QCD EOS near the critical point

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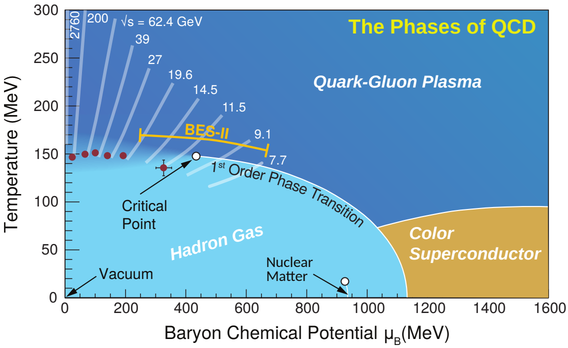
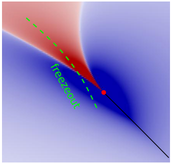
with M. Pradeep, 1905.13247

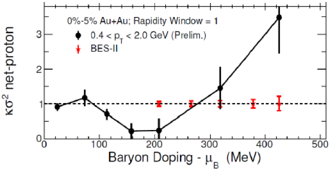
Intriguing hints and the goal of BESII

Where on the QCD phase boundary is the CP?



Equilibrium κ_4 vs T and μ_B:

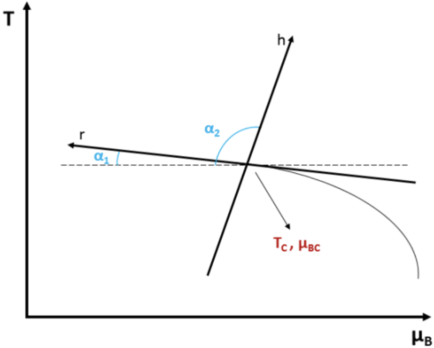




"intriguing hint" (2015 LRPNS)

Motivation for phase II of BES at RHIC and BEST topical collaboration.

Equation of state near CP



EOS is an essential input for hydro.

Universality: QCD pressure is as singular as the Ising model Gibbs free energy:

$$P_{\mathrm{QCD}}(\mu, T) = -G_{\mathrm{Ising}}(h, r)$$
$+$ less singular terms ,

(from Parotto et al.)

$$h(\mu, T) = h_T \Delta T + h_\mu \Delta\mu = -\frac{\cos\alpha_1 \Delta T + \sin\alpha_1 \Delta\mu}{w T_c \sin(\alpha_1 - \alpha_2)};$$
$$r(\mu, T) = r_T \Delta T + r_\mu \Delta\mu = \frac{\cos\alpha_2 \Delta T + \sin\alpha_2 \Delta\mu}{\rho w T_c \sin(\alpha_1 - \alpha_2)},$$

$G_{\mathrm{Ising}}(h, r)$ is universal and known, but α_1 and α_2 are not.

Mean field EOS

$$P(\mu, T) = -\min_{\phi} \Omega(\phi, \mu, T)\,.$$

$$\Omega(\phi, \mu, T) = \Omega_0 - h\phi + \frac{r}{2}\phi^2 + \frac{u}{4}\phi^4 + \dots\,.$$

Note reparametrization invariance.

Note the Z_2 symmetry:

$$\phi \to -\phi, \quad h \to -h, \quad r \to r\,.$$

and scaling:

$$\phi \sim r^{1/2}, \quad h \sim r^{3/2}, \quad \Omega - \Omega_0 \sim r^2\,.$$

Subtlety of mapping r

$$\begin{aligned} h(\mu, T) &\sim \cos\alpha_1 \Delta T + \sin\alpha_1 \Delta\mu \\ r(\mu, T) &\sim \cos\alpha_2 \Delta T + \sin\alpha_2 \Delta\mu \end{aligned}$$

While mapping of h is straightforward: $h = 0$ is the transition line,

the mapping of r is a little tricky. Since $h \sim r^{1+1/2} \ll r$, admixture of h in r is impossible to discern without considering also subleading order scaling terms.

In $\Omega \sim r^2$ that means $\sim r^{2+1/2}$, i.e., ϕ^5.

In addition, mixing h and r violates Z_2 symmetry, and so does ϕ^5.

The effect of ϕ^5

$$\Omega = \Omega_0 - \bar{h}\phi + \frac{1}{2}\bar{r}\phi^2 + \frac{u}{4}\phi^4 + vu\phi^5 + O(\phi^6),$$

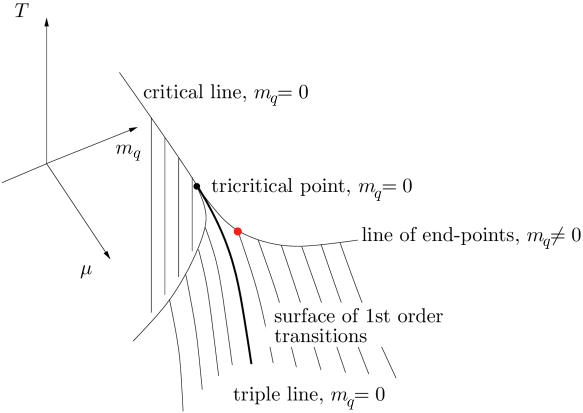
This (nonlinear) reparametrization removes ϕ^5:

$$\phi \to \phi + v\phi^2 + \mathrm{const}$$

and modifies h and r:

$$\begin{aligned} h &= \bar{h} + \mathcal{O}(\bar{r}^2), \\ r &= \bar{r} + 2v\bar{h}, \end{aligned}$$

Critical point near tricritical point



Mean-field theory near QCD tricritical point

$$P(\mu, T, m_q) = -\min_{\Phi} \Omega(\Phi, \mu, T, m_q)$$

where

$$\Omega(\Phi, \mu, T, m_q) = V(\Phi, \mu, T) - m_q \Phi$$

$$V(\Phi, \mu, T) = V_0 + \frac{a}{2}\Phi^2 + \frac{b}{4}\Phi^4 + \frac{c}{6}\Phi^6 + \dots\,,$$

Scaling:

$$\Phi \sim a^{1/4}, \quad b \sim a^{1/2}, \quad m_q \sim a^{5/4} \quad V - V_0 \sim a^{3/2}\,.$$

TCP at $\Phi = 0$: $a = b = 0$.
CP at $V' = m_q$, $V'' = V''' = 0$:

$$\Phi_c = \left(\frac{3m_q}{8c}\right)^{1/5}, \quad a_c = 5c\Phi_c^4, \quad b_c = -\frac{10c}{3}\Phi_c^2.$$

Expand $\Phi = \Phi_c + \phi$:

$$\Omega = \left(\Omega_c + \bar{h}\phi + \frac{\bar{r}}{2}\phi^2 + \frac{u}{4}\phi^4 + vu\phi^5\right) + \Delta b\Phi_c\phi^3 + \frac{c}{6}\phi^6 + \dots\,,$$

where

$$\begin{aligned} \bar{h} &= -(\Delta a + \Delta b\Phi_c^2)\Phi_c\,, \\ \bar{r} &= \Delta a + 3\Delta b\Phi_c^2\,, \\ u &= \frac{20\Phi_c^2}{3}\,, \quad v = \frac{3}{20\Phi_c}\,. \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} h &= -\left(\Delta a + \Delta b\Phi_c^2\right)\Phi_c, \\ r &= \frac{7}{10}\Delta a + \frac{27}{10}\Delta b\Phi_c^2. \end{aligned}$$

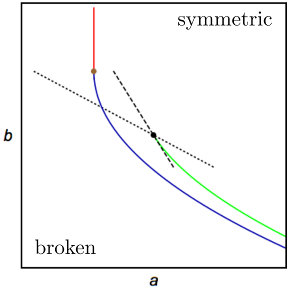
$$\begin{aligned} \tan\alpha_1 &= -\left(\frac{dT}{d\mu}\right)_{h=0} = \frac{h_\mu}{h_T} = \frac{a_\mu + b_\mu\Phi_c^2}{a_T + b_T\Phi_c^2}\,; \\ \tan\alpha_2 &= -\left(\frac{dT}{d\mu}\right)_{r=0} = \frac{r_\mu}{r_T} = \frac{a_\mu + 27b_\mu\Phi_c^2/7}{a_T + 27b_T\Phi_c^2/7}\,. \end{aligned}$$

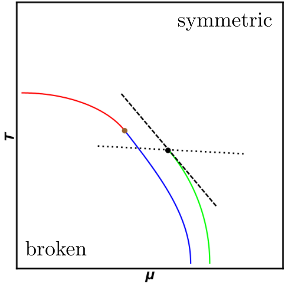
As $\Phi_c \sim m_q^{1/5} \to 0$: $\alpha_1 - \alpha_2 \sim m_q^{2/5} \to 0$.

Slope difference

$$\tan\alpha_1 - \tan\alpha_2 = \left(\frac{dT}{d\mu}\right)_{r=0} - \left(\frac{dT}{d\mu}\right)_{h=0} \quad = \quad \frac{20}{7a_T^2}\frac{\partial(a,b)}{\partial(\mu,T)}\Phi_c^2 + \mathcal{O}(\Phi_c^4)$$

$$\sim \quad m_q^{2/5}\,.$$





$\alpha_1 > \alpha_2$

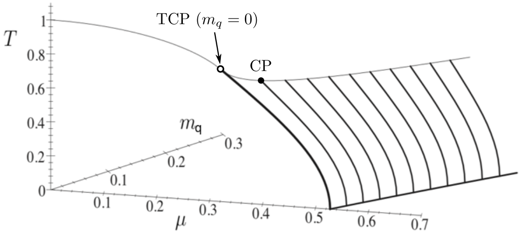
RMM of Halasz *et al*

$$D = \begin{pmatrix} 0 & X + T + i\mu \\ X^\dagger + T + i\mu & 0 \end{pmatrix}$$

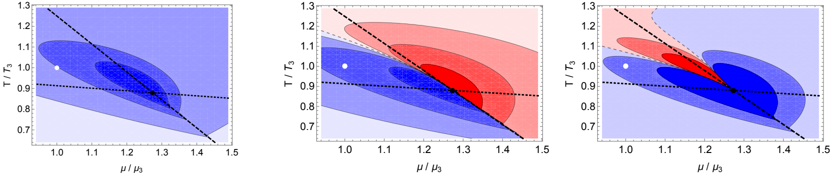
$$P(\mu, T, m_q) = -\mathcal{N} \min_{\phi} \Omega^{RMM}(\Phi; \mu, T, m_q)\,,$$

where

$$\Omega^{RMM} = \Phi^2 - \frac{1}{2}\ln\left\{\left[(\Phi+m_q)^2 - (\mu+iT)^2\right].\left[(\Phi+m_q)^2 - (\mu-iT)^2\right]\right\}$$



Baryon number susceptibilities $\chi_2 \equiv P_{\mu\mu}$, $\chi_3 \equiv P_{\mu\mu\mu}$, $\chi_4 \equiv P_{\mu\mu\mu\mu}$:



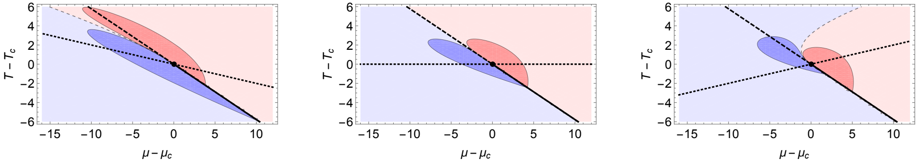
For $m_q = 5$ MeV: $\alpha_1 \sim 13^\circ, \quad \alpha_2 \sim 1^\circ\,.$

Observaions:

- both slopes are negative
- the sign of χ_3 is negative on crossover line

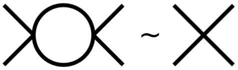
The slope of h-axis and the sign of χ_3

The sign of χ_3 on crossover ($h = 0$) line
is related to the slope of $r = 0$ line (i.e., α_2):



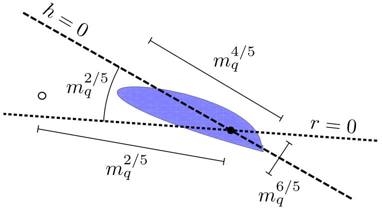
Ginzburg region

Breakdown of mean-field approximation, Ginzburg criterion:



$$
\begin{aligned}
u^2\xi^\epsilon &\sim u \\
\xi &\sim m_q^{-2/5\epsilon}
\end{aligned}
$$

Ginzburg region:



Beyond mean-field

In $d = 4$ there are two Z_2-odd perturbations of dimension 5:

$$\phi^5 \text{ and } \phi^2\nabla^2\phi.$$

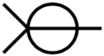
Only one linear combination can be represented by $r - h$ mixing:

$$V_3 = u\phi^5 - \phi^2\nabla^2\phi \quad \left(= \phi^2 \frac{\delta\Omega}{\delta\phi}\right).$$

Indeed, $\Delta_3 = \beta\delta - 1 = 1/2 + \mathcal{O}(\epsilon^2)$, same as of h/r.

Another linear combination is

$$V_5 = u\phi^5 - (10S_5/3)\phi^2\nabla^2\phi, \text{with } S_5 = \mathcal{O}(\epsilon)$$

due to mixing  . $\Delta_5 = 1/2 + \epsilon + \mathcal{O}(\epsilon^2)$.

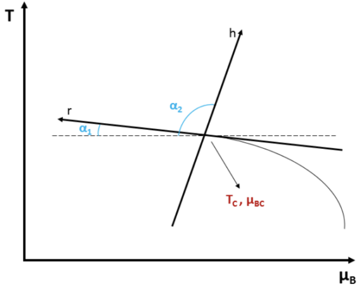
More irrelevant, because in $d = 3$, $\Delta_5 \approx 1.3 - 1.6 > \Delta_3 \approx 0.56$.

Beyond mean-field

$$
\begin{aligned}
\tan\alpha_1 - \tan\alpha_2 &= \left(\frac{dT}{d\mu}\right)_{r=0} - \left(\frac{dT}{d\mu}\right)_{h=0} \\
&= \frac{7}{10} C_{\rm MF}\left(1 + S_5(\epsilon) + O(\epsilon^2)\right) m_q^{2/5} + \mathcal{O}(m_q^{4/5})
\end{aligned}
$$

$$
P_{\textsf{sing}}(\mu, T) = -r^{2-\alpha}\left(g(hr^{-\beta\delta}) + v_5 r^{\Delta_5} g_5(hr^{-\beta\delta})\right) ,
$$

Conclusion



- The slope difference $\alpha_1 - \alpha_2$ vanishes as $m_q^{2/5}$ in the chiral limit.
- For small m_q the slope of h-axis is *negative*: $0 < \alpha_2 < \alpha_1$. Confirmed by RMM.
- $\chi_3 < 0$ and is experimentally measurable.
- $\alpha_2 \sim 0$ enhances CP effects on cumulants of baryon number: e.g., $\partial^2 G/\partial h^2 \sim r^{-\gamma}$ vs $\partial^2 G/\partial r^2 \sim r^{-\alpha}$, where $\gamma \approx 1$ and $\alpha \ll 1$.