Universidad Nacional de La Plata

Asymptotic safety and noncommutativeness

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Outline

NC QFT

Why NC Quantum Field Theories?

Initially a method to regularize infinities in QFT

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Quantized Space-Time

HARTLAND S. SNYDER
Department of Physics, Northwestern University, Evanston, Illinois

(Received May 13, 1946)

It is usually assumed that space-time is a continuum. This assumption is not required by Lorentz invariance. In this paper we give an example of a Lorentz invariant discrete space-time.

THE problem of the interaction of matter and fields has not been satisfactorily solved to this date. The root of the trouble in present field theories seems to lie in the assumption of point interactions between matter and fields. trary procedures, and neither process has yet been formulated in a relativistically invariant manner. It may not be possible to do this.

It is possible that the usual four-dimensional continuous space-time does not provide a suitable

NC QFT

$$[\hat{x}_1, \hat{x}_2] = 2i\theta.$$

NC QFT

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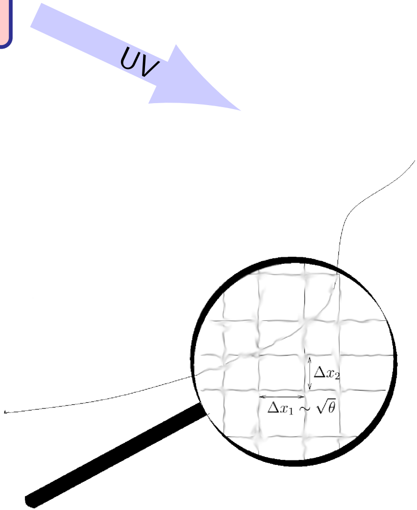

NC QFT

More recent approaches:

Connes geometric perspective

Connes & Lott, Nucl. Phys. Proc. Suppl. 18B (1991), pag. 29-47;

As effective field theories of Strings

Seiberg & Witten, hep-th/9908142;

Agrees with expected general features of effective field theories of Quantum Gravity

for example Doplicher, Fredenhagen, Roberts hep-th/0303037 .

NC QFT

Maybe the simplest approach is to deform the algebra of functions (fields):

$$(f \star g)(x) = \boxed{e^{i\Theta_{ab}\partial_a^f \partial_b^g} f(x)\, g(x)}$$

$\star$ is NC & associative; indeed

$$[x_a, x_b]_\star = 2i\Theta_{ab}.$$

NC QFT: $\lambda\varphi_\star^4$

Now one can define a QFT by replacing products with $\star$. For the quartic theory

$$S = \int d^4x \, (\partial\varphi_\star)^2 + m^2\varphi_\star^2 + \frac{\lambda}{4!}\varphi_\star^4.$$

where $\lambda\varphi_\star^4 = \lambda(\varphi\star\varphi\star\varphi\star\varphi)$ and $\star$ is the Moyal product.

Because of the Moyal product, we have three different contributions:

$$\Gamma^{NC}_{1-loop} = \frac{1}{2}\log \mathrm{Det}\Big\{ -\partial^2 + m^2 + \frac{\lambda}{3}\left[L(\phi^2(x)) + R(\phi^2(x)) + L(\phi)R(\phi)\right]\Big\},$$

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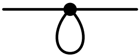
where we have used the notation

$$L(\phi(x))g(x) = \phi(x)\star g(x), \qquad R(\phi(x))g(x) = g(x)\star\phi(x).$$

Because of the Moyal product, we have three different contributions:

$$L(\phi^2(x)), R(\phi^2(x)) \equiv$$



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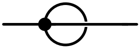


NC QFT

Planar contribution

$$L(\phi^2(x)) + R(\phi^2(x)) \to$$

$$\Gamma^P_{1-loop} = \frac{\lambda}{3!}\frac{m^{d-2}}{(2\pi)^{d/2}}\Gamma(1-d/2, m^2/\Lambda^2)\int_{\mathbb{R}^d}\phi^2$$

NC QFT

Nonplanar contribution

$L(\phi)R(\phi) \to$

$$\Gamma^{NP}_{1-loop} = \frac{\lambda}{12}(16\,\pi^3)^{-d/2} m^{d-2} \int d^d\bar{p}\; \tilde{\phi}^*(p)\tilde{\phi}(p) \cdot \Sigma_{NP}(p).$$

A Smearing function is generated by the geometry

$$\Sigma_{NP}(\hat{p}) := 2(m|\Theta\hat{p}|)^{1-d/2} K_{d/2-1}(2m|\Theta\hat{p}|)$$

$$\Gamma^{NP}_{1-loop} \sim \int_{\mathbb{R}^d} d\hat{x}\,\phi(\hat{x}) \int_{\mathbb{R}^d} d\hat{y}\,\frac{1}{\det\Theta}\; e^{\beta(\hat{x}-\hat{y})\Theta^{-2}(\hat{x}-\hat{y})}\; \phi(\hat{y}).$$

NC QFT

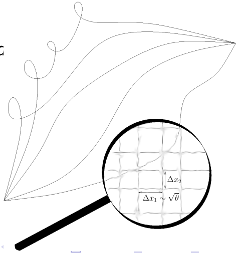
Nonplanar contribution

$L(\phi)R(\phi) \rightarrow$

$$\Gamma^{NP}_{1-loop} = \frac{\lambda}{12}(16\,\pi^3)^{-d/2} m^{d-2} \int d^d\bar{p}\ \tilde{\phi}^*(p)\tilde{\phi}(p) \cdot \Sigma_{NP}(p).$$

A Smearing function is generated by the geometry

$$\Sigma_{NP}(\hat{p}) := 2(m|\Theta\hat{p}|)^{1-d/2}K_c$$



NC QFT

$$\Sigma_{NP}(\hat{p}) := 2(m|\Theta\hat{p}|)^{1-d/2}K_{d/2-1}(2m|\Theta\hat{p}|)$$

$$\Downarrow$$

UV/IR mixing.

Minwalla, Van Raamsdonk, Seiberg, hep-th/9912072

Craig, Koren: hep-ph/1909.01365 $\rightarrow$ possibility to generate small scales in Standard model

Grosse-Wulkenhaar model

These problems are not shared by every NC QFT.

Consider for example the "minor" change introduced by Grosse & Wulkenhaar ('03)

$$S = \int d^4x \phi \star \left(\Box + m^2\right)\phi + \lambda \phi_\star^4 + \text{¡¡} \; \omega^2 x^2 \phi^2 \; \text{!!} \tag{1}$$

It is renormalizable to all-orders in a perturbative expansion (Grosse & Wulkenhaar '04)

Grosse-Wulkenhaar model

- Computation of the one-loop beta function (Grosse & Wulkenhaar, '04)

$$\beta_\lambda = \frac{\lambda^2}{48\pi^2} \frac{1-\omega^2\theta^2}{(1+\omega^2\theta^2)^3} + \cdots \tag{2}$$

- Two- and three-loops beta function (Disertori & Rivasseau, '06)

- All order beta function (Disertori, Gurau, Magnen and Rivasseau, '06)

$$\Downarrow$$

Asymptotic safety

Grosse-Wulkenhaar model

Responsible for this:

→ Langman-Szabo duality ('02);

$$S = \int d^4x \phi \star \left(\Box + m^2 \right) \phi + \lambda \phi_\star^4 + \omega^2 x^2 \phi^2 \qquad (3)$$

→ easy solution because of a sort of Ward identity (unitary transformation, similar to those of Luttinger liquid).

Snyder-de Sitter model

Can we construct some connection with cosmology?

Consider de Sitter (or FRW) noncommutative geometries.

- construction of fuzzy de Sitter via representations of Lie algebras (Burić, Latas, Nenadović, '18, '19);
- curved k-Minkowski spaces from Quantum Groups (Gutierrez-Sagredo, Ballesteros, Gubitosi, Herranz, '19);

Snyder-de Sitter model

Snyder-de Sitter (SFV & Mignemi hep-th/1909.xxxxx, 1806.11467):

$$[\hat{x}_i, \hat{x}_j] = i\beta^2 J_{ij}, \qquad [\hat{p}_i, \hat{p}_j] = i\alpha^2 J_{ij},$$

$$[\hat{x}_i, \hat{p}_j] = i[\delta_{ij} + \alpha^2 \hat{x}_i \hat{x}_j + \beta^2 \hat{p}_j \hat{p}_i + \alpha\beta(\hat{x}_j \hat{p}_i + \hat{p}_i \hat{x}_j)].$$

$$(\Lambda = 3\alpha^2 \sim 10^{-66}\mathrm{eV}^{-2})$$

Motivated by unmodified Poincaré symmetries:

$$[J_{ij}, J_{kl}] = i(\delta_{ik} J_{il} - \delta_{il} J_{jk} + \delta_{jk} J_{il} - \delta_{jl} J_{ik}),$$

$$[J_{ij}, \hat{p}_k] = i(\delta_{ik} \hat{p}_j - \delta_{kj} \hat{p}_i),$$

$$[J_{ij}, \hat{x}_k] = i(\delta_{ik} \hat{x}_j - \delta_{kj} \hat{x}_i).$$

Snyder-de Sitter model

Consider a ϕ^4 action in Snyder-de Sitter:

$$S = \int d^D x \sqrt{g} \left\{ \phi \left(\hat{p}^2 + \frac{\alpha^2}{2} \hat{J}^2 + m^2 \right) \phi + \phi_\star^4 \right\},$$

One can make a transformation of the coordinates

$$\hat{x}_i =: X_i + \lambda P_i, \qquad \hat{p}_i =: (1-\lambda) P_i - \frac{\alpha}{\beta} X_i,$$

$$\Downarrow$$

$$[X_i, X_j] = i\beta^2 J_{ij}, \qquad [P_i, P_j] = 0, \qquad [X_i, P_j] = i(\delta_{ij} + \beta^2 P_i P_j),$$

This is Snyder space.

Snyder-de Sitter model

$$P_i =: p_i = -i\partial_i, \quad X_i =: x_i + \beta^2 x_j p_j p_i = x_i - \beta^2 x_j \partial_j \partial_i.$$

$$\Downarrow$$

Canonical commuting variables!
If we choose projective coordinates in de Sitter,

$$g = \frac{1}{(1+\alpha^2 \hat{x}^2)^{(D+1)}},$$

we can expand the quadratic part of the action for small α

$$S^{(2)} = \int d^D x\, \phi \left(p^2 + m^2 - \frac{D(D+1)}{2}\alpha^2 + \frac{\alpha^2}{\beta^2}x^2 + \alpha^2 (xp)(px) + \frac{(1-D)}{2}\alpha^2 x^2 p^2 - \frac{(D+1)}{2}\frac{\alpha^4}{\beta^2}x^4 + \mathcal{O}(\theta^4) \right)\phi.$$

Snyder-de Sitter model

Consider the one-loop divergent terms:

$$\int \phi^4 \left[-\frac{\lambda^2}{128\pi^2\varepsilon} - \frac{\alpha^2\lambda^2 m_{eff}^2}{64\pi^2\varepsilon\omega^2} - \frac{\beta^2\lambda^2 m_{eff}^2}{32\pi^2\varepsilon} \right] + \phi^4 x^2 \left[-\frac{3\alpha^2\lambda^2}{256\pi^2\varepsilon} - \frac{\beta^2\lambda^2\omega^2}{32\pi^2\varepsilon} \right] - \frac{\beta^2\lambda^2\phi^4_{(1),\star}}{128\pi^2\varepsilon} + \phi(\delta m^2 + x^2\delta\omega^2 + \text{higher powers of x})\phi$$

$$\delta m^2 = \frac{\lambda\, \text{polynomial}(\alpha,\beta,\omega)}{240\pi^2\varepsilon\omega^4}$$

$$\delta\omega^2 = \frac{\lambda\left(\omega^2\left(\omega - 416\alpha^2\right) + 96\alpha^2 m^4 + 6\beta^2 m^2\omega^3\right)}{16\pi^2\varepsilon\omega}$$

$$\text{hp of x} = \frac{7\alpha^2\lambda x^6\omega^3}{3\pi^2\varepsilon} + \frac{\lambda x^4\omega\left(\beta^2\omega^3 + 40\alpha^2 m^2\right)}{8\pi^2\varepsilon} + \frac{\lambda\beta^2 x^2\omega^3(-\partial_y^2)}{96\pi^2\varepsilon\omega}$$

Closing remarks

- connection of Grosse-Wulkenhaar's term with geometry;
- need to study the 1-loop renormalization flow;
- possibility of generating the cosmological constant scale through RG flow?
- relation to swampland AdS conjecture;
- implications/bounds from $\frac{\alpha^2}{\beta^2} \sim 10^{-9}\mathrm{eV}^2$?
- existence of Ward identity?