# Different perspectives on background independent RG flows

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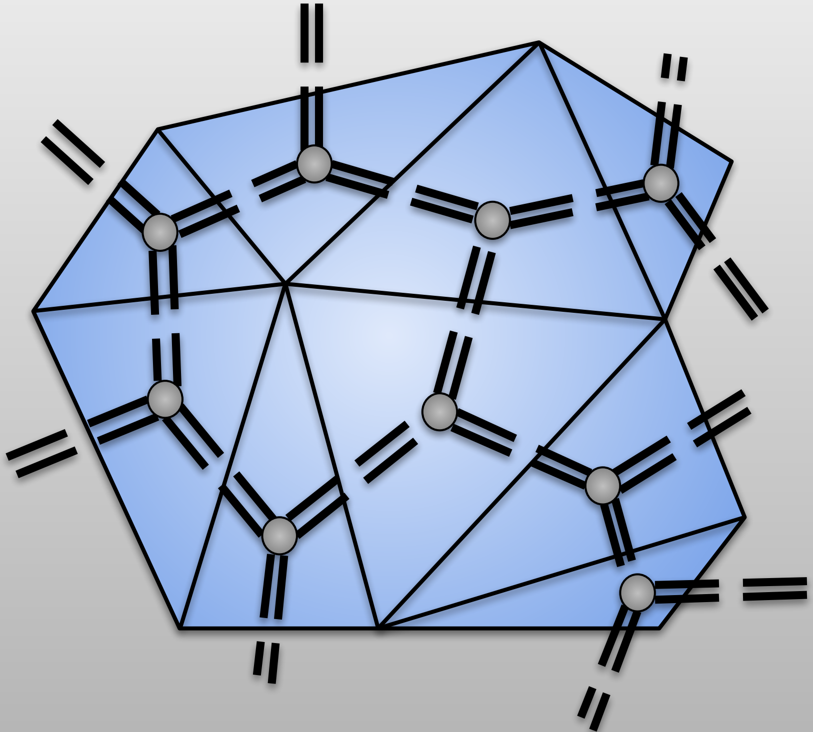


***Based on:***

- **Phys.Rev. D97 (2018) no.12, 126018,**
- **Classical and Quantum Gravity (2019), 36, 15,**
- **Universe 5 (2019) no.2, 53**
- **arXiv:1904.07042**

**In collaboration with: J. Ben Geloun, A. Eichhorn, T. Koslowski, J. Lumma and D. Oriti**

# Outline:

- Introduction and motivation
- Coarse-graining in quantum gravity
- Background-independent coarse-graining
- Recent applications
- Outlook

# Introduction and motivation:

We want to make sense of the path integral for quantum gravity:

$$\mathcal{Z} = \sum_{\mathcal{T}} \int [\mathcal{D}(\text{geometries})] \, [\mathcal{D}\Phi] \, \mathrm{e}^{iS_{\text{grav}} + iS_{\text{matter}}[\Phi]}$$

Sum over different topologies?

What is the correct measure?

Matter fields

What is the microscopic action?

# Approach I: continuum QFT

● Treating quantum gravity within the perturbative continuum quantum field theory paradigm requires the introduction of a UV cutoff,

$$\int_{p^2<\Lambda_{UV}^2} [\mathcal{D}h_{\mu\nu}]\, e^{-S_{\mathrm{EH}}(\bar{g}+h)} \qquad \text{with} \qquad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

● Introduction of a background: scales are obtained from fiducial background metric.

● Question: Can we meaningfully take the continuum limit?

$$\int_{p^2<\Lambda_{\mathrm{UV}}^2} [\mathcal{D}h_{\mu\nu}]\, \mathrm{e}^{-S_{\mathrm{EH}}} \overset{?}{\to} \int_{p^2<\Lambda_{\mathrm{UV}=\infty}^2} [\mathcal{D}h_{\mu\nu}]\, \mathrm{e}^{-S_{\mathrm{EH}}}$$

● The existence of a non-trivial fixed point, i.e., a regime where the theory becomes scale invariant ensures a well-defined continuum limit.

Asymptotic Safety!

Introduce a background

“Momentum scale”: eigenvalues of the background Laplace operator

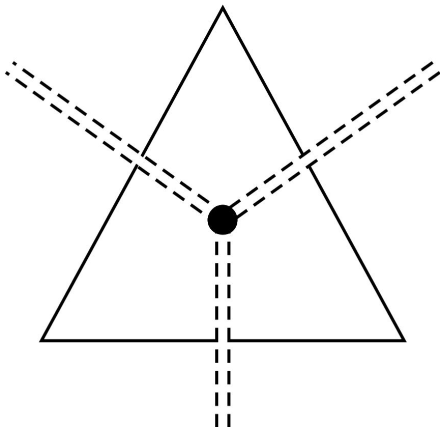
Standard coarse-graining: integrate out modes shell-wise

# Approach II: “be wise - discretize!”

$$\sum_{\mathcal{T}} \int [\mathcal{D}(\text{geometries})] \rightarrow \sum_{\text{tessellations}}$$

● In 2d, sum of random geometries is implemented in a partition function for random matrices (matrix models).

$$\mathrm{Tr}\phi^3 \sim$$

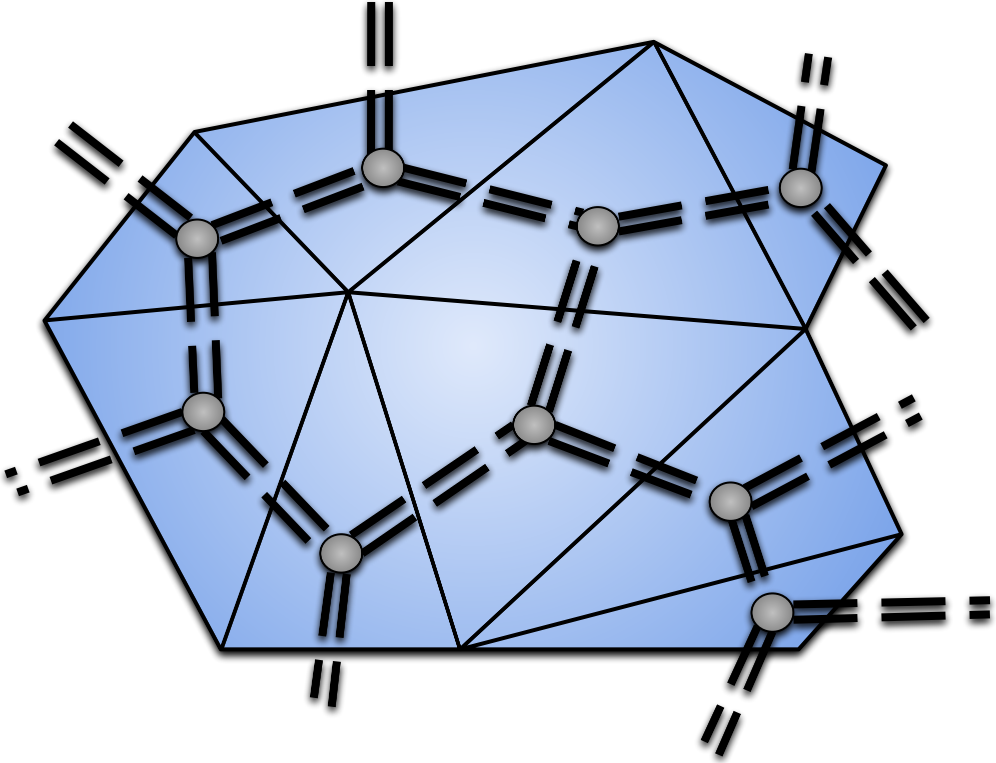


$$Z \sim \int [d\phi]\, \mathrm{e}^{-\frac{1}{2}\mathrm{Tr}\phi^2 + \frac{g}{\sqrt{N}}\mathrm{Tr}\phi^3}$$

Size of matrix

$$\mathcal{Z} = \sum_h N^{2-2h} Z_h$$

● Physical quantities are obtained in the continuum limit: number of triangles grows while their area shrinks



Generalization to higher dimensions: (C)DT, Tensor models, spinfoams

Path integral for quantum gravity

- Make sense of QG as a continuum quantum field theory for the metric

- Find a consistent QFT which is predictive and valid to all scales

- Asymptotic safety

?

- Discretize the path integral

- Suitable for non-perturbative calculations

- Physical (quantum) spacetime is obtained after a suitable continuum limit

# Coarse-graining in quantum gravity with the FRG:

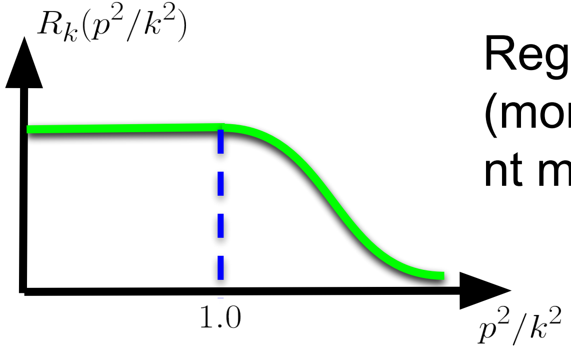
## Functional renormalization group:

Euclidean path integral:

$$\int [\mathcal{D}\varphi]\, \mathrm{e}^{-S(\varphi)}$$

Introduce a regulator term and perform the path integral momentum-shell-wise:

$$\int [\mathcal{D}\varphi]\, \mathrm{e}^{-S(\varphi) - \int_p \varphi(p) R_k(p^2) \varphi(-p)}$$




Regulator term (momentum-depende nt mass term)

The flowing action satisfies an exact flow equation (FRG equation)

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[ \left( \Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right]$$

$$\partial_t = k \partial_k$$

$$\Gamma_k = \sum_i \bar{g}_i(k) \mathcal{O}^i(\phi)$$

Canonical dimension: dimensional analysis

$$\bar{g}_i = k^{d_{\bar{g}_i}} g_i$$

Beta functions

$$\partial_t \Gamma_k = \sum_i k^{d_{\bar{g}_i}} (d_{\bar{g}_i} g_i + \beta_i) \mathcal{O}^i(\phi)$$

# QFT for metric:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \quad \Longrightarrow \quad \Gamma[g_{\mu\nu}] = \Gamma[\bar{g}_{\mu\nu} + h_{\mu\nu}]$$

Regulator: $\int_x \sqrt{\bar{g}}\, h_{\mu\nu} R_k^{\mu\nu\alpha\beta}(\bar{\nabla}^2) h_{\alpha\beta}$

Background structure: 1) Assignment of canonical dimensions to couplings;
2) Separate “fast” and “slow” modes.

# Background independence holds “indirectly”.

$$\frac{\delta \Gamma}{\delta h} - \frac{\delta \Gamma}{\delta \bar{g}} = \Delta$$

● Challenging to  implement such relation to the flowing action;

● Symmetry-breaking induced terms;

● Fate of BRST symmetry: gauge-fixing and Gribov copies;

# Matrix models:

● There is no background. No definition of momentum scale.

● Implement the "Wilsonian integration" by integrating out rows and columns of matrices. [Brézin, Zinn-Justin '92]

● Coarse-graining parameter: size of the matrix (# of dofs).

The continuum limit with contributions from all h's is obtained by the so-called double-scaling limit.

$$N(g - g_c)^{(2-\gamma_{\mathrm{str}})/2} = \mathrm{const} \qquad N \to \infty \text{ and } g \to g_c$$

The double-scaling limit equation can be written as the linearized flow of the coupling g around a fixed point.

The renormalization group parameter is the matrix size N.

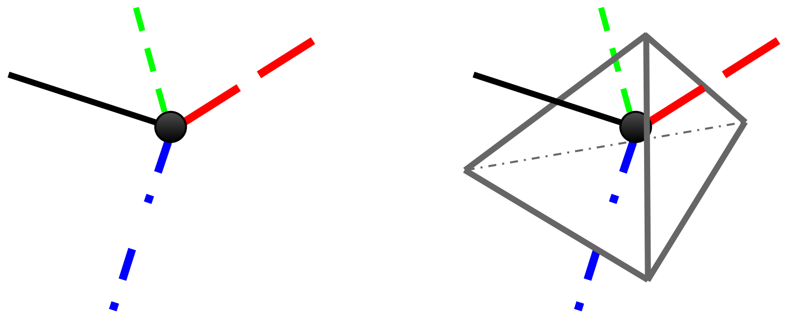
$$g(N) = g_c + \mathrm{const.}' N^{\frac{-2}{2-\gamma_{\mathrm{str}}}}$$

Set up the FRG

[Brézin, Zinn-Justin '92; Eichhorn, Koslowski '13; Eichhorn, Koslowski, **ADP** '18]

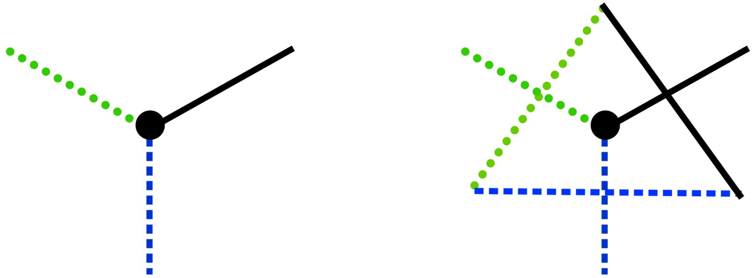
# Background-independent coarse-graining in tensor models:

● We are interested in writing the path integral for 4d quantum gravity. Natural generalization of the matrix-model program: tensor models [Ambjorn, Durhuus, Jonsson ‘91; Sasakura ‘91; Godfrey, Gross ‘91]



● Technical obstacle: these models, in general, do not feature a large-N expansion

● Introduction of (un)colored models [Gurau ‘09, ‘10, ‘11; Bonzom, Gurau, Rivasseau ‘12]



● Feynman amplitudes are dual to pseudo-manifolds

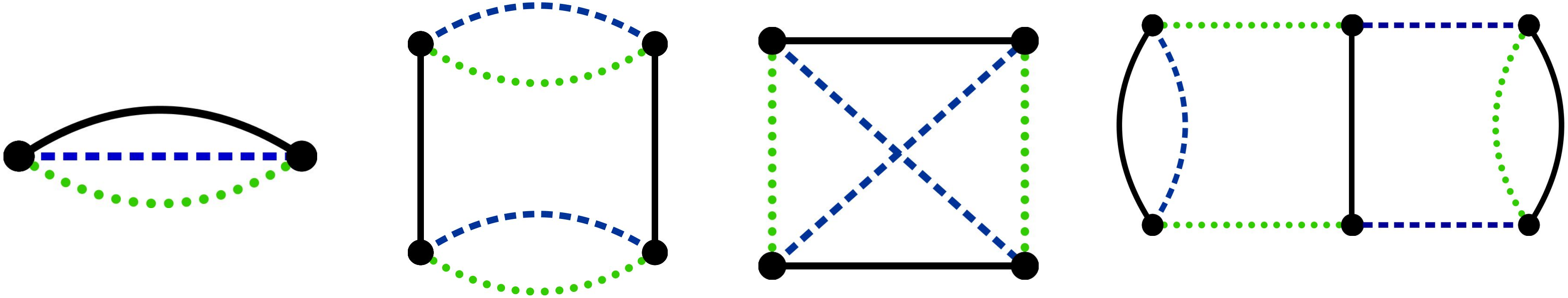
- The interactions are invariant under the following transformations

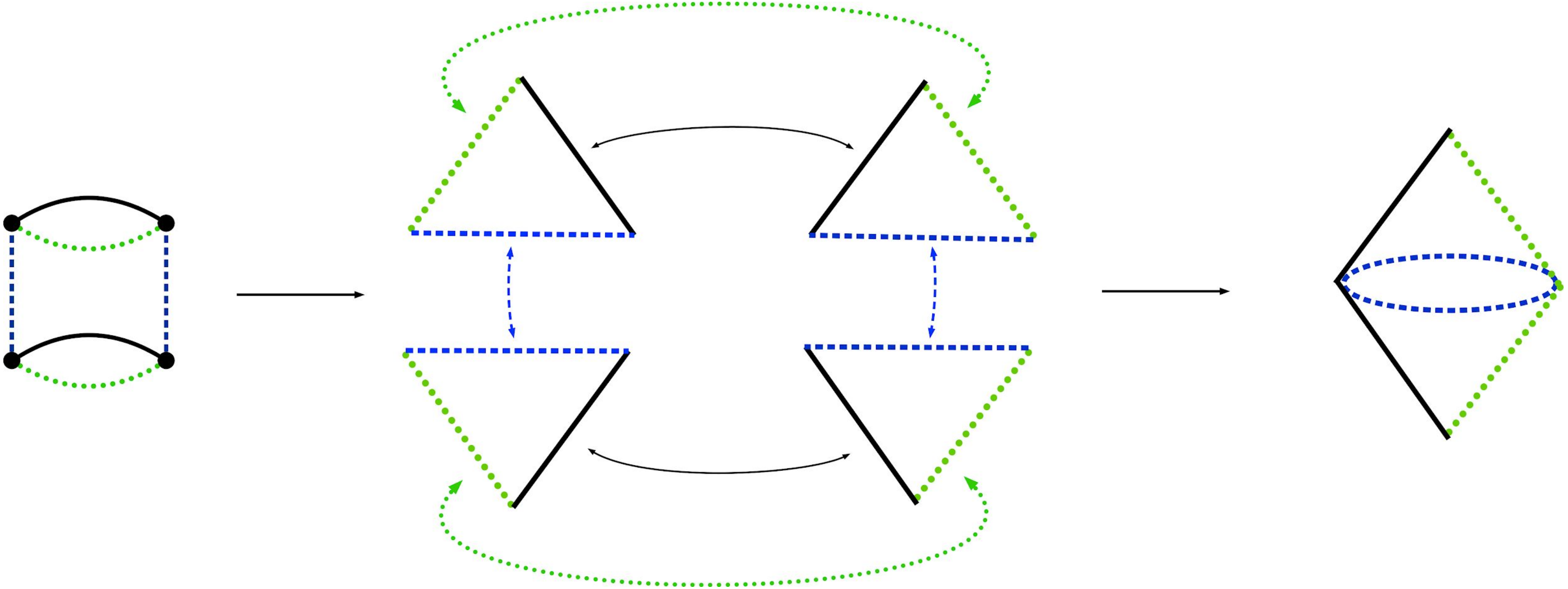
$$\left\{ T_{a_1 \ldots a_d} \to T'_{a_1 \ldots a_d} = O^{(1)}_{a_1 b_1} \ldots O^{(d)}_{a_d b_d} T_{a_1 \ldots a_d} \right\}$$

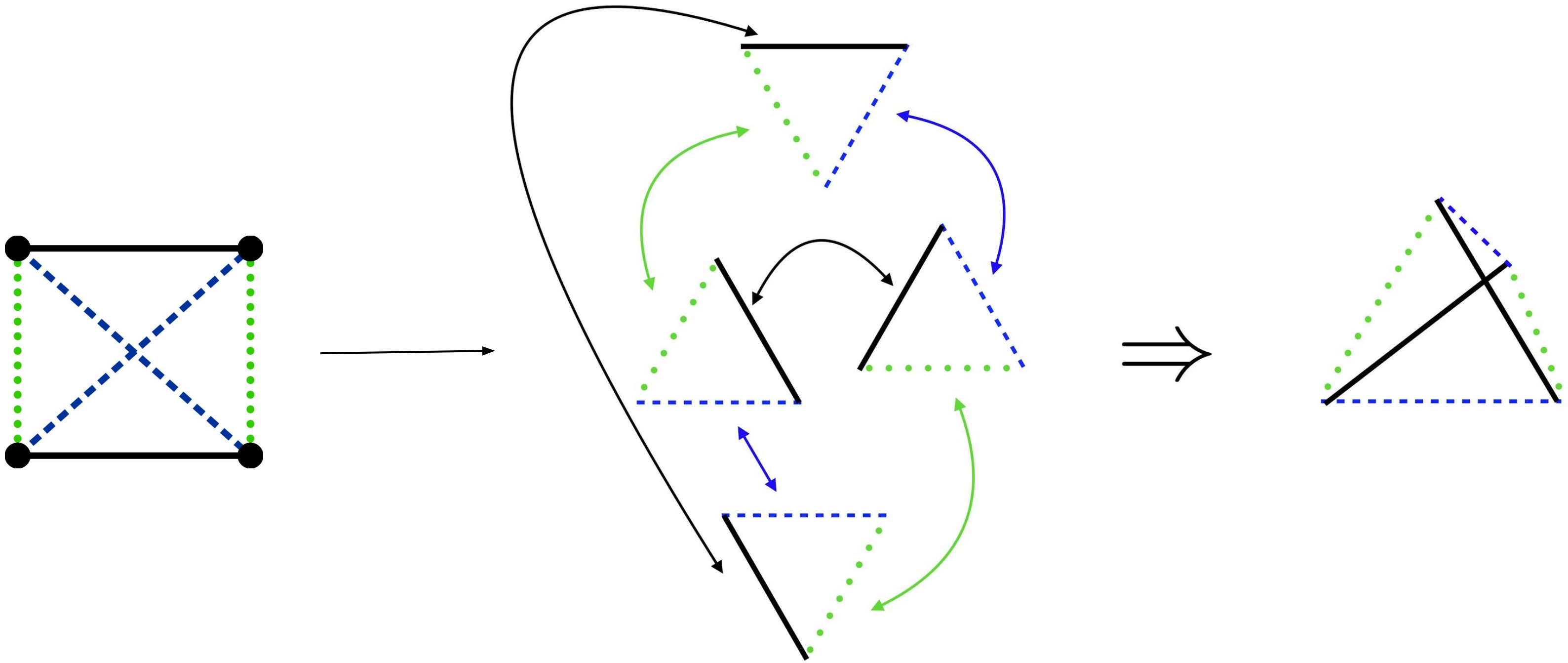
*Real models*

$$\left\{ \begin{array}{l} T_{a_1 \ldots a_d} \to T'_{a_1 \ldots a_d} = U^{(1)}_{a_1 b_1} \ldots U^{(d)}_{a_d b_d} T_{a_1 \ldots a_d} \\ \bar{T}_{a_1 \ldots a_d} \to \bar{T}'_{a_1 \ldots a_d} = \bar{U}^{(1)}_{a_1 b_1} \ldots \bar{U}^{(d)}_{a_d b_d} \bar{T}_{a_1 \ldots a_d} \end{array} \right\}$$

*Complex models*







- Setting up a flow equation for rank-d tensor models [Eichhorn, Koslowski '13; Eichhorn, Koslowski, **ADP** '18]:

$$Z_N = \int [dT]_{N'}\, \mathrm{e}^{-S(T)+\mathrm{Tr} J\cdot T - \frac{1}{2}\mathrm{Tr} T\cdot R_N \cdot T}$$

$$N\partial_N \Gamma_N = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_N}{\delta T_{a_1\dots a_d}\delta T_{b_1\dots b_d}} + R_N(a_1,\dots,a_d)\delta_{a_1 b_1}\dots\delta_{a_d b_d}\right)^{-1} N\partial_N R_N(a_1,\dots a_d)\right]$$

- Flow equation dictates how the effective average action flows with the number of degrees of freedom N.

● Regulator/Suppression term is chosen as: [Eichhorn, Lumma, Koslowski, **ADP** ‘18; Eichhorn, Koslowski, **ADP** ‘18]

$$R_N(\{a_i\}) = \left(\frac{N^r}{\sum_i a_i^p} - 1\right) \theta \left(\frac{N^r}{\sum_i a_i^p} - 1\right)$$

● Unlike in the case of theories that are defined on a background, we cannot fix r and p by simple dimensional analysis: different choices lead to different scalings.

● How to assign scaling dimensions in this context? **Ultimately, the scaling with N is fixed by the dual geometric interpretation.** [Eichhorn, Lumma, Koslowski, **ADP** ‘18; Eichhorn, Koslowski, **ADP** ‘18]

● Relating the tensor model couplings with those present in the Regge action leads to r/p=1 [Eichhorn, Koslowski, **ADP** ‘18]

● In practice, we need to solve the flow equation within some approximation. Setting up truncations.

$$\Gamma_N(T) = \sum_i \bar{g}_i \mathcal{O}_i(T)$$

Scale derivative of the effective average action. Read off beta functions.

Dimensionful couplings

Include all operators up to a given canonical dimension value

$$N\partial_N \Gamma_N(T) = \sum_i \left( [\bar{g}_i]\, \bar{g}_i + N^{[\bar{g}_i]} \beta_i \right) \mathcal{O}_i(T)$$

● However… how to use the canonical dimensions as a guiding principle if we do not know them a priori?

● Two ways: i) Derive the system of beta functions and fix the canonical dimensionality by demanding a well-defined large-N limit (Iterate this procedure until all couplings of a given canonical dimension are collected). e.g. [Ben Geloun, Koslowski, Oriti, **ADP** ‘18; Eichhorn, Koslowski, Lumma, **ADP** ‘18; Eichhorn, Koslowski, Lumma, **ADP** ‘18];

ii) Derive an expression for the canonical dimension which is compatible with the large-N expansion *and* with the FRG [Eichhorn, Koslowski, Lumma, **ADP** *To appear*]

● Compute the zeros of the system of beta function and check stability. Fixed points must be stable.

Strategy i)

$$\eta = \frac{18N\left(2\lambda_N^{4;2}+3\lambda_N^{4;1}(2N+\ldots)\right)}{3\left((m_N+N)^2-2\lambda_N^{4;3}-12\lambda_N^{4;2}N\right)-2\lambda_N^{4;1}\left(27N^2+\ldots\right)}$$

$$\partial_t m_N = -\frac{N}{(m_N+N)^2}\Big[\Big(9\lambda_N^{4;1}\,(6N^2+\ldots) + \lambda_N^{4;3}\,(36N^3+\ldots)$$
$$+3\lambda_N^{4;2}(6N+\ldots)\Big) + \eta\left(\lambda_N^{4;3}\,(9N^3+\ldots) + \lambda_N^{4;1}\,(18N^2+\ldots) + 9\lambda_N^{4;2}N\right)\Big]$$
$$-\eta m_N$$

$$m_N \to N\bar{m}_N \qquad \lambda_N^{4;1} \to \bar{\lambda}_N^{4;1} \qquad \lambda_N^{4;2} \to N^{\alpha}\bar{\lambda}_N^{4;2} \qquad \lambda_N^{4;3} \to N^{-1}\bar{\lambda}_N^{4;3}$$

Strategy ii)

Derive the relation between the scaling dimension and the combinatorial properties of the interactions. [Eichhorn, Koslowski, Lumma, **ADP** *To appear*]

$$d(\gamma) = 3 - \frac{1}{2}(3p(\gamma) + f(\gamma))$$

Colored graph

Number of tensor pairs

Number of faces

Assignment of scaling dimensions independently of the truncation.

● In summary: Tensor models can be used as a generalization of matrix models to higher dimensions; Their (un)colored incarnation features a 1/N expansion; The Feynman diagrams are dual to pseudo-manifolds…

● Search for universal continuum limit that resembles our universe. **How?**

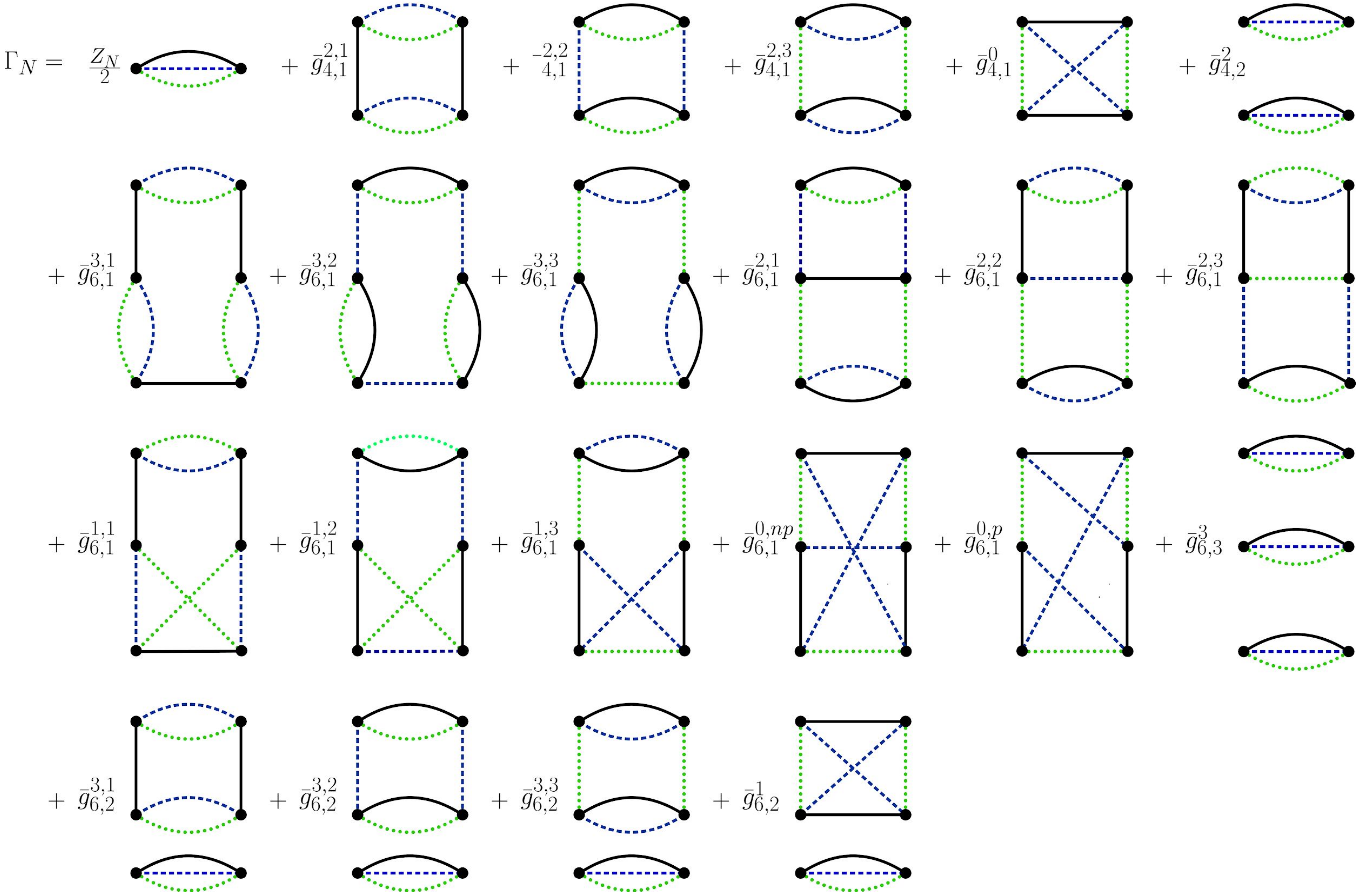
● Explore different continuum limits. **Does exist a universality class that matches other universality class of any QG approach, e.g., asymptotic safety?**

**Proposal:** use the FRG as an exploratory/discovery tool to address these questions.

# Recent developments:

- Application of the previous toolbox to 3-dimensional (Abelian) group field theories; [Ben Geloun, Koslowski, Oriti, **ADP** '18]
- Characterization of different universality classes for rank-3 real tensor models; [Eichhorn, Koslowski, Lumma, **ADP** '18]
- First steps towards 4d tensor models; [Eichhorn, Koslowski, **ADP** '18][Eichhorn, Lumma, Sikandar, **ADP** *To appear*]
- Foundational formal work on the renormalization group for background independent theories; [Eichhorn, Koslowski, Lumma, **ADP** *To appear*]

# • Rank-3 model



$\Gamma_N = \frac{Z_N}{2}$
$+ \bar{g}^{2,1}_{4,1}$
$+ \bar{g}^{2,2}_{4,1}$
$+ \bar{g}^{2,3}_{4,1}$
$+ \bar{g}^{0}_{4,1}$
$+ \bar{g}^{2}_{4,2}$
$+ \bar{g}^{3,1}_{6,1}$
$+ \bar{g}^{3,2}_{6,1}$
$+ \bar{g}^{3,3}_{6,1}$
$+ \bar{g}^{2,1}_{6,1}$
$+ \bar{g}^{2,2}_{6,1}$
$+ \bar{g}^{2,3}_{6,1}$
$+ \bar{g}^{1,1}_{6,1}$
$+ \bar{g}^{1,2}_{6,1}$
$+ \bar{g}^{1,3}_{6,1}$
$+ \bar{g}^{0,np}_{6,1}$
$+ \bar{g}^{0,p}_{6,1}$
$+ \bar{g}^{3}_{6,3}$
$+ \bar{g}^{3,1}_{6,2}$
$+ \bar{g}^{3,2}_{6,2}$
$+ \bar{g}^{3,3}_{6,2}$
$+ \bar{g}^{1}_{6,2}$

➤ System of beta functions has a rich set of roots

➤ Many of them are just truncation artifact

➤ Two sets of fixed points were found:

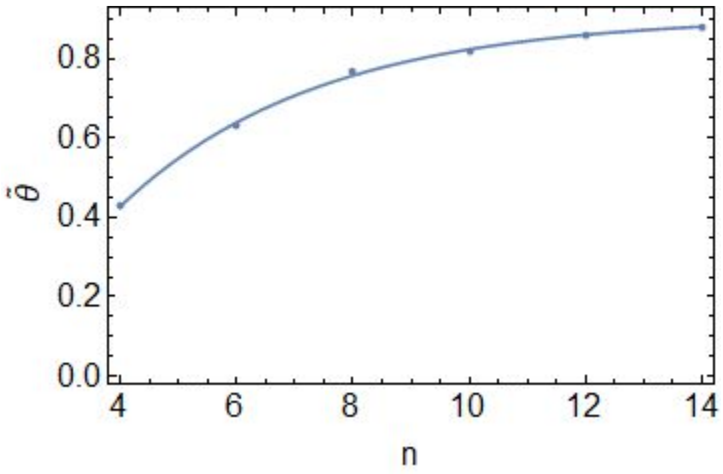
1) One which corresponds to dimensional reduction to 2d;

2) Two fixed points which do not feature dimensional reduction and can potentially be associated with 3d quantum gravity

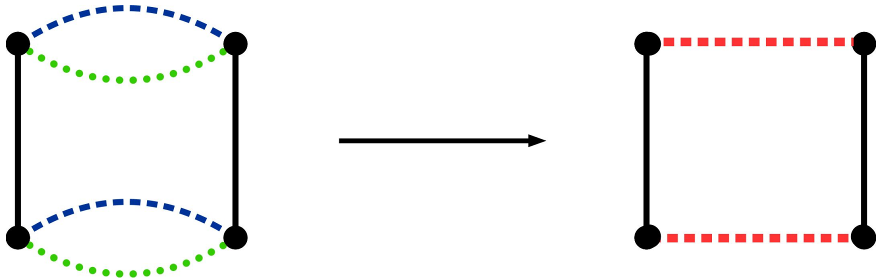
| scheme | $g^{0\,*}_{4,1}$ | $g^{2,i\,*}_{4,1}$ | $g^{2\,*}_{4,2}$ | $g^{0,np\,*}_{6,1}$ | $g^{0,p\,*}_{6,1}$ | $g^{1,i\,*}_{6,1}$ | $g^{2,i\,*}_{6,1}$ | $g^{3,i\,*}_{6,1}$ | $g^{1\,*}_{6,2}$ | $g^{3,i\,*}_{6,2}$ | $g^{3\,*}_{6,3}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| full | $\pm$ 0.42 | -0.50 | 3.43 | 0 | 0 | 0 | -5.33 | 4.19 | $\pm$ 2.65 | -2.26 | 35.27 | |
| pert. | $\pm$ 0.46 | -0.54 | 3.68 | 0 | 0 | 0 | -6.33 | 4.96 | $\pm$ 3.18 | -2.56 | 39.82 | |
| $\eta = 0$ | $\pm$ 0.60 | -0.71 | 5.09 | 0 | 0 | 0 | -10.67 | 8.20 | $\pm$ 7.16 | -4.35 | 67.32 | |

| scheme | $\theta_{1,2}$ | $\theta_{3,4}$ | $\theta_{5,6}$ | $\theta_{7,8}$ | $\theta_{9,10}$ | $\theta_{11,12}$ | $\theta_{13,14}$ | $\theta_{15,16,17}$ | $\theta_{18,19}$ | $\theta_{20}$ | $\theta_{21}$ | $\eta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| full | 1.35 $\pm$ i 1.56 | 0.13 | -0.02 $\pm$ 5.10 i | -0.08 $\pm$ 5.35 i | -0.08$\pm$ 5.35 i | -0.88 $\pm$ i 1.33 | -1.40 | -1.43 | -2.00 | -3.04 | -4.80 | -0.33 |
| pert. | 1.46 $\pm$ $i$ 1.39 | 0.15 | -0.11 $\pm$ i 4.83 | -0.10 $\pm$ i 5.42 | -0.10 $\pm$ i 5.42 | -0.99 $\pm$ i 1.34 | -1.41 | -1.46 | -2.04 | -2.95 | -5.36 | -0.32 |
| $\eta = 0$ | 1.95 $\pm$ 0.69 i | 0.38 | -0.03 $\pm$ 5.96 i | -0.29 $\pm$ 7.06 i | -0.29 $\pm$ 7.06 i | $-1.74$ | -1.89 $\pm$ 2.72 i | -2.07 | -3 | -5.70+1.57i | -5.70-1.57 i | 0 |

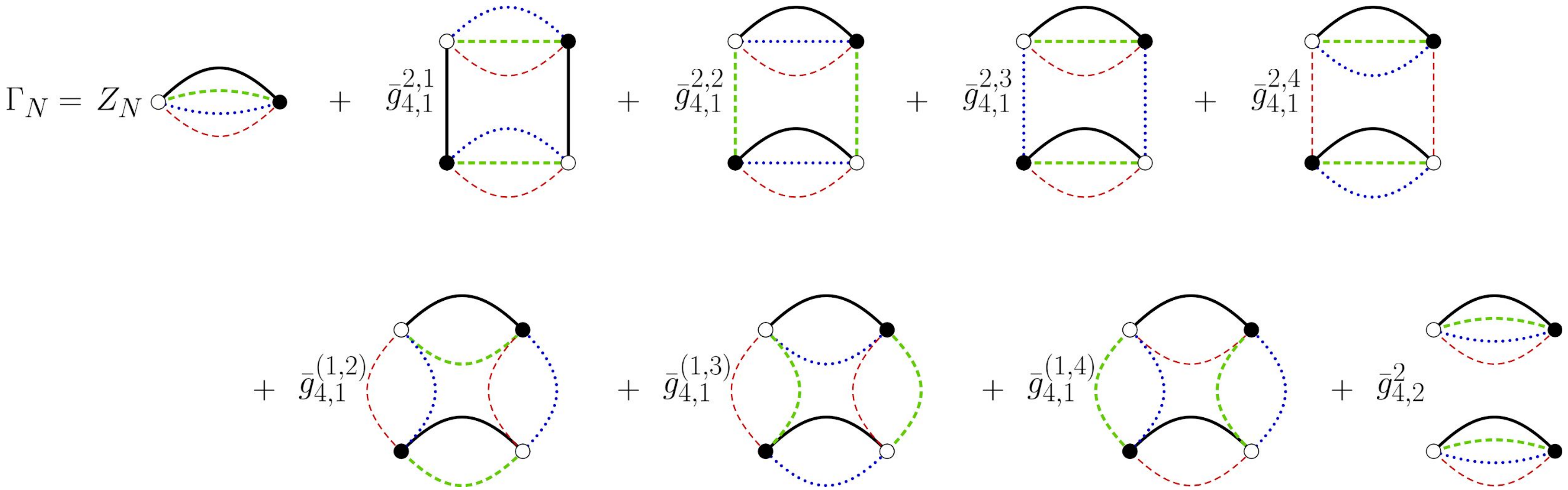
➤ In the case of dimensional reduction, the FRG is able to reproduce the exact results of matrix models with good precision in simple truncations.




[Eichhorn, Koslowski, **ADP** '18]



- **Rank-4 model**

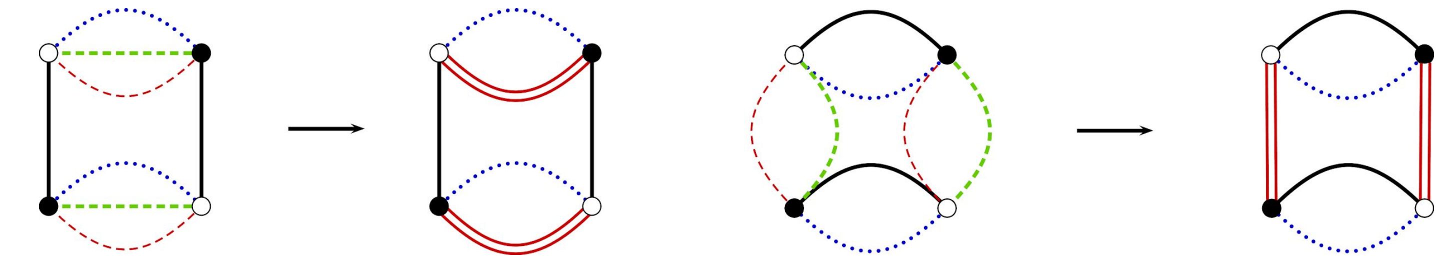

$\Gamma_N = Z_N$
$+ \ \bar{g}^{2,1}_{4,1}$
$+ \ \bar{g}^{2,2}_{4,1}$
$+ \ \bar{g}^{2,3}_{4,1}$
$+ \ \bar{g}^{2,4}_{4,1}$
$+ \ \bar{g}^{(1,2)}_{4,1}$
$+ \ \bar{g}^{(1,3)}_{4,1}$
$+ \ \bar{g}^{(1,4)}_{4,1}$
$+ \ \bar{g}^{2}_{4,2}$

➢ As for the rank-3, fixed points which correspond to matrix models are found.

➢ It is possible to have dimensional reduction to rank-3 tensor models, but  all zeros of the system of beta functions that could feature that are not stable.

➢ A fixed point which does not feature dimensional reduction exists. This is a potential candidate for the continuum limit of 4d quantum gravity.

| $g_{4,1}^{2,i}$ | $g_{4,1}^{(1,j)}$ | $g_{4,2}^{2}$ | $\eta$ | $\theta_1$ | $\theta_{2,3}$ | $\theta_{4,5}$ | $\theta_{6,7,8}$ |
|---|---|---|---|---|---|---|---|
| $-0.09$ | $-0.07$ | $-5.70$ | $-0.91$ | $3.44$ | $0.18$ | $-0.40 \pm 0.07\,i$ | $-0.56$ |



➤ Fixed points in 3d and 4d feature four and three relevant directions, respectively.

➤ Considering the systematic error of the present calculations, the results are compatible with the results obtained for asymptotic safety.

➤ More sophisticated computations are needed.

➤ FRG as a discovery/exploratory tool for (new) universality classes.

*Converging to quantum gravity from different directions?*

# Outlook:

● Understanding the path integral for quantum gravity might be key for further developments in this field.

● Different approaches to quantum gravity might belong to the same universality class.

● Symbiotic relation between different approaches, e.g.,

*Tensor models*

● Background independent

● Emergent geometry (?)/ CDT

↔

*Asymptotic Safety*

● Suitable for easier connection with SM physics

● Pheno

- *Next steps:*

  ➢ Characterization of universality classes for rank-4 tensor models

  ➢ Construction of a tensor model which generates the same configuration space as CDT

  ➢ Understand the role of matter degrees of freedom in this program: are they emergent or not?

  ➢ Does the renormalization group provide insights on (quantum) cosmology (cf. GFT cosmology)?

# Thank you