Prospects and limitations of the functional RG approach in Quantum Gravity

Alfio Bonanno
INAF Catania Astrophysical Observatory, INFN - Catania

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Outline

- Intro on Functional RG
- Einstein-Hilbert result
- Beyond Einstein-Hilbert
- Issues with Quadratic Gravity
- Physical content of the theory
- Conclusions

Effective action Γ in scalar field theory

- start: generic action $S_{\hat{k}}[\chi]$

$$S_{\hat{k}}[\chi] = \int d^d x \left\{ \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2} m^2 \chi^2 + \text{interactions} \right\}$$

- generating functional for connected Green functions

$$W[J] = \ln \int \mathcal{D}\chi \exp\left\{ -S_{\hat{k}}[\chi] + \int d^d x \, J \, \chi \right\}$$

- classical field

$$\phi = \langle \chi \rangle = \frac{\delta W[J]}{\delta J}$$

effective action $\Gamma[\phi]$ = Legendre transform of $W[J]$

$$\Gamma[\phi] = \int d^d x \, J\phi - W[J]$$

Effective average action Γ_k in scalar field theory

- start: generic action $S_{\hat{k}}[\chi]$

$$S_{\hat{k}}[\chi] = \int d^d x \left\{ \tfrac{1}{2}(\partial_\mu \chi)^2 + \tfrac{1}{2} m^2 \chi^2 + \text{interactions} \right\}$$

- introduce scale-dependent mass term $\Delta_k S[\chi]$ in $W[J]$

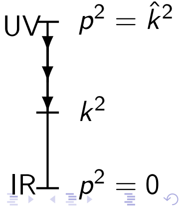
$$W_k[J] = \ln \int \mathcal{D}\chi \exp\left\{ -S_{\hat{k}}[\chi] - \Delta_k S[\chi] + \int d^d x \, J \, \chi \right\}$$

$$\Delta_k S[\chi] = \tfrac{1}{2} \int d^d x \, \chi \, \mathcal{R}_k(-\partial^2) \, \chi$$

- discriminate between low/high-momentum modes

$$\mathcal{R}_k(p^2) = \begin{cases} k^2 & p^2 \ll k^2 \\ 0 & p^2 \gg k^2 \end{cases}$$

  - high momentum modes: integrated out
  - low momentum modes: suppressed by mass term



Effective average action Γ_k for scalars

- start: generic action $S_{\hat{k}}[\chi]$

$$S_{\hat{k}}[\chi] = \int d^d x \left\{ \tfrac{1}{2}(\partial_\mu \chi)^2 + \tfrac{1}{2} m^2 \chi^2 + \text{interactions} \right\}$$

- introduce scale-dependent mass term $\Delta_k S[\chi]$ in $W[J]$

$$W_k[J] = \ln \int \mathcal{D}\chi \exp \left\{ -S_{\hat{k}}[\chi] - \Delta_k S[\chi] + \int d^d x \, J \, \chi \right\}$$

- classical field

$$\phi = \langle \chi \rangle = \frac{\delta W_k[J]}{\delta J}$$

define $\tilde{\Gamma}_k[\phi]$ = Legendre transform of $W_k[J]$

$$\tilde{\Gamma}_k[\phi] = \int d^d x \, J\phi - W_k[J]$$

- effective average action

$$\Gamma_k[\phi] = \tilde{\Gamma}_k[\phi] - \Delta_k S[\phi]$$

Properties of the effective average action

- Definition:

$$\Gamma_k[\phi] = \tilde{\Gamma}_k[\phi] - \Delta_k S[\phi]$$

- k-dependence governed by Functional RG Equation (FRGE)

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left[\left(\delta^2\Gamma_k + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

  - Formally: exact equation $\Leftrightarrow$ no approximations in derivation
  - independent of "fundamental theory" $\Leftrightarrow$ $S_{\hat{k}}$ enters as initial condition

- Limits: Γ_k interpolates continuously between:
  - $k \to \infty$ $\simeq$ bare/classical action S
  - $k \to 0$ $=$ ordinary effective action Γ

- Theory: specified by RG trajectory

$$k \mapsto \Gamma_k[\phi]$$

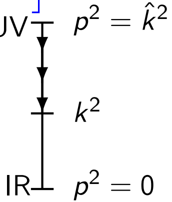
Effective average action for gravity

- starting point: generic diff inv action $S^{\mathrm{grav}}[\gamma_{\mu\nu}]$
- perform background gauge fixing $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$$S^{\mathrm{gf}} = \frac{1}{2}\int d^4x\sqrt{\bar{g}}\,F_\mu Y^{\mu\nu} F_\nu$$

$$F_\mu = \bar{D}^\mu h_{\mu\nu} - \frac{1+\rho}{4}\bar{D}_\mu h\,, \quad Y^{\mu\nu} = \left[\frac{\alpha}{16\pi G_N} + \beta\bar{D}^2\right]\bar{g}^{\mu\nu}$$

  - gauge choices: harmonic gauge ($\rho = 1$), geometric, ...
  - add ghost term: $S^{\mathrm{gh}}[h, C, \bar{C}, \bar{b}, b; \bar{g}]$
- IR cutoff $\Delta_k S$: expand $h_{\mu\nu}$, C_μ, $\bar{C}^\mu$ in $\bar{D}^2$-eigenmodes
  - $-\bar{D}^2$-eigenvalues $\gg k^2$: integrated out
  - $-\bar{D}^2$-eigenvalues $\ll k^2$: suppressed by mass term



- exact RG equation for Γ_k:

$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\left(\delta^2\Gamma_k + \mathcal{R}_k(-\bar{D}^2)\right)^{-1} k\partial_k\mathcal{R}_k(-\bar{D}^2)\right] + \text{ghosts}$$

Truncating the theory space

- Caveat: FRGE cannot be solved exactly
- non-perturbative approximation scheme: truncation of $\Gamma_k[\Phi]$

$$\Gamma_k[\Phi] = \sum_{i=1}^{N} \bar{u}_i(k)\, \mathcal{O}_i[\Phi]$$

  - $\Longrightarrow$ substitute into FRGE
  - $\Longrightarrow$ projection of flow gives β-functions for $\bar{u}_i(k)$

$$k\partial_k \bar{u}_i(k) = \beta_i(\bar{u}_i; k)$$

- A first truncations:

$$\Gamma_k[g, C, \bar{C}, b; \bar{g}] = \Gamma_k^{\mathrm{grav}}[g] + \underbrace{\widehat{\Gamma}_k[g - \bar{g}; \bar{g}]}_{\text{truncate}} + S^{\mathrm{gf}} + S^{\mathrm{gh}}$$

  - suggested by WT-identities: $\widehat{\Gamma}_k[g - \bar{g}; \bar{g}] \approx 0$

Truncating the theory space

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$$k\partial_k \bar{u}_i(k) = \beta_i(\bar{u}_i; k)$$

- Example: Einstein-Hilbert truncation:

$$\Gamma_k^{\text{grav}}[g] \approx \frac{1}{16\pi G_k} \int d^4x \sqrt{g}\, \{-R + 2\Lambda_k\}$$

  - β-functions for dimless couplings $g_k \equiv G_k k^2$, $\lambda_k \equiv \Lambda_k k^{-2}$

The Einstein-Hilbert truncation

- Einstein-Hilbert truncation $\Longleftrightarrow$ two "running" couplings: $G(k), \Lambda(k)$

$$\Gamma_k^{\rm grav}[g] = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g}\,\{-R + 2\Lambda(k)\}$$

- project FRGE onto truncation subspace
- result: non-perturbative β-functions for dimensionless couplings

$$g_k := k^2 G_k\,, \quad \lambda_k := \Lambda_k k^{-2}$$

- Particular choice of $\mathcal{R}_k$ (sharp cutoff)

$$k\,\partial_k g_k = (\eta_N + 2) g_k\,,$$

$$k\,\partial_k\,\lambda_k = -\,(2 - \eta_N)\,\lambda_k - \tfrac{g_k}{\pi}\Big[5\ln(1 - 2\,\lambda_k) - 2\zeta(3) + \tfrac{5}{2}\eta_N\Big]$$

anomalous dimension of Newton's constant:

$$\eta_N = -\frac{2g_k}{6\pi + 5g_k}\Big[\tfrac{18}{1-2\lambda_k} + 5\ln(1 - 2\lambda_k) - \zeta(2) + 6\Big]$$

The Einstein-Hilbert truncation

- non-perturbative β-functions for dimensionless couplings

$$k\,\partial_k g_k = \beta_g(g_k, \lambda_k)\,, \quad k\,\partial_k \lambda_k = \beta_\lambda(g_k, \lambda_k)$$

- β-functions have NGFP:

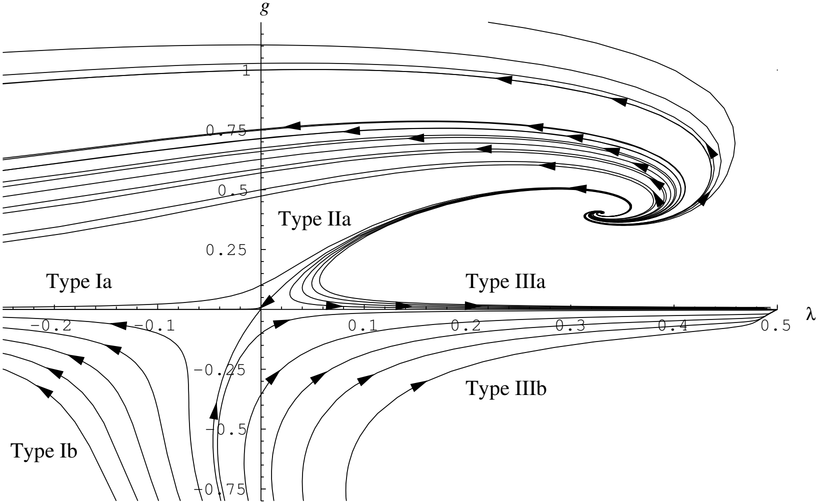
$$\beta_g(g^*, \lambda^*) = \beta_\lambda(g^*, \lambda^*) = 0\,, \quad g^* > 0, \quad \lambda^* > 0$$

  - UV attractive in g_k, λ_k
  - quantum physics: anomalous dim. of Newtons constant: $\eta_N = -2$!

$$G(k) = g^*\,k^{-2}, \quad \Lambda(k) = \lambda^* k^2$$

- If present in full theory: NGFP provides UV completion of gravity

Phase diagram of quantum gravity in the EH-truncation



M. Reuter, F. Saueressig, Phys. Rev. D **65** (2002) 065016 [hep-th/0110054]

Questions raised by the Einstein-Hilbert results:

Einstein-Hilbert truncation leads to natural questions:

- Does the NGFP also exist in higher-dimensional truncations?
- How many couplings are relevant?
- What does it happen at $k = 0$?
- What is the physical content of the theory?

Exploring the theory space of gravity ($\Gamma_k^{\mathrm{grav}}[g]$)

$\vdots$			
R^8	$\dots$		
R^7	$\dots$		
R^6	$\dots$		
R^5	$\dots$		
R^4	$\dots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$	$R \Box R$	$+$ 7 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
$\mathbb{1}$			

Exploring the theory space of gravity ($\Gamma_k^{\text{grav}}[g]$)

$\vdots$

Einstein-Hilbert truncation

R^8	$\dots$		
R^7	$\dots$		
R^6	$\dots$		
R^5	$\dots$		
R^4	$\dots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$	$\Box R$	+ 7 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
$\mathbb{1}$			

Exploring the theory space of gravity ($\Gamma_k^{\mathrm{grav}}[g]$)

Einstein-Hilbert truncation

R^2-truncation

$\vdots$			
R^8	$\dots$		
R^7	$\dots$		
R^6	$\dots$		
R^5	$\dots$		
R^4	$\dots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$	$\Box R$	+ 7 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
$\mathbb{1}$			

Exploring the theory space of gravity ($\Gamma_k^{\mathrm{grav}}[g]$)

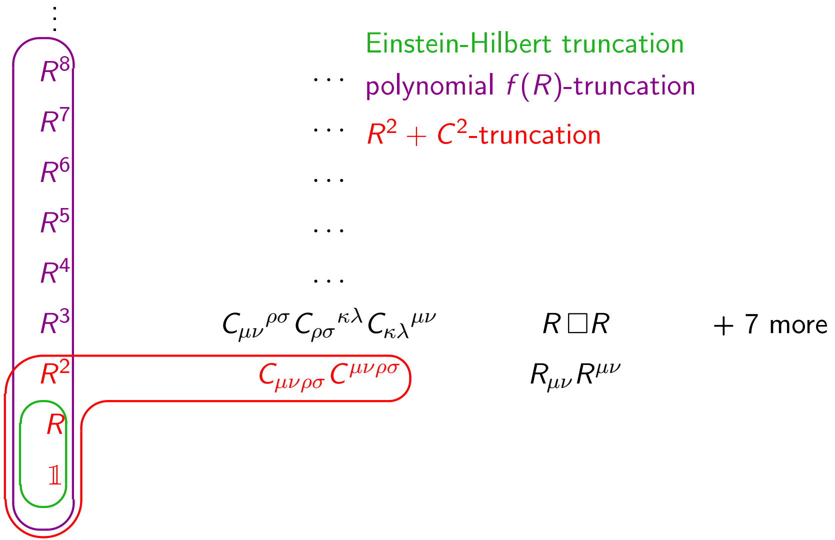
Einstein-Hilbert truncation

polynomial $f(R)$-truncation

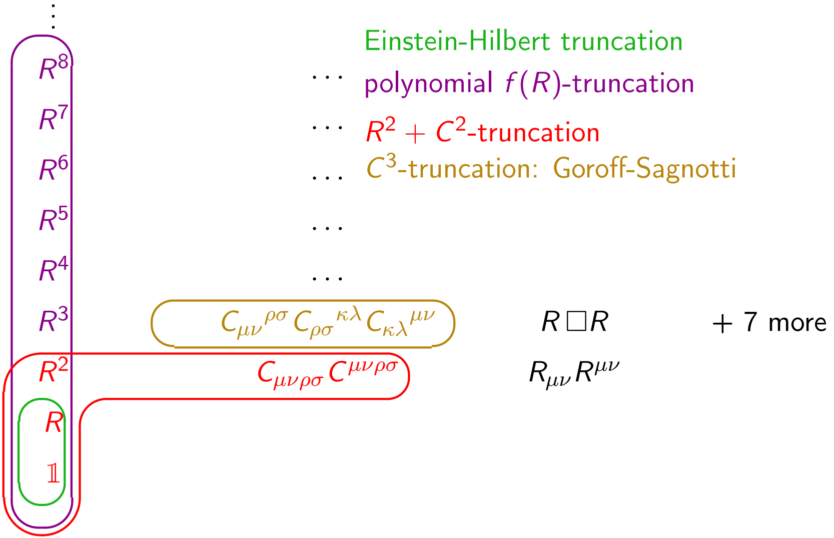
$\vdots$

R^8	$\cdots$		
R^7	$\cdots$		
R^6	$\cdots$		
R^5	$\cdots$		
R^4	$\cdots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$	$R \Box R$	$+$ 7 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
$\mathbb{1}$			

Exploring the theory space of gravity ($\Gamma_k^{\text{grav}}[g]$)



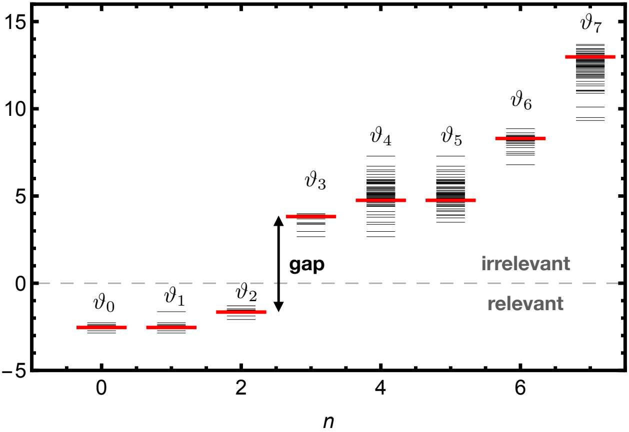
Exploring the theory space of gravity ($\Gamma_k^{\text{grav}}[g]$)



Increasing evidence for a finite dimensional UV-manifold

- ...
- A. Codello, C. Rahmede, R. Percacci, 2007, Phys.Rev.Lett
- D. Benedetti, F.Saueressig, P.Machado, 2009.
- AB, M.Reuter, JHEP 2005
- D. Benedetti, F. Caravelli, JHEP 2012 $f(R)$
- Gies et al, 2017 PRL , C^3
- de Brito, G.P. et al, f(R) reparametrization
- ...(gravity + matter, unimodular gravity, tensor models)

- Falls, Litim, Schroeder, PRD 2019



Issues in Quadratic Gravity

Gravitational action of the type

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\gamma R - \alpha C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} + \beta R^2 \right] \qquad (0)$$

leads to a renormalizable theory in $D = 4$ dimensions

- Unitarity is lost due to the presence of a spin-2 ghost of mass $m_2^2 = \frac{\gamma}{2\alpha}$ in the spectrum of physical states
  K. Stelle PRD 16, 953, 1977
    - see M. Piva's talk for the fakeon solution
- What is the phase diagram from the functional RG approach?
    - Benedetti, Machado, Saueressig, MPA 2009
    - Hamada & Yamada JHEP 2017

Issues in Quadratic Gravity

The starting action is

$$\Gamma_k = \Gamma_k^{\text{gravity}} + S_{\text{gf}} + S_{\text{gh}}. \tag{1}$$

The higher-derivative gravity action is parameterised with

$$\begin{aligned}\Gamma_k^{\text{gravity}} &= \int d^4x\,\sqrt{g}\,\left[\lambda - \xi R + aR^2 + bR_{\mu\nu}^2\right] \\ &= \int d^4x\,\sqrt{g}\left[\lambda - \xi R + \left(a + \frac{b}{3}\right)R^2 + \frac{b}{2}C^2 - \frac{b}{2}E\right] \\ &= \int d^4x\,\sqrt{g}\left[\lambda - \xi R + \left(a + \frac{b}{4}\right)R^2 + \frac{b}{4}R_{\mu\nu\rho\sigma}^2 - \frac{b}{4}E\right],\end{aligned} \tag{2}$$

where we define the Gauss-Bonnet term $E = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ which is topological invariant in $d =$, and $C^2 = E + 2R_{\mu\nu}R^{\mu\nu} - 2/3R^2 = R_{\mu\nu\rho\sigma}^2 - 2R_{\mu\nu}^2 + 2/3R^2$ is the squared Weyl tensor.

The gauge and ghost action are given by

$$S_{\rm gf} = \frac{1}{2\alpha} \int d^4x \sqrt{\bar{g}}\, \bar{g}^{\mu\nu} \Sigma_\mu \Sigma_\nu \,, \tag{3}$$

$$S_{\rm gh} = - \int d^4x \sqrt{\bar{g}}\, \bar{C}_\mu \left[\bar{g}^{\mu\nu} \bar{\nabla}^2 + \frac{1-\beta}{2} \bar{\nabla}^\mu \bar{\nabla}^\nu + \bar{R}^{\mu\nu} \right] C_\nu \,, \tag{4}$$

where C_μ and $\bar{C}_\mu$ are the ghost and anti-ghost fields and

$$\Sigma_\mu = \bar{\nabla}^\nu h_{\nu\mu} - \frac{\beta+1}{4} \bar{\nabla}_\mu h \,, \tag{5}$$

with $h = \bar{g}^{\mu\nu} h_{\mu\nu}$.

- Employ the York decomposition

$$
\begin{aligned}
h_{\mu\nu} &= h^{\perp}_{\mu\nu} + \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu + \left(\bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{1}{4} \bar{g}_{\mu\nu} \bar{\Box} \right) \sigma + \frac{1}{4} \bar{g}_{\mu\nu} h, \\
C_\mu &= C^{\perp}_\mu + \bar{\nabla}_\mu \tilde{C}, \\
\bar{C}_\mu &= \bar{C}^{\perp}_\mu + \bar{\nabla}_\mu \tilde{\bar{C}}
\end{aligned}
$$

- Lichnerowicz Laplacians

$$
\begin{aligned}
\bar{\Delta}_{L0} S &:= -\bar{\Box}\, S, \\
\bar{\Delta}_{L1} \xi_\mu &:= -\bar{\Box}\, \xi_\mu + \bar{R}_\mu{}^\nu \xi_\nu, \\
\bar{\Delta}_{L2} h_{\mu\nu} &:= -\bar{\Box}\, h_{\mu\nu} + \bar{R}_\mu{}^\rho h_{\rho\nu} + \bar{R}_\nu{}^\rho h_{\mu\rho} - 2 \bar{R}_\mu{}^\alpha{}_\nu{}^\beta h_{\alpha\beta}.
\end{aligned}
$$

- The Hessian of Γ_k are given as follows:

$$\Gamma_k^{(TT)} = \frac{\xi}{2}\bar{\Delta}_{L2} - a\bar{R}\left(\bar{\Delta}_{L2} - \frac{\bar{R}}{2}\right) + \frac{b}{2}\left(\bar{\Delta}_{L2}^2 - \frac{3\bar{R}}{2}\bar{\Delta}_{L2} + \frac{\bar{R}^2}{2}\right) - \frac{\lambda}{2}, \tag{6}$$

$$\Gamma_k^{(\xi\xi)} = \left[\frac{\xi}{2}\bar{R} - \lambda - \frac{1}{\alpha}\left(\bar{\Delta}_{L1} - \frac{\bar{R}}{2}\right)\right]\left(\bar{\Delta}_{L1} - \frac{\bar{R}}{2}\right), \tag{7}$$

$$\Gamma_k^{(SS)} = \begin{pmatrix} \Gamma_k^{(\sigma\sigma)} & \Gamma_k^{(\sigma h)} \\ \Gamma_k^{(h\sigma)} & \Gamma_k^{(hh)} \end{pmatrix}, \tag{8}$$

$$\Gamma_k^{(\sigma\sigma)} = \left[-\frac{3\xi}{16}\left(\bar{\Delta}_{L0} - \bar{R}\right) + \frac{9a}{8}\bar{\Delta}_{L0}^2 + \frac{3b}{8}\bar{\Delta}_{L0}^2 - \frac{3\lambda}{8} - \frac{9}{16\alpha}\left(\bar{\Delta}_{L0} - \frac{\bar{R}}{3}\right)\right]\bar{\Delta}_{L0} \quad (9)$$

$$\Gamma_k^{(\sigma h)} = \Gamma_k^{(h\sigma)} = \left(-\frac{3\xi}{16} + \frac{9a}{8}\bar{\Delta}_{L0} + \frac{3b}{8}\bar{\Delta}_{L0}\right)\bar{\Delta}_{L0}\left(\bar{\Delta}_{L0} - \frac{\bar{R}}{3}\right) - \frac{3\beta}{16\alpha}\bar{\Delta}_{L0}\left(\bar{\Delta} \quad (10)$$

$$\Gamma_k^{(hh)} = -\frac{3\xi}{16}\bar{\Delta}_{L0} + \frac{9a}{8}\left(\bar{\Delta}_{L0} - \frac{\bar{R}}{3}\right)\bar{\Delta}_{L0} + \frac{3b}{8}\left(\bar{\Delta}_{L0} - \frac{\bar{R}}{3}\right)\bar{\Delta}_{L0} + \frac{\lambda}{8} - \frac{\beta^2}{16\alpha}\bar{\Delta} \quad (11)$$

It may be useful to use regulators such that the regulator such that the Lichnerowicz Laplacians are replaced to k^2. That is,

$$\mathcal{R}_k^{(\mathrm{TT})}(z) = \left(\Gamma_k^{(\mathrm{TT})}(k^2) - \Gamma_k^{(\mathrm{TT})}(z)\right)\theta\left(k^2 - z\right) \quad \text{with } z = \bar{\Delta}_{L2}, \tag{12}$$

$$\mathcal{R}_k^{(\xi\xi)}(z) = \left(\Gamma_k^{(\xi\xi)}(k^2) - \Gamma_k^{(\xi\xi)}(z)\right)\theta\left(k^2 - z\right) \quad \text{with } z = \bar{\Delta}_{L1}, \tag{13}$$

$$\mathcal{R}_k^{(\mathrm{SS})}(z) = \left(\Gamma_k^{(\mathrm{SS})}(k^2) - \Gamma_k^{(\mathrm{SS})}(z)\right)\theta\left(k^2 - z\right) \quad \text{with } z = \bar{\Delta}_{L0}. \tag{14}$$

These regulators are the so-called type-II cutoff. We can choose the type-I cutoff or hybrid of both types.

The flow equation

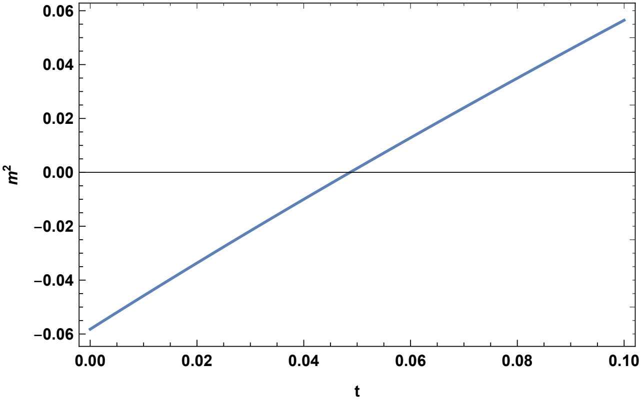
After long manipulations, the flow equation reads

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}_{(2\mathrm{TT})} \left[\frac{\partial_t R_k^{h^\perp h^\perp}}{\Gamma^{(2)}_{h^\perp h^\perp} + R_k^{h^\perp h^\perp}} \right] + \frac{1}{2} \mathrm{Tr}_{(0)} \left[\frac{\partial_t R_k^{hh}}{\Gamma^{(2)}_{hh} + R_k^{hh}} \right] + \eta_k^{(1)} + \eta_k^{(0)} \,. \tag{15}$$

One can easily count the degrees of freedom: from the TT mode (5 degrees of freedom) and *h*-mode (1 degrees of freedom), there are 6 degrees of freedom, while degrees of freedom of $\eta_k^{(1)}$ and $\eta_k^{(0)}$ have 3 and 1, respectively. $\eta_k^{(1)}$ and $\eta_k^{(0)}$ have a minus sign, so that the total degrees of freedom is $6 - 4 = 2$.

Fixed points of the beta functions

- Pure quadratic gravity
  - $a = 0.00509116$ $b = -0.0112065$ $\xi = 0.0112348$ $\lambda = 0.0043599$
  - The masses of the spin-2 and spin-0 field are: $m_2^2 = -1.00252$ $m_0^2 = -0.0112347663$ are imaginary near the cutoff



Sperically symmetric solutions ($m_2^2 > 0$)

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \tag{16}$$

Define:

$$h(r) = 1 + V(r) \quad f(r) = 1 + W(r) \tag{17}$$

Defining a new function $Y(r) = r^{-2}\,(rW(r))'$ one finds, for large r

$$\begin{aligned} &\left(\nabla^2 - m_0^2\right)\left(\nabla^2 V + 2Y\right) = 0 \\ &\nabla^2 \left(\left(\nabla^2 - \frac{3m_2^2 m_0^2}{m_2^2 + 2m_0^2} \right) V + 2\frac{m_2^2 - m_0^2}{m_2^2 + 2m_0^2} Y \right) = 0. \end{aligned} \tag{18}$$

Non tachyonic case

$$
\begin{aligned}
h(r) &= 1 + C_t - \frac{2M}{r} + 2Y_2^- \frac{\mathrm{e}^{-m_2 r}}{r} + 2Y_2^+ \frac{\mathrm{e}^{m_2 r}}{r} + \\
&+ Y_0^- \frac{\mathrm{e}^{-m_0 r}}{r} + Y_0^+ \frac{\mathrm{e}^{m_0 r}}{r} \\
f(r) &= 1 - \frac{2M}{r} + Y_2^- \frac{\mathrm{e}^{-m_2 r}}{r}(1 + m_2 r) + \\
&+ Y_2^+ \frac{\mathrm{e}^{m_2 r}}{r}(1 - m_2 r) - Y_0^- \frac{\mathrm{e}^{-m_0 r}}{r}(1 + m_0 r) + \\
&- Y_0^+ \frac{\mathrm{e}^{m_0 r}}{r}(1 - m_0 r)
\end{aligned}
\tag{19}
$$

Behavior near $r = 0$

Frobenious analysis

$$h(r) = h_0 r^t \left(1 + \sum_{n=1}^{N} h_n r^n\right) + \mathcal{O}\left(r^{t+N+1}\right)$$
$$f(r) = r^s \left(1 + \sum_{n=0}^{N} h_n r^n\right) + \mathcal{O}\left(r^{s+N+1}\right)$$

- $(0, 0)$ class: Minkowski solution and de Sitter solution. Regular metric at $r = 0$
- $(-1, -1)$ class: Schwarzschild solutions
- $(2, -2)$ class: new family of solutions (first discovered by Bob Holdom).

Behavior near $r = r_0 \neq 0$

$$h(r) = h_0(r - r_0)^t \left(1 + \sum_{n=1}^{N} h_n(r - r_0)^n\right) + \mathcal{O}\left(r^{t+N+1}\right)$$

$$f(r) = (r - r_0)^s \left(1 + \sum_{n=0}^{N} h_n(r - r_0)^n\right) + \mathcal{O}\left(r^{s+N+1}\right)$$

- $(1, 0)$ class: Wormholes: radial divergent metric and regular temporal component of the metric
- $(1, 1)$ class: regular horizons

Behavior near horizon - Einstein-Weyl theory

C_{2-} decreasing Yukawa - $C_{2,0}$ coefficient of $1/r$.

$$h(r) = h_1 \left((r - r_H) + h_2 (f_1, r_H) (r - r_H)^2 + h_3 (f_1, r_H) (r - r_H)^3 + \mathcal{O} \left((r - r\right.\right.$$
$$f(r) = f_1 (r - r_H) + f_2 (f_1, r_H) (r - r_H)^2 + f_3 (f_1, r_H) (r - r_H)^3 + \mathcal{O} \left((r - r_H)^4\right.$$

We "shoot" from large r towards an interior point and we impose continuity of the function and the derivatives

$$\begin{aligned} F_1 (h_1, f_1, C_{2,0}, C_{2-}) &= h_L(r_f) - h_R(r_f) \\ F_2 (h_1, f_1, C_{2,0}, C_{2-}) &= h'_L(r_f) - h'_R(r_f) \\ F_3 (h_1, f_1, C_{2,0}, C_{2-}) &= f_L(r_f) - f_R(r_f) \\ F_4 (h_1, f_1, C_{2,0}, C_{2-}) &= f'_L(r_f) - f'_R(r_f) \end{aligned}$$

Phase diagram

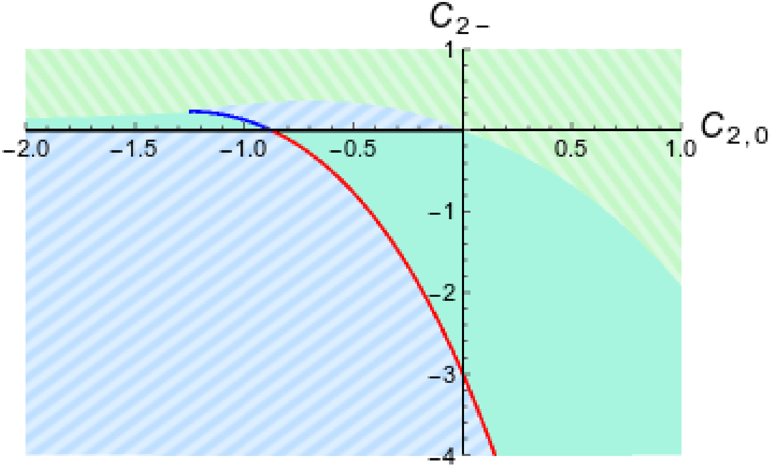


Figura: solid: vanishing metric, green diagonal: singular metric , blue diagonal WHs

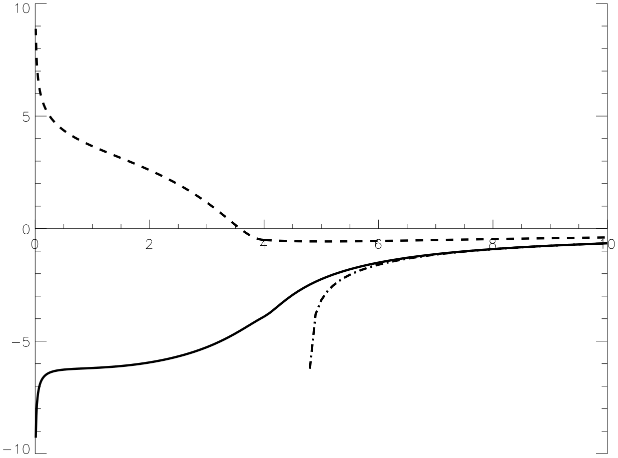


Figura: Holdom-type of solution obtained for $M = 4.79$ and $u = -23.77$. Radial profile of $-\ln(A(r)$ (dashed) and $\ln B(r)$ (solid) for $a_2 = 1.5$ and $a_3 = a_4 = b_2 = 1$. Dot-dashed line represents the corresponding Schwarzschild

Critical point

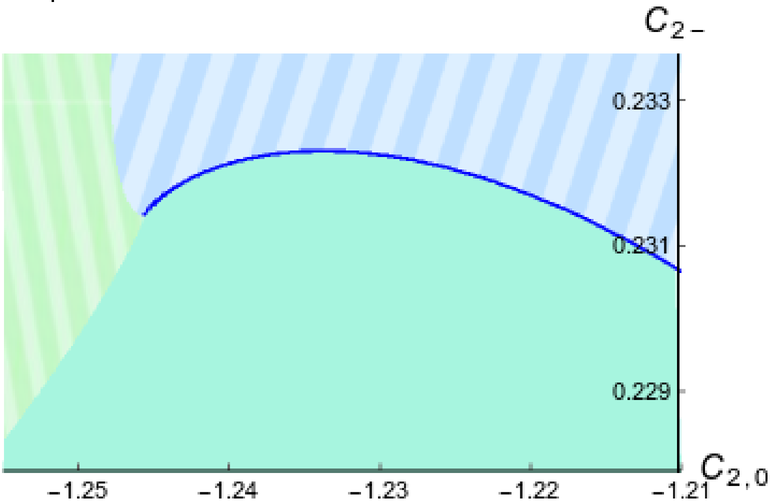
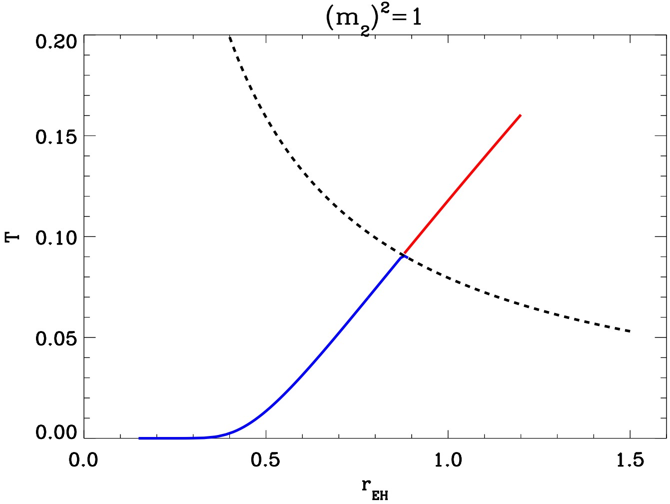


Figura: Coexistence of the three solutions


$(m_2)^2 = 1$
0.20
0.15
0.10
0.05
0.00
T
0.0
0.5
1.0
1.5
r_{EH}

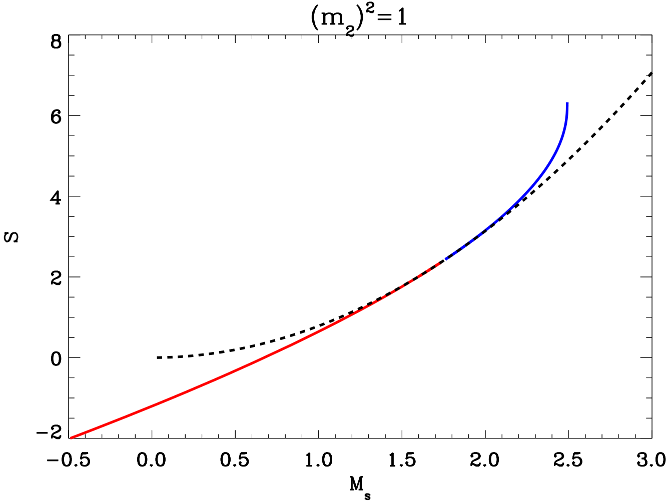
Entropy

- Bekenstein-Hawking entropy

$$S_{BH} = \frac{1}{4}A$$

- Wald entropy

$$S = -\frac{1}{8\pi}\int_{\mathrm{Horizon}} d\theta d\phi \frac{\delta \mathcal{S}}{\delta R_{rtrt}}\sqrt{-g_{tt}g_{rr}}$$



$(m_2)^2=1$
S
M_s
8
6
4
2
0
−2
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Tachyonic branch: $m_2^2 < 0$

- AB S.Silveravalle, PRD R 2019

$$
\begin{aligned}
h(r) &= 1 + C_t - \frac{2M}{r} + 2A_2 \frac{\cos\left(|m_2| r + \varphi_2\right)}{r} + A_0 \frac{\cos\left(|m_0| r + \varphi_0\right)}{r} \\
f(r) &= 1 - \frac{2M}{r} + A_2 \frac{\cos\left(|m_2| r + \varphi_2\right)}{r} + A_2 |m_2| \sin\left(|m_2| r + \varphi_2\right) \\
&- A_0 \frac{\cos(|m_0| r + \varphi_0)}{r} - A_0 |m_0| \sin\left(|m_0| r + \varphi_0\right)
\end{aligned}
$$

which also depends on six unknown (four coefficients and two phases, φ_0 and φ_2). However, the spacetime is no longer asymptotically flat and we must require $A_0|m_0| \ll 1$ and $A_2|m_2| \ll 1$ for our linearised solution to be valid at large values of the radial coordinate r.

It is interesting to look at the behavior of the curvature invariant near $r = 0$ for $M = 0$. It is not difficult to show that regular behavior at $r = 0$ can be obtained if $\varphi_0 = \varphi_2 = \pi/2 + 2\pi n$ so that

$$\begin{aligned} R &= -3m_0^3 A_0 \\ R_{\mu\nu}R^{\mu\nu} &= \frac{1}{3}(7m_0^6 A_0^2 - 2m_0^3|m_2|^3 A_0 A_2 + 4m_2^6 A_2^2) \\ K &= \frac{1}{3}(5m_0^6 A_0^2 - 4m_0^3|m_2|^3 A_0 A_2 + 8m_2^6 A_2^2) \end{aligned} \tag{20}$$

We have thus found a new class of "rippled" solutions of the field equations in the weak field regime which are everywhere regular. One can argue that these solutions are the Lorentzian counterpart of the kinetic condensate solutions which stabilize the conformal factor in $R + R^2$ (AB, M.Reuter, PRD 2013)

Numerical integration of the EOM

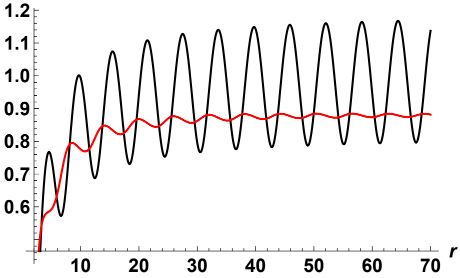


Figura: $f(r)$ (black) and $h(r)$ (red) obtained by solving the EOM of the quadratic theory as a function of the radial coordinate (in Planck units).

Euclidean Quantum Gravity

- Gravitational instanton

$$Z = \int D[g] e^{-\mathcal{S}_{\mathrm{E}}}$$

- lower bound

$$\mathcal{S}_{\mathrm{E}} \approx -4.07\pi\beta$$

where $\beta > 0$ is the coefficient of the R^2 term.

Conclusions

- According to AS pure gravity seems to finite and predictive
- UV critical manifold has finite dimensionality
- The tensor structure of the resulting theory is problematic
  - the quantum theory does not represent reality
  - a miracle happens in the untruncated theory