One-loop Effective Action Of Quantum Gravity With Fakeons And Its Phenomenology

Marco Piva

Villazzano, September 18$^{\text{th}}$, 2019
FRGIM – Functional and Renormalization-Group Methods

Unitarity

$$SS^{\dagger} = 1, \qquad S = 1 + iT.$$

Unitarity equation (optical theorem)

$$2\mathrm{Im}T = TT^{\dagger}.$$

Cutting equations

$$\Downarrow$$

Pseudo-unitarity equation

$$2\mathrm{Im}T = THT^{\dagger}.$$

$$H = \mathrm{diag}(\ldots, 1, \ldots, 1, -1, \ldots, -1, \ldots).$$

Renormalizability and unitarity in QG

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$$\Gamma^{(\mathrm{ct})}_{\mathrm{HE}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\big[c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R^{\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha\beta}_{\mu\nu} + \underbrace{\ldots}_{\infty}\big].$$

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Stelle action: renormalizable but not unitary theory

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\big[\gamma R + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2 + 2\Lambda_{\mathrm{C}}\big].$$

$$\Gamma^{(\mathrm{ct})}_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\big[a_\gamma R + a_\alpha R_{\mu\nu}R^{\mu\nu} + a_\beta R^2 + 2a_{\mathrm{C}}\big].$$

Prescription for the propagator $\frac{\pm 1}{k^2 - m^2}$

Prescription for the amplitude $\mathcal{A}(p)$

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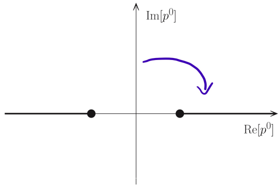
Standard particle : $$\frac{1}{k^2 - m^2 + i\epsilon}.$$

Ghost : $$\frac{-1}{k^2 - m^2 + i\epsilon}.$$

Prescription for the amplitude $\mathcal{A}(p)$



$$\mathcal{A}_+(p) = \mathcal{A}(p + i\epsilon)$$

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Standard particle : $$\frac{1}{k^2 - m^2 + i\epsilon}.$$

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Fake particle (fakeon):

$$\pm \frac{k^2 - m^2}{(k^2 - m^2)^2 + \mathcal{E}^4}.$$

+ integration domain deformations

(see D. Anselmi and MP, JHEP 1706 (2017) 066.)

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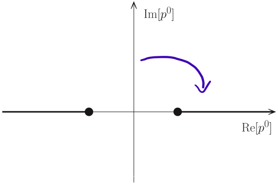

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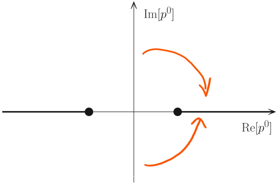

$$\mathcal{A}_{\rm AV}(p) = \frac{1}{2}\left[\mathcal{A}_+(p) + \mathcal{A}_-(p)\right].$$

A theory of particles and fakeons is unitary.

D. Anselmi and MP, PRD 96 (2017) 045009. D. Anselmi, JHEP 02 (2018) 141.

$G(k^2, m^2, \mathcal{E}) = \frac{k^2 - m^2}{(k^2 - m^2)^2 + \mathcal{E}^4}$.

Minkowski models

$$i\mathcal{M}(p) = c \int_{\mathbb{R}} \frac{\mathrm{d}k^0}{2\pi} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 k}{(2\pi)^3} G(p - k, m_1, \mathcal{E}) G(k, m_2, \mathcal{E}).$$

Not unitary, nonlocal and non-Hermitean divergences (Aglietti and Anselmi).

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Incomplete prescription, inconsistencies in diagrams, violates Lorentz invariance. (Anselmi and MP, Lee and Wick, Nakanishi)

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Unitary, unambiguous and Lorentz invariant (Anselmi and MP).

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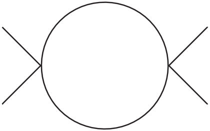
Unitary, unambiguous and Lorentz invariant (Anselmi and MP).
The fakeon prescription is not the Cauchy principal value.

$$\lim_{\mathcal{E}\to 0} \frac{k^2 - m^2}{(k^2-m^2)^2 + \mathcal{E}^4} = \mathcal{P}\frac{1}{k^2 - m^2} = \lim_{\epsilon \to 0^+} \frac{1}{2}\left(\frac{1}{k^2 - m^2 + i\epsilon} + \frac{1}{k^2 - m^2 - i\epsilon}\right).$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4!}\varphi^4.$$

One-loop bubble diagram



Compute the Euclidean amplitude $\mathcal{A}(p)$ and then use the prescription.

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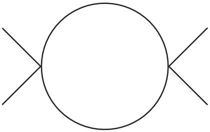

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After renormalizing the UV divergence

Feynman prescription

$$\mathcal{A}_+(p) = \frac{1}{2(4\pi)^2}\ln\frac{-p^2 - i\epsilon}{\mu^2}.$$

New presciption

$$\mathcal{A}_{\mathrm{AV}}(p) = \frac{1}{4(4\pi)^2}\ln\frac{(p^2)^2}{\mu^4}.$$

The theory of quantum gravity and fakeons

D. Anselmi, JHEP 1706 (2017) 086.

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2} \int \sqrt{-g} \left[2\Lambda_C + \zeta R + \alpha \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) - \frac{\xi}{6} R^2 \right].$$

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The propagator in De Donder gauge ($\Lambda_{\mathrm{C}} = 0,\ \alpha = \xi,\ g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$) is

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_0 = \frac{i}{2p^2\left(\zeta - \alpha p^2\right)}\mathcal{I}_{\mu\nu\rho\sigma} = \left\{\frac{1}{p^2} - \frac{1}{p^2 - \zeta/\alpha}\right\}\frac{i}{2\zeta}\mathcal{I}_{\mu\nu\rho\sigma},$$

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With the new prescription it turns into

$$\left\{\frac{1}{p^2 + i\epsilon} - \frac{p^2 - \zeta/\alpha}{\left[(p^2 - \zeta/\alpha + i\epsilon)^2 + \mathcal{E}^4\right]_{\mathrm{AV}}}\right\}\frac{i}{2\zeta}\mathcal{I}_{\mu\nu\rho\sigma}.$$

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Renormalization

The fakeon prescription does not spoil renormalizability.

Using the Batalin-Vilkovisky formalism we computed the beta functions in a new method by renormalizing the sources of the BRST tf's.

D. Anselmi and MP, JHEP 05 (2018) 027.

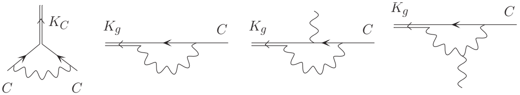
$$S_{\text{count}} = \frac{\mu^{-\varepsilon}}{(4\pi)^2\varepsilon}\int \sqrt{-g}\left[2\Delta\Lambda_C + \Delta\zeta R + \Delta\alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\Delta\xi}{6}R^2\right] + (S, \mathcal{F}),$$

$$\Delta\alpha = -\frac{133}{10}, \qquad \Delta\xi = \frac{5}{6} + \frac{5\xi}{\alpha} + \frac{5\xi^2}{3\alpha^2}, \qquad \Delta\zeta = \zeta\left(\frac{5}{6\xi} + \frac{5\xi}{3\alpha^2} + A\right),$$

$$\Delta\Lambda_C = \Lambda_C\left(-\frac{5}{\alpha} + \frac{2}{\xi} - 2A\right) - \frac{5\zeta^2}{4\alpha^2} - \frac{\zeta^2}{4\xi^2}.$$

The beta functions are

$$\beta_\alpha = -\frac{2\kappa^2}{(4\pi)^2}\Delta\alpha, \qquad \beta_\xi = -\frac{2\kappa^2}{(4\pi)^2}\Delta\xi, \qquad \beta_\zeta = -\frac{2\kappa^2}{(4\pi)^2}\Delta\zeta, \qquad \beta_{\Lambda_C} = -\frac{2\kappa^2}{(4\pi)^2}\Delta\Lambda_C.$$



Absorptive part of graviton self energy

$\Lambda_C = 0$

D. Anselmi and MP, JHEP 11 (2018) 021.

Equivalent action:
auxiliary fields ϕ, $\chi_{\mu\nu}$ + Weyl transformation + field redefinitions.

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$$S_{\mathrm{QG}}(g,\phi,\chi,\Phi) = S_{\mathrm{H}}(g) + S_{\chi}(g,\chi) + S_{\phi}(\tilde{g},\phi) + S_m(\tilde{g}e^{\kappa\phi},\Phi),$$
$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + 2\chi_{\mu\nu}.$$

$$S_{\mathrm{H}} = -\frac{\zeta}{2\kappa^2}\int\sqrt{-g}R, \quad S_{\phi}(g,\phi) = \frac{3\zeta}{4}\int\sqrt{-g}\left[\nabla_\mu\phi\nabla^\mu\phi - \frac{m_\phi^2}{\kappa^2}(1-e^{\kappa\phi})^2\right].$$

$$S_{\chi}(g,\chi) = S_{\mathrm{H}}(\tilde{g}) - S_{\mathrm{H}}(g) - 2\int\chi_{\mu\nu}\frac{\delta S_{\mathrm{H}}(\tilde{g})}{\delta g_{\mu\nu}} + \frac{\zeta^2}{2\alpha\kappa^2}\int\sqrt{-g}\,(\chi_{\mu\nu}\chi^{\mu\nu} - \chi^2)\big|_{g\to\tilde{g}}.$$

$$S_m = \text{Standard Model},$$

Graviton multiplet: $G_A = \{h_{\mu\nu}, \phi, \chi_{\rho\sigma}\}$.

$$\{G, F, F\}, \qquad \{G, S, F\}, \qquad \{G, S, Gh\}.$$

Computation of the absorptive parts $M_{AB} = \langle G_A G_B \rangle_{\text{abs}}^{\text{1-loop}}$

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$$\Gamma_{\text{abs}}^{hh} = -\int \frac{\delta S_{\text{QG}}}{\delta g_{\mu\nu}} \Delta g_{\mu\nu},$$

$$\Gamma_{\text{abs}}^{\Phi} = \frac{i}{16\pi} \int \sqrt{-g} R_{\mu\nu}\, \theta(r_\Phi)\theta(1-r_\Phi)\sqrt{1-r_\Phi} \left[P_\Phi(r_\Phi) \left(R^{\mu\nu} - \frac{1}{3} g^{\mu\nu} R \right) + Q_\Phi(r_\Phi) g^{\mu\nu} R \right],$$

$$r_\Phi = -4m_\Phi^2/\Box, \quad P_\Phi,\ Q_\Phi = \text{polynomials}.$$

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Contributions to the absorptive part

$$\Gamma_{\text{abs}}^{\text{GFF}} = \Gamma_{\text{abs}}^{hh} + \Gamma_{\text{abs}}^{m},$$

$$\Gamma_{\text{abs}}^{\text{GSF}} = \Gamma_{\text{abs}}^{hh} + \Gamma_{\text{abs}}^{\phi h} + \Gamma_{\text{abs}}^{\phi\phi} + \Gamma_{\text{abs}}^{m},$$

$$\Gamma_{\text{abs}}^{\text{GSGh}} = \Gamma_{\text{abs}}^{\text{GSF}} + \Gamma_{\text{abs}}^{\chi h} + \Gamma_{\text{abs}}^{\chi\phi} + \Gamma_{\text{abs}}^{\chi\chi}.$$

Violation of microcausality

Resumming the self energies the corrected χ propagator at the peak m_χ is

$$\langle \chi_{\mu\nu}(p)\chi_{\rho\sigma}(-p)\rangle_{s\sim\bar{m}_\chi^2} = -\frac{i\kappa^2}{\zeta}\frac{Z_\chi}{s-\bar{m}_\chi^2+i\bar{m}_\chi\Gamma_\chi}\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s),$$

$$\Gamma_\chi = -\frac{m_\chi^3}{M_{\text{Pl}}^2}C, \qquad C = \frac{N_s+6N_f+12N_v}{120}, \qquad \Gamma_\chi < 0.$$

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Breit-Wigner distribution

$$\frac{i}{E-m+i\frac{\Gamma}{2}} \qquad \longrightarrow \qquad \text{sgn}(t)\theta(\Gamma t)\text{exp}\left(-imt-\frac{\Gamma t}{2}\right).$$

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$$\text{Duration} \sim 1/|\Gamma_\chi|$$

$$\text{If } m_\chi \sim 10^{11}\text{GeV then } 1/|\Gamma_\chi| \sim 7\cdot 10^{-17}\text{s}$$

$$\text{If } m_\chi \sim 10^{12}\text{GeV then } 1/|\Gamma_\chi| \sim 4\cdot 10^{-20}\text{s}$$

For time intervals of the order $1/|\Gamma_\chi|$ past, present and future, as well as cause and effect lose meaning.

The classical limit

At classical level the fakeon prescription is just

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Example: HD particle in an external force

$$m\ddot{x} + m\tau^2 \ddot{\ddot{x}} = \left(1 + \tau^2 \frac{\mathrm{d}^2}{\mathrm{d}t^2}\right) m\ddot{x} = F_{\mathrm{ext}}(t).$$

$$\Downarrow$$

$$m\ddot{x} = \int_{-\infty}^{\infty} G_{\mathrm{F}}(t-t')F_{\mathrm{ext}}(t')\mathrm{d}t' \equiv \left\langle F_{\mathrm{ext}} \right\rangle.$$

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Analogously the classical limit of QG leads to

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \langle \mathrm{T}_{\mu\nu} \rangle .$$

For the explicit case of FLRW metric see D. Anselmi, JHEP 1904 (2019) 061.

Summary

- A new quantization prescription gives a unitary and renorm. QFT of gravity.
  - Predictive and possibly testable in the near future.
  - Quantization of cosmological perturbations (E. Bianchi and MP, in preparation).
- New type of degrees of freedom (fakeons).
- New phenomenology due to the presence of fakeons.