RMT in Sub Atomic Physics and Beyond

Conference in Honor of

Jac Verbaarschot's 65th Birthday

Correlation Matrices in States of Financial Markets

Where is RMT???

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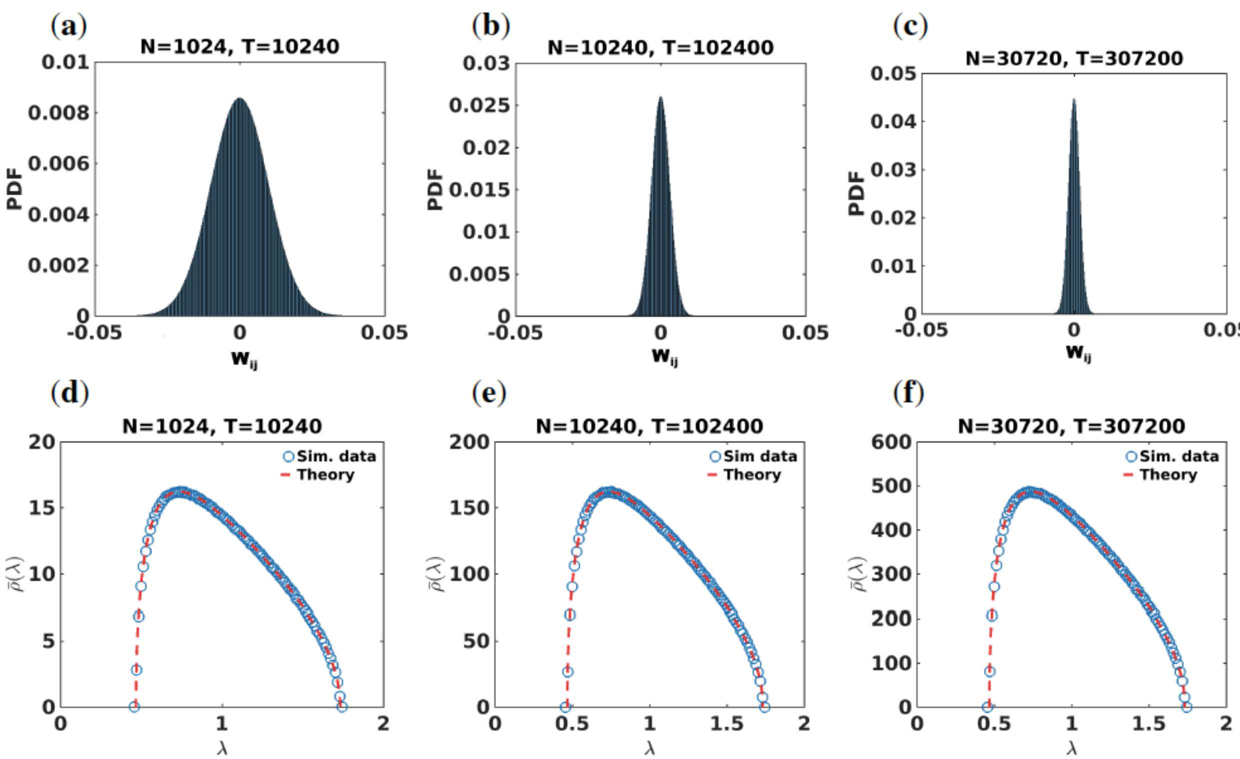
Introduction

— Time series and the scope of time series; multivariate analysis.
— Correlations versus covariance
— The random prior; white noise and Wishart ensembles
— Stationary processes are well worked but unrealistic
— Can we learn something from N time series of length T with k=N/T large; many time series
— Zero subspace and power map
— An ensemble of data with k~1 (typically slightly smaller)
— 2-D Ising model; Critical behaviour
— Zero subspace and power map
— SP 500 (New York) and Ni 225 (Tokio)
— Why financial time series; why precisely the stock market?
— Clustering and market states;
— Zero subspace and power map
— RMT clusters
— spectral properties

See Black board for reference

White Noise: Marchenko Pastur density; Q = T/N small

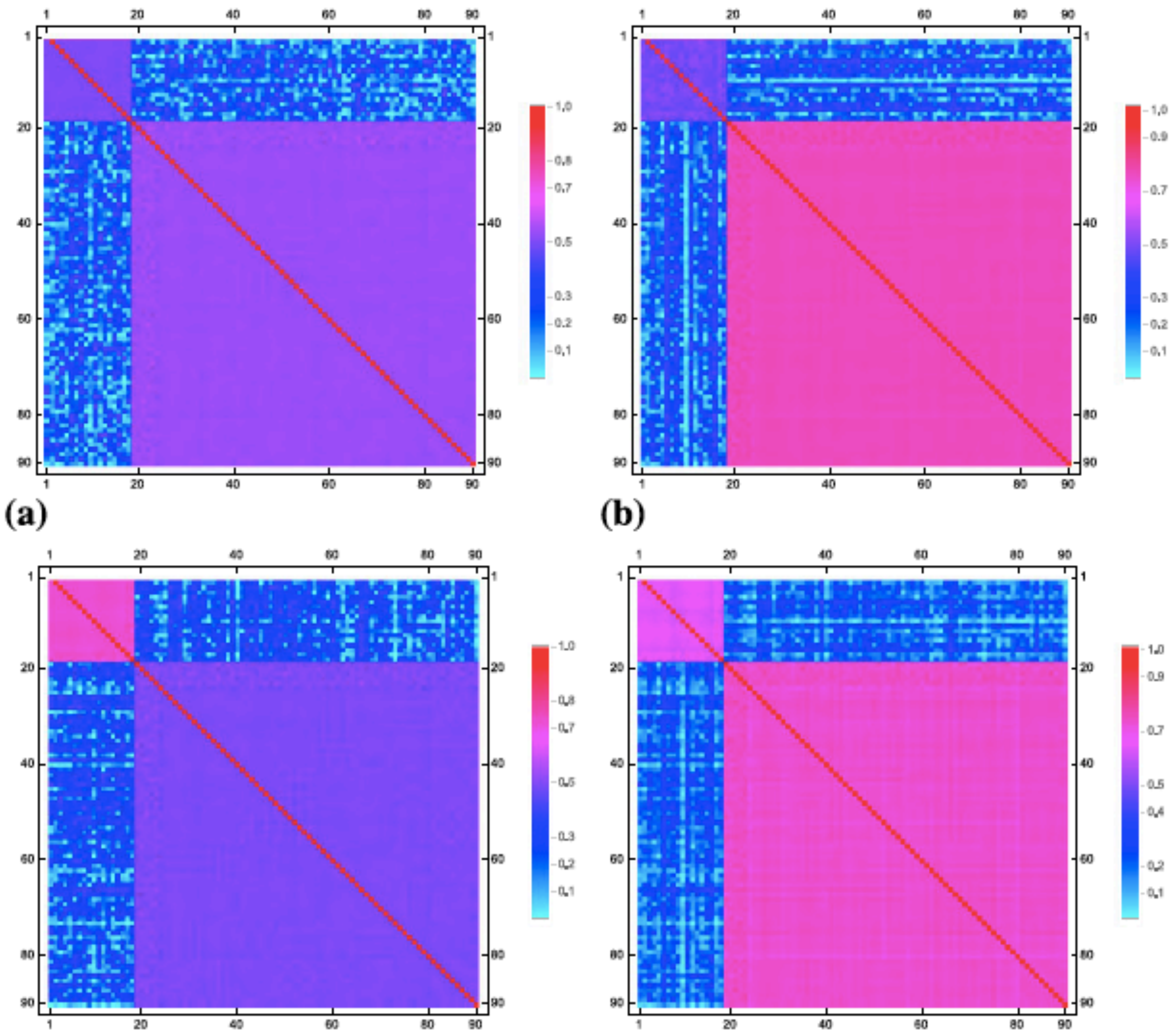
$$\bar{\rho}(\lambda) = \frac{Q}{2\pi\sigma^2} \frac{\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{min})}}{\lambda} + (1 - Q)\delta(\lambda).$$



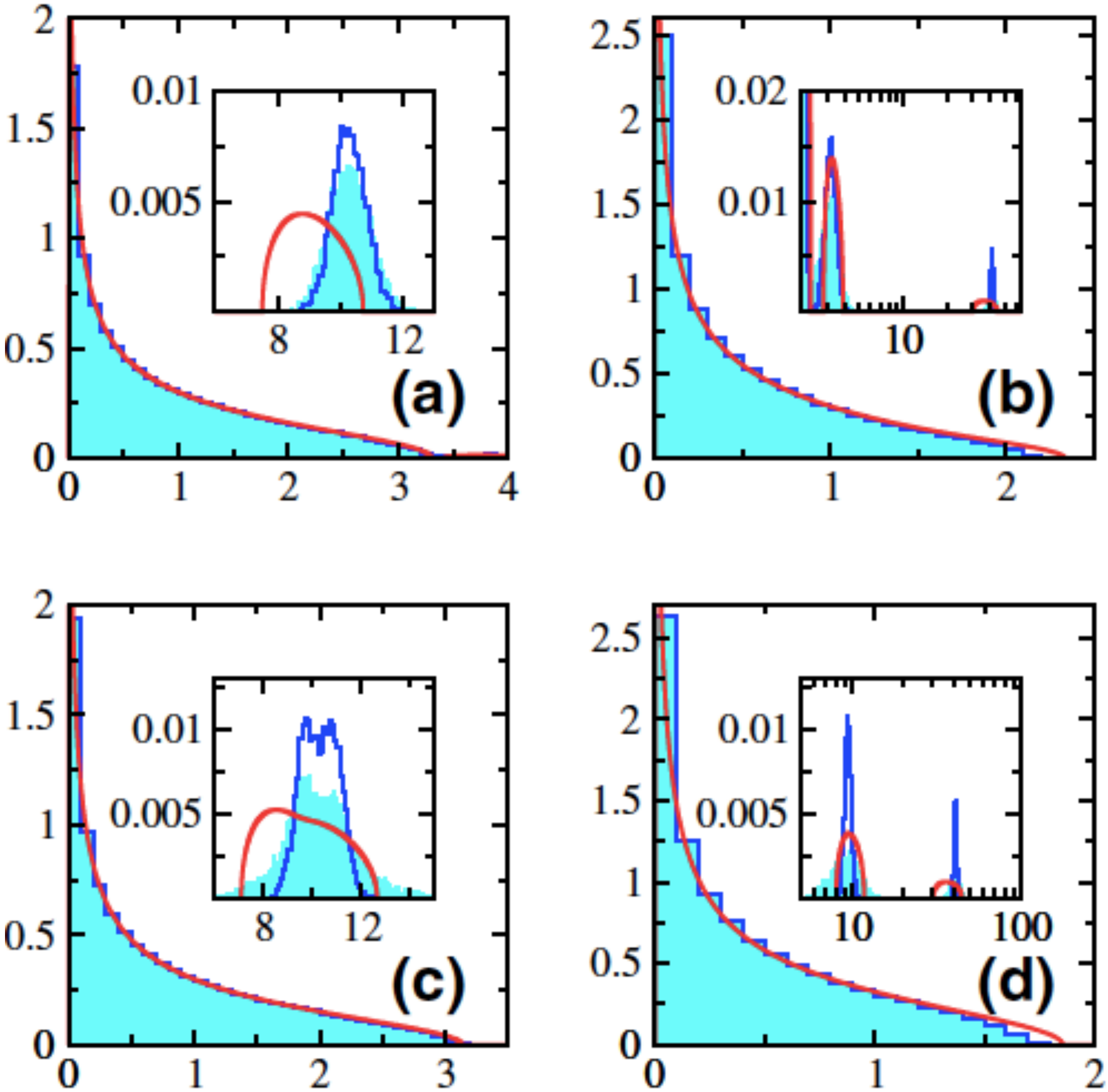
Results for Data ensembles

Block correlations ensemble average

(a) $\upsilon_1 = \upsilon_2 = 0.1$ **(b)** $\upsilon_1 = 0.1, \upsilon_2 = 0.5$; **(c)** $\upsilon_1 = 0.5$, $\upsilon_2 = 0.1$ and **(d)** $\upsilon_1 = \upsilon_2 = 0.5$.



Distribucion of eigenvalues for the Data ensemble



Time series in the Ising Model with Metropolis algorithm

$$H_{\text{Ising}} = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j$$

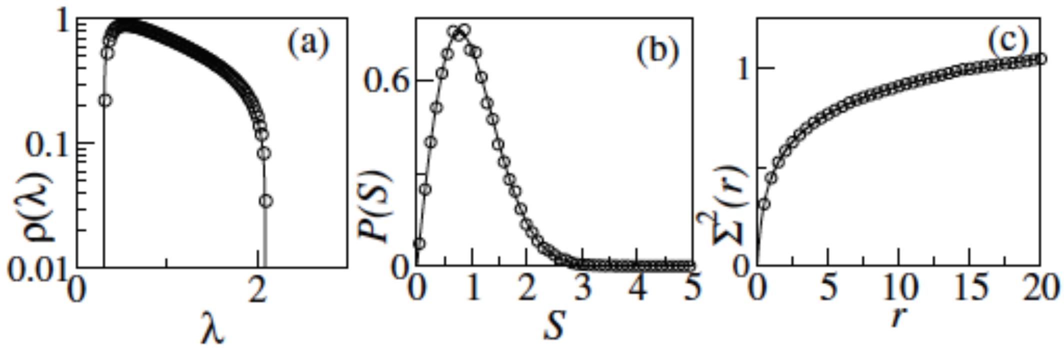


Fig. 1: Spectral statistics of C at a high temperature, $2J/T = 0.001$, for $L = 192$. In (a) we compare the spectral density with the Marćenko-Pastur formula. In (b) and (c), we compare the nearest-neighbor spacing distribution, $P(S)$, and the number variance, $\Sigma^2(r)$, with those of RMT. Numerics are shown by open circles and RMT results are shown by solid lines.

The analytic result for the power ζ of a spectral Zipfplot with

power θ in a d-dimensional space of time series leading to the correlation

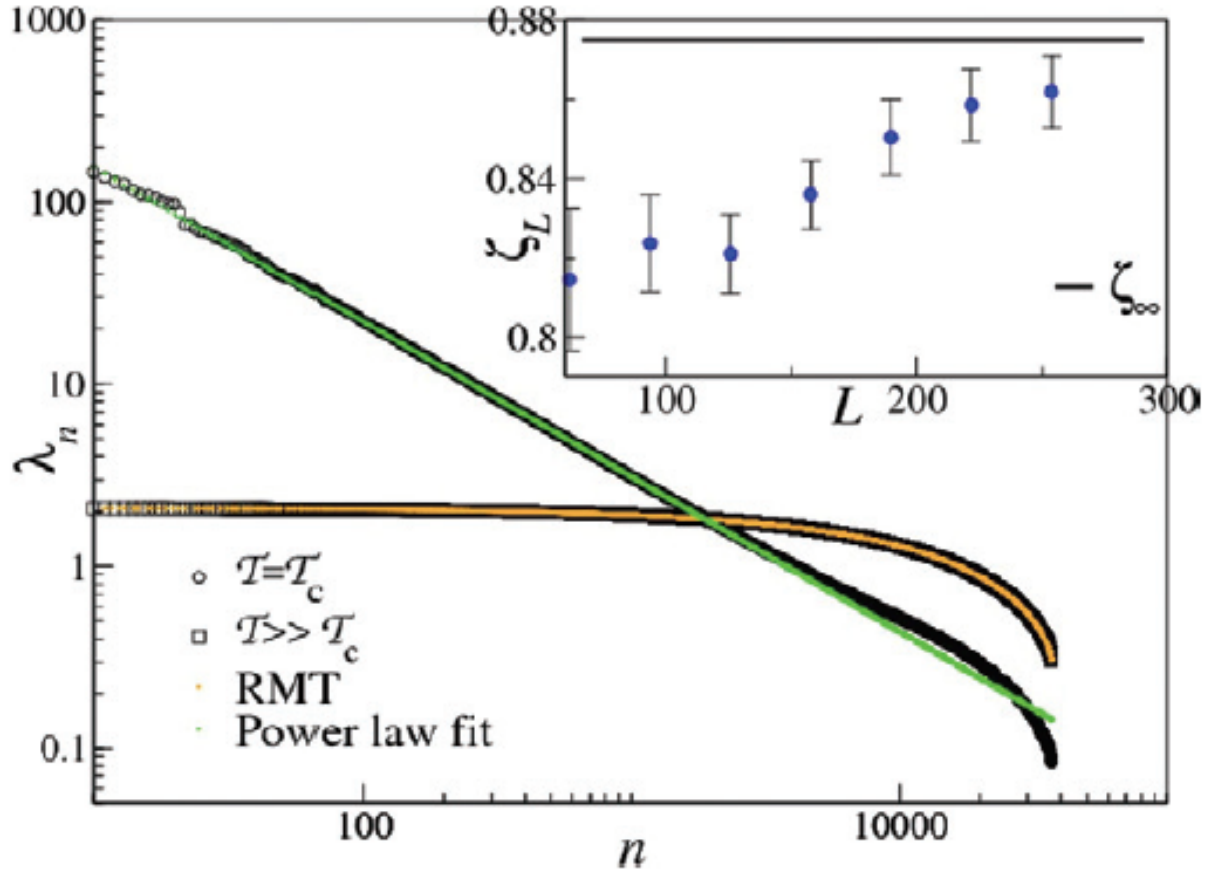
$$C_{\vec{n},\vec{m}} \approx \frac{c}{|\vec{n}-\vec{m}|^{\theta}}, \quad \text{for} \quad 1 \ll |\vec{n}-\vec{m}| \ll L.$$

Is

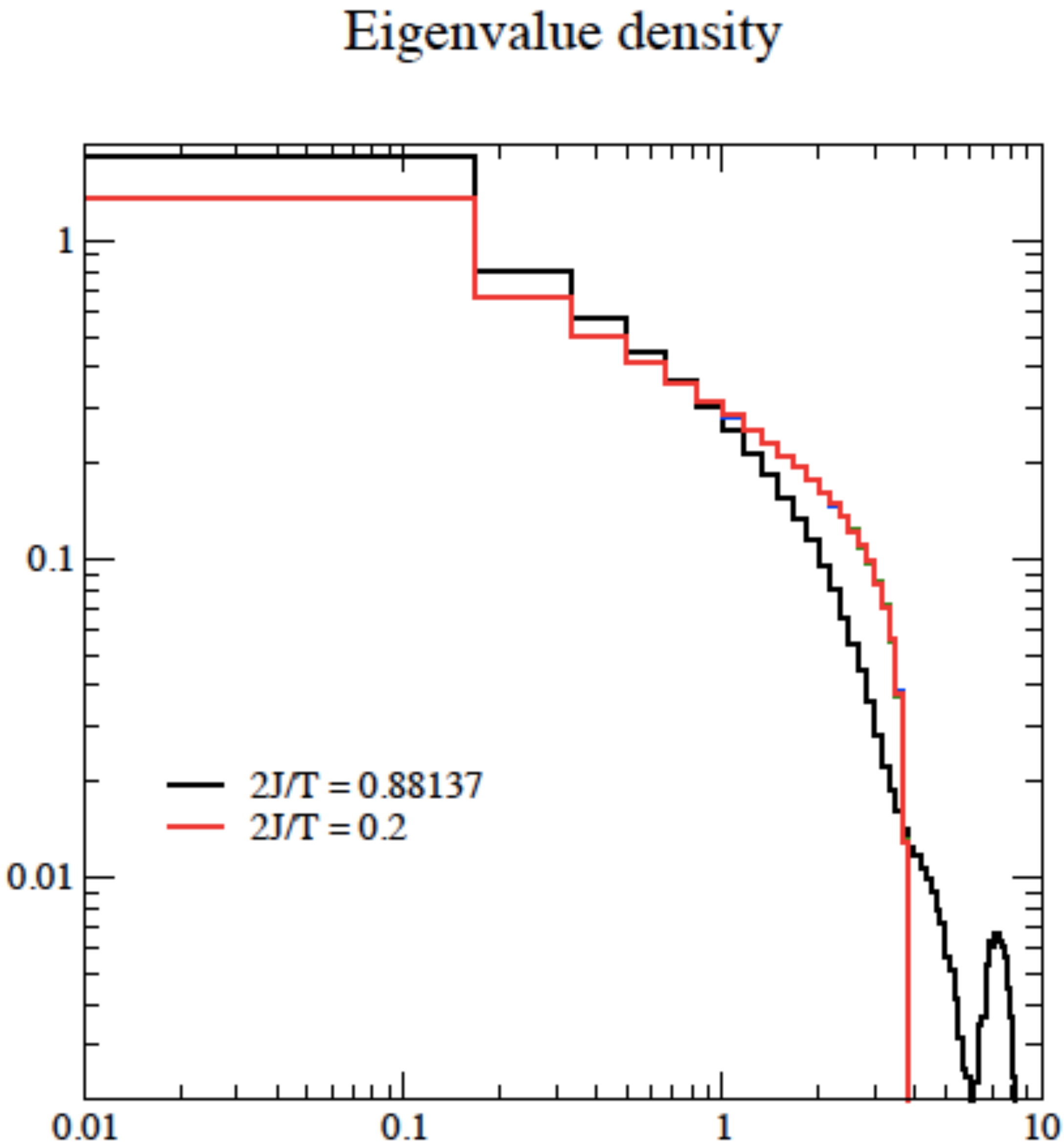
$$\zeta = \frac{d-\theta}{d};$$

From Reference: Prosen, T., B. Buča, and T. H. Seligman. "Spectral analysis of finite-time correlation matrices near equilibrium phase transitions." *EPL (Europhysics Letters)* 108.2 (2014): 20006.

Zipfplot or ranking at critical and high temperature for T>>N

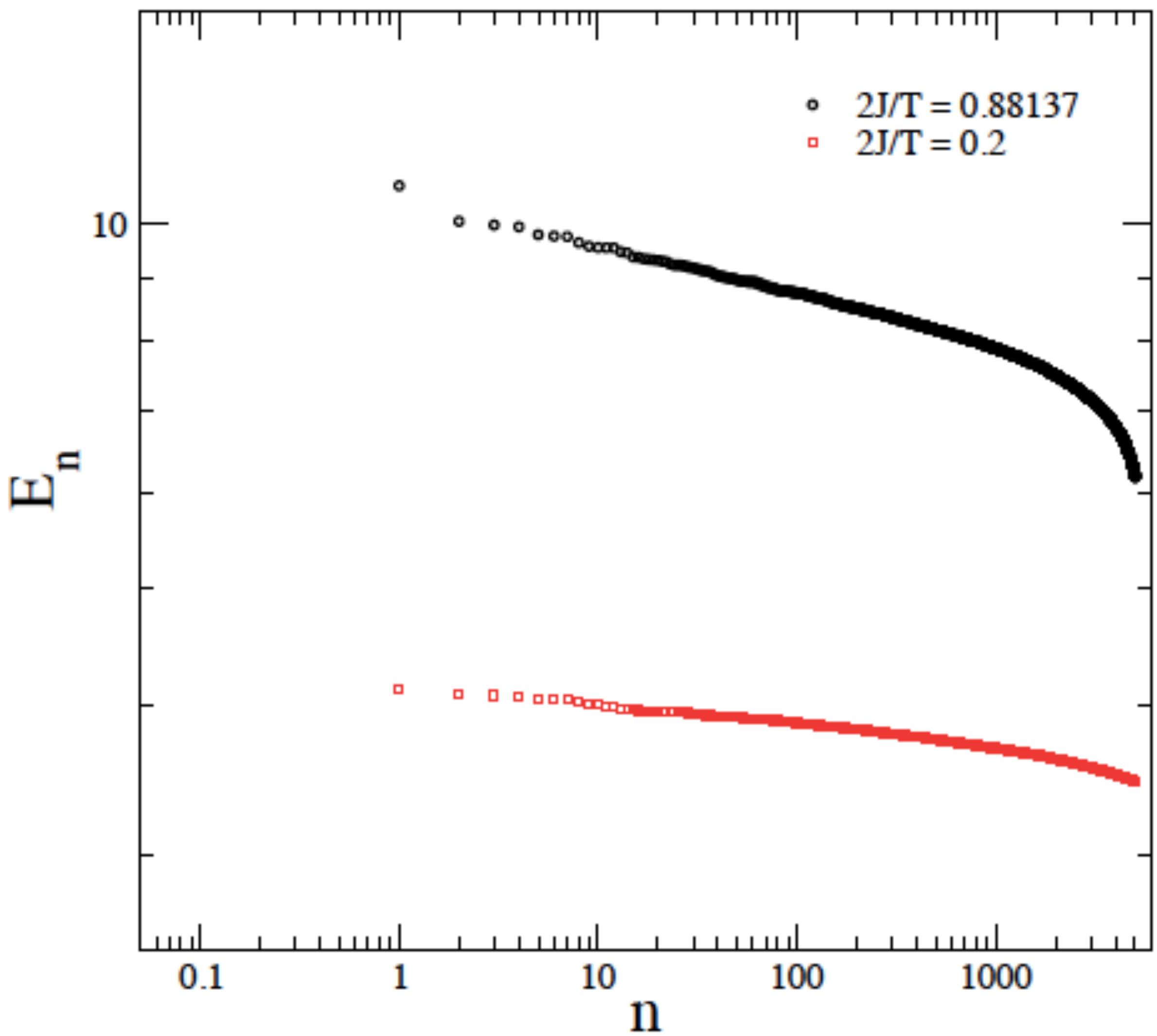


Data Matrix method



T<<N

The Zipfplot for the highest eigenvalue at the critical point



Stylized Clusters

The Power Map

is originally a noise reduction technique,
and later leads to the so called emerging spectrum.

$$C_{kl}^{(q)} = \operatorname{sign} C_{kl} \left|C_{kl}\right|^{q}$$

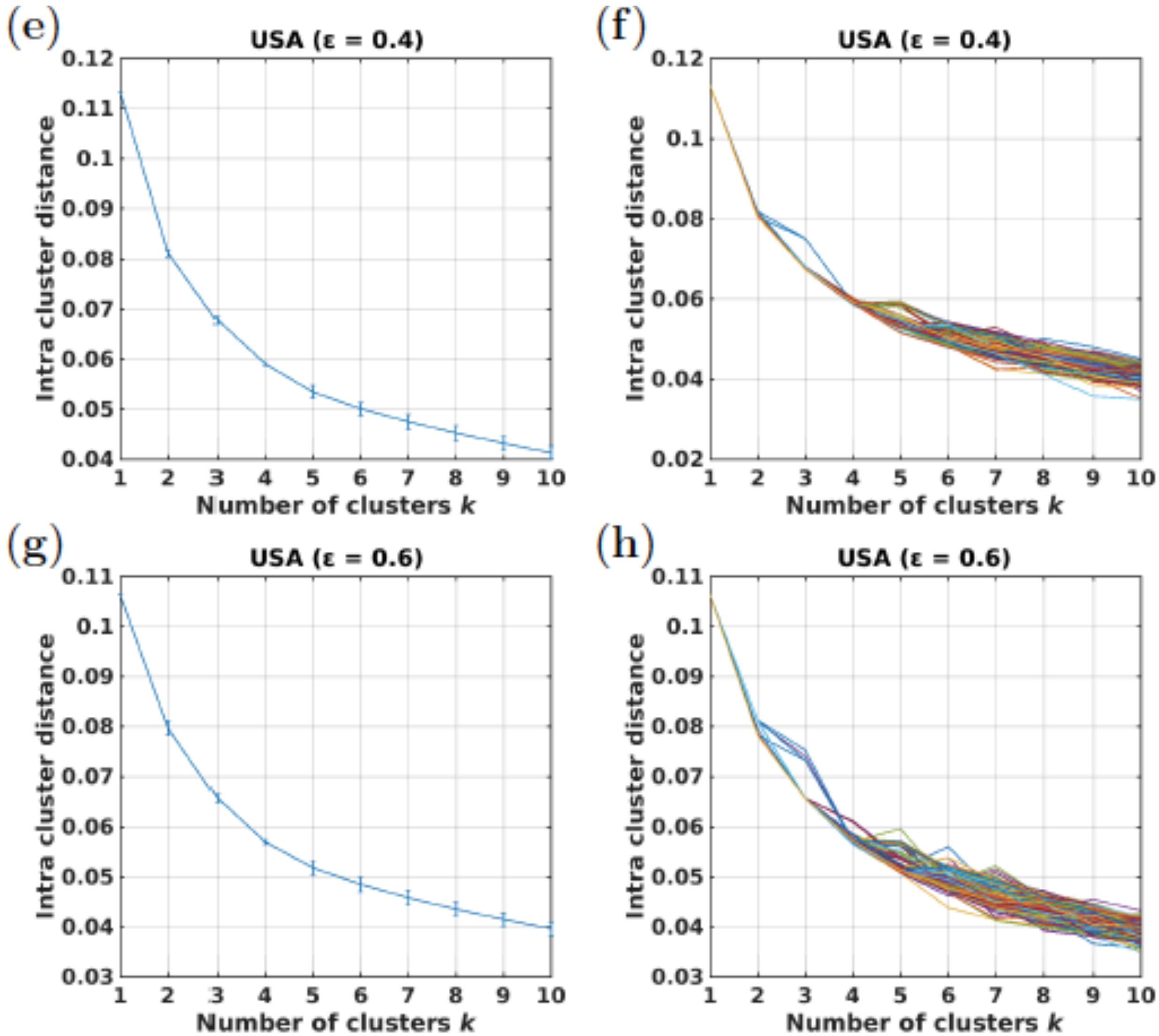
Schäfer, Rudi, Nils Fredrik Nilsson, and Thomas Guhr. "Power mapping with dynamical adjustment for improved portfolio optimization." *Quantitative Finance* 10.1 (2010): 107-119.

Typical values of q are 1.5 - 1.8
More recently values of q between 1.001 and 1.2 where used to obtain more information about the zero modes in terms of the *Emerging spectrum,* well separated from the bulk, when T<<N.

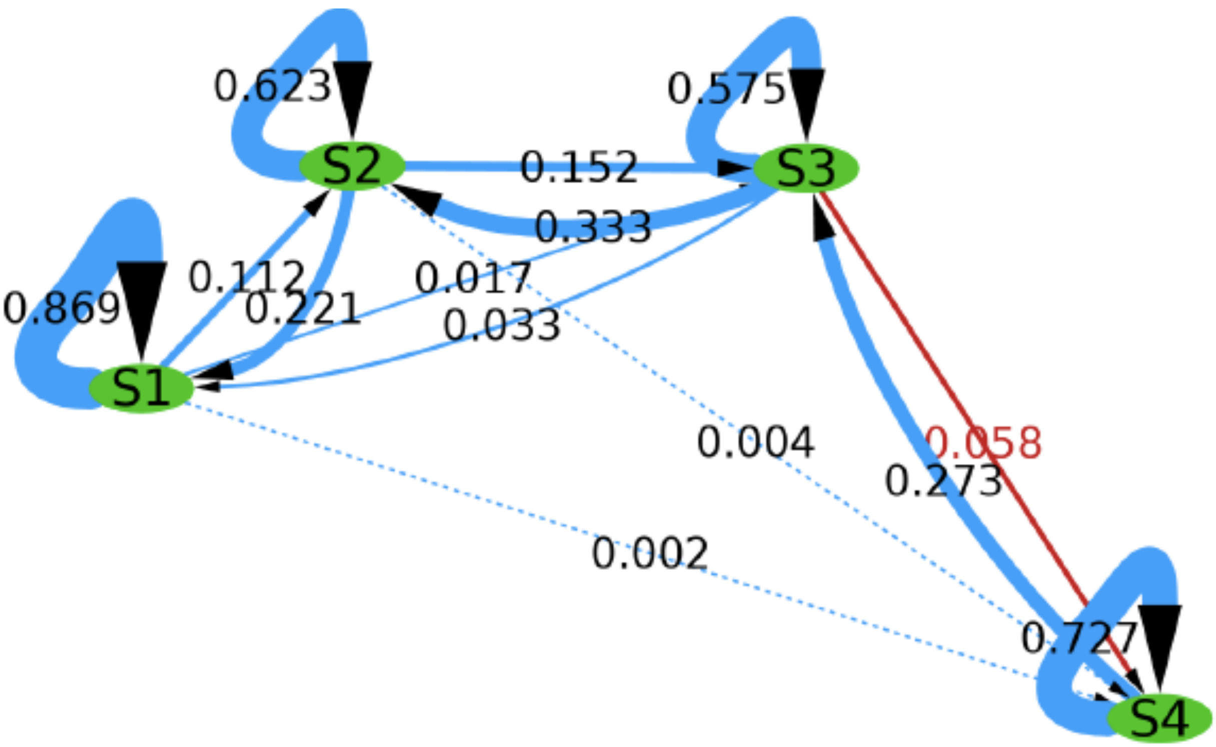
Vinayak, Schäfer, Rudi, and Thomas H. Seligman. "Emerging spectra of singular correlation matrices under small power-map deformations." *Physical Review E* 88.3 (2013): 032115.

Clustering techniques and market states

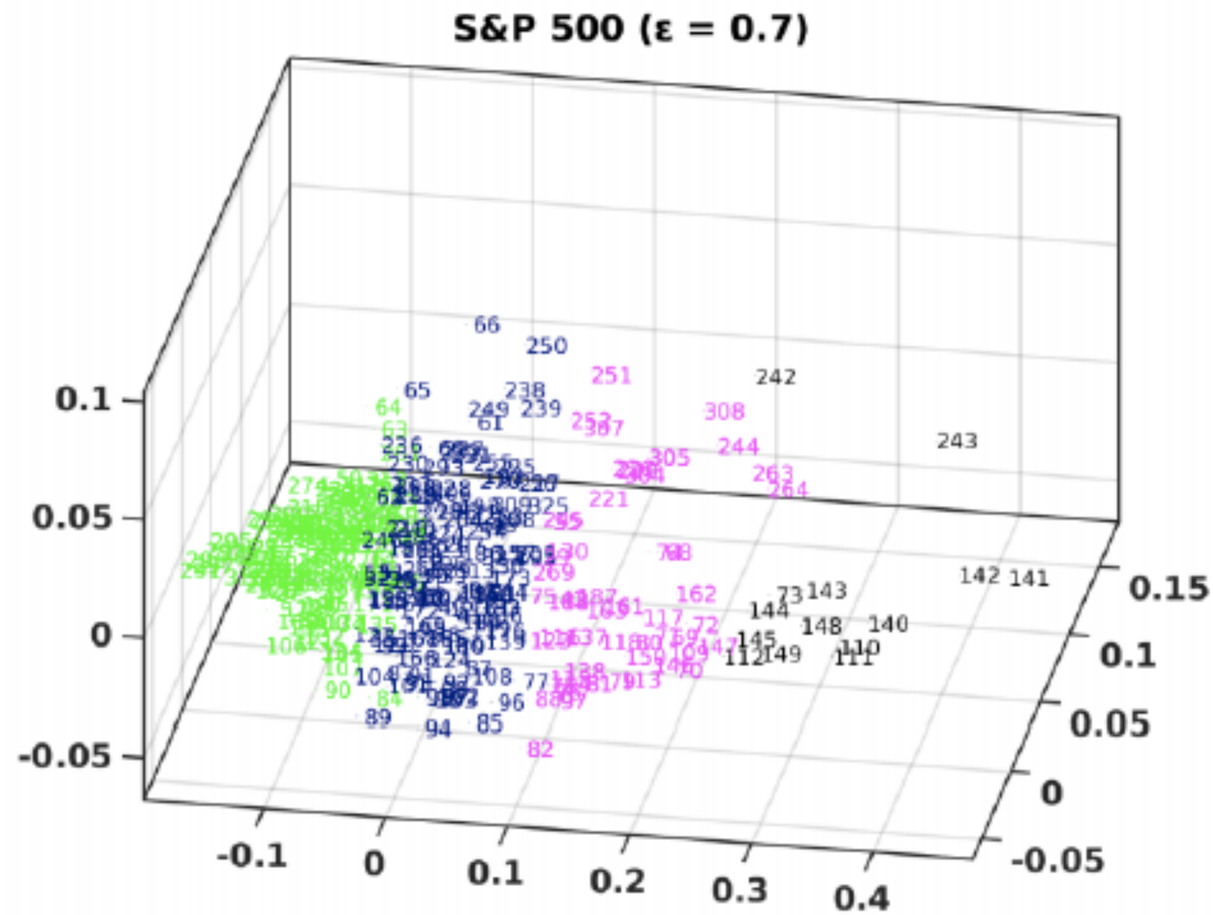
We use a standard k menas code and dimensional scaling to three dimensions,



Transfer Graph SP500



A view of clustering



Stylized clusters

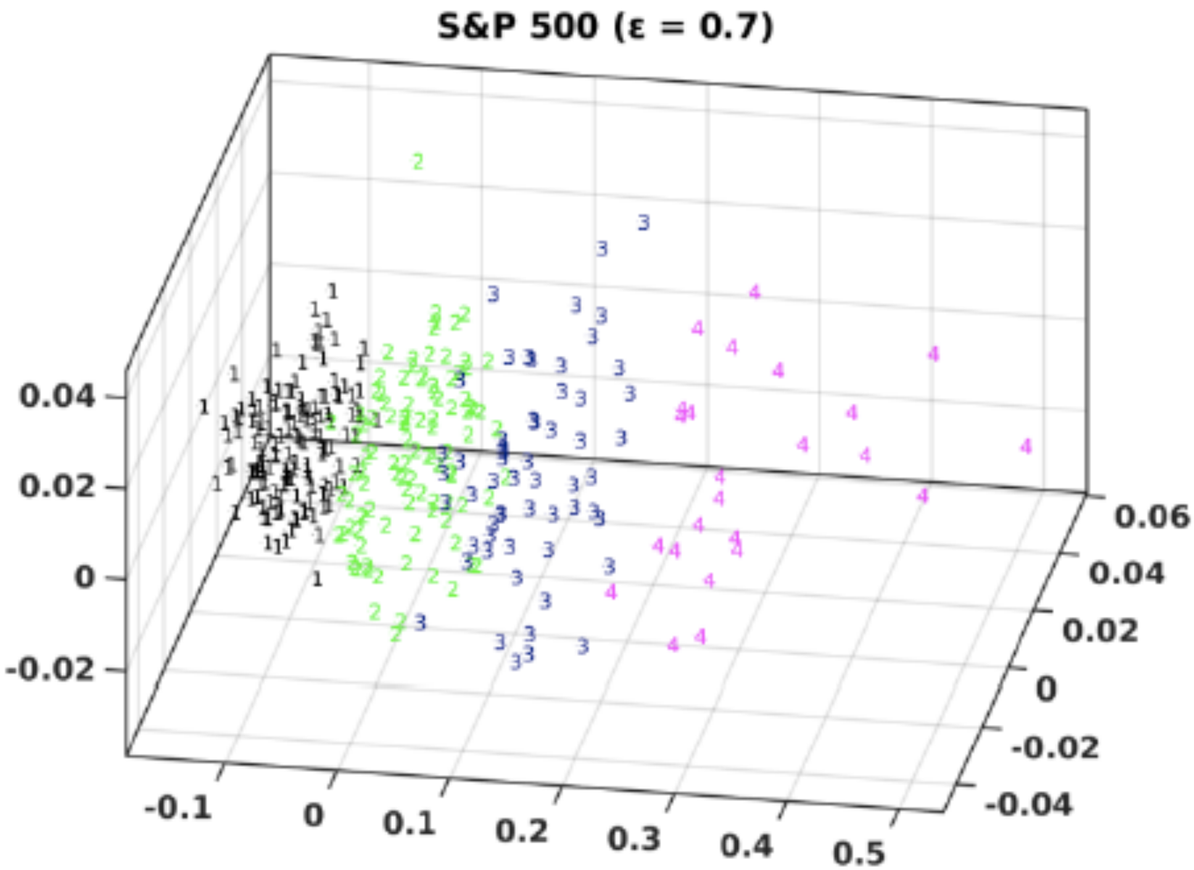
Construct a RMT cluster as a correlated Wishart ensemble with the average matrix for each state and apply the same power map used in the original

Use nI white noise data matrices D and construct the stylized ensemble for the cluster I

$$\text{Cluster I} = 1/nI \ \ \mathrm{sqrt}(\langle SI \rangle)D \ (\mathrm{sqrt}(\langle SI \rangle \ D)'$$

where nI corresponds to the number nI of frames or correlation matrices SI in the cluster I and D is an nI x T normalized white noise matrix,

The stylized clusters



As the results are quite new, we have having fun but:

No conclusion!

Happy birthday Jac !!!!

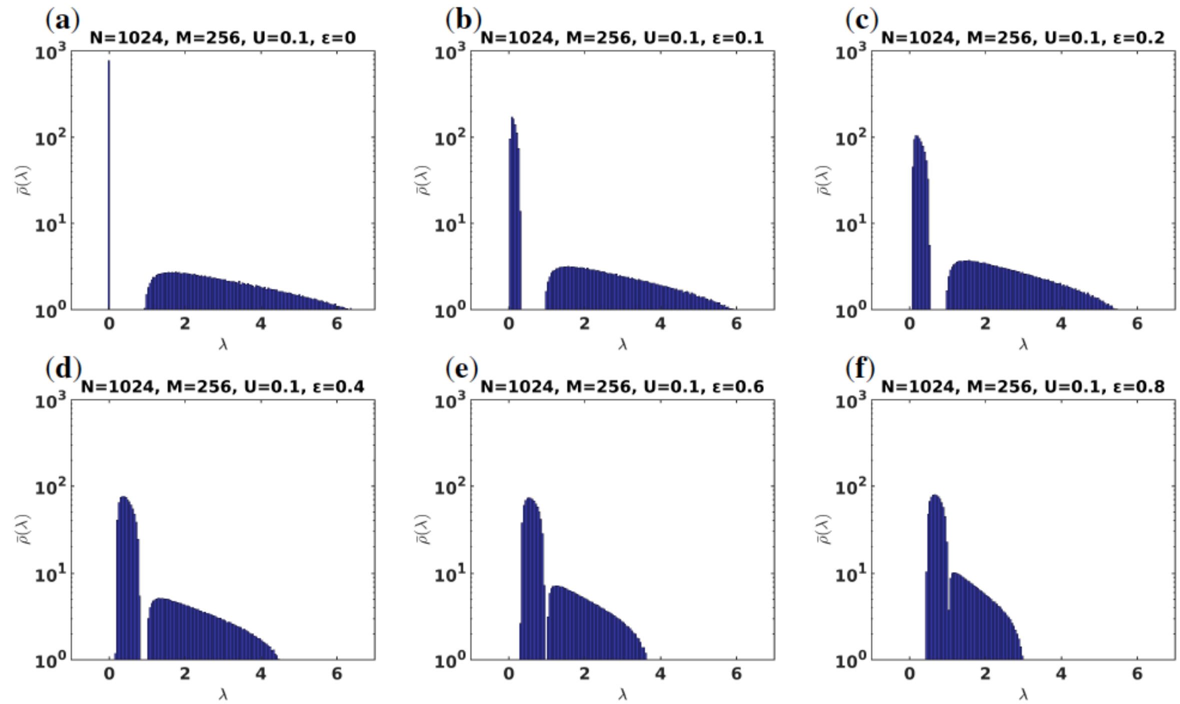


Fig. 4 Semi-log plots of the eigenvalue spectra for the correlated Wishart ensemble $\boldsymbol{W}$ with parameters $N = 1024$ and $M = 256$ at a constant correlation with $U = 0.1$, and distortion parameters of: (a) $\varepsilon = 0$, (b) $\varepsilon = 0.1$, (c) $\varepsilon = 0.2$, (d) $\varepsilon = 0.4$, (e) $\varepsilon = 0.6$, and (f) $\varepsilon = 0.8$. For $\varepsilon = 0.1$, the emerging spectrum is well separated from non-zero eigenvalues but with the increase of the distortion parameter ε the emerging spectrum starts moving towards the remaining non-zero eigenvalues spectra, and eventually merges with it at higher values, e.g., $\varepsilon = 0.8$.

Transfer matrix

Upper triangle shifts to higher states: Lower triangle shifts back

	S1	S2	S3	S4
S1	0.7444	0.2331	0.0150	0.0075
S2	0.2381	0.6111	0.1429	0.0079
S3	0.0588	0.3725	0.5098	0.0588
S4	0	0	0.3571	0.6429

Full power map spectrum for correlated Wishart ensembles with constant correlation 0.1