Scheme Independent Asymptotic Freedom for Gauged-Higgs-Yukawa Models

Presented by: **Alessandro Ugolotti***

based on Refs. [arXiv: 1804.09688, 1901.08581]

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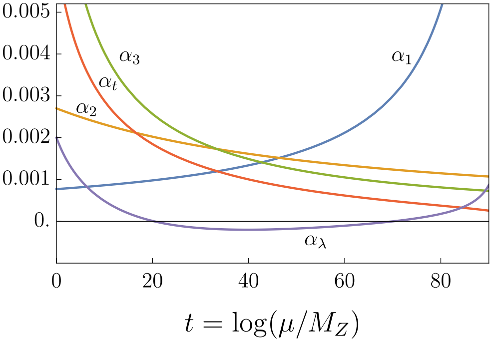
Overview on the Problematics of the SM

The gauge couplings seem to unify at the GUT scale but then divergences appear.



[Fabbrichesi,Percacci et al. JHEP 11 ('18)]

- Landau-pole in the U(1)-gauge sector as well as for the Higgs-scalar sector. Triviality problem.
- $\lambda < 0$ at a relative "small" energy scale. Instability of $V(\phi)$.

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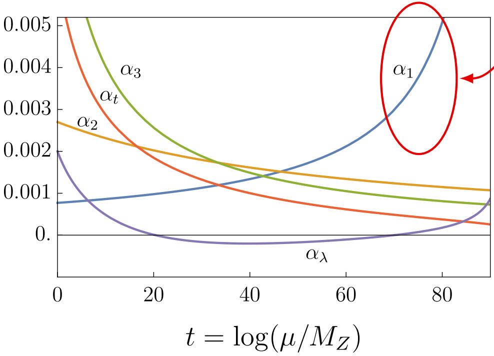

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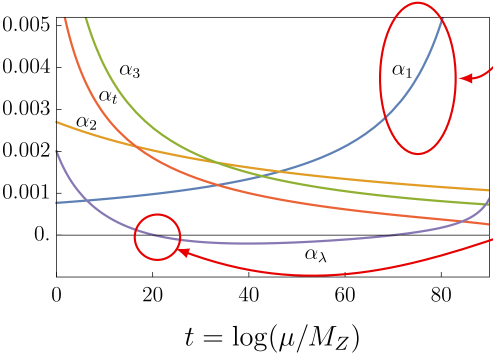

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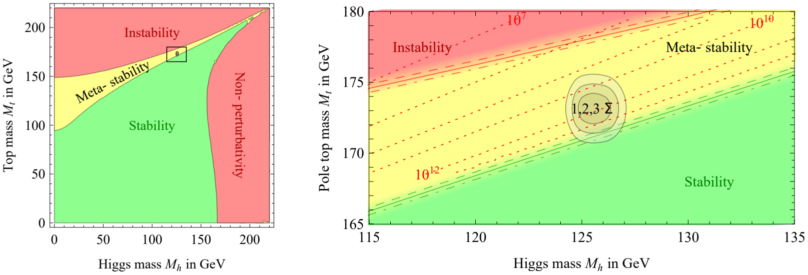
- Landau-pole in the U(1)-gauge sector as well as for the Higgs-scalar sector. Triviality problem.
- $\lambda < 0$ at a relative "small" energy scale. Instability of $V(\phi)$.

- The SM is an *Effective Theory* and can be considered valid up to a certain scale Λ, typically assumed to be $M_{\rm Pl}$.
- Given this Λ one can draw a phase diagram for the SM Higgs potential.



[Degrassi, Giudice, Isidori, Strumia et al. JHEP 08 ('12)]

- Absolute stability of the Higgs potential is excluded at 98% C.L. for $M_h < 126$ GeV. [Degrassi, Giudice, Isidori, Strumia et al. JHEP 08 ('12)]

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- Stability: Is it possible to have $\lambda\phi^4 > 0 \ \forall \ \Lambda$ and $\forall \ \phi$?
- Triviality: Is it possible to have a UV-complete theory?

Models under Investigation

Solve the $\mathrm{U}(1)$ is cumbersome → `We neglect it`

↓

Only the non-Abelian gauge group of SM is retain → `The Coleman-Gross Theorem garanties that AF trajectories exist`

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$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L$ gauge-group coupled to ϕ and ψ_{top}

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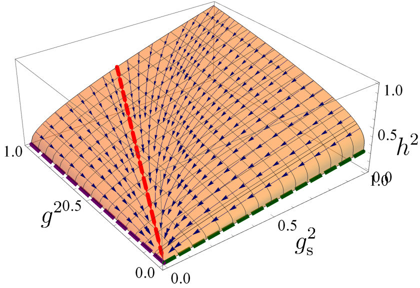
Within standard perturbation theory the couplings retained are 4:

- g, the charge of $\mathrm{SU}(2)_L$
- g_{s}, the charge of $\mathrm{SU}(3)_c$
- h, the top-Yukawa coupling
- λ, the ϕ^4 coupling

Total Asymptotic Freedom: known solutions

Already at 1-Loop, perturbative renormalizabile analysis is capable to reveal asymptotic freedom in all the couplings (g, g_s, h_top, λ). [Gross,Wilczek '73; Cheng,Eichten,Li '74; Chang '74; Fradkin,Kalashnikov '75; Chang,Perez-Mercader '78; Bais,Weldon '78; Callaway '88; Giudice,Isidori,Salvio,Strumia '15; Holdom,Ren,Zhang '15]



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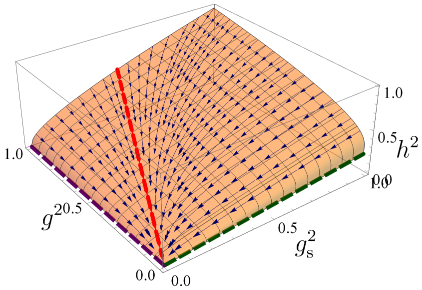

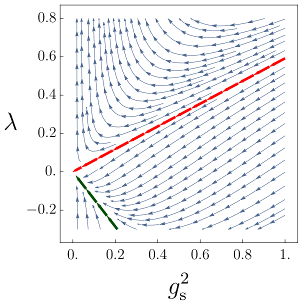

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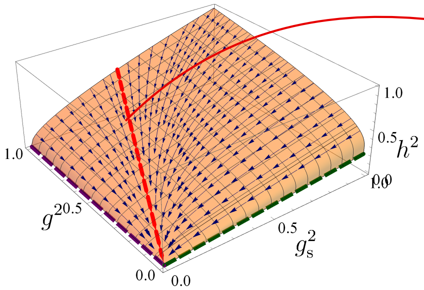

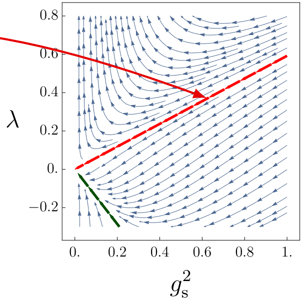

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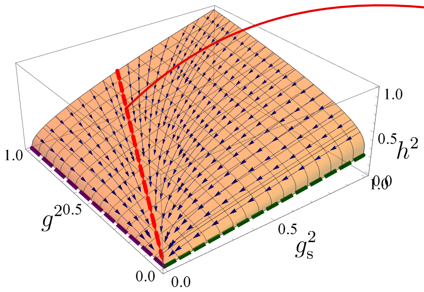
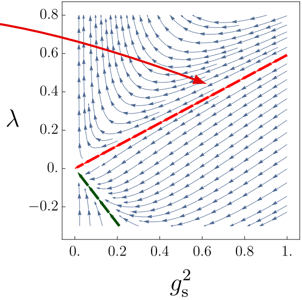




Red trajectories: $h_{\rm top}^2 \sim g^2 \sim g_{\rm s}^2 \sim \lambda$, or vice versa when $g_{\rm s}^2 \to 0$

- $g^2/g_{\rm s}^2 \to \#_g$
- $\lambda/g_{\rm s}^2 \to \#_\lambda$
- $h_{\rm top}^2/g_{\rm s}^2 \to \#_{\rm top} < \infty$

Going beyond Perturbative Renormalizability

- HOW?

New Ingredients in the Scalar Sector

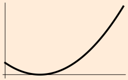
Higher-dimensional (irrelevant) operators are included

$\phi^6, \phi^8, \cdots$ into $V(\phi)$

Functional RG for the full $V(\phi)$

The "Asymptotic Symmetry" assumption is relaxed

vev can be $\neq 0$ $\forall$ momentum fluctuations



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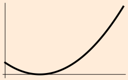

- MOTIVATION: asymptotic freedom can be found in models which are not AF in standard perturbation theory [Gies,Zambelli PR D92 ('15), PR D96 ('17)]

Implementing the functional set-up

- As we observed: $\lambda \sim g_{\mathrm{s}}^2$ in the UV limit.
- How can be achieved that at a functional level?
  We need a "smart" field rescaling:

$$x \equiv g_{\mathrm{s}}^{2\boldsymbol{P}}(\phi^\dagger\phi), \qquad f(x) = u(\phi^\dagger\phi), \qquad f'(\boldsymbol{x}_0) = 0.$$

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For example in a polynomial truncation the quartic interaction reads:

$$u = \cdots + \frac{1}{2}\lambda(\phi^\dagger\phi)^2 + \cdots = \cdots \frac{1}{2}\underbrace{\frac{\lambda}{g_{\mathrm{s}}^{4P}}}_{\xi_2}\underbrace{g_{\mathrm{s}}^{4P}(\phi^\dagger\phi)^2}_{x^2}\cdots = f(x).$$

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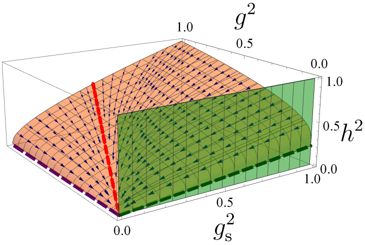
- Projecting the Exact RG equation onto constant field configuration:

$$\partial_t f(x) = -4f(x) + (2+\eta_x)xf'(x) + \text{loops}$$

Novel AF Solutions for a Toy Model

Consider the limiting $\mathbb{Z}_2$-Yukawa-QCD case ($g^2 = 0$):

$$\text{loops} = \underbrace{\frac{1/(32\pi^2)}{1 + 3\xi_2 g_s^{2P} x}}_{\text{scalar loop}} - \underbrace{\frac{27/(8\pi^2)}{9 + 2g_s^{2-2P} x}}_{\text{fermion loop}}$$



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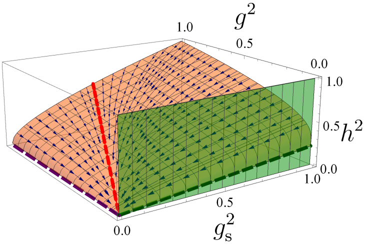

The FP equation $\partial_t f(x) = 0$ has an analytic solution:

$$f(x) = \boldsymbol{C_f}\, x^{\frac{4}{2+\eta_x}} + \frac{1}{128\pi^2}\, {}_2F_1\left[1, -\frac{4}{2+\eta_x}, \frac{-2+\eta_x}{2+\eta_x}, -3\xi_2 g_\mathrm{s}^{2\boldsymbol{P}} x\right]$$
$$- \frac{3}{32\pi^2}\, {}_2F_1\left[1, -\frac{4}{2+\eta_x}, \frac{-2+\eta_x}{2+\eta_x}, -\frac{2}{9} g_\mathrm{s}^{2-2\boldsymbol{P}} x\right].$$

Parameters

$P,\, x_0,\, \xi_2,\, C_f$

Conditions

$f'(x_0) = 0,\; f''(x_0) = \xi_2$

$$P \in \left[\tfrac{1}{4}, \tfrac{1}{2}\right],\; \xi_2 > \xi_2^{\min}$$

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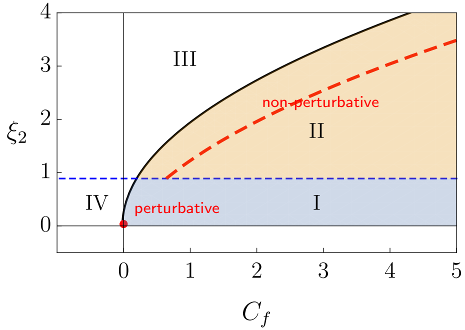
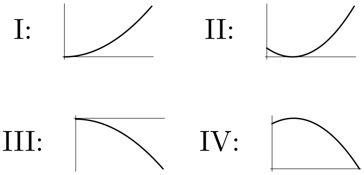
- Example for $P = 1/2$: Line of new AF and stable solutions!



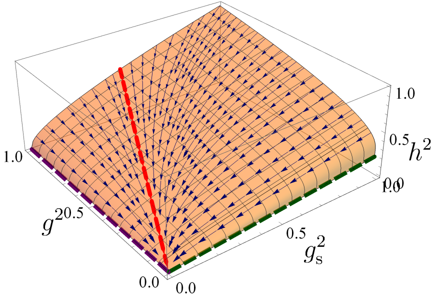


Scheme (in)dependence within FRG?

- We need to access ALL possible Mass-dependent Scheme! How?

Let us go back to the more general $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(3)_c$ model:

$$\text{loops} = \frac{1}{16\pi^2}\left[\underbrace{l_0^{(\mathrm{H})}(z_\mathrm{H})}_{\text{Higgs loop}} + \underbrace{3 l_0^{(\theta)}(z_\theta)}_{\text{Goldstone loop}} + \underbrace{9 l_0^{(\mathrm{W})}(z_\mathrm{W})}_{\text{Gauge loop}} - \underbrace{12 l_0^{(\mathrm{F})}(z_\mathrm{F})}_{\text{Fermion loop}}\right],$$



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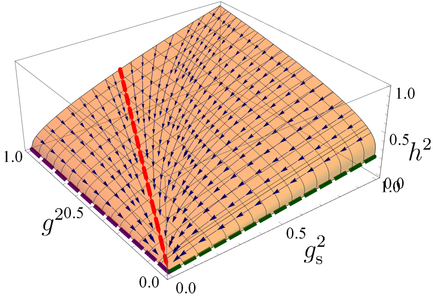

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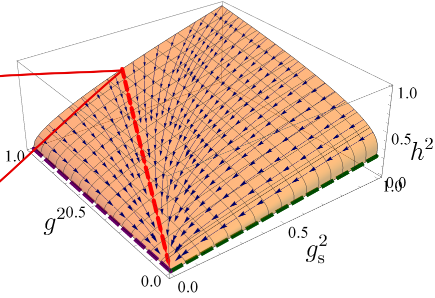

Assumption in the UV

$\boxed{\lim_{g_{\rm s}^2 \to 0} z_{(\dots)} = 0}$ $\longrightarrow$ Taylor expansion of $l_0^{(\dots)}(z)$

At the linear order:

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The coefficients $\mathcal{A}_\Phi > 0$ encode for ALL possible FRG regulators. Indeed

$$\mathcal{A}_\Phi = -\frac{1}{16\pi^2}\left[\partial_z l_0^{(\Phi)}(z)\right]_{z=0} = \frac{1}{2k^2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{\tilde{\partial}_t P_\Phi(p^2)}{\left[P_\Phi(p^2)\right]^2},$$

where P_Φ is the regularized kinetic term.

Simple example: $P = 1/4$

- Only the scalar loops contribute in the UV limit

$$\partial_t f = -4f + (2+\eta_x)xf' - g_{\mathrm{s}}^{1/2}\left[\mathcal{A}_{\mathrm{H}}(f' + 2xf'') + 3\mathcal{A}_\theta f'\right].$$

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possesses a non-trivial minimum !

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Conclusions & Outlook

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$\overline{\text{MS}}$	$P = 1$, $Q, \hat{\kappa}$	$P = 1$, $Q, \hat{\kappa}$	$P = 1/2$, $Q, \breve{\kappa}$
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- Stability of $V(\phi)$ for any amplitude of the fluctuation field.
- Threshold effects do invalidate conventional DER analysis.
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Conclusions & Outlook

	$\mathrm{SU(2)_L \times SU(3)_c}$	$\mathbb{Z}_2$-Yukawa-QCD	non-Abelian Higgs
$\overline{\text{MS}}$	$P = 1$, $Q, \hat{\kappa}$	$P = 1$, $Q, \hat{\kappa}$	$P = 1/2$, $Q, \breve{\kappa}$
FRG	$P \in [1/4, 1/2]$, ξ_2	$P \in [1/4, 1/2]$, ξ_2	$P \in (0, +\infty)$, ξ_2

- Existence of two-parameters (P, ξ_2) family of NEW asymptotically free solutions.
- Stability of $V(\phi)$ for any amplitude of the fluctuation field.
- Threshold effects do invalidate conventional DER analysis.
- This is a scheme-independent phenomenon.
- A change of scheme induces a map of the coupling space of initial conditions

$\Rightarrow$ Solve the RG flow down to $k = 0$ to get a prediction of $M_{\mathrm{H}}/M_{\mathrm{W}}$.

$\Rightarrow$ Inclusion of the U(1)

Thank you for your attention!

Simple example: $P = 1/2$

- All loops contribute at the leading order in the g_s-expansion of $\partial_t f(x)$.
- The Fixed Point solution is still a quadratic polynomial:

$$f_\mathrm{FP}(x) = \frac{\boldsymbol{\xi}_2}{2}\, x^2 - \frac{3g_\mathrm{s}}{4}\, x \left[2\boldsymbol{\xi}_2(\mathcal{A}_\mathrm{H} + \mathcal{A}_\theta) + 3\mathcal{A}_\mathrm{W}\#_g - 8\mathcal{A}_\mathrm{F}\#_\mathsf{top}\right],$$

which possesses a non-trivial minimum !

$$x_0 = \frac{3g_\mathrm{s}}{4\xi_2}\, [2\xi_2(\mathcal{A}_\theta + \mathcal{A}_\mathrm{H}) + 3\mathcal{A}_\mathrm{W}\#_g - 8\mathcal{A}_\mathrm{F}\#_\mathsf{top}] = g_\mathrm{s}\, \boldsymbol{\kappa},$$

positive if

$$\frac{8\mathcal{A}_\mathrm{F}\hat{h}_*^2 - 3\mathcal{A}_\mathrm{W}\hat{g}_*^2}{2(\mathcal{A}_\theta + \mathcal{A}_\mathrm{H})} < \xi_2$$

Regulator dependent!

$\overline{\text{MS}}$ Scheme

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The $\overline{\text{MS}}$ scheme can be formulated in a functional way [O'Dwyer,Osborn AnnPhys. 323 ('08); Codello,Safari,Vacca,Zanusso EPJ C ('17)]

$$l_0^{(\overline{\text{MS}})d=4}(z) = \frac{z^2}{2} \quad \Longrightarrow \quad \mathcal{A}_\Phi^{\overline{\text{MS}}} = 0.$$

It seems that NO AF solutions are present in this scheme.

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REASON: $z \not\to 0$ as $g_\mathrm{s} \to 0$. Thus the correct definition is:

$$\mathcal{A}_\Phi^{\overline{\text{MS}}}(x_0) = -\frac{1}{16\pi^2}\left[\partial_z l_0^{(\overline{\text{MS}})}(z)\right]_{x=x_0} = \frac{z_0}{16\pi^2}.$$

$\overline{\text{MS}}$ Scheme. Example $P = 1$.

Only the gauge-boson and fermion loops contribute to β_f:

$$\partial_t f(x) = -4f(x) + 2xf'(x) + \frac{9}{32\pi^2} z_{\mathrm{W}}^2 - \frac{3}{8\pi^2} z_{\mathrm{F}}^2$$

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$$f_{\mathrm{FP}}(x) = C_f x^2 - \frac{3(3\#_g^2 - 16\#_{\text{top}}^2)}{256\pi^2} x^2 \ln(x)$$

The consistency conditions $f'(x_0) = 0$ and $f''(x_0) = \xi_2$ implies that $C_f = C_f(x_0)$ such that:

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$$\xi_2 = \frac{3(16\#_{\mathrm{top}}^2 - 3\#_g^2)}{128\pi^2} > 0$$

The positivity of ξ_2 is fulfilled by the SM ! x_0 remains unconstraint

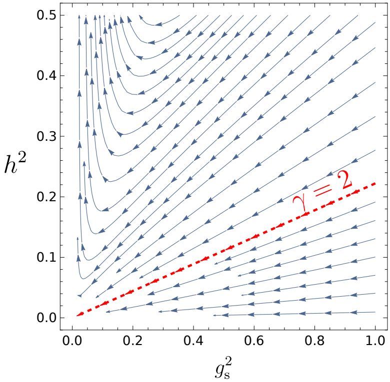
Cheng-Eichten-Li solution PR D9, 2259 (1974)

The $\mathbb{Z}_2$-**Yukawa-QCD** model represents a toy model for the SM sub-sector retaining only **Higgs** (ϕ), **Top quark** (ψ) and **Gluons** (A_i^μ)

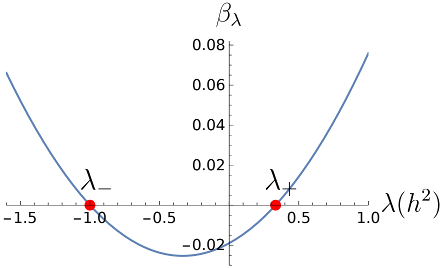
$$S = \int_x \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\bar{m}}{2}\phi^2 + \frac{\bar{\lambda}}{8}\phi^4 + \bar{\psi}\mathrm{i}\not{D}\psi + \frac{\mathrm{i}\bar{h}}{\sqrt{2}}\phi\bar{\psi}\psi \right.$$
$$\left. + \frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} + \frac{1}{2\alpha}(\partial_\mu A_i^\mu)^2 + \bar{\eta}^i\partial^\mu\nabla^{ij}_\mu\eta^j \right].$$

In the Deep Euclidean Regime $\bar{m} \to 0$: $\boldsymbol{h}$, $\boldsymbol{\lambda}$ and $\boldsymbol{g}_\mathrm{s}$ (perturbative renormalized coupling).

Cheng-Eichten-Li solution



AF region: $h^2 \underset{g_s \to 0}{\sim} g_s^{2\gamma}$



CEL solution: λ_+

Cheng-Eichten-Li solution

Quasi-fixed point (QFP) criteria[a]

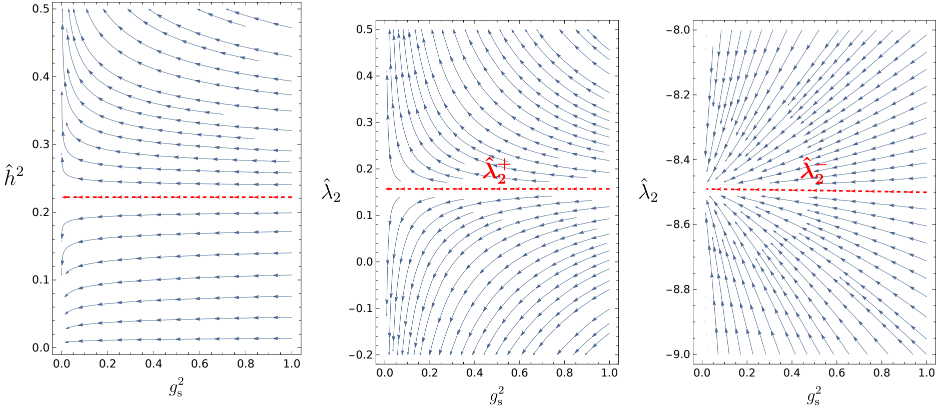
[a] D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973)

- Asymptotically Free (AF) trajectories can be detected via a suitable rescaling of the couplings

$$\left.\begin{array}{l} h^2 \underset{g_{\rm s}^2\to 0}{\sim} \# \, g_{\rm s}^2 \\ \lambda \underset{g_{\rm s}^2\to 0}{\sim} \# \, g_{\rm s}^2 \end{array}\right\} \quad \Longrightarrow \quad \left\{\begin{array}{l} \hat{h}^2 \equiv \dfrac{h^2}{g_{\rm s}^2} \xrightarrow[g_{\rm s}^2\to 0]{} \# \\ \hat{\lambda}_2 \equiv \dfrac{\lambda}{g_{\rm s}^2} \xrightarrow[g_{\rm s}^2\to 0]{} \# \end{array}\right.$$

Cheng-Eichten-Li solution

- The RG flows of $\hat{h}^2(g_s^2)$ and $\hat{\lambda}_2(g_s^2)$ have **constant QFP solutions**.



Effective field theory analysis including thresholds

The functional RG flow equation for the full dimensionless renormalized potential in $d = 4$, $N_{\text{color}} = 3$ and $N_{\text{flavor}} = 6$ is

$$\partial_t u = \underbrace{-4u(\rho) + (2+\eta_\phi)\rho u'(\rho)}_{\text{scaling part}} \underbrace{+\frac{(32\pi^2)^{-1}}{1+u'(\rho)+2\rho u''(\rho)}}_{+I_0^{(B)}(\omega)} \underbrace{-\frac{3(8\pi^2)^{-1}}{1+h^2\rho}}_{-I_0^{(F)}(\omega_1)},$$

where $\rho \sim \phi^2/2$ and $\omega = u'(\rho) + 2\rho u''(\rho)$.

Effective field theory analysis including thresholds

1. **Polynomial truncation** assuming to be in the SSB regime

$$u(\rho) = \sum_{n=2}^{N_p} \frac{\lambda_n}{n!}(\rho - \kappa)^n \, .$$

2. **Generalized boundary conditions**[1] are introduced
   - Arbitrary rescaling P for λ_2,
   - λ_{N_p+1} as a free parameter

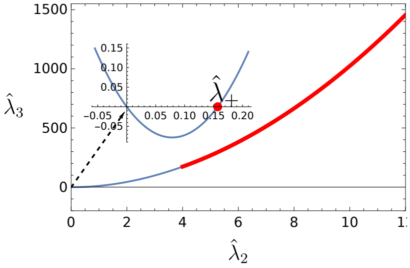
$$\hat{\lambda}_2 \equiv \frac{\lambda_2}{\boldsymbol{h^{4P}}} \, , \qquad \hat{\lambda}_{n>2} \equiv \frac{\lambda_n}{h^{2P_n}} \, .$$

[1] **H. Gies and L. Zambelli, Phys. Rev. D96, 025003 (2017)**

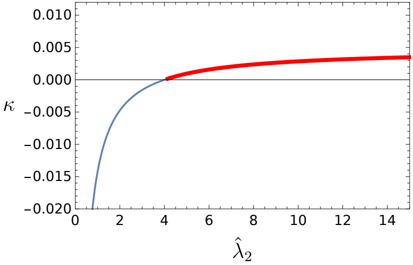
Effective field theory analysis including thresholds

SOLUTIONS for $P = 1/2$ ($P_3 = 2$) and $N_p = 2$

1. CEL solution: $\hat{\lambda}_2^+$



2. **New solutions**: $\kappa \neq 0$



Functional Approach

Quasi-fixed point (QFP) criteria

1. Its functional implementation requires a field rescaling

$$x = \boldsymbol{h^{2P}}\rho\,, \qquad f(x) = u(\rho)\,.$$

   For example: $\xi_2 = \lambda_2 h^{-4P}$.

2. We need to solve the non linear differential equation

$$0 = \partial_t f(x) = \partial_t u(\rho) - \boldsymbol{P\frac{\partial_t h^2}{h^2}} x f'(x)$$

Functional Approach

ϕ^4-DOMINANCE APPROXIMATION

3. **Assumption:** in the UV the scalar fluctuations are dominated by the ϕ^4-interaction.

$$u(\rho) \sim \frac{\lambda_2}{2}\rho^2 \quad \Longrightarrow \quad l_0^{(B)}(\omega) \rightsquigarrow \frac{1}{32\pi^2}\frac{1}{1+3h^{2P}\xi_2 x}$$

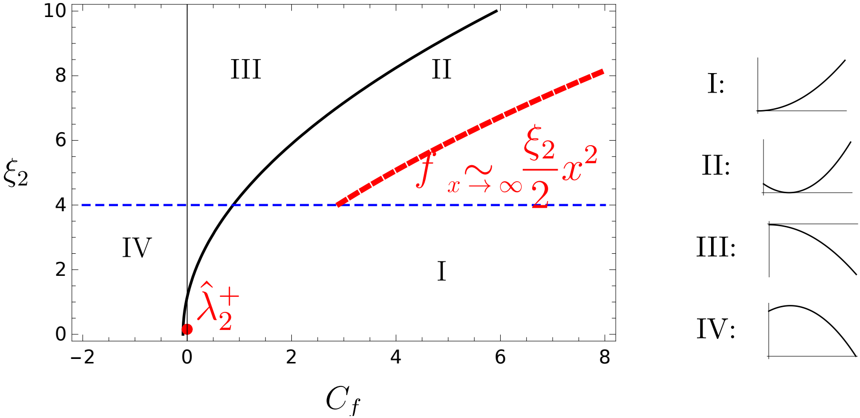
4. $\partial_t f(x) = 0$ has an **Analytic Solution:**

$$\begin{aligned} f(x) = C_f\, x^{4/d_x} &+ \#\, {}_2F_1\left[1, b(h), c(h), -3\xi_2 h^{2P} x\right] \\ &- \#\, {}_2F_1\left[1, b(h), c(h), -h^{2-2P} x\right] \end{aligned}$$

5. **Conditions:** $f'(x_0) = 0$ and $f''(x_0) = \xi_2$.

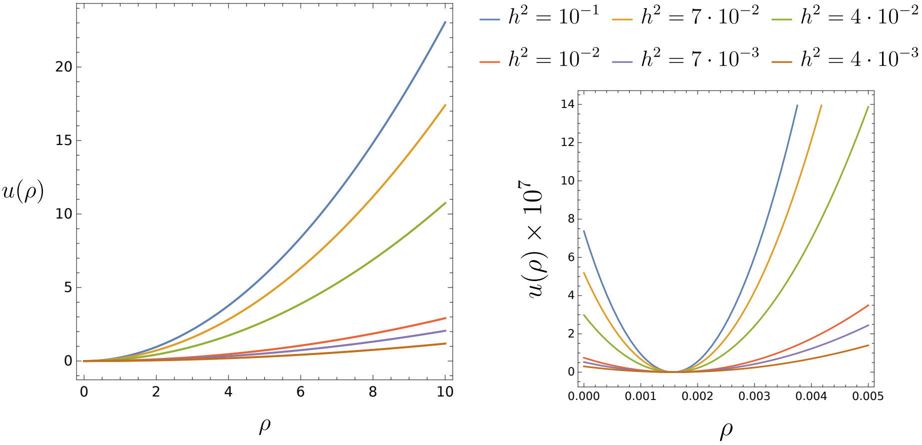
Functional Approach

SOLUTIONS for $P = 1/2$ in the $h^2 \to 0$ limit



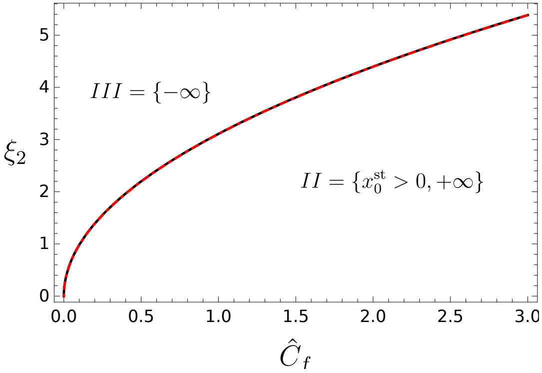
Functional Approach

NEW SOLUTIONS for $P = 1/2$ with $\kappa \neq 0$



ϕ^4-dominance approximation: $P \in (1/4, 1/2)$

- In the (C_f, ξ_2)-plane there is a one-parameter family of QFPs solutions



$$f(x) \underset{x\to\infty}{\sim} 0$$