BRST-invariant RG flows

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Introduction

Exact Renormalization Group Equation

Physical systems at different energy scales, are described by different descriptions.

Renormalization Group (RG) transformations, transform us between these different descriptions of a physical system at different energies.

By means of Wilsonian RG & specific regularization (Cut-off on Momenta),
We'll have a RG differential Equation (RGE), which is Exact.

Gauge Symmetry

What will happen to the gauge symmetry, in this procedure?

- Cut-off on Momenta $\rightarrow$ Breaking of Gauge Symmetry
- Gauge Symmetry through the flow

Outline

- Exact Renormalization Group Equation, ERGE (Wetterich)
- The special way of regularization $\longrightarrow$ Breaking of gauge symmetry
- Recovering the symmetry: (4-d gauge-fixed Yang-Mills theory)

  Regularized (mass-like regularization) & BRST symmetric

  - Faddeev-Popov quantization with Fourier noise
  - BRST symmetry and master equation
  - Modified master equation
  - flow equations and gauge invariance
  - Non-linear gauge and mass-like regularization
  - flow of the effective average action and compatibility with the master equation

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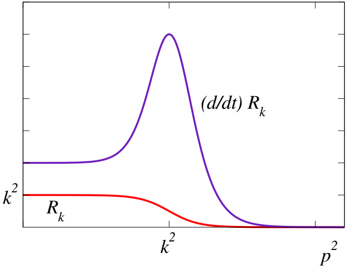
Wetterich ERGE

(Wetterich'93, Gies'06)

$$e^{W_k[J]} \equiv \mathcal{Z}_k[J] := \exp\left(-\Delta S_k\left[\frac{\delta}{\delta J}\right]\right)\mathcal{Z}$$

$$= \int_\Lambda \mathcal{D}\varphi\, e^{-S[\varphi]-\Delta S_k[\varphi]+\int J\varphi},$$

$$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^D q}{(2\pi)^D}\varphi(-q)R_k(q)\varphi(q)$$



$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\partial_t R_k\left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1}\right]$$

$$\Gamma_k[\phi] = \sup_J\left(\int J\phi - W_k[J]\right) - \Delta S_k[\phi]$$

$t = ln\frac{t}{\Lambda}$, $\partial_t = k\frac{d}{dk}$, $\phi(x) = \langle\varphi(x)\rangle_J = \frac{\delta W_k[J]}{\delta J(x)}$

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Breaking of gauge symmetry by regularization

Adding a mass-like regulator term to gauge-fixed Yang-Mills theory:

$$\Delta S_k = \frac{1}{2}\int \frac{d^D p}{(2\pi)^D}\, A^a_\mu(-p)\,(R_{k,A})^{ab}_{\mu\nu}(p)\, A^b_\nu(p) + ghosts$$

Manifest BRST invariance is certainly lost. For having a BRST symmetric RG flow:

- Starting with a gauge symmetric regularized effective action
- Compatibility of the flow equation with the gauge symmetry

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Faddeev-Popov Quantization (Off-shell formulation)

(Zinn-Justin'74)

$$\mathcal{Z} = \int \mathcal{D}A \, e^{-S_{YM}[A]}$$

$$\mathcal{Z} = \int \mathcal{D}A e^{-S_{YM}[A]} \delta\Big(F[A] - n(x)\Big) \, \Delta^{ab}_{FP}$$

Using Fourier transform of $\delta\Big(F[A] - n(x)\Big)$, introducing Nakanishi-Lautrup field and writing $\Delta^{ab}_{FP} = \frac{\delta F^a[A]}{\delta A^c_\mu} \frac{\delta A^{\omega\, c}_\mu}{\delta \omega^b}\Big|_{\omega=0} = \frac{\delta F^a[A]}{\delta A^c_\mu} D^{cb}_\mu$ in terms of ghosts:

$$\mathcal{Z} = \int \mathcal{D}A \mathcal{D}b \mathcal{D}\bar{c} \mathcal{D}c \, e^{-S_{YM}[A] - b^a(x)\left(F^a[A] - n^a(x)\right) - S_{gh}[\bar{c}, c, A]}$$

Choosing a Gaussian weight for the noise field $e^{\frac{-n^a n^a}{2\xi}}$ results in:

$$S_{NL} = \frac{\xi}{2} b^a b^a$$

$$S_{gf} = \frac{1}{2\xi} F^a[A] F^a[A]$$

Choosing a Fourier weight for the noise field $e^{\frac{-v^a n^a}{2\xi}}$ results in:

$$e^{-S_{NL}} = \delta(b^a - v^a)$$

$$S_{gf} = v^a F^a[A]$$

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BRST symmetry

(Zinn-Justin'74)

For any S_{NL}, the action, is invariant under the following BRST symmetry:

$$S_{YM}[A]+S_{gf}[A,b]+S_{NL}[b]+S_{gh}[A,\bar{c},c]$$

where $S_{gf}[A,b]=b^aF^a[A]$.

$$\begin{aligned} sA^a_\mu &= D^{ab}_\mu c^b \\ sc^a &= \frac{g}{2}f^{abc}c^bc^c \\ s\bar{c}^a &= b^a \\ sb^a &= 0\,. \end{aligned}$$

In the case of Fourier noise after integration over b^a, we'll left with:

$$\begin{aligned} sA^a_\mu &= D_\mu c^a\,, \\ sc^a &= \frac{g}{2}f^{abc}c^bc^c\,, \\ s\bar{c}^a &= v^a\,, \\ sv^a &= 0\,. \end{aligned}$$

$$S_{gf}[A,v]=v^aF^a[A]$$

S_{gf} remains linear in $F^a[A]$

BRST symmetry and Zinn-Justin Master equation

(Zinn-Justin'74)

Zinn-Justin Master Equation:

$$e^{W[J,\eta,\bar{\eta},v,K,L]} = \int \mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c}\, e^{-S[A,c,\bar{c},v]-S_{\rm so}}\,.$$

$$\begin{aligned} S_{\rm so} = & - J_a^\mu A_\mu^a - \bar{\eta}^a c^a - \bar{c}^a \eta^a \\ & + K_\mu^a\,(D^\mu c)^a + L^a \frac{1}{2} g f^{abc} c^b c^c\,. \end{aligned}$$

Using $s^2 = 0$ and the Legendre transform:

$$\Gamma[A,c,\bar{c},b,K,L] = \sup_{J,\eta,\bar{\eta}} \left\{ J_a^\mu A_\mu^a + \bar{\eta}^a c^a + \bar{c}^a \eta^a - W[J,\eta,\bar{\eta},K,L] \right\},$$

$$\frac{\delta\Gamma}{\delta A_a^\mu}\frac{\delta\Gamma}{\delta K_\mu^a} + \frac{\delta\Gamma}{\delta c^a}\frac{\delta\Gamma}{\delta L^a} + v^a \frac{\delta\Gamma}{\delta \bar{c}^a} = 0\,.$$

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Modified Master Equation

(Ellwanger'94; Reuter, Wetterich'94)

By adding the mass-like regulator term:

$$\tilde{\Gamma}_k = \Gamma_k + \Delta S_k, \qquad \Delta S_k = \frac{1}{2}\int A^a_\mu\, R_{\mu\nu}\, A^a_\nu + ghosts$$

$$\frac{\delta\tilde{\Gamma}_k}{\delta A^\mu_a}\frac{\delta\tilde{\Gamma}_k}{\delta K^a_\mu} + \frac{\delta\tilde{\Gamma}_k}{\delta c^a}\frac{\delta\tilde{\Gamma}_k}{\delta L^a} + b^a\frac{\delta\tilde{\Gamma}_k}{\delta\bar{c}^a} - \mathrm{Tr} R_{\mu\nu}\left(\frac{\delta^2\tilde{\Gamma}_k}{\delta A^a_\mu \delta\Phi}\right)^{-1}\frac{\delta^2\tilde{\Gamma}_k}{\delta\Phi^\dagger\delta K^{a\mu}} + \cdots = 0\,.$$

where:

$$\Phi^i = \begin{pmatrix} A^{a\mu} \\ c^a \\ -\bar{c}^a \end{pmatrix}$$

concrete calculations possible but tedious

(Ellwanger,Hirsch,Weber'96'98)
(Gies,Jaeckel,Wetterich'04)

modified master equation preserve under the flow.

(Ellwanger'94; Litim, Pawlowski'99;Pawlowski'95)

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Flow equations and gauge invariance

- Use of background field methods

(Reuter, Wetterich'94)

True quantum gauge invariance encoded in modified symmetry identities (Nielsen identities, Shift-Ward identity) (...)

- Manifestly gauge invariant RG

(Morris'98)
(Arnone,Gatti,Morris'02;Morris,Rosten'06)

Embedding into $SU(N \mid N)$ gauge theory, no gauge fixing, no ghosts

- Geometric effective action and Wilsonian flows

(Pawlowski'03)
(Branchina,Meissner,Veneziano'03;Donkin,Pawlowski'12)

Vilkovisky-DeWitt framework, modified Nielsen identities

- Gauge invariant flow equation

(Wetterich'16'17)

Projection onto physical modes, bootstrap construction of action and fields

BRST Symmetry:

- Quantum BRST transformation which is scale dependent $BRST_k$

(Sonoda'07)
(Igarashi,Itoh,Sonoda'07'08'10; Igarashi,Itoh,Morris'19)

- the regulator as a part of gauge fixing

$$\Delta S_k + S_{gf} \to S_{gf,k}$$

(Sh.A,Gies,Zambelli'19)

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Non-Linear gauge and mass-like regularization

(Sh.A,Gies,Zambelli'19)

In the case of Fourier weight for the noise field, $S_{gf} = v^a F^a[A]$; we choose a non-linear gauge:

$$\mathrm{F}^a[A] = A^{b\mu} \mathrm{Q}^{abc}_{\mu\nu} A^{c\nu} + \mathrm{L}^{ab}_{\mu} A^{a\mu}$$

$$\mathrm{Q}^{abc}_{\mu\nu} = \frac{v^a}{2|v|^2} \mathrm{Q}_{\mu\nu} \delta^{bc} ,$$

$$\mathrm{L}^{ab}_{\mu} = \left(1 + r_{\mathrm{gh}}(-\partial^2)\right) \partial_\mu \delta^{ab}$$

$$\mathrm{Q}_{\mu\nu} = R_{\mu\nu}(\partial) - \frac{1}{\xi} \partial_\mu \partial_\nu .$$

$$R^{\mu\nu}(\partial) = R_{\mathrm{L}}(-\partial^2)\Pi^{\mu\nu}_{\mathrm{L}} + R_{\mathrm{T}}(-\partial^2)\Pi^{\mu\nu}_{\mathrm{T}}$$

$$R_{\mathrm{gh}}(\partial) = (-\partial^2) r_{\mathrm{gh}}(-\partial^2)$$

$$S_{gf} = \frac{1}{2} A^a_\mu \mathrm{Q}^{\mu\nu} A^a_\nu + v^a (1 + r_{\mathrm{gh}}(-\partial^2)) \partial^\mu A^a_\mu ,$$

$$S_{gh} = -\bar{c}^a (1 + r_{\mathrm{gh}}(-\partial^2)) \left(\partial^\mu D_\mu c\right)^a$$

$$- \frac{v^a}{2|v|^2} \bar{c}^a \left(\left(\mathrm{Q}^{\mu\nu} A^b_\nu \right) (D_\mu c)^b + A^b_\mu \left(\mathrm{Q}^{\mu\nu} D_\nu c \right)^b \right)$$

Breaking of global color symmetry, non-local vertex, new and v-dependent ghost-gluon vertices.

BRST-symmetric and regularized effective action

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ERGE

(Sh.A,Gies,Zambelli'19)

Obtaining the Exact RG flow equation:

$$e^{W_k[J,\eta,\bar{\eta},v,K,L]} = \int \mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c}\, e^{-S[A,c,\bar{c},v]-S_{\rm so}}$$

$$S_{\rm so} = -J^{a\mu}A^a_\mu - \bar{\eta}^a c^a - \bar{c}^a\eta^a + K^a_\mu (D^\mu c)^a + L^a \frac{g}{2} f^{abc} c^b c^c - \frac{v^a}{|v|^2}\bar{c}^a A^b_\mu I^{b\mu} - \frac{v^a}{|v|^2}\bar{c}^a (D_\mu c)^b M^{b\mu}\,.$$

$$\mathcal{J}_i = \begin{pmatrix} J^a_\mu \\ \bar{\eta}^a \\ \eta^a \end{pmatrix}, \Phi^i = \begin{pmatrix} A^{a\mu} \\ c^a \\ -\bar{c}^a \end{pmatrix}, \mathcal{I}_i = \begin{pmatrix} K^a_\mu \\ L^a \\ M^a_\mu \\ I^a_\mu \end{pmatrix}. \widetilde{\Gamma}[\Phi, v, \mathcal{I}] = \sup_{\mathcal{J}_i} \left\{ \mathcal{J}_i^\dagger \Phi^i - W[\mathcal{J}, v, \mathcal{I}] \right\}$$

$$\partial_t \widetilde{\Gamma} = \frac{1}{2}\left(\partial_t R_{\mu\nu}\delta^{ab}\right)\left(\left(\widetilde{\Gamma}^{(2)-1}\right)_{A^a_\mu A^b_\nu} + \widetilde{\Gamma}^{(2)}_{M^a_\mu \Phi^j}\left(\widetilde{\Gamma}^{(2)-1}\right)_{\Phi^j A^b_\nu} + \widetilde{\Gamma}^{(2)}_{K^b_\nu I^a_\mu} + \frac{\delta\widetilde{\Gamma}}{\delta M^a_\mu} A^{b\nu} - \left(\frac{\delta}{\delta K^b_\nu}\widetilde{\Gamma}\right)\left(\widetilde{\Gamma}\frac{\overleftarrow{\delta}}{\delta I^a_\mu}\right)\right)$$

$$+ \left(\partial_t r_{\rm gh}\partial_\mu\delta^{ab}\right)\left(\widetilde{\Gamma}^{(2)}_{K^a_\mu \Phi^j}\left(\widetilde{\Gamma}^{(2)-1}\right)_{\Phi^j(-\bar{c}^b)} - \bar{c}^b\left(\frac{\delta}{\delta K^a_\mu}\widetilde{\Gamma}\right)\right) + \partial_t \Delta S_{gf}\,,$$

$$\partial_t \Delta S_{gf} = \left(\partial_t r_{\rm gh}\partial_\mu\delta^{ab}\right) v^b A^{a\mu} + \frac{1}{2}\left(\partial_t R_{\mu\nu}\delta^{ab}\right) A^{a\mu}A^{b\nu}\,,$$

ERGE

(Sh.A,Gies,Zambelli'19)

$$\Gamma[\Phi, v, \mathcal{I}] = \widetilde{\Gamma}[\Phi, v, \mathcal{I}] - \Delta S[\Phi, v]$$

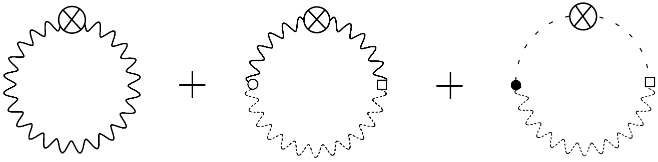
For a family of truncations:

$$\Gamma_k[\Phi, v, \mathcal{I}] = \Gamma[\Phi, v] + S^{\mathrm{brst}}_{\mathrm{so}}[\Phi, v, \mathcal{I}]$$

$$S_{gf} = \frac{1}{2} A^a_\mu \mathrm{Q}^{\mu\nu} A^a_\nu + v^a (1 + r_{\mathrm{gh}}(-\partial^2)) \partial^\mu A^a_\mu,$$

$$S_{gh} = -\bar{c}^a (1 + r_{\mathrm{gh}}(-\partial^2)) (\partial^\mu D_\mu c)^a - \frac{v^a}{2|v|^2} \bar{c}^a \left(\left(\mathrm{Q}^{\mu\nu} A^b_\nu \right) (D_\mu c)^b + A^b_\mu (\mathrm{Q}^{\mu\nu} D_\nu c)^b \right)$$

$$\partial_t \Gamma[\Phi, v] = \frac{1}{2} \left(\partial_t R_{\mu\nu} \delta^{ab} \right) \left(\widetilde{\Gamma}^{(2)-1} \right)_{A^a_\mu A^b_\nu} + \frac{1}{2} \left(\partial_t R_{\mu\nu} \delta^{ab} \right) \widetilde{\Gamma}^{(2)}_{M^a_\mu \Phi^j} \left(\widetilde{\Gamma}^{(2)-1} \right)_{\Phi^j A^b_\nu}$$
$$+ \left(\partial_t r_{\mathrm{gh}} \partial_\mu \delta^{ab} \right) \widetilde{\Gamma}^{(2)}_{K^a_\mu \Phi^j} \left(\widetilde{\Gamma}^{(2)-1} \right)_{\Phi^j (-\bar{c}^b)}.$$



Master equation and BRST symmetry through the RG flow

(Sh.A,Gies,Zambelli'19)

$$\Sigma[\widetilde{\Gamma}] = \frac{\delta\widetilde{\Gamma}}{\delta A^a_\mu}\frac{\delta\widetilde{\Gamma}}{\delta K^{a\mu}} + \frac{\delta\widetilde{\Gamma}}{\delta c^a}\frac{\delta\widetilde{\Gamma}}{\delta L^a} + v^a\frac{\delta\widetilde{\Gamma}}{\delta \bar{c}^a} + M^a_\mu\frac{\delta\widetilde{\Gamma}}{\delta K^a_\mu} + A^a_\mu I^{a\mu} + \frac{\delta\widetilde{\Gamma}}{\delta M^a_\mu} I^a_\mu .$$

$\Sigma = 0$ is the master equation. We will have BRST-symmetry through the flow if the master equation and flow equation are compatible with each other:

$$\partial_t \mathcal{Z} = \mathcal{G}_t \mathcal{Z}$$

$$\mathcal{G}_t = \left(\partial_t r_{\mathrm{gh}}\partial_\mu\delta^{ab}\right)\left(v^b\frac{\delta}{\delta J^a_\mu} - \frac{\delta^2}{\delta\eta^b\delta K^a_\mu}\right) - \frac{1}{2}\left(\partial_t R_{\mu\nu}\delta^{ab}\right)\left(\frac{\delta^2}{\delta J^a_\mu\delta J^b_\nu} - \frac{\delta^2}{\delta M^a_\mu\delta J^b_\nu} - \frac{\delta^2}{\delta I^a_\mu\delta K^b_\nu}\right).$$

$$\mathcal{D} = -J^a_\mu\frac{\delta}{\delta K^a_\mu} + \bar{\eta}^a\frac{\delta}{\delta L^a} + v^a\eta^a - M^\mu_a\frac{\delta}{\delta K^a_\mu} + I^a_\mu\frac{\delta}{\delta J^a_\mu} - I^\mu_a\frac{\delta}{\delta M^\mu_a},$$

$$[\mathcal{D}, \mathcal{G}_t] = 0.$$

Summary

- Wetterich Exact Renormalization Group Equation
- Breaking gauge symmetry and modified master equation(non-linear and second-order master equation)
- Regularized and BRST symmetric effective action (bilinear master equation)
  - Fourier noise and Nonlinear gauge fixing
  - Breaking of the global color symmetry, an external Nakanishi-Lautrup field, a non-local vertex, new regulator-dependent ghost-gluon vertices
- Compatibility of the flow equation with the master equation

BRST symmetric RG flow

Future Works

- Simplest perturbatively renormalizable truncation
- Proper-time regulator Renormalization Group flow

Thanks for Your Attention

Backup

For any gauge-fixing functional $F^a[A]$:

$$S[A, c, \bar{c}, v] = S_{YM}[A] + S_{gf}[A, v] + S_{\mathrm{gh}}[A, c, \bar{c}, v]$$

can be written as usual:

$$S[A, c, \bar{c}, v] = S_{YM}[A] + s\Psi\,,$$

where

$$\Psi = \bar{c}^a \mathrm{F}^a[A]\,.$$