BRST in the Exact RG

FRGIM - Functional and Renormalisation-Group Methods

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Tim Morris,
Physics & Astronomy,
University of Southampton, UK.

Y Igarashi, K Itoh & TRM, 1904.08231, to be publ in Prog. Theor. Exp. Phys.

Motivation:

Quantum gravity has power law UV divergences, which are discarded in dimensional regularisation but which are crucial in defining a perturbative continuum limit.

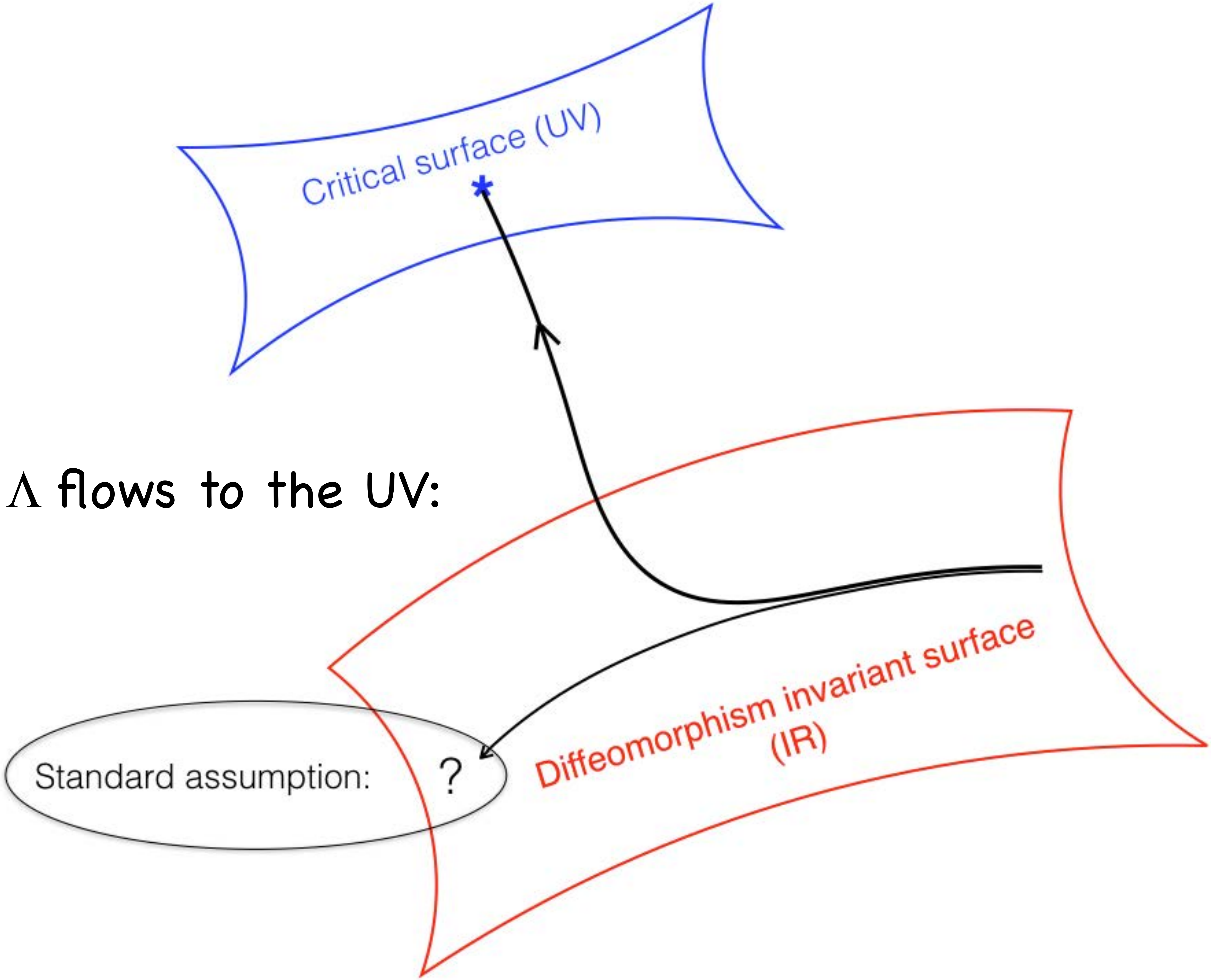
Need to make these visible by UV regularisation Λ. This breaks diffeomorphism (BRST) invariance. Then:

Need to distinguish between breaking by the regularisation and breaking which is inherent to the quantisation.

Need to develop streamlined procedures for solving for physical amplitudes when BRST invariance is (only) broken by the cutoff.

TRM JHEP 1808 (2018) 024 [1802.04281], Int J Mod Phys D [1804.03834], SciPost Phys. 5 (2018) no.4, 040 [1806.02206]; TRM & MP Kellett, Class.Quant.Grav. 35 (2018) no.17, 175002 [1803.00859]; TRM, MP Kellett & A Mitchell-Lister, to appear.

Λ flows to the UV:



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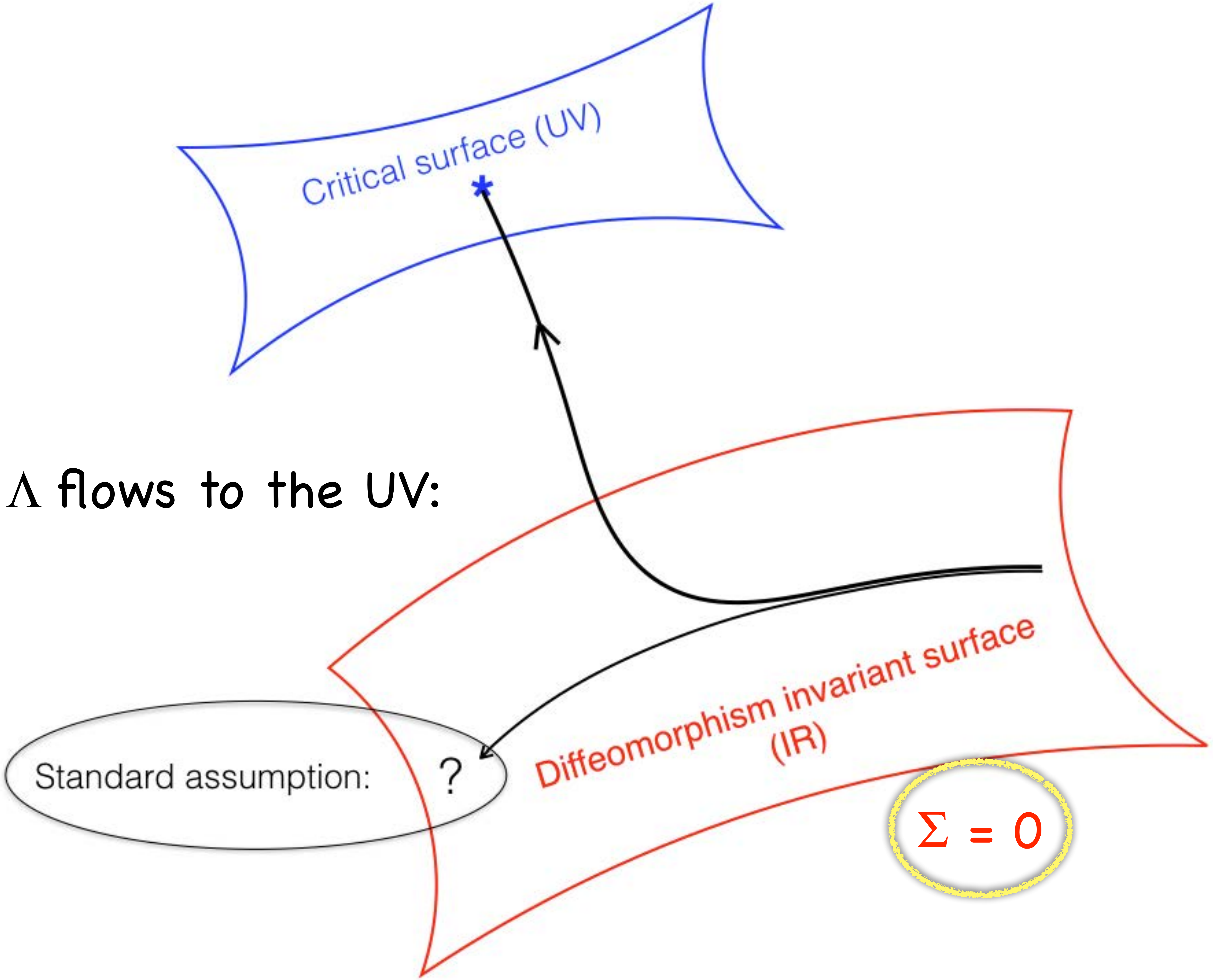

Wilsonian effective action:

UV cutoff function $K(p^2/\Lambda^2)$

$$S = S_0 + S_I$$

$$S_0[\phi,\phi^*] = \tfrac{1}{2}\phi^A K^{-1} \triangle^{-1}_{AB}\, \phi^B + \phi^*_A K^{-1} R^A{}_B \phi^B$$

Free BRST transformations: $Q_0\, \phi^A = R^A{}_B\, \phi^B$

Full BRST transformations: $Q\phi^A = (\phi^A, S) = K \dfrac{\partial_l S}{\partial \phi^*_A}$

Antibracket: $$(X,Y) = \frac{\partial_r X}{\partial \phi^A} K \frac{\partial_l Y}{\partial \phi^*_A} - \frac{\partial_r X}{\partial \phi^*_A} K \frac{\partial_l Y}{\partial \phi^A}$$

Measure operator: $$\Delta X = (-)^{A+1} \frac{\partial_r}{\partial \phi^A} K \frac{\partial_r}{\partial \phi^*_A} X$$

Wilsonian effective action:

Quantum Master Equation: $\Sigma = 0$

$$\Sigma = \frac{1}{2}(S,S) - \Delta S$$

Polchinski Equation:

$$\dot{S}_I = -\frac{1}{2}\partial^r_A S_I \dot{\triangle}^{AB} \partial^l_B S_I + \frac{1}{2}(-)^A \dot{\triangle}^{AB} \partial^l_B \partial^r_A S_I = \frac{1}{2}a_0[S_I, S_I] - a_1[S_I]$$

IR cutoff propagator $\bar{K} = 1 - K$

Compatibility: $\dot{\Sigma} = a_0[S_I, \Sigma] - a_1[\Sigma]$

The gauge invariant surface is an invariant subspace: once you're in you never leave...

Wilsonian effective action:

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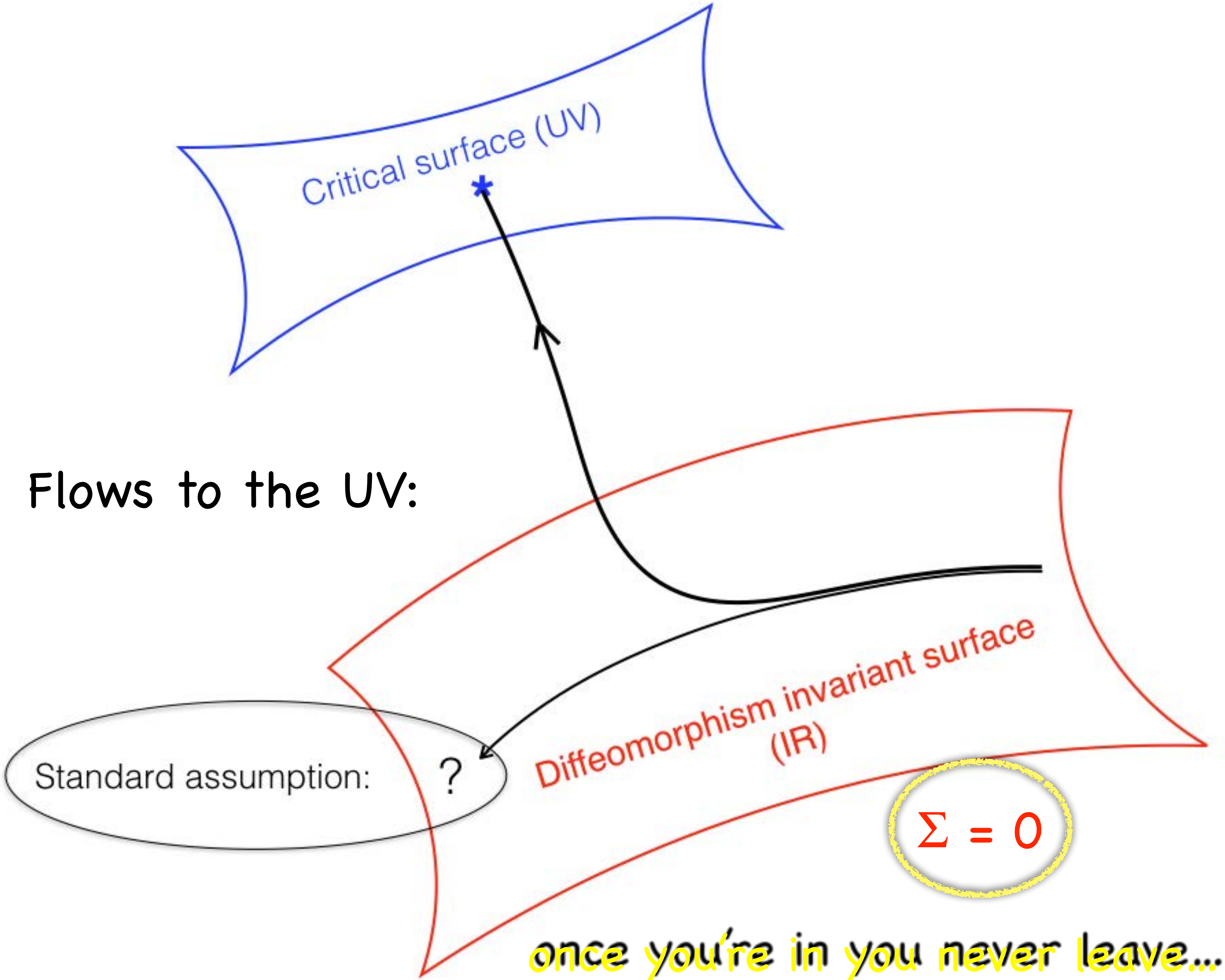
Flows to the UV:



Legendre effective action:

Legendre transformation:

$$\Gamma_I[\Phi, \Phi^*] = S_I[\phi, \phi^*] - \tfrac{1}{2}\,(\phi - \Phi)^A\,\bar{\triangle}^{-1}_{AB}\,(\phi - \Phi)^B$$

$\Phi^* = \phi^*$

$\Rightarrow$

$$\dot{\Gamma}_I = -\tfrac{1}{2}\,\mathrm{Str}\left(\dot{\bar{\triangle}}\bar{\triangle}^{-1}\left[1 + \bar{\triangle}\Gamma^{(2)}_I\right]^{-1}\right)$$

$$\Gamma^{(2)}_{I\;AB} = \frac{\overrightarrow{\partial}}{\partial\Phi^A}\Gamma_I\frac{\overleftarrow{\partial}}{\partial\Phi^B}$$

with IR cutoff Λ

Legendre effective action:

Effective average action:

$$\Gamma = \Gamma_0 + \Gamma_I\,, \qquad \Gamma_0 = \tfrac{1}{2}\,\Phi^A \triangle^{-1}_{AB}\Phi^B + \Phi^*_A R^A{}_B \Phi^B$$

Antibracket:

$$(\Xi, \Upsilon) = \frac{\partial_r \Xi}{\partial \Phi^A}\,\frac{\partial_l \Upsilon}{\partial \Phi^*_A} - \frac{\partial_r \Xi}{\partial \Phi^*_A}\,\frac{\partial_l \Upsilon}{\partial \Phi^A}$$

$$\left(\Gamma^{(2)}_{I*}\right)^A{}_B = \frac{\overrightarrow{\partial}}{\partial \Phi^*_A}\Gamma_I \frac{\overleftarrow{\partial}}{\partial \Phi^B}$$

⇨ $$\Sigma = \tfrac{1}{2}(\Gamma, \Gamma) - \mathrm{Tr}\left(K \Gamma^{(2)}_{I*}\left[1 + \bar{\triangle}\Gamma^{(2)}_I\right]^{-1}\right)$$

These give modified Slavnov-Taylor identities

U Ellwanger, Phys Lett B335 (1994) 364

⇨ Zinn-Justin identities as $\Lambda \to 0$

Legendre effective action:

Gives direct access to physics in limit $\Lambda \to 0$

Is one-particle irreducible

But BRST invariance obscured at $\Lambda > 0$:

$$\Sigma = \frac{1}{2}(\Gamma,\Gamma) - \mathrm{Tr}\left(K \Gamma^{(2)}_{I*} \left[1 + \bar{\triangle} \Gamma^{(2)}_{I} \right]^{-1} \right)$$

Wilsonian effective action:

Is entirely equivalent

UV regularisation makes Δ well defined

Unbroken quantum BRST invariance: $\Sigma = \frac{1}{2}(S,S) - \Delta S = 0$

But $\Delta S \neq 0$ leads to Λ-dependent mass terms etc..

In interacting case neither equation has local solutions:

UV cutoff function $K(p^2/\Lambda^2)$

$$\Sigma = \frac{1}{2}(S,S) - \Delta S = 0$$

$$\Sigma = \frac{1}{2}(\Gamma,\Gamma) - \mathrm{Tr}\left(K\Gamma^{(2)}_{I*}\left[1+\bar{\Delta}\Gamma^{(2)}_{I}\right]^{-1}\right)$$

so no local bare action that satisfies regularised QME or mST

But don't need a bare action.
Can solve these equations directly for the renormalized trajectory to all orders in derivative expansion.

Many explicit examples at tree-level & 1 loop in:

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Here concentrate on general framework…

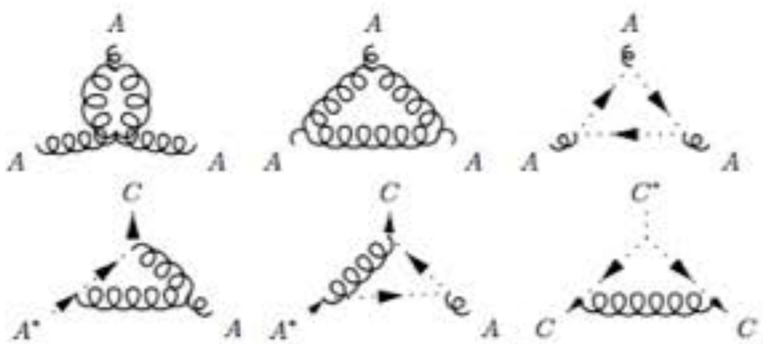


Figure 5.1: One-loop vertices in Γ_3. In gauge invariant basis, A^* also plays the rôle of the antighost.

The one-loop vertices as a whole have to satisfy the mST identities (3.43). Apart from the freedom to choose the integration constants according to (4.11), there is no further flexibility, and thus the body of such a solution (3.42), as defined by the momentum integrals, must already satisfy these mST identities as we noted in C4 and at the end of sec. 3.5, and will confirm shortly. We can write the vertices as

$$\Gamma_3 = -i\int_{p,q,r} A_\mu(p)[A_\nu(q),A_\lambda(r)]\,\Gamma^{AAA}_{\mu\nu\lambda}(p,q,r) - i\int_{p,q,r} A^*_\mu(p)[A_\nu(q),C(r)]\,\Gamma^{A^*AC}_{\mu\nu}(p,q,r) - i\int_{p,q,r} C^*(p)C(q)C(r)\,\Gamma^{C^*CC}(p,q,r)\,, \quad (5.24)$$

where $(2\pi)^4\delta(p+q+r)$ is understood to be included in the measure. The correction terms in the mST can similarly be written:

$$\mathrm{Tr}\left(-K\Gamma^{(2)}_{1*}\bar\Delta\Gamma^{(2)}_{2,\mathrm{cl}} + K\Gamma^{(2)}_{1*}\bar\Delta\Gamma^{(2)}_{1}\bar\Delta\Gamma^{(2)}_{1}\right) = \int_{p,q,r} C(p)[A_\mu(q),A_\nu(r)]\,\Delta^{CAA}_{\mu\nu}(p,q,r) + \int_{p,q,r} A^*_\mu(p)C(q)C(r)\,\Delta^{A^*CC}_{\mu}(p,q,r)\,. \quad (5.25)$$

We compute that

$$-(\Gamma_1,\Gamma_2) = -iC_A\left\{\left(\{A^*_\mu,C\} + [\partial_\mu A_\nu - \partial_\nu A_\mu, A_\nu]\right)B\,\partial_\mu C + \partial_\mu A^*_\mu B C^2 - A_\mu\,\mathcal{A}_{\mu\nu}[A_\nu,C]\right\}\,. \quad (5.26)$$

and thus for example from the A^*CC part of (3.43) we find one must have:

$$p_\mu\Gamma^{C^*CC}(p,q,r) + 2q_\nu\Gamma^{A^*AC}_{\mu\nu}(p,q,r) = -C_A\left[p_\mu B(p) + r_\mu B(r)\right] + \Delta^{A^*CC}_\mu(p,q,r)\,. \quad (5.27)$$

The Λ-integration constants satisfy the LHS alone and correspond to the freedom to change the normalization of the bracketed pair in (4.11). The rest of the above equation can be viewed as defining the longitudinal part $\Gamma^{A^*AC}_{\mu\alpha}(p,q,r)P^L_{\alpha\nu}(q)$ of A^*AC vertex. Similarly, other mST relations define either longitudinal or transverse parts, via Q_0A_ν or $Q_0^-A^*_\mu$ respectively.

41

Inspecting fig. 5.1, we see that there is only one diagram that contributes to C^*CC piece in (4.11). This is therefore the easiest way to extract γ_g which in turn will give us the one-loop β-function. We find:

$$-\left(\gamma_g + \frac{\gamma_C}{2}\right) = \dot\Gamma^{C^*CC}(0,0,0) = \frac{C_A}{2}\frac{\partial}{\partial t}\int_q \bar\Delta^2(q)q_\mu q_\nu\bar\Delta_{\mu\nu}(q) = \frac{\xi C_A}{(4\pi)^2}\int_0^\infty\! du\,\frac{\partial}{\partial u}\bar K^3(u) = \frac{\xi C_A}{(4\pi)^2}\,. \quad (5.28)$$

To extract the β-function, we absorb the Z_g^{-1} in (4.11) into the coupling

$$g(\Lambda) \equiv g_{(r)} = Z_g^{-1}g\,, \quad (5.29)$$

which thus runs. To one loop, the β-function is then

$$\beta(g) = \Lambda\partial_\Lambda g(\Lambda) = -\dot g(\Lambda) = \gamma_g\,g^3(\Lambda)\,, \quad (5.30)$$

where from (5.28), (5.17) and (5.12) we recover the famous result, here as a flow in Λ:[13]

$$\gamma_g = \left(\gamma_g + \tfrac12\gamma_C\right) + \tfrac12\left(\gamma_A - \gamma_C\right) - \tfrac12\gamma_A = -\frac{11}{3}\frac{C_A}{(4\pi)^2}\,. \quad (5.31)$$

There are two diagrams that contribute to the A^*AC vertex in (4.11), but otherwise the computation offers almost as straightforward a route to the β-function:

$$\begin{aligned}-\left(\gamma_g + \frac{\gamma_C}{2}\right)\delta_{\mu\nu} &= \dot\Gamma^{A^*AC}_{\mu\nu}(0,0,0)\\ &= \frac{C_A}{2}\frac{\partial}{\partial t}\int_q \bar\Delta\left[\bar\Delta q_\rho q_\nu\bar\Delta_{\mu\rho} + q_\rho q_\sigma\bar\Delta_{\rho\sigma}\bar\Delta_{\mu\nu} + q_\rho q_\sigma\bar\Delta_{\mu\rho}\bar\Delta_{\nu\sigma} - 2q_\nu q_\sigma\bar\Delta_{\mu\rho}\bar\Delta_{\rho\sigma}\right]\\ &= \delta_{\mu\nu}\left(\xi + \xi(\xi+3) + \xi^2 - 2\xi^2\right)\frac{C_A}{4(4\pi)^2}\int_0^\infty\! du\,\frac{\partial}{\partial u}\bar K^3(u) = \frac{\xi C_A}{(4\pi)^2}\delta_{\mu\nu}\,, \quad (5.32)\end{aligned}$$

where the propagators are all evaluated at q. The end result agrees with (5.28) and thus again we get the famous one-loop β function coefficient (5.31). Verification of the wavefunction renormalization dependence of the other vertices in (4.11) proceeds in a similar if somewhat longer fashion. We note that through such relations, *cf.* (5.12), (5.17), (5.28) and (5.32), we can confirm that the Slavnov-Taylor identities (4.23) are indeed satisfied.

We finish by confirming explicitly that the body of the momentum integrals do automatically satisfy (5.27). The A^*C vertex in (3.39), *cf.* (5.13), can be written as

$$-C_A\bar\Delta_{\mu\nu}A^*_\nu\partial_\mu\bar\Delta C\,. \quad (5.33)$$

As we have seen, care is needed in defining the integration constant $B_0\partial_\nu$ in this vertex, but here we will be interested in putting this to one side and demonstrating that the bulk of the integral

[13] The β-function under flow in Λ, specialised to SU(2) and Feynman-gauge, was computed in ref. [89].

42

Wilsonian effective action: operators

Koszul-Tate charge: $Q^-\phi^*_A = (\phi^*_A, S) = -K\,\dfrac{\partial_l S}{\partial \phi^A}$

If $S + \varepsilon\,\mathcal{O}$ satisfies $\Sigma = 0$: $\hat{s}\,\mathcal{O} = 0$

$\hat{s}^2 = 0$

$$\hat{s}\,\mathcal{O} = (\mathcal{O}, S) - \Delta\mathcal{O} = (Q + Q^- - \Delta)\,\mathcal{O}$$

Therefore: $\mathcal{O} = \hat{s}\,\mathcal{K} = (\mathcal{K}, S) - \Delta\mathcal{K}$ satisfies $\Sigma = 0$

But this just corresponds to field and source redefinitions:

$$\delta\phi^A = -\varepsilon K\,\frac{\partial_r \mathcal{K}}{\partial \phi^*_A}\,, \qquad \delta\phi^*_A = \varepsilon K\,\frac{\partial_r \mathcal{K}}{\partial \phi^A}$$

Therefore want: $\hat{s}\,\mathcal{O} = 0$ such that $\mathcal{O} \neq \hat{s}\,\mathcal{K}$

Quantum BRST cohomology

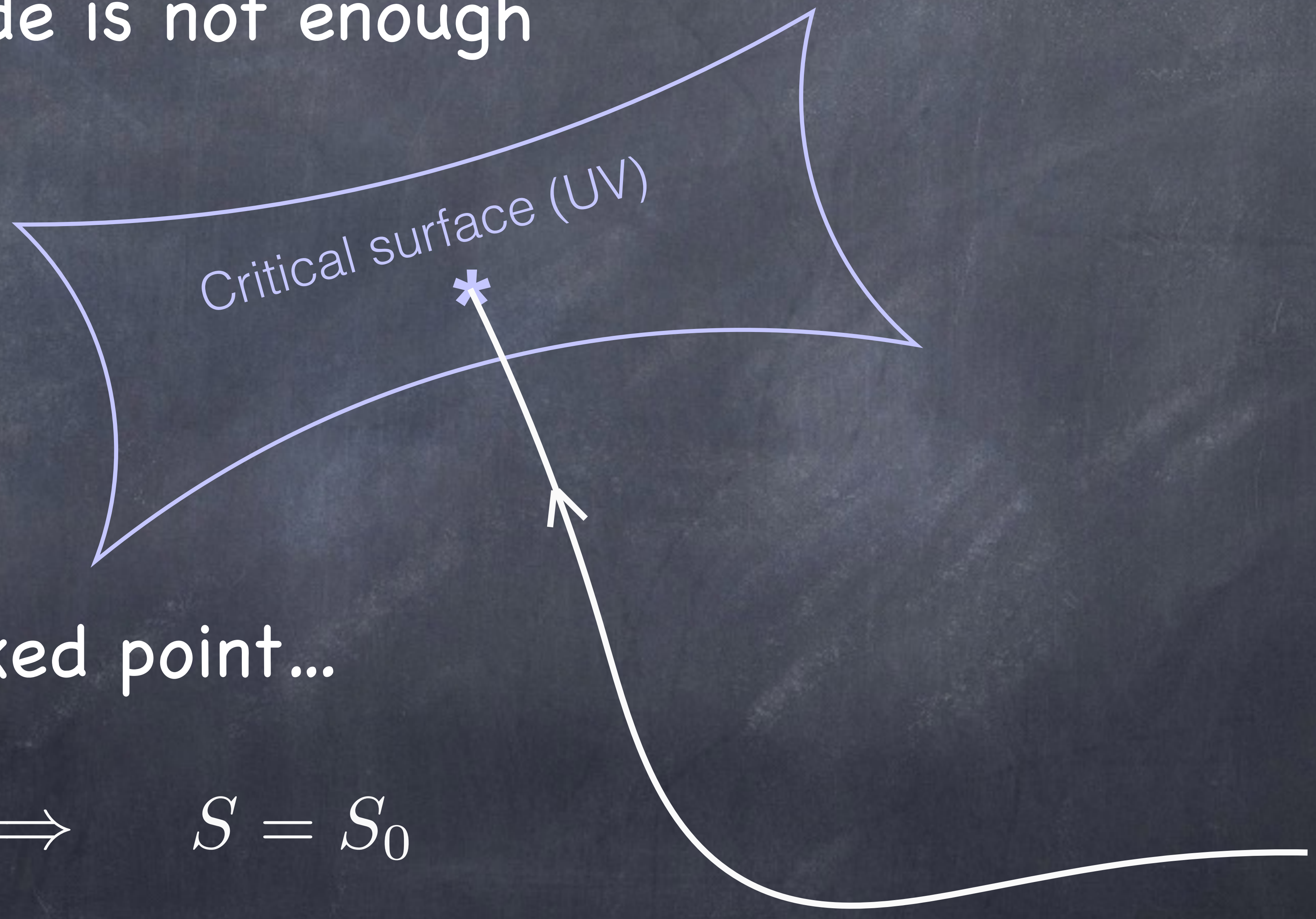
Δ regularised

derivative expansion

Renormalized trajectory inside $\Sigma = 0$

$$\dot{\Sigma} = a_0[S_I, \Sigma] - a_1[\Sigma]$$

Fixed point inside is not enough



E.g. Gaussian fixed point...

$$S_I = 0 \quad \Longrightarrow \quad S = S_0$$

$$\Sigma = \Sigma_0 = \tfrac{1}{2}(S_0, S_0) - \Delta S_0 = 0$$

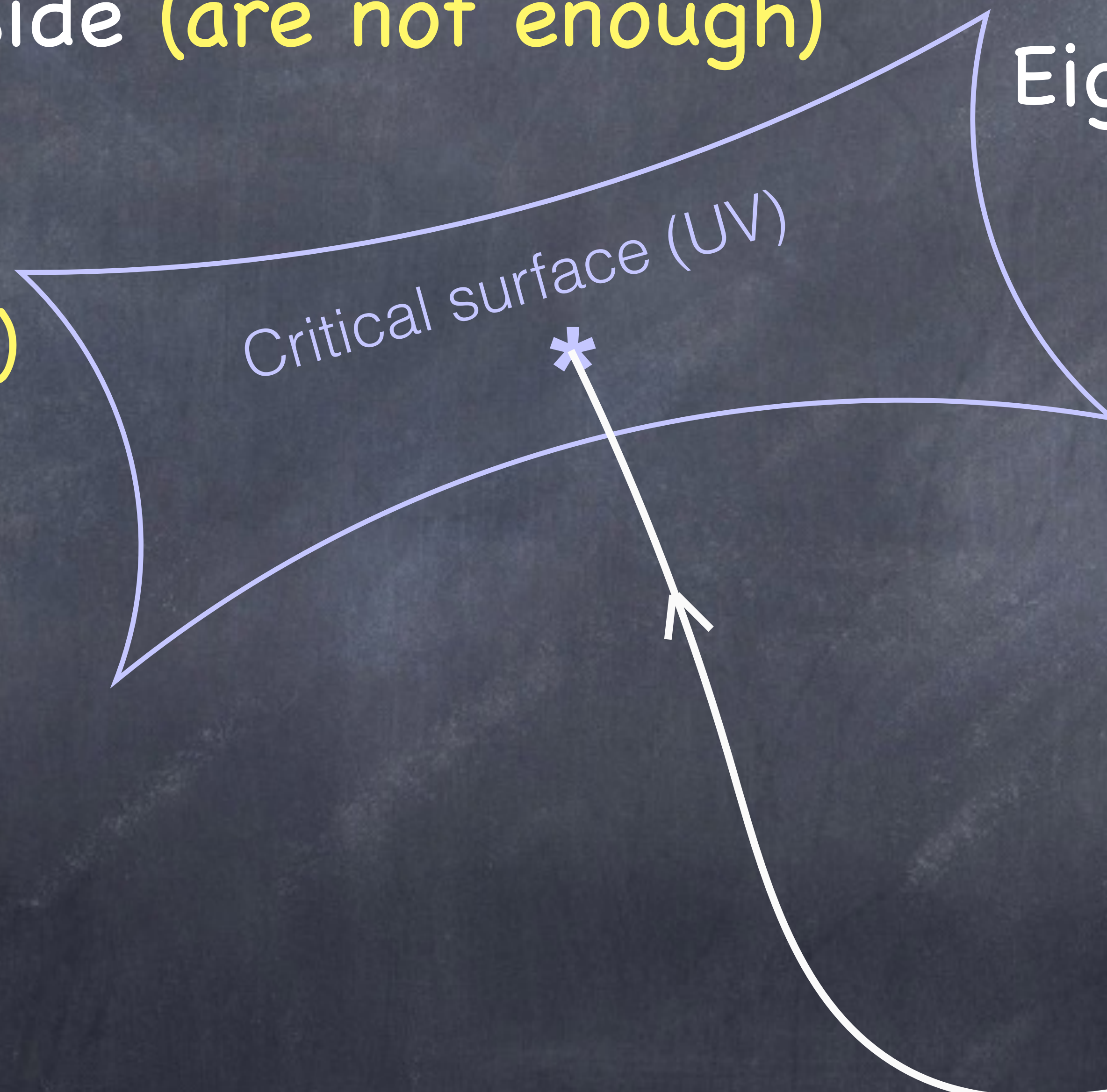
Renormalized trajectory inside $\Sigma = 0$

$$\dot{\Sigma} = a_0[S_I, \Sigma] - a_1[\Sigma] \qquad \dot{S}_I = \tfrac{1}{2} a_0[S_I, S_I] - a_1[S_I]$$

Eigenoperators inside (are not enough)

$$S = S_0 + g S_1$$

(close to fixed point)



Eigenoperator eqn:

$$\dot{S}_1 = -a_1[S_1]$$

General soln is expansion over eigenoperators with constant coefficients.

Renormalized trajectory inside $\Sigma = 0$

$$\dot{\Sigma} = a_0[S_I, \Sigma] - a_1[\Sigma] \qquad \dot{S}_I = \tfrac{1}{2}a_0[S_I, S_I] - a_1[S_I]$$

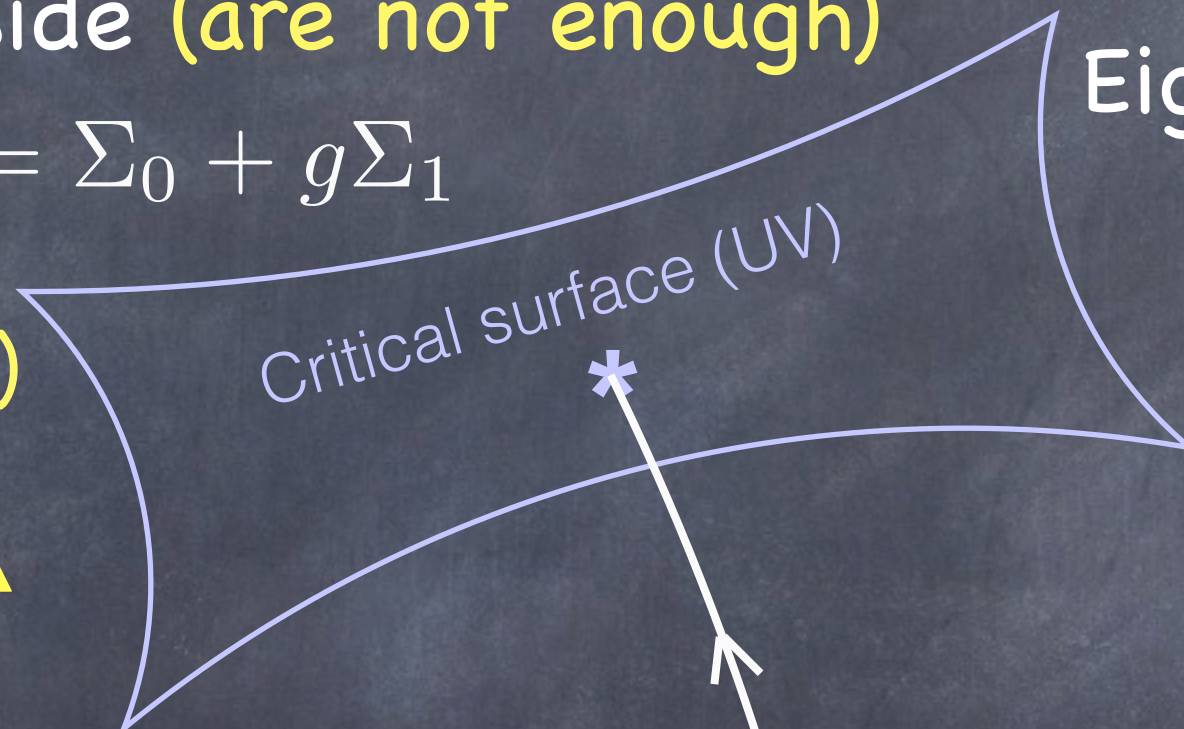
Eigenoperators inside (are not enough)

$$S = S_0 + gS_1 \quad \Sigma = \Sigma_0 + g\Sigma_1$$

(close to fixed point)

$$\hat{s}_0 = Q_0 + Q_0^- - \Delta$$

$$\Sigma_1 = \hat{s}_0\, S_1$$



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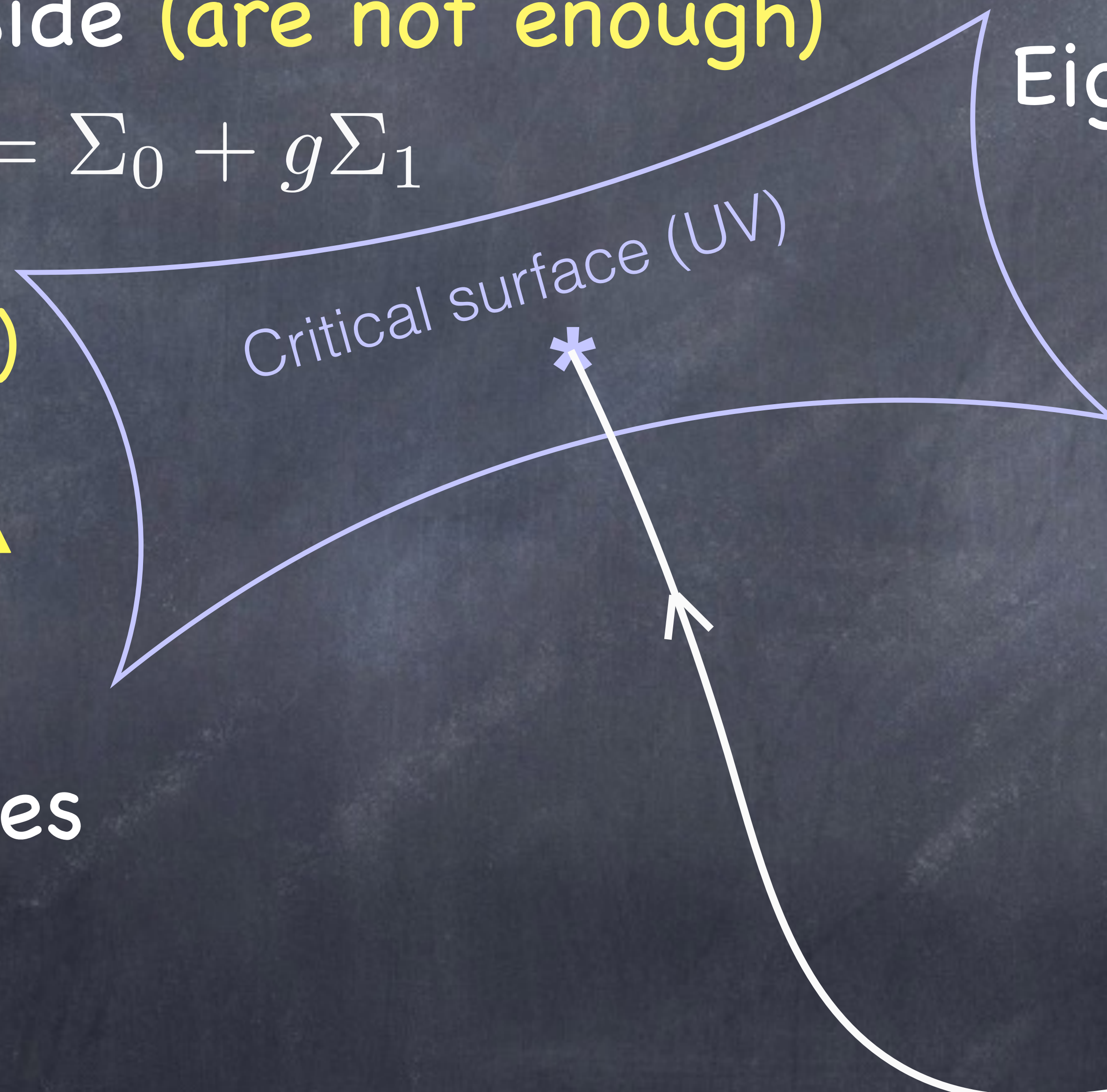

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Renormalized trajectory inside $\Sigma = 0$

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Eigenoperators inside (are not enough)

$$S = S_0 + gS_1 \quad \Sigma = \Sigma_0 + g\Sigma_1$$

(close to fixed point)

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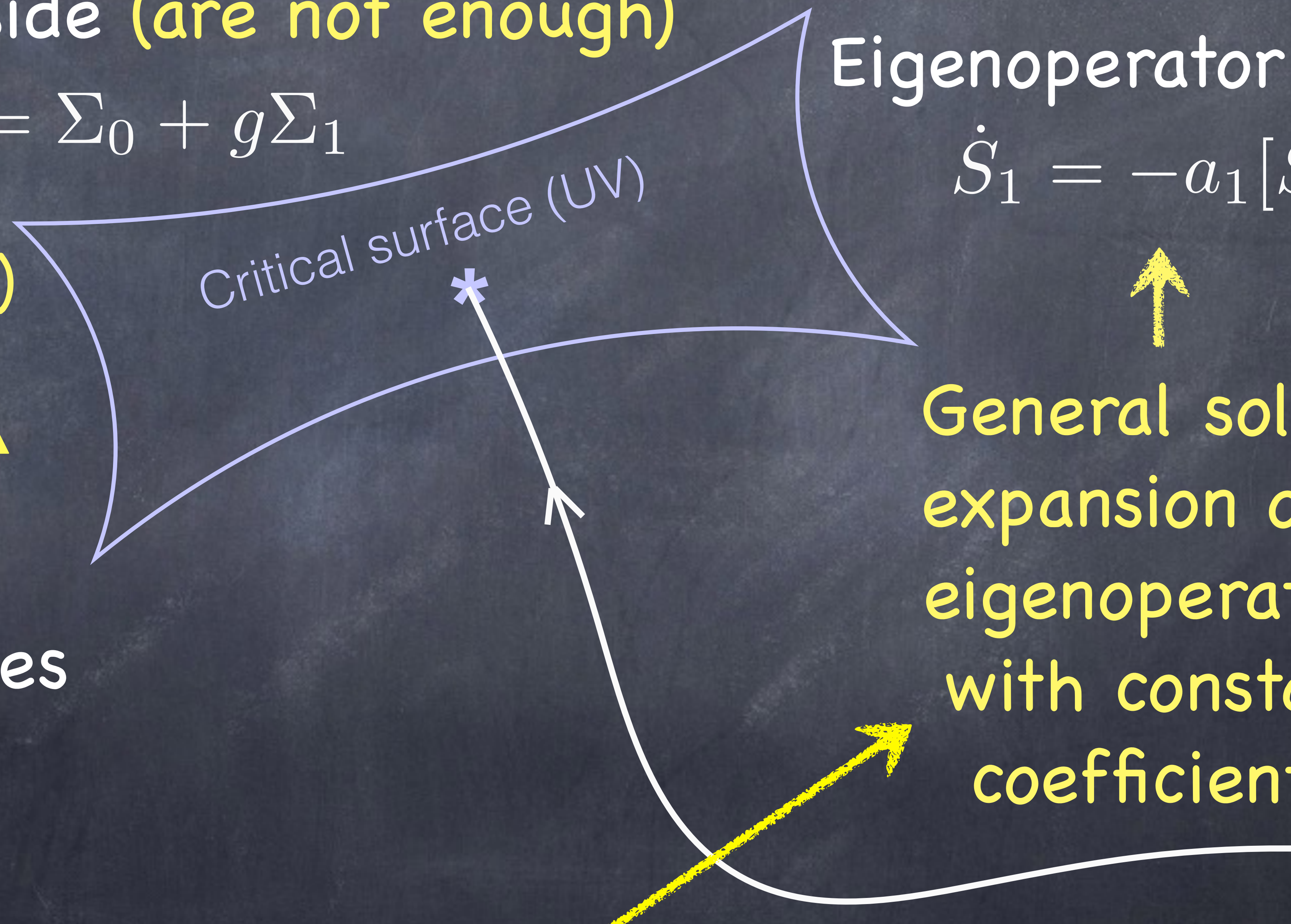

Eigenoperator eqn:

$$\dot{S}_1 = -a_1[S_1]$$

General soln is expansion over eigenoperators with constant coefficients.

Quantum BRST cohomology in this space

Renormalized trajectory inside $\Sigma = 0$

$$S = S_0 + gS_1 + g^2 S_2 + \cdots \qquad \dot{S}_n = \frac{1}{2}\sum_{m=1}^{n-1} a_0[S_{n-m}, S_m] - a_1[S_n]$$

(close to fixed point)

$$\Sigma = \Sigma_0 + g\Sigma_1 + g^2\Sigma_2 + \cdots \qquad \dot{\Sigma}_n = \sum_{m=1}^{n-1} a_0[S_{n-m}, \Sigma_m] - a_1[\Sigma_n]$$

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If QME already solved up to $\Sigma_{m<n} = 0$

then $\dot{\Sigma}_n = -a_1[\Sigma_n]$

Renormalized trajectory inside $\Sigma = 0$

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If QME already solved up to $\Sigma_{m<n} = 0$

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$\Longrightarrow \Sigma_n$ can only be violated by a linear combination of eigenoperators

Renormalized trajectory inside $\Sigma = 0$

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If QME already solved up to $\Sigma_{m<n} = 0$

then $\dot{\Sigma}_n = -a_1[\Sigma_n]$

$\Longrightarrow \Sigma_n$ can only be violated by a linear combination of eigenoperators

$$\Sigma_n = \hat{s}_0\, S_n + \frac{1}{2}\sum_{m=1}^{n-1}(S_{n-m}, S_m)$$

Renormalized trajectory inside $\Sigma = 0$

$$S = S_0 + gS_1 + g^2 S_2 + \cdots \qquad \dot{S}_n = \cancel{\frac{1}{2}\sum_{m=1}^{n-1} a_0[S_{n-m}, S_m]} - a_1[S_n]$$

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If QME already solved up to $\Sigma_{m<n} = 0$

then $\dot{\Sigma}_n = -a_1[\Sigma_n]$

$\implies \Sigma_n$ can only be violated by a linear combination of eigenoperators

$$\Sigma_n = \hat{s}_0\, S_n + \cancel{\frac{1}{2}\sum_{m=1}^{n-1}(S_{n-m}, S_m)}$$

to be repaired by a linear combination of eigenoperators

Renormalized trajectory inside $\Sigma = 0$

$$S = S_0 + gS_1 + g^2 S_2 + \cdots \qquad \dot{S}_n = \frac{1}{2}\sum_{m=1}^{n-1} a_0[S_{n-m}, S_m] - a_1[S_n]$$

$$\Sigma = \Sigma_0 + g\Sigma_1 + g^2\Sigma_2 + \cdots \qquad \dot{\Sigma}_n = \sum_{m=1}^{n-1} a_0[S_{n-m}, \Sigma_m] - a_1[\Sigma_n]$$

follows from perturbative development of the BRST cohomology

$\Longrightarrow \Sigma_n$ can only be violated by a linear combination of eigenoperators

$$\Sigma_n = \hat{s}_0\, S_n + \frac{1}{2}\sum_{m=1}^{n-1}(S_{n-m}, S_m)$$

to be repaired by a linear combination of eigenoperators

Renormalized trajectory inside $\Sigma = 0$

Only freedom is to change coeffs in linear sum over eigenoperators

Soln of flow eqn (body of the physical amplitudes) guaranteed correct up to this linear combination of eigenoperators

Fix remaining freedom with renormalization conditions

Causes coupling constants to run.

Loop expansion

space-time dimension

Eigenoperator eqn: $\dot{S}_1 = -\cancel{a_1[S_1]}$

Relevant operators built from local terms with dimension < 4

Only freedom is in these Λ-constant local terms (counter-terms)

N.B. derivative expansion property at finite Λ, crucial.

Legendre effective action

Effective average action:

$$\Gamma = \Gamma_0 + \Gamma_I\,, \qquad \Gamma_0 = \tfrac{1}{2}\,\Phi^A \triangle^{-1}_{AB}\Phi^B + \Phi^*_A R^A{}_B \Phi^B$$

Antibracket:

$$(\Xi, \Upsilon) = \frac{\partial_r \Xi}{\partial \Phi^A}\,\frac{\partial_l \Upsilon}{\partial \Phi^*_A} - \frac{\partial_r \Xi}{\partial \Phi^*_A}\,\frac{\partial_l \Upsilon}{\partial \Phi^A}$$

$$\left(\Gamma^{(2)}_{I*}\right)^A{}_B = \frac{\vec{\partial}}{\partial \Phi^*_A}\Gamma_I \frac{\overleftarrow{\partial}}{\partial \Phi^B}$$

$$\Sigma = \tfrac{1}{2}(\Gamma,\Gamma) - \mathrm{Tr}\left(K\Gamma^{(2)}_{I*}\left[1+\bar{\triangle}\Gamma^{(2)}_I\right]^{-1}\right)$$

$$\Gamma = \Gamma_0 + g\Gamma_1 + g^2\Gamma_2 + g^3\Gamma_3 + \cdots$$

$$\hat{s}_0\,\Gamma_1 = (Q_0 + Q_0^- - \Delta)\,\Gamma_1 = 0 \qquad\qquad \Delta\Gamma = \mathrm{Tr}\left(K\Gamma^{(2)}_*\right)$$

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

$$\Longrightarrow$$

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

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Ghost numbers

Yang-Mills

$2\,\mathrm{tr}\int_x \{\cdots\}$

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

$$\Longrightarrow$$

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

0

Ghost numbers

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

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0 1

Ghost numbers

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

$$\Longrightarrow$$

+1

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

0 1

Ghost numbers

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

$$\Longrightarrow \qquad -1$$

+1

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

0 1

Ghost numbers

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

$$\Gamma_0 = \tfrac{1}{2}\,(\partial_\mu A_\nu)^2 - \tfrac{1}{2}\,(\partial\cdot A)^2 + A^*_\mu \partial_\mu C$$

$$\Longrightarrow$$

+1   +1   -1

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

0   1

Ghost numbers

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

$$\Longrightarrow$$

-1

+1   +1

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

0   1   -2

Ghost numbers

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

$$\Gamma_0 = \tfrac{1}{2}\,(\partial_\mu A_\nu)^2 - \tfrac{1}{2}\,(\partial\cdot A)^2 + A^*_\mu \partial_\mu C$$

$$\Longrightarrow$$

+1 +1 -1

$$Q_0\,A_\mu = \partial_\mu C\,, \qquad Q_0^-\,A^*_\mu = \Box\,A_\mu - \partial_\mu\partial\cdot A\,, \qquad Q_0^-\,C^* = -\partial\cdot A^*$$

0 1 -2

Ghost numbers Dimension

Yang-Mills

$2\,\mathrm{tr}\int_x \{\cdots\}$

$$\Gamma_0 = \tfrac{1}{2}\,(\partial_\mu A_\nu)^2 - \tfrac{1}{2}\,(\partial\cdot A)^2 + A^*_\mu \partial_\mu C$$

+1

+1

$\Longrightarrow$ -1

+1

$$Q_0\,A_\mu = \partial_\mu C\,, \qquad Q_0^-\,A^*_\mu = \Box\,A_\mu - \partial_\mu\partial\cdot A\,, \qquad Q_0^-\,C^* = -\partial\cdot A^*$$

0 1 -2

Ghost numbers Dimension

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

2

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

+1

+1

$\Longrightarrow$ -1

+1

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

0 1 -2

Ghost numbers Dimension

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + \overset{2}{A^*_\mu}\partial_\mu C$$

+1 +1 +1 +1 $\Longrightarrow$ -1

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

0 1 -2

Ghost numbers Dimension

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

2

$$\Gamma_0 = \tfrac{1}{2}\,(\partial_\mu A_\nu)^2 - \tfrac{1}{2}\,(\partial\cdot A)^2 + A^*_\mu \partial_\mu C$$

+1
+1

+1
+1

$\Longrightarrow$

-1

2

$$Q_0\,A_\mu = \partial_\mu C\,, \qquad Q_0^-\,A^*_\mu = \Box\,A_\mu - \partial_\mu\partial\cdot A\,, \qquad Q_0^-\,C^* = -\partial\cdot A^*$$

0 1

-2

Ghost numbers Dimension

Yang-Mills

$2\,\mathrm{tr}\int_x \{\cdots\}$

2

$$\Gamma_0 = \tfrac{1}{2}\,(\partial_\mu A_\nu)^2 - \tfrac{1}{2}\,(\partial\cdot A)^2 + A^*_\mu \partial_\mu C$$

+1 +1 +1 +1 $\Longrightarrow$ -1 2

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

0 1 -2

Ghost numbers

Dimension

Anti-field number: treats pieces differently!

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

2

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

+1 +1 +1 +1 $\Longrightarrow$ -1 2

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

+0 0 1 -1 1 -2

0 0

Ghost numbers

Dimension

Anti-field number: treats pieces differently!

Yang-Mills

$2\,\mathrm{tr}\int_x \{\cdots\}$

2

$$\Gamma_0 = \tfrac{1}{2}\,(\partial_\mu A_\nu)^2 - \tfrac{1}{2}\,(\partial\cdot A)^2 + A^*_\mu\partial_\mu C$$

+1
+1

+1
+1

$\Longrightarrow$

-1

2

$$Q_0\,A_\mu = \partial_\mu C\,, \qquad Q_0^-\,A^*_\mu = \Box\,A_\mu - \partial_\mu\partial\cdot A\,, \qquad Q_0^-\,C^* = -\partial\cdot A^*$$

+0 0 1 -1 1

-2
2

0 0

Ghost numbers

Dimension

Anti-field number:
treats pieces differently!

Yang-Mills

$2\,\mathrm{tr}\int_x \{\cdots\}$

2

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

+1
+1

+1
+1

$\Longrightarrow$

-1

2

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

+0 0 1 -1 1

-2
2

0 0

Ghost numbers

Dimension

Anti-field number: treats pieces differently!

$$\hat{s}_0\, \Gamma_1 = 0$$

Want a solution up to $\Gamma_1 \mapsto \Gamma_1 + \hat{s}_0 \mathcal{K}$

Yang-Mills

$2\,\mathrm{tr}\int_x \{\cdots\}$

2

$$\Gamma_0 = \tfrac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \tfrac{1}{2}\left(\partial\cdot A\right)^2 + A^*_\mu \partial_\mu C$$

+1 +1

+1 +1 $\Longrightarrow$ -1 2

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

+0 0 1 -1 1 -2 2

0 0

Ghost numbers Dimension Anti-field number: treats pieces differently!

$$\hat{s}_0\, \Gamma_1 = 0$$

Want a solution up to $\Gamma_1 \mapsto \Gamma_1 + \cancel{\hat{s}_0 \mathcal{K}}$

But $\mathcal{K}$ must be Λ independent, max dimension 3, ghost number -1, and have 3 fields!

Yang-Mills

$$2\,\mathrm{tr}\int_x \{\cdots\}$$

2

$$\Gamma_0 = \tfrac{1}{2}\,(\partial_\mu A_\nu)^2 - \tfrac{1}{2}\,(\partial\cdot A)^2 + A^*_\mu \partial_\mu C$$

+1 +1 $\Longrightarrow$ -1 2

+1 +1

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

+0 0 1 -1 1 -2

0 0 2

Ghost numbers Dimension Anti-field number: not conserved by action!

$$\hat{s}_0\, \Gamma_1 = 0$$

For same reasons, Γ_1 has maximum anti-field number 2

where it is unique: $\Gamma_1^2 = -iC^*C^2$

Thus: $\Gamma_1 = \Gamma_1^2 + \Gamma_1^1 + \Gamma_1^0$

is the unique extension of Maxwell theory

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

$$\hat{s}_0\, \Gamma_1 = (Q_0 + Q_0^-)\, \Gamma_1 = 0 \qquad \Gamma_1^2 = -iC^*C^2$$

Descendents:

$$Q_0 \Gamma_1^2 = 0 \qquad \checkmark$$

$$Q_0 \Gamma_1^1 = -Q_0^- \Gamma_1^2$$

$$i\partial\cdot A^* C^2 = -iA^*_\mu\{\partial_\mu C, C\} = -iA^*_\mu\{Q_0 A_\mu, C\} = iQ_0\left(A^*_\mu[A_\mu, C]\right)$$

$$\Longrightarrow \quad \Gamma_1^1 = -iA^*_\mu[A_\mu, C]$$ unique `deformation' to $A^*_\mu D_\mu C$

$$Q_0 \Gamma_1^0 = -Q_0^- \Gamma_1^1$$

$$\Longrightarrow \quad \Gamma_1^0 = -i\partial_\mu A_\nu [A_\mu, A_\nu]$$ unique cubic interaction

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

$$\hat{s}_0\, \Gamma_1 = (\,Q_0 + Q_0^-\,)\, \Gamma_1 = 0 \qquad \Gamma_1^2 = -iC^*C^2$$

Descendents:

$$Q_0 \Gamma_1^2 = 0 \qquad \checkmark$$

$$Q_0 \Gamma_1^1 = -Q_0^- \Gamma_1^2$$

$$i\partial\cdot A^* C^2 = -iA^*_\mu \{\partial_\mu C, C\} = -iA^*_\mu \{Q_0 A_\mu, C\} = iQ_0 \left(A^*_\mu [A_\mu, C]\right)$$

$\Longrightarrow \quad \Gamma_1^1 = -iA^*_\mu [A_\mu, C]$ unique `deformation' to $A^*_\mu D_\mu C$

$$Q_0 \Gamma_1^0 = -Q_0^- \Gamma_1^1$$

$\Longrightarrow \quad \Gamma_1^0 = -i\partial_\mu A_\nu [A_\mu, A_\nu]$ unique cubic interaction

$$s_0\, \Gamma_{2,\mathrm{cl}} = (Q_0 + Q_0^-\,)\, \Gamma_{2,\mathrm{cl}} = -\tfrac{1}{2}(\Gamma_1, \Gamma_1) = Q_0 \left(-\tfrac{1}{4}[A_\mu, A_\nu]^2\right)$$

unique quartic interaction

$$Q_0\, A_\mu = \partial_\mu C\,, \qquad Q_0^-\, A^*_\mu = \Box\, A_\mu - \partial_\mu \partial\cdot A\,, \qquad Q_0^-\, C^* = -\partial\cdot A^*$$

<u>Define</u> coupling to be coefficient of unique:

$$g(\Lambda)\,\Gamma_1 = Z_g^{-1}(\Lambda)\, g\,\Gamma_1$$

Only other possibility are s_0-closed two-point vertices.

Uniquely two options:

$$\tfrac{1}{2} z_A (Q_0 + Q_0^-)(A^*_\mu A_\mu) = -\tfrac{1}{2} z_A \left\{ (\partial_\mu A_\nu)^2 - (\partial\cdot A)^2 \right\} + \tfrac{1}{2} z_A A^*_\mu \partial_\mu C$$

$$\tfrac{1}{2} z_C (Q_0 + Q_0^-)(C^* C) = \tfrac{1}{2} z_C\, A^*_\mu \partial_\mu C$$

But these are s_0-exact, so just canonical reparametrisation:

$$\mathcal{K} = Z_E^{\frac{1}{2}} \Phi^*_E \Phi^E_{(r)}$$

$$\Phi^E = \frac{\partial_l}{\partial \Phi^*_E} \mathcal{K}[\Phi_{(r)}, \Phi^*]\,, \;\; \Phi^*_{(r)E} = \frac{\partial_r}{\partial \Phi^E_{(r)}} \mathcal{K}[\Phi_{(r)}, \Phi^*]$$

$$A_\mu = Z_A^{\frac{1}{2}} A_{(r)\mu}\,, \quad A^*_\mu = Z_A^{-\frac{1}{2}} A^*_{(r)\mu}\,, \quad C = Z_C^{\frac{1}{2}} C_{(r)}\,, \quad C^* = Z_C^{-\frac{1}{2}} C^*_{(r)}$$

Thus RG flow generates flow in the freely variable parts:

$$\tfrac{1}{2} Z_A^{-1} A_\mu(-\Box\delta_{\mu\nu} + \partial_\mu\partial_\nu)A_\nu \;+\; Z_A^{\frac{1}{2}} Z_C^{-\frac{1}{2}} A^*_\mu \partial_\mu C$$

$$-\, igZ_g^{-1} Z_C^{-\frac{1}{2}} \left(C^* C^2 + A^*_\mu[A_\mu, C]\right) - igZ_g^{-1} Z_A^{-\frac{3}{2}} \partial_\mu A_\nu [A_\mu, A_\nu]$$

$$-\;\frac{1}{4} g^2 Z_g^{-2} Z_A^{-2} [A_\mu, A_\nu]^2$$

Standard parameterisation:

$$Z_3 = Z_A\,, \;\; \tilde{Z}_3 = Z_C^{\frac{1}{2}} Z_A^{-\frac{1}{2}}\,, \;\; Z_1 = Z_g Z_A^{\frac{3}{2}}\,, \;\; \tilde{Z}_1 = Z_g Z_C^{\frac{1}{2}}\,, \;\; Z_4 = Z_g^2 Z_A^2$$

$$\frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3} = \frac{Z_4}{Z_1} = Z_g Z_A^{\frac{1}{2}}$$

Slavnov-Taylor identities!

Conclusions.

- Despite breaking by cutoff BRST invariance still very much present in flow equation.
- Regularisation of Δ plus existence of derivative expansion, defines quantum BRST cohomology.
- Together yields a formalism that is still elegant and not much harder than Dim Reg.