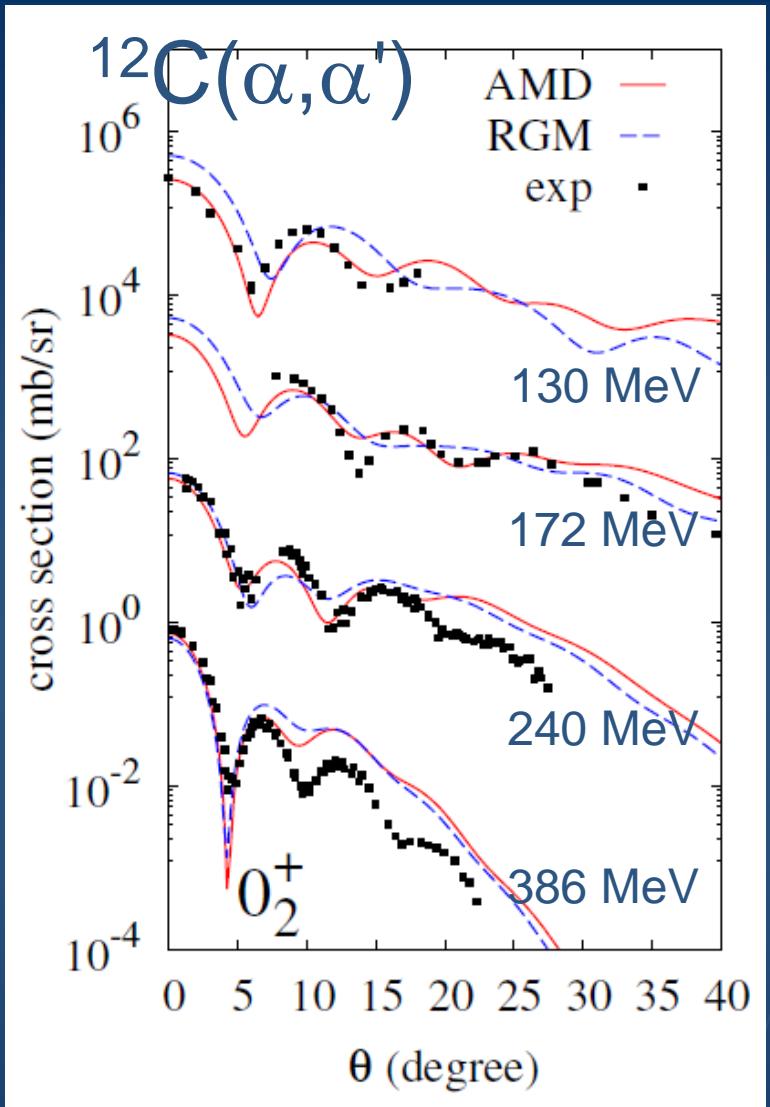
Cross sections are successfully described:  
Alpha inelastic scattering can be a good probe  
for searching new cluster states

# What we can learn from $\alpha, \alpha'$ scattering?

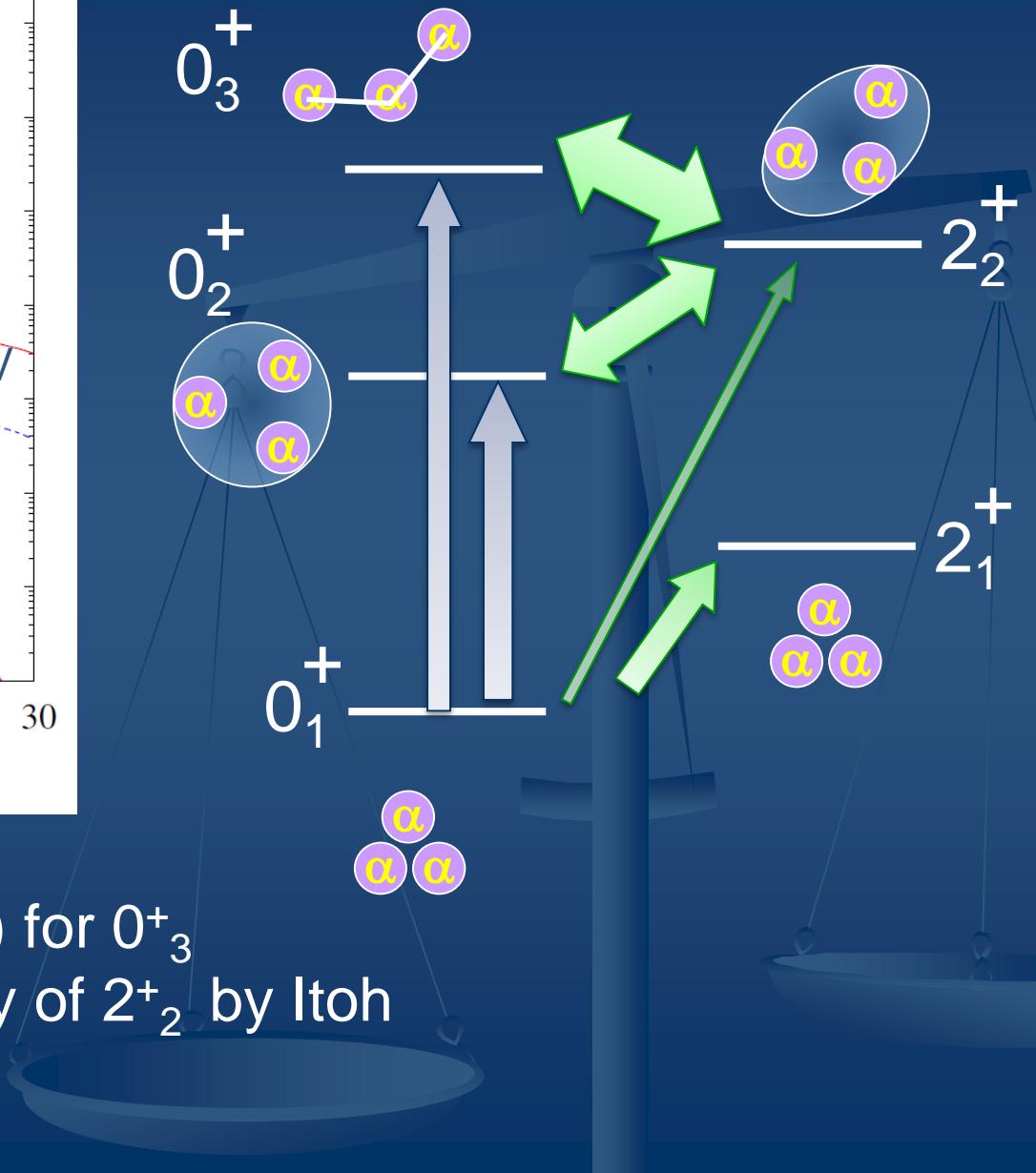
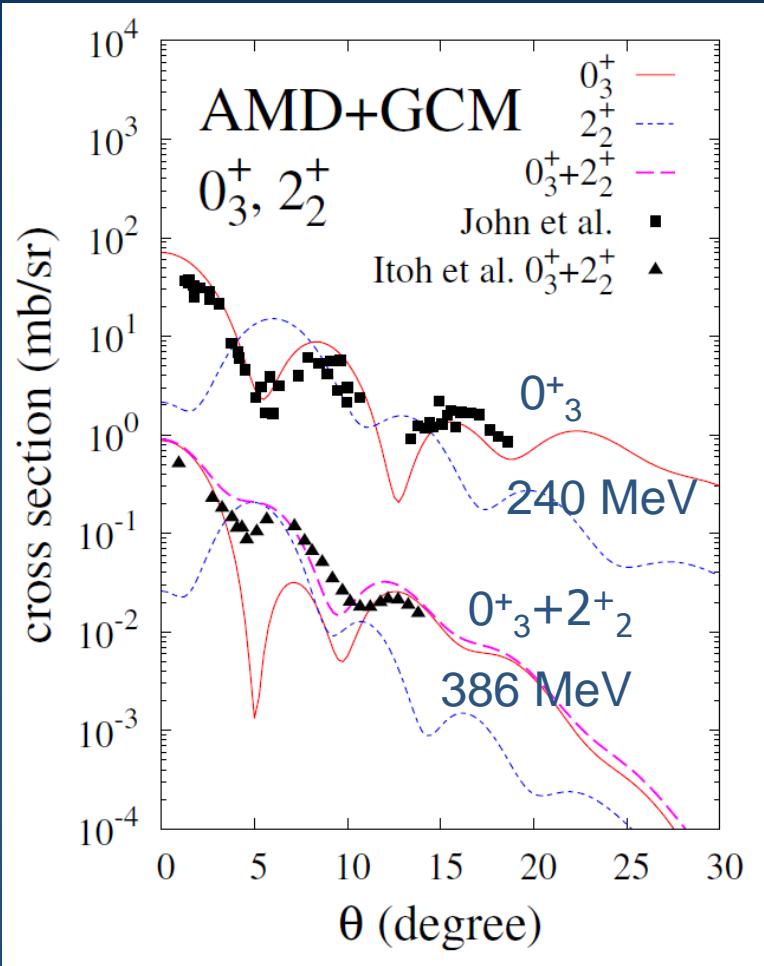
- Search for new cluster states
- Determination of  $B(IS0)$  and  $B(IS1)$
- Information of transitions between excited states

# Rotational band from cluster gas





AMD is better  
 RGM: too strong coupling  
 with  $2_2^+$



- Determination of  $B(E0)$  for  $0_3^+$
- Experimental discovery of  $2_2^+$  by Itoh

# Topic 2-2

Cluster states in  $^{16}\text{O}$   
with sAMD+GCM

Ikeda diagram  
 $4\alpha$  gas?



$^{12}\text{C} + \alpha$ ?



7.2

$^{16}\text{O}_{\text{gs}}$



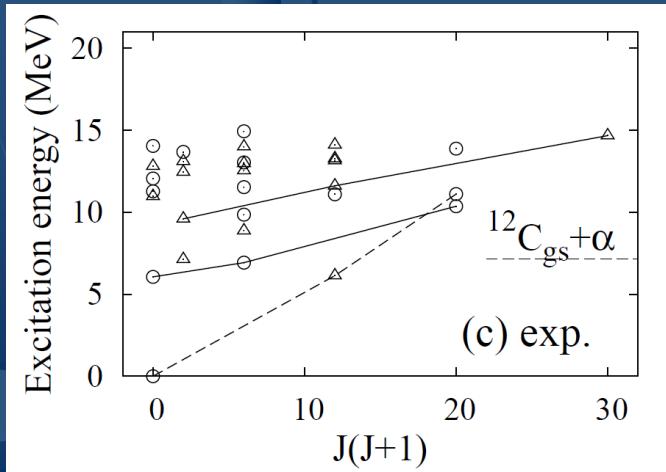
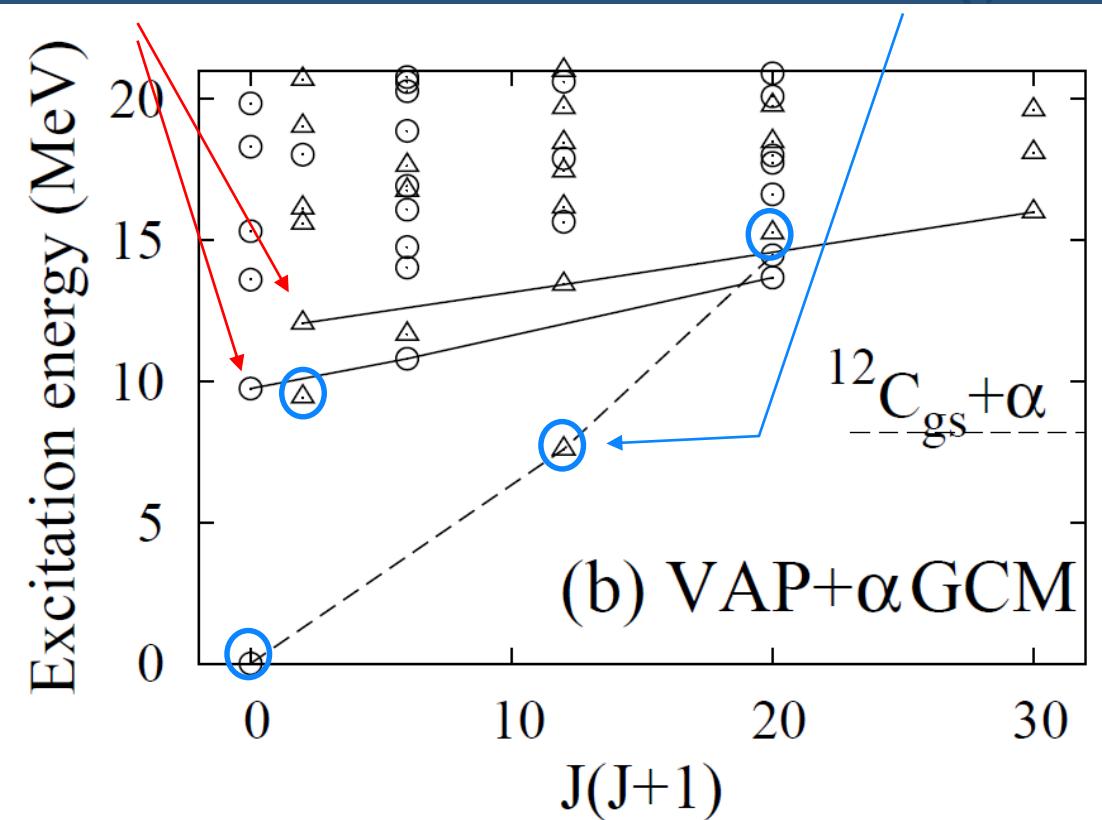
# Band structures of $^{16}\text{O}$

large amplitude cluster states

$^{12}\text{C}(0+)_1 + \alpha$  bands  
 $K=0+$  ( $0^+_1, 2^+_1, 4^+_1$ ),  
 $K=0-$  ( $1^-_2, 3^-_2, 5^-_1$ )

small amplitude modes

Ground band( $0^+_1, 3^-_1, 4^+_2$ ),  
vib(TD)  $1^-_1$

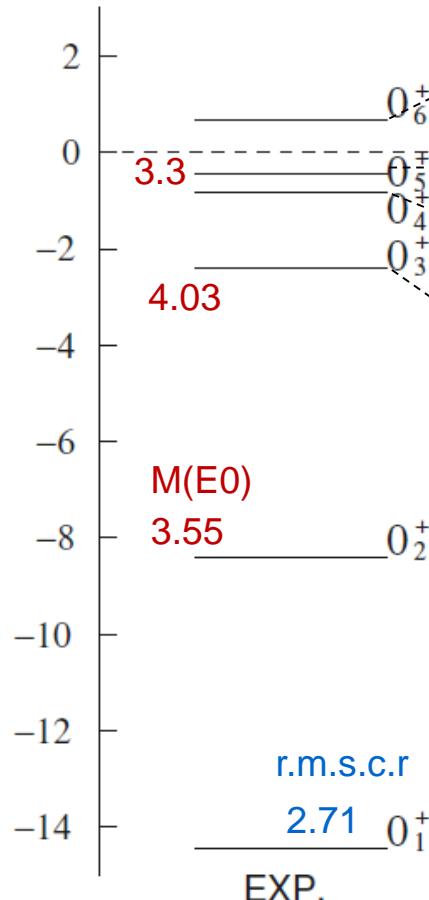


# Monopole excitations in $^{16}\text{O}$

$4\alpha$ -OCM

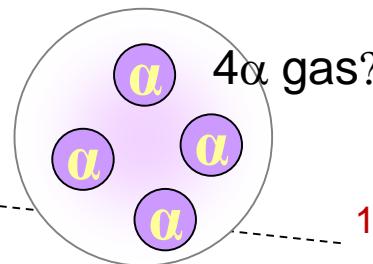
Funaki et al. PRL 101, 082502 (2008)

[MeV]



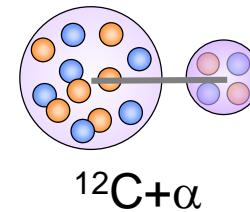
(a)

$^{12}\text{C(AMD)} + \alpha$ GCM



4 $\alpha$  gas:  
 $^{12}\text{C}^*(0+_2) + \alpha$

$^{12}\text{C}^*(0+_2) + \alpha$



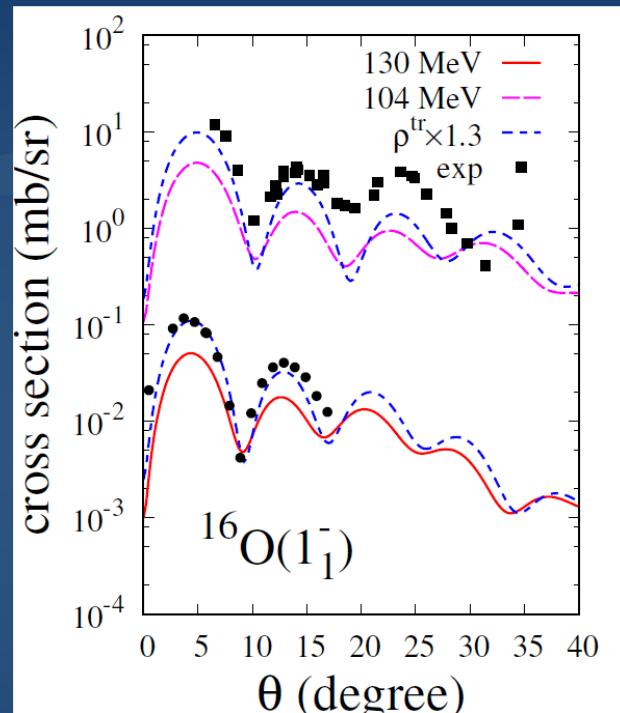
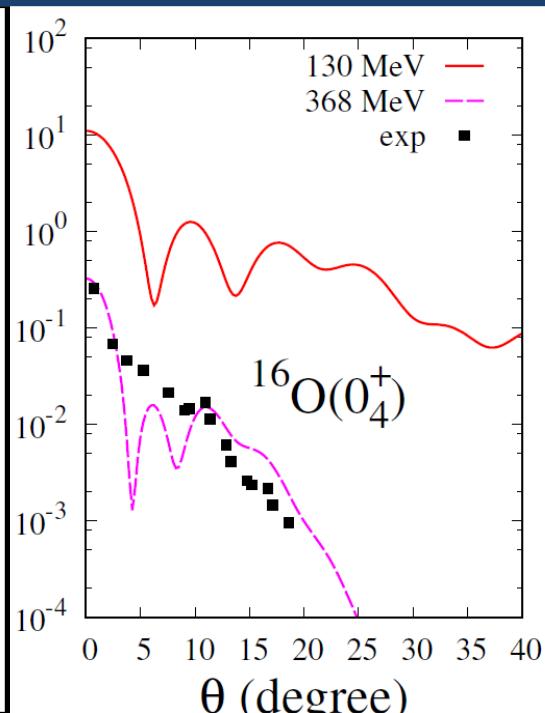
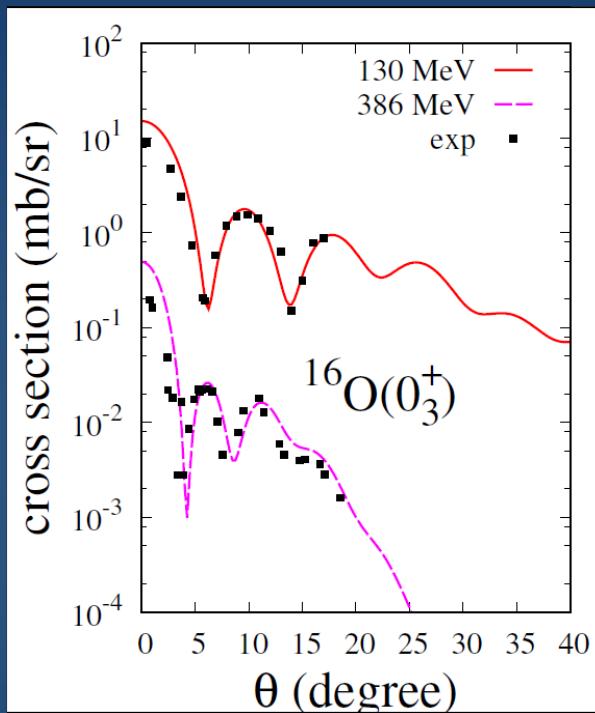
$^{12}\text{C} + \alpha$

$^{12}\text{C}(0+_1) + \alpha$

$^{12}\text{C}(0+_1) + \alpha$

2.9       $0_1^+$

# $\alpha$ scattering off $^{16}\text{O}$ (first MCC calc.)

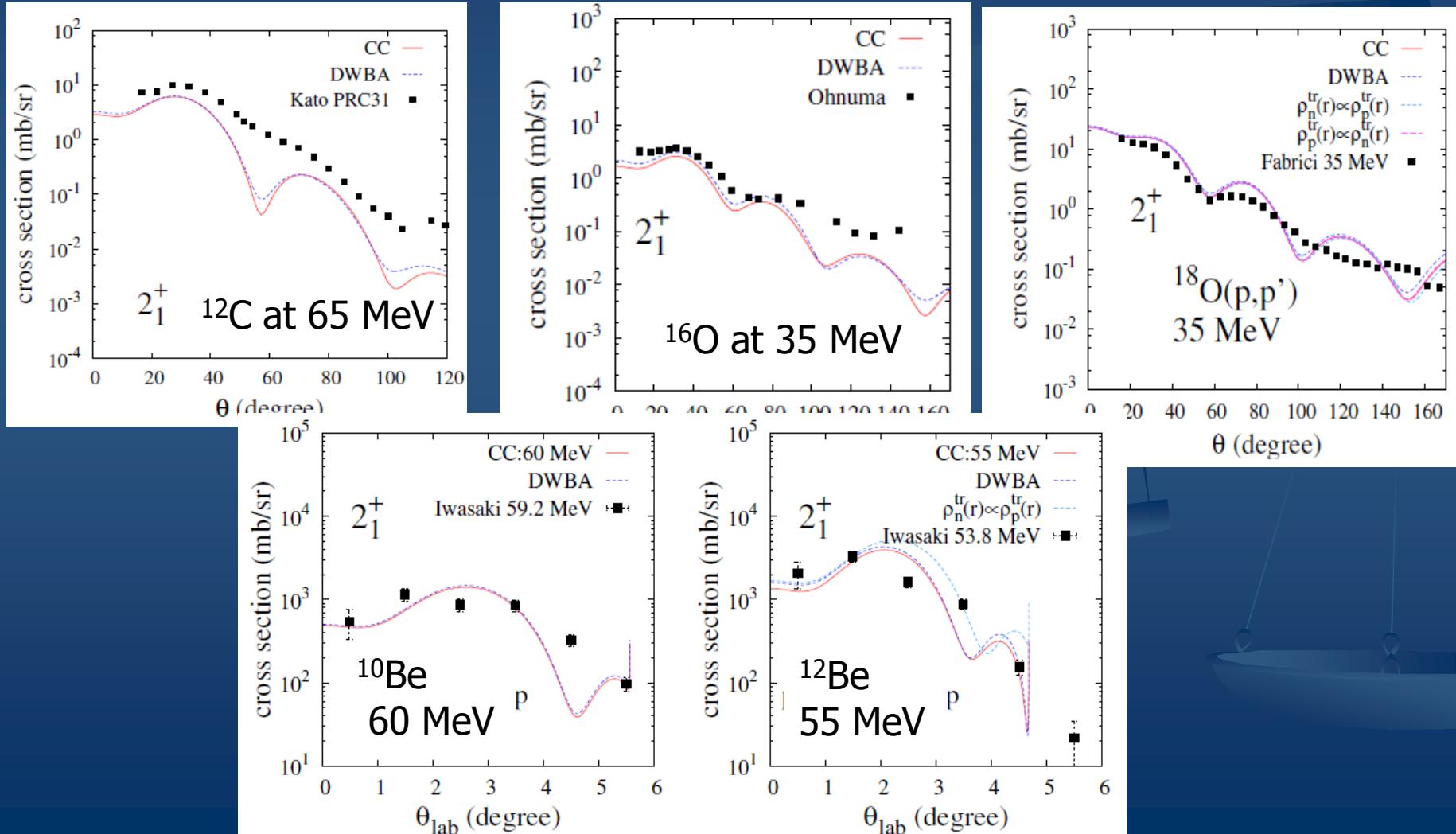


0+ states: large amp. cluster modes  
successfully reproduces 0+ cross sections  
1- state: small amp. mode  
Larger  $B(\text{IS}1)$  by a factor of 1.3 is favored

# Application to unstable nuclei (p,p') scattering for $2^+_1$ transitions

to measure quadrupole deformation

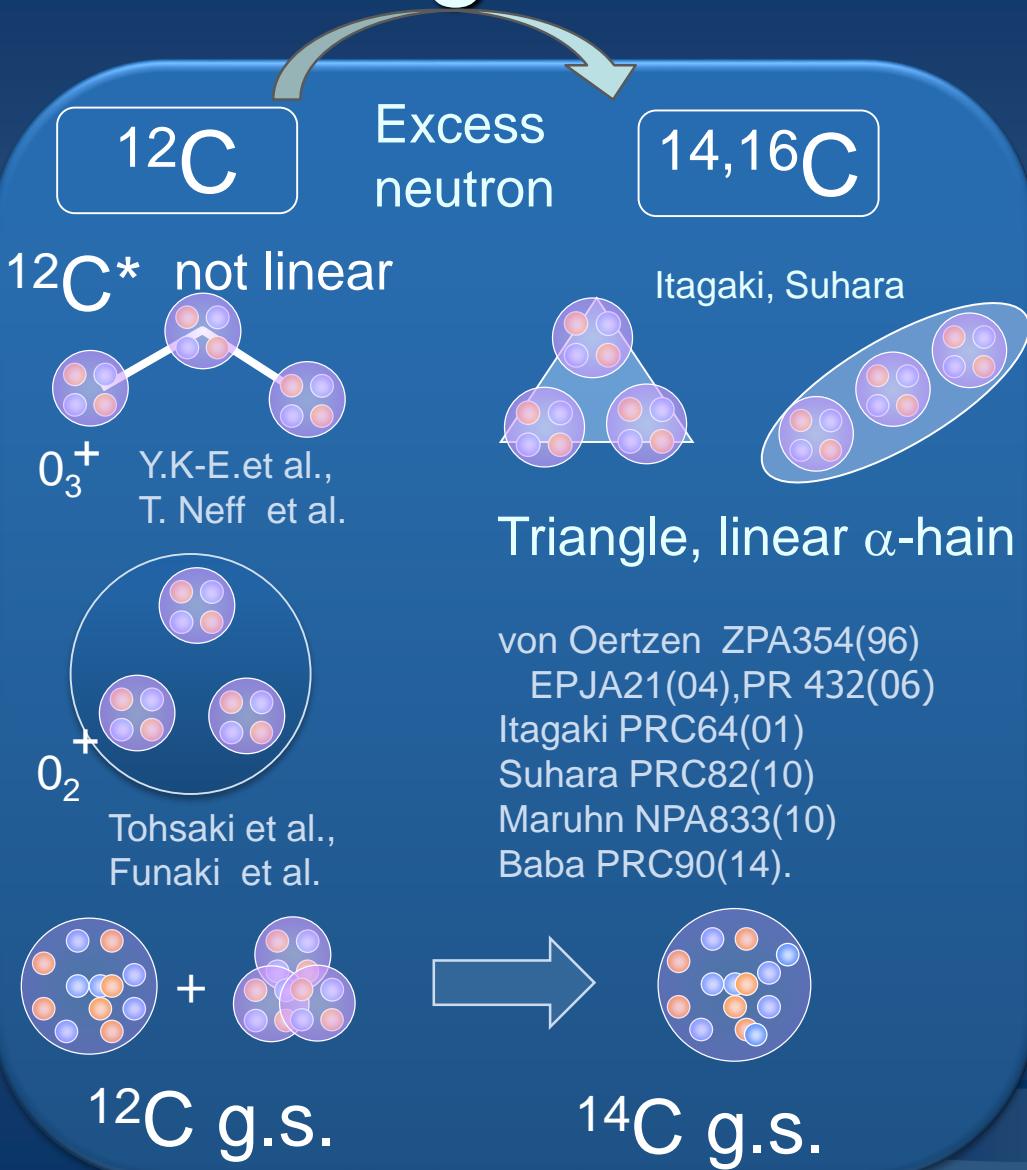
Proton/neutron differences (Mn/Mp ratio) can be discussed



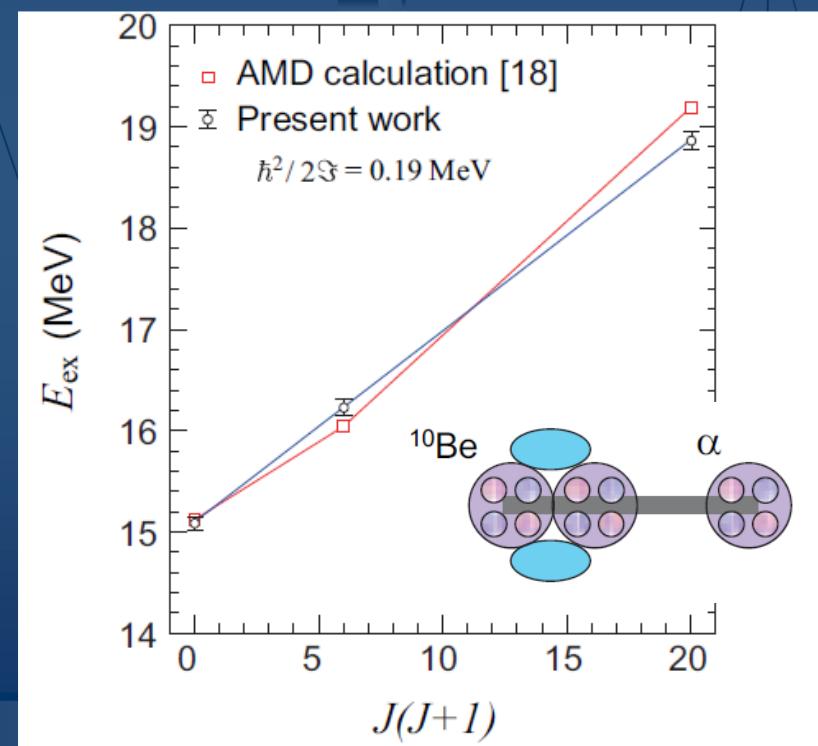
# Topic 3

Cluster states in  $^{14}\text{C}$   
with AMD+(3 $\alpha$ +nn)GCM

# Triangle / Linear states in $^{14}\text{C}^*$



**$^{14}\text{C}$  3 $\alpha$  linear chain**  
 theor. Suhara.Y.K-E, PRC82 (2010)  
 exp. Yamaguchi et al, PLB766(2017)



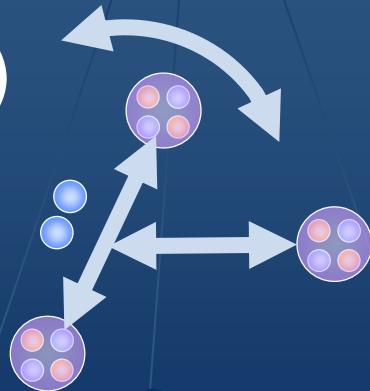
# $^{14}\text{C}:\text{AMD} + (\text{3}\alpha + \text{nn})\text{GCM}$

AMD  $^{14}\text{C}(0^+_{1,2}, 2^+_{1,2})$



combined with cluster config.

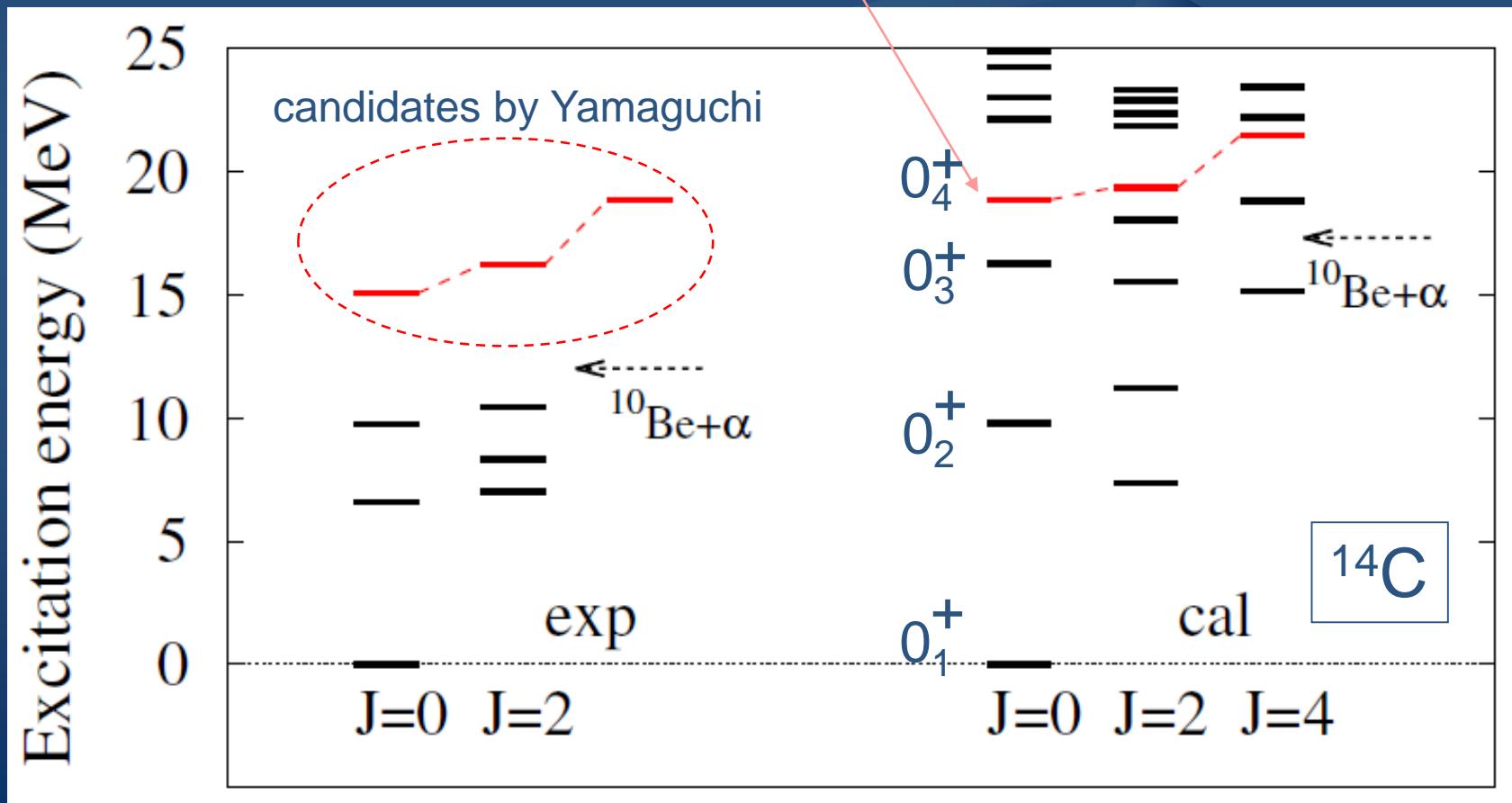
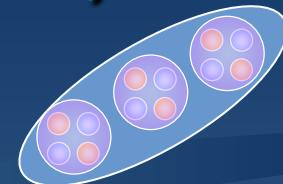
GCM( $\text{3}\alpha + \text{nn}$ )



3-alpha dynamics with nn

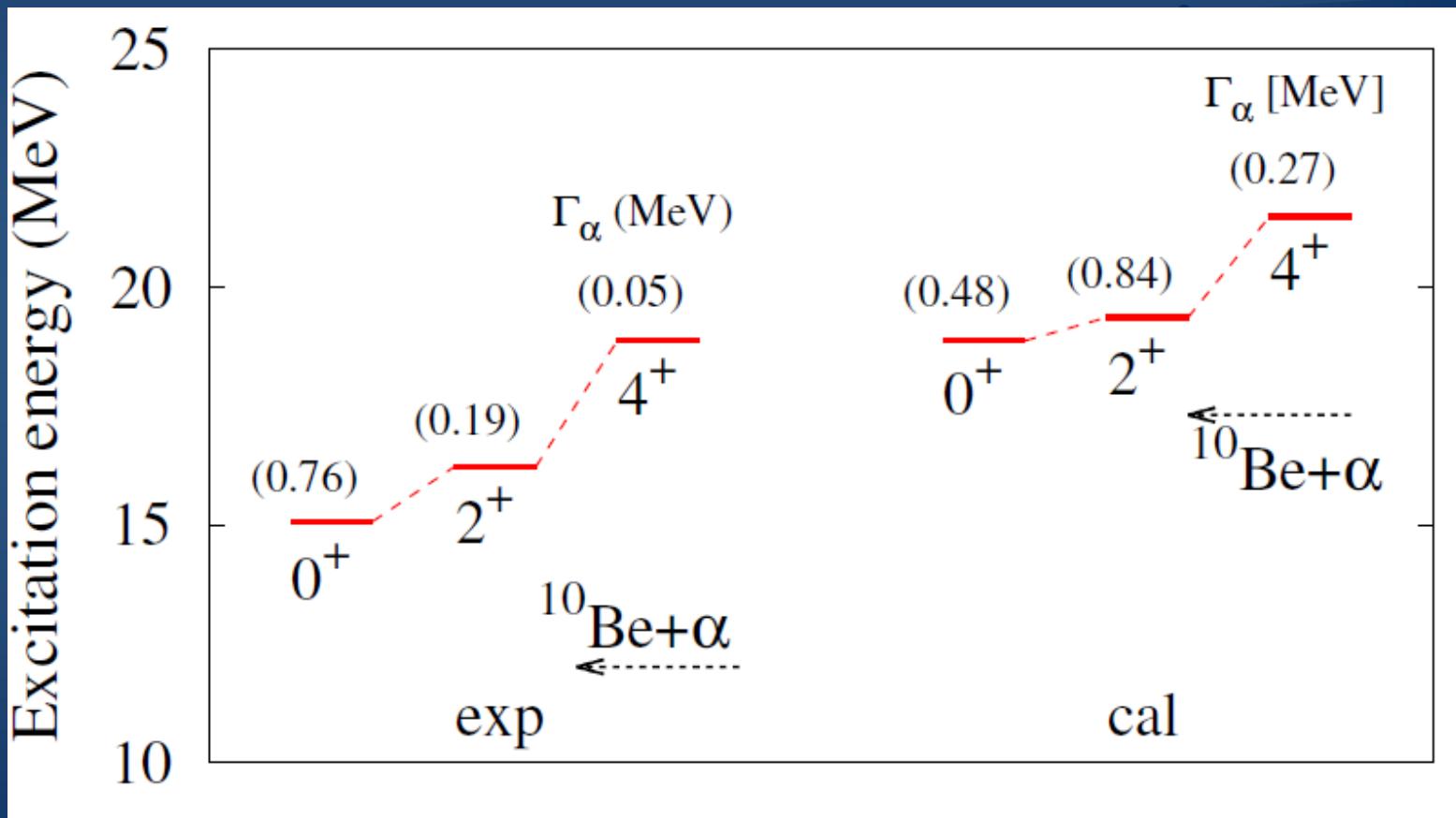
# $^{14}\text{C}$ results : AMD+(3 $\alpha$ +nn)GCM

3 $\alpha$  linear chain



# $^{14}\text{C}$ : linear chain states

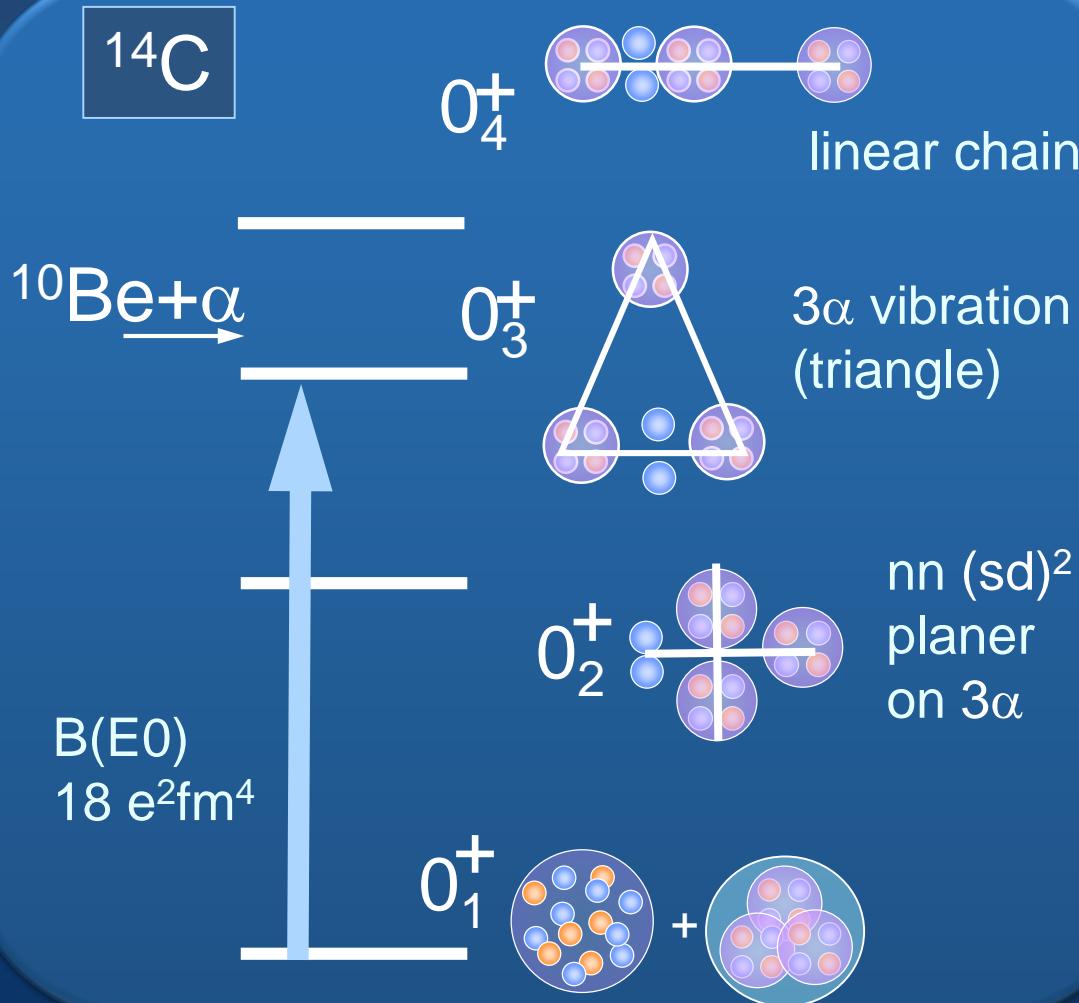
## comparison with obsevation



exp: Yamaguchi et al  
PLB766(2017)

# $0^+$ : monopole excitations in $^{14}\text{C}$

$3\alpha$  dynamics with nn: some differences

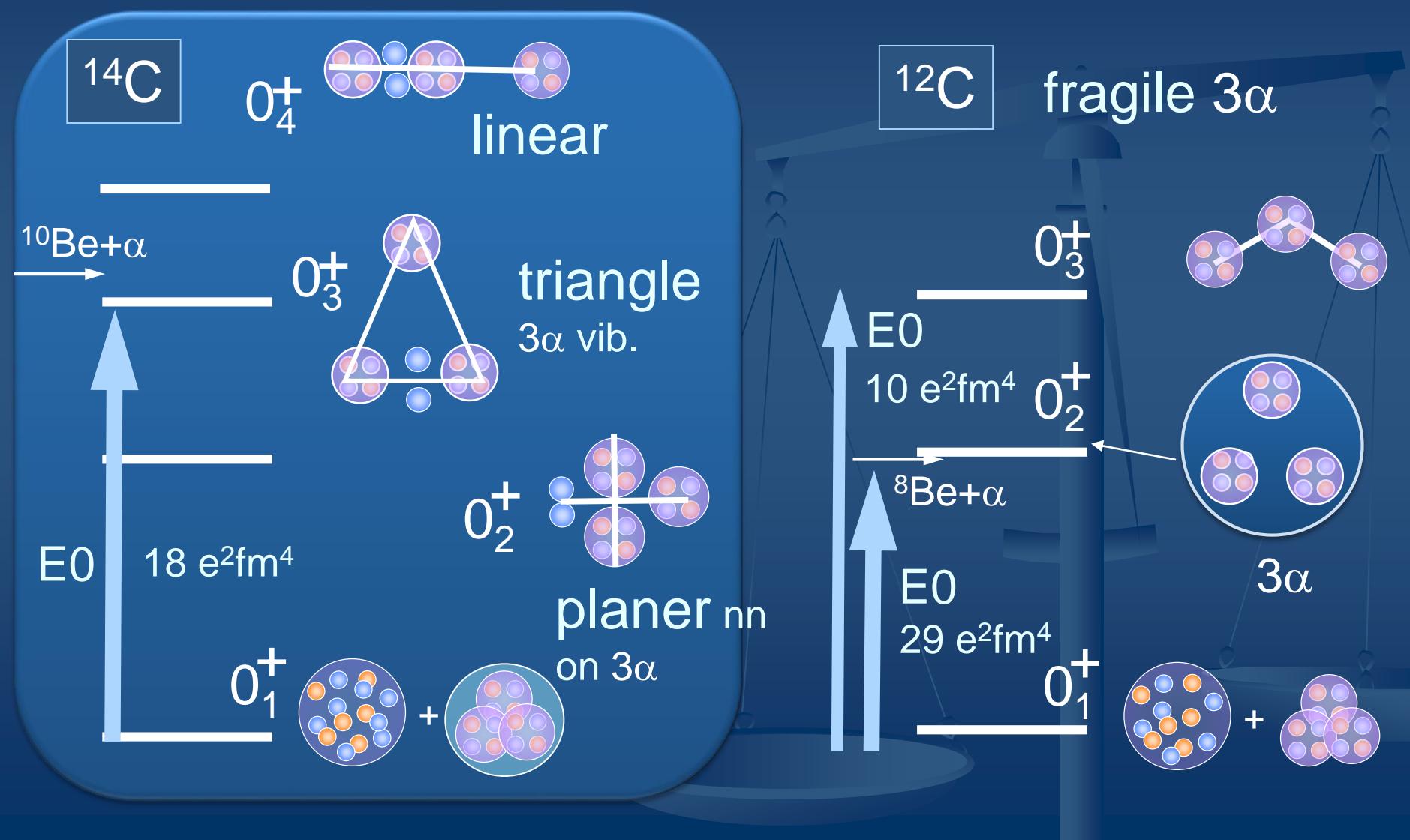


- $2\alpha$  bonded by nn
- new configurations because of nn



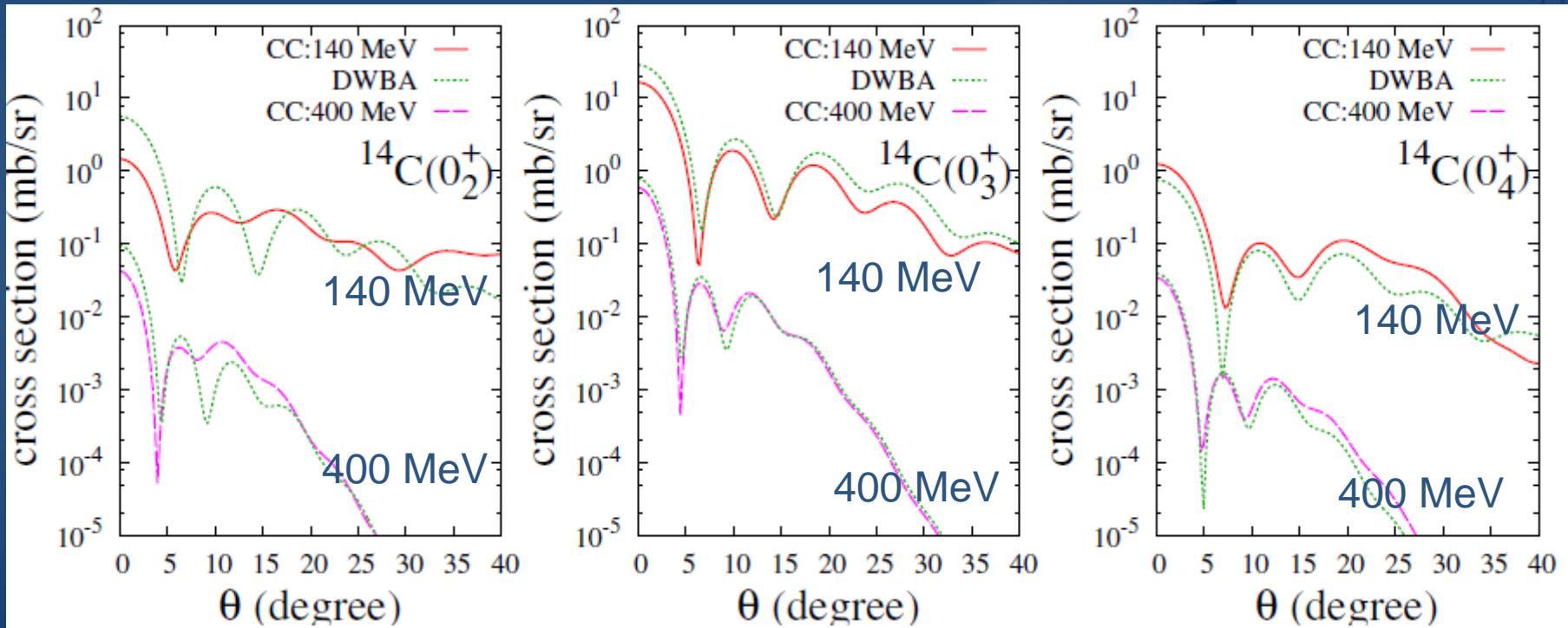
# monopole excitations in $^{14}\text{C}$ , $^{12}\text{C}$

3 $\alpha$  dynamics with and w/o nn: differences



# $^{14}\text{C}(\alpha, \alpha')$

- E <sub>$\alpha$</sub> =140 MeV and 400 MeV



$^{14}\text{C}(0_3^+)$  can be observed by  $(\alpha, \alpha')$

# Summary

- (s)AMD+GCM was applied to monopole&dipole excitations in  $^{12}\text{C}$ ,  $^{16}\text{O}$ , and  $^{10}\text{Be}$
- Various LE modes: cluster, vib, vortical modes
- alpha inelastic scattering
- cluster states in  $^{14}\text{C}$ : Triangle vib., linear-chain 3a

Various modes arises in monopole & dipole excitations

Physics behind:

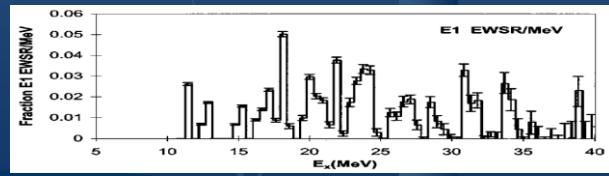
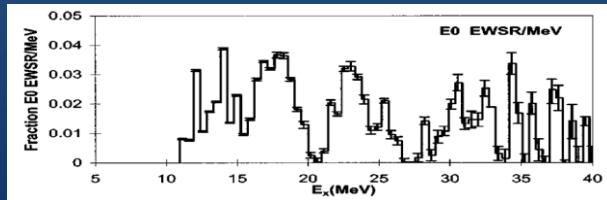
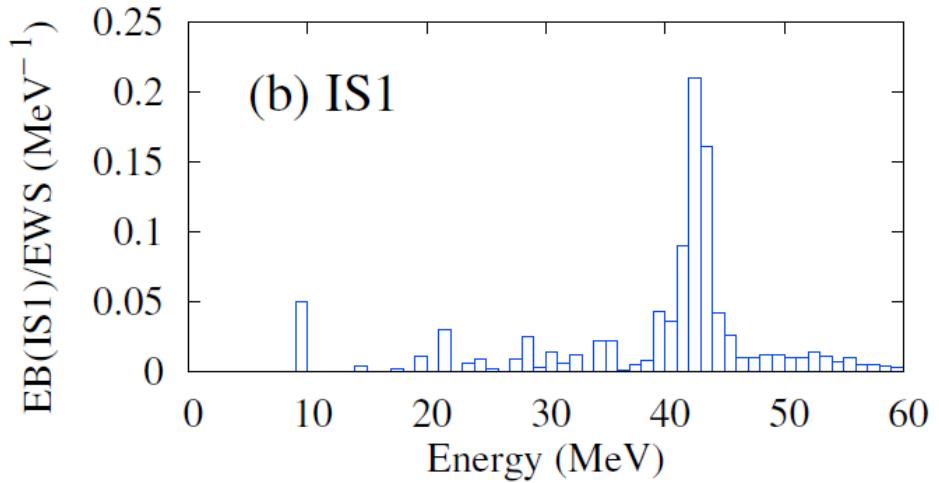
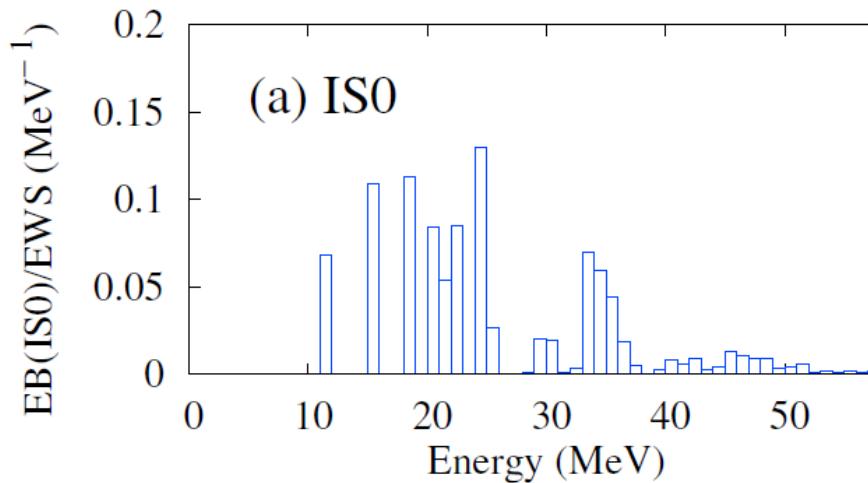
small amplitude modes & large amplitude cluster dynamics in nuclear phenomena

# References

- (p,p') arXiv:1908.03
- (a,a') off 16O Phys.Rev. C99 (2019) no.6, 064608
- (a,a') off 12C Phys.Rev. C99 (2019) no.6, 064601
- monpole&dipole in 16O  
Phys.Rev. C89 (2014) 024302, Phys.Rev. C100 (2019)  
014301
- monpole&dipole in 12C  
Phys.Rev. C97 (2018) no.1, 014303, Phys.Rev. C93  
(2016) no.5, 054307

# EB(IS0,IS1) in $^{16}\text{O}$

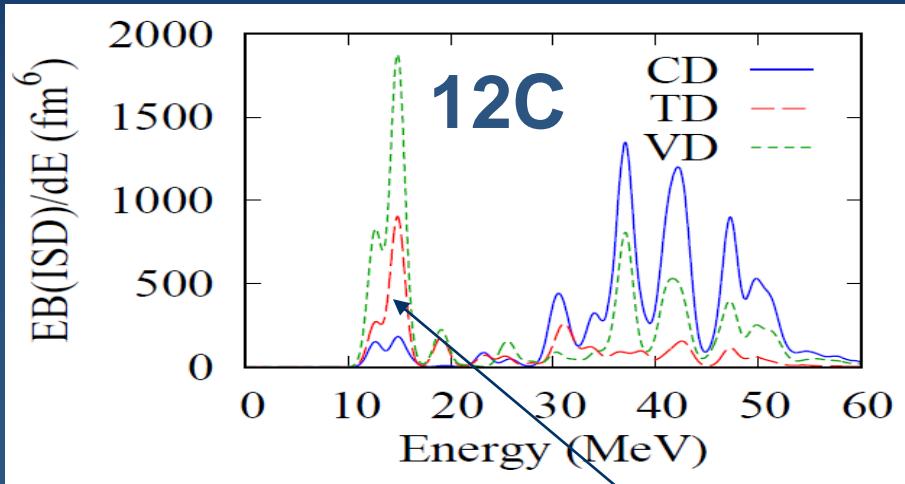
## ■ sAMD+GCM



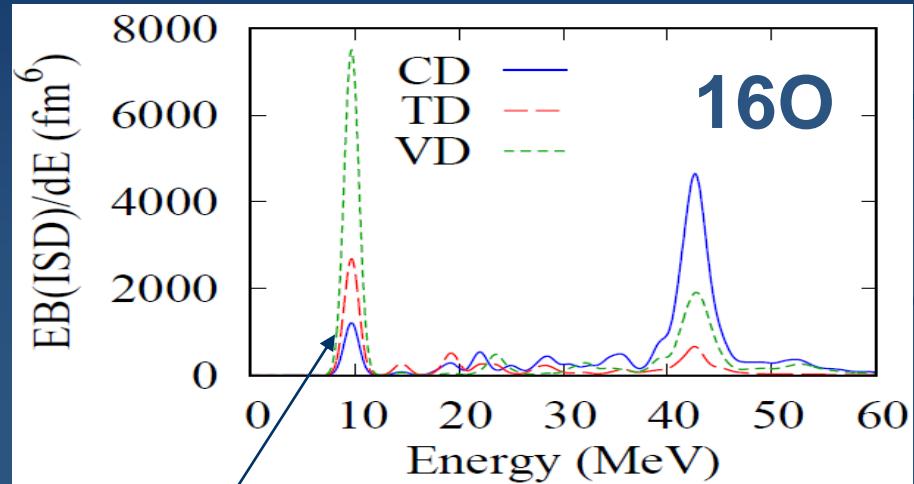
(a,a') exp: Lui et al. PRC64(2001)

# IS-LED in $^{12}\text{C}$ , $^{16}\text{O}$

3% of EWSR



5% of EWSR

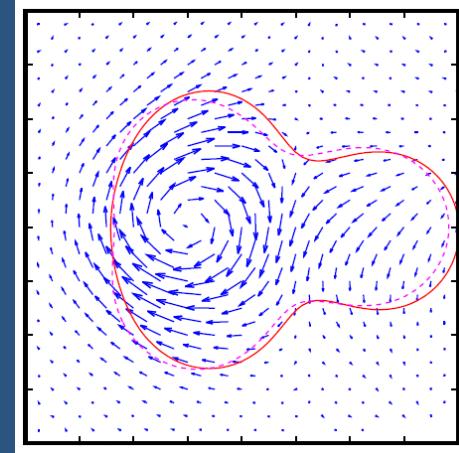
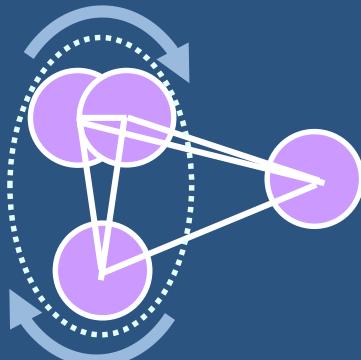


TD dominant LED !

$^{12}\text{C}(1^-_1)$   
 $^{16}\text{O}(1^-_2)$   
Developed  
Clustering:  
 $3\alpha$ ,  $^{12}\text{C}+\alpha$



$^{12}\text{C}(1^-_2)$   
 $^{16}\text{O}(1^-_1)$   
1p1h



# Effective nuclear interactions

$$H^{\text{eff}} = \sum_{i=1} t_i + \sum_{i < j} v_{ij}^{\text{eff}} + \sum_{i < j < k} v_{ijk}^{\text{eff}} \quad i, j, k = 1 \cdots A$$

Central force: MV1 force  
two-range Gaussian 2-body + zero-range 3-body forces

Is force: term of G3RS force  
two-range Gaussian 2-body ( ${}^3\text{O}$ )  
Coulomb force: 7-range Gaussians

Matter properties of MV1 force (case-1 with  $m=0.62$ ,  $b=h=0$ )  
 $\rho_0=0.192 \text{ fm}^{-2}$ ,  $E_0/A=17.9 \text{ MeV}$ ,  $K=245 \text{ MeV}$ ,  $m^*=0.59m$

	$\alpha$	${}^{12}\text{C}$	${}^{16}\text{O}$	${}^2\text{C}+\alpha$ thres.
Cal. (MeV)	27.8	87.6	123.5	8.2
Exp. (MeV)	28.3	92.2	127.6	7.16

# Effective interactions

- Central force : MV1 parameterization
  - two-range Gaussian 2-body+zero-range 3-body
  - similar to Gogny central force in a sense
- LS force : two-range Gaussian 2-body from G3RS
- Coulomb force is also added.
- Matter properties:  
 $\rho_0=0.192 \text{ fm}^{-2}$ ,  $E_0/A=17.9 \text{ MeV}$ ,  $K=245 \text{ MeV}$ ,  $m^*=0.59m$
- B.E. of nuclei:

	$\alpha$	$^{12}\text{C}$	$^{16}\text{O}$	$^2\text{C}+\alpha$ thres.
Cal. (MeV)	27.8	87.6	123.5	8.2
Exp. (MeV)	28.3	92.2	127.6	7.16