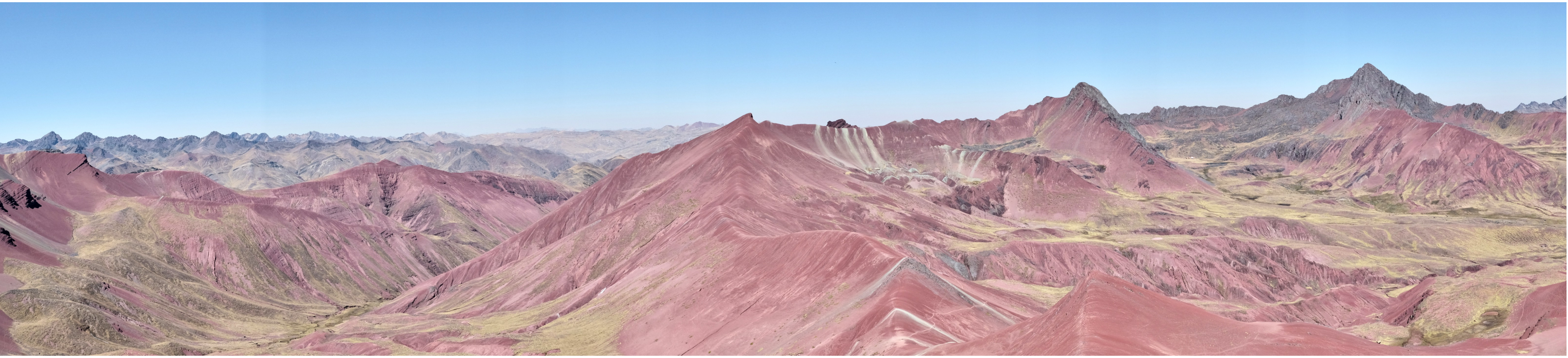
Kardar-Parisi-Zhang Equation with Temporally Correlated Noise: a Non-Perturbative RG Approach



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[based on arXiv:1907.02256]

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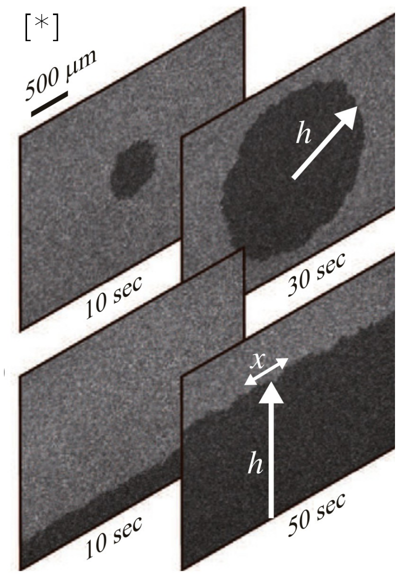
Summary

- Kardar-Parisi-Zhang (KPZ) Equation and Universality Class
- Non-perturbative Renormalization Group (NPRG) and its Application to KPZ Equation
- KPZ Equation with Time-Correlated Microscopic Dynamics: state of the art
- KPZ Equation with Time-Correlated Microscopic Dynamics: NPRG

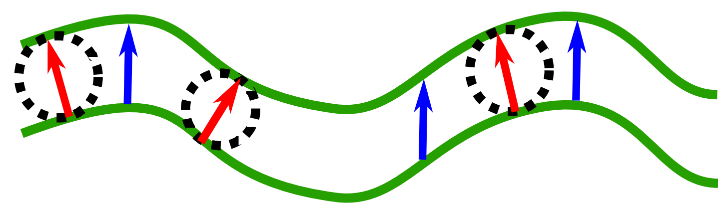
The Kardar-Parisi-Zhang Equation

Kardar-Parisi-Zhang[•] (1986):

$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2}(\vec{\nabla} h)^2 + \eta$$



with random noise $\langle \eta(\mathbf{x})\eta(\mathbf{x}')\rangle = 2D\delta(\mathbf{x}-\mathbf{x}'), \quad \mathbf{x} = (t, \vec{x})$



Describes flat-to-rough transition in growing surfaces

$h(\mathbf{x})$ as the Free-Energy of directed polymer in random media $h = \dfrac{2\nu}{\lambda} \ln Z$

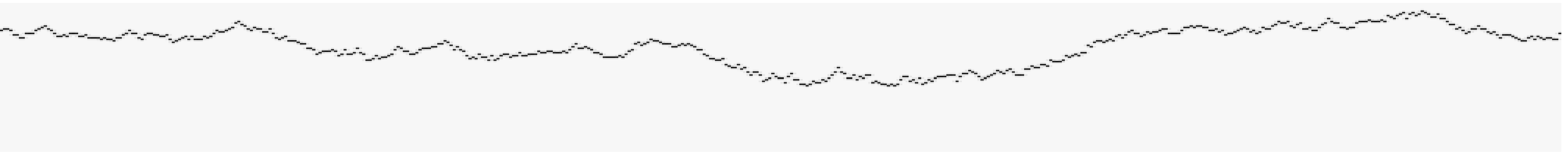
Velocity field $v = -\nabla h$ evolves under the (noisy) Burgers equation:

$$\partial_t v + \lambda v \cdot \nabla v = D\nabla^2 v + f\,, \qquad f = -\nabla \eta$$

[•] Kardar, Parisi and Zhang, *PRL* **56** (1986), [*] Takeuchi, *JSTAT* **1** (2013)

Universal Features of KPZ Equation

Self-organized criticality $\Rightarrow$ No fine-tuning needed



In a finite system of size L fluctuations will saturate to a value $w(L) \sim L^{\chi}$ in a time $T_s \sim L^z$

$$w^2(t,L) = \left\langle \frac{1}{L}\int_x h^2(\mathbf{x}) - \left(\frac{1}{L}\int_x h(\mathbf{x})\right)^2 \right\rangle$$

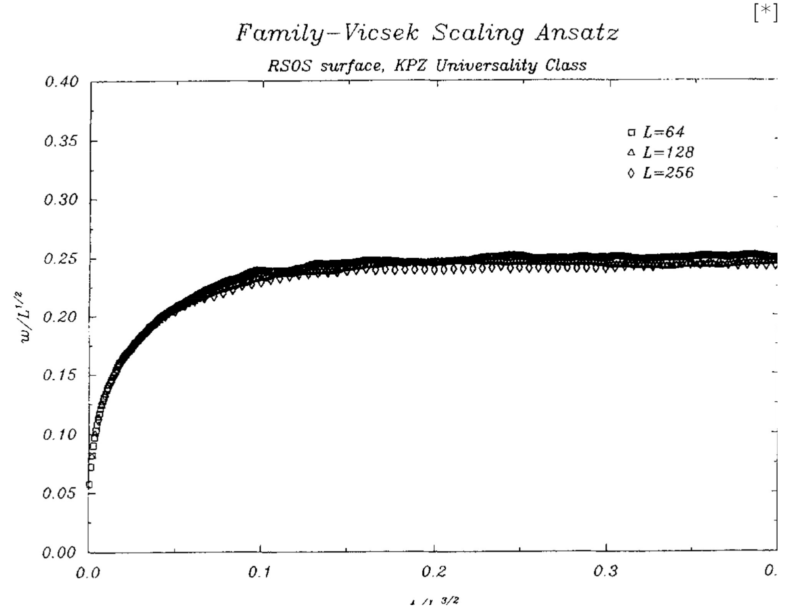
χ → Roughness Exponent
z → Dyamical Exponent

Family-Vicsek scaling ansatz

$$w(t,L) = L^{\chi} f_w\left(\frac{t}{L^z}\right)$$

$\chi, z, f_w(\cdot)$ are universal

[*]



[*]Halpin-Healy & Zhang, *Physics Reports* **254**, (1995)

Beyond Scaling in d=1: Predictions

- The scaling of w suggests $h(t,x) \overset{t\to\infty}{\sim} v_\infty t + (\Gamma t)^{\chi/z}\, \tilde{h}(x,t)$

$P[\tilde{h}]$ depends on the initial condition $h(t=0,x)$

Flat	Curved	Stationary
$P[\tilde{h}] \sim F_{1,GOE}$	$P[\tilde{h}] \sim F_{2,GUE}$	$P[\tilde{h}] \sim F_{0,BR}$
Calabrese and Le Doussal (2011,2012)	Amir *et al.* (2011) Sasamoto and Spohn (2010) Calabrese *et al.*(2010)	Imamura and Sasamoto (2012,2013)

- ...not only scaling for w but also for $C(\mathbf{q}) = \langle h(\mathbf{q}) h(-\mathbf{q}) \rangle$

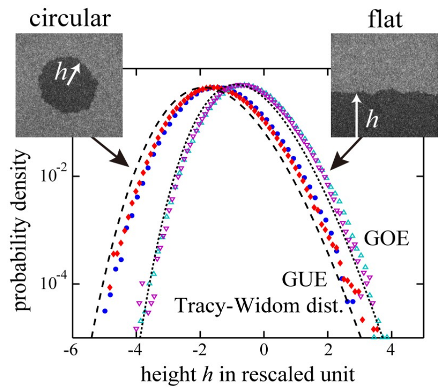
$$C(\mathbf{q}) = q^{-(d+z+2\chi)} F_C\left(\frac{\omega}{q^z}\right)$$

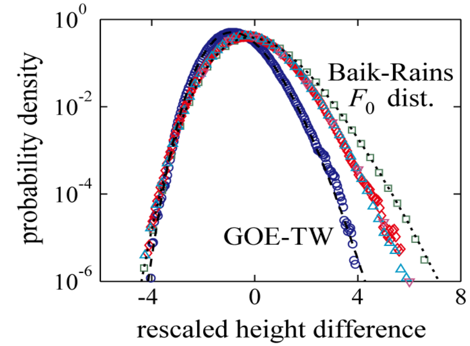
Prähofer and Spohn (2004)

Beyond Scaling in d=1: Evidences

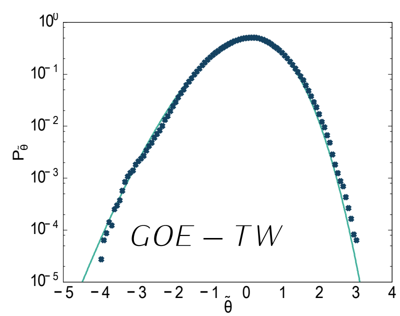
- Geometry dependent sub-classes

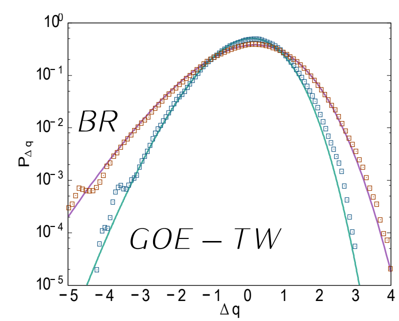
Experimental observation in turbulent liquid crystals (Takeuchi and Sano 2010,2012)





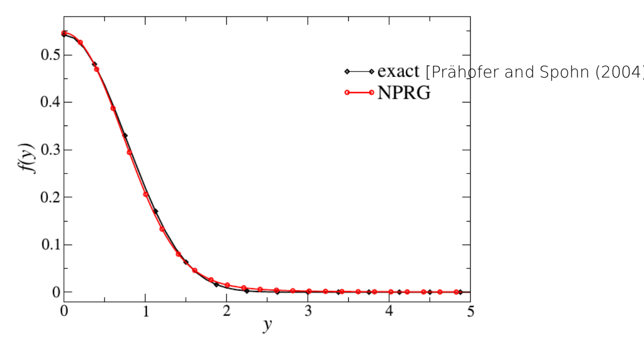
Numerical evidence in Exciton-Polaritons (DS, Canet and Minguzzi 2018)





- Scaling function related to F_C :

Non-perturbative RG (Canet *et al.*)



Symmetries

i) Symmetry under infinitesimal tilt: exact, holds in all dimensions

$$\vec{v}(\mathbf{x}) = \vec{\nabla} h(\mathbf{x})$$

$$\vec{v} \to \vec{v} + \vec{\epsilon}\,, \quad \vec{x} \to \vec{x} + \lambda \vec{\epsilon} t$$

$\Rightarrow$ $\boxed{\lambda \text{ does not renormalize}}$

$$\boxed{z + \chi = 2}$$

ii) Time-reversal symmetry: accidental, holds only in one dimension

$$h(t, \vec{x}) \to -h(-t, \vec{x})$$

$\Downarrow$

$\boxed{\text{Fluctuation-Dissipation Theorem (FDT)}}$ $\boxed{\chi = 1/2}$

---

i) + ii) $\Rightarrow$ $\boxed{\chi = 1/2\,, \quad z = 3/2 \text{ in one dimension}}$

The Strong-Coupling KPZ Fixed Point: $d \geq 2$

One-loop perturbative RG flow of $g = \dfrac{\lambda^2 D}{\nu^3}$: $\quad \dfrac{\mathrm{d}g(\ell)}{\mathrm{d}\ell} = (2-d)g + K_d \dfrac{2d-3}{2d} g^2$

$$g^*_{EW} = 0 \qquad\qquad g^*_{KPZ} = K_d^{-1}(d-2)\frac{2d}{2d-3}$$

$$\frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{\mathrm{d}g(\ell)}{\mathrm{d}\ell}\right)\bigg|_{g^*_{EW}} = 2-d \qquad\qquad \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{\mathrm{d}g(\ell)}{\mathrm{d}\ell}\right)\bigg|_{g^*_{KPZ}} = d-2$$

$$\Downarrow$$

Change in the stability at $d = 2$: in $d > 2$ two possible fixed point

Flat to rough out-of-equilibrium phase transition

...however the rough phase is not reachable at any order of perturbation[•]

Need for non-perturbative techniques

[•]K. J. Wiese, *PRE* **56** (1997)

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Non-Perturbative RG (NPRG) and KPZ equation

Exact RG equation for the Effective Average Action Γ :

$$\boxed{\partial_\kappa \Gamma_\kappa = \frac{1}{2}\mathrm{Tr}\left\{\partial_\kappa R_\kappa \left[\Gamma_\kappa^{(2)} + R_\kappa\right]^{-1}\right\}}$$

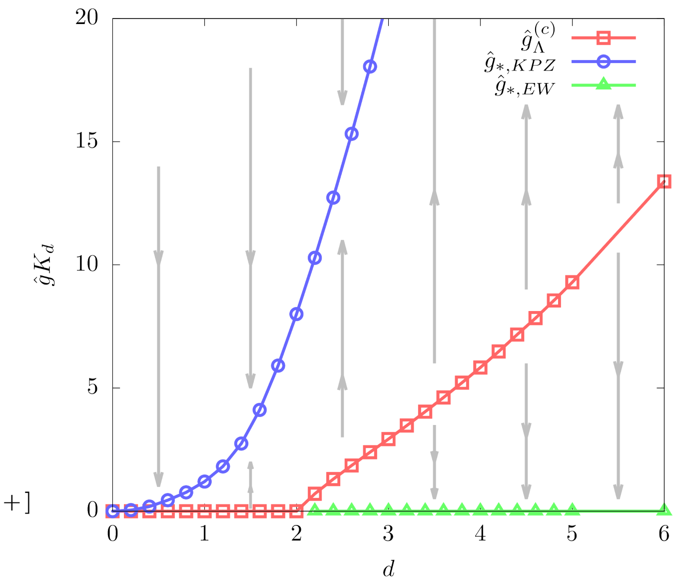
allows for smooth integration of the d.o.f

The Local Potential Approximation for KPZ,

$$\Gamma_{\kappa,LPA}[\varphi,\tilde{\varphi}] = \int_{\mathbf{q}} \tilde{\varphi}\left(\partial_t \varphi - \nu_\kappa \nabla^2 \varphi - \lambda_\kappa |\vec{\nabla}\varphi|^2\right) - D_\kappa \tilde{\varphi}^2$$

is already enough to find good qualitative results...

...but fails to give good quantitative predictions for $d > 2$ [+]



[+]L. Canet, arXiv0509541 (2006)

The Full Quadratic Ansatz[*]

Derivative non-linearity $\Longrightarrow$ Important to be functional in the momenta

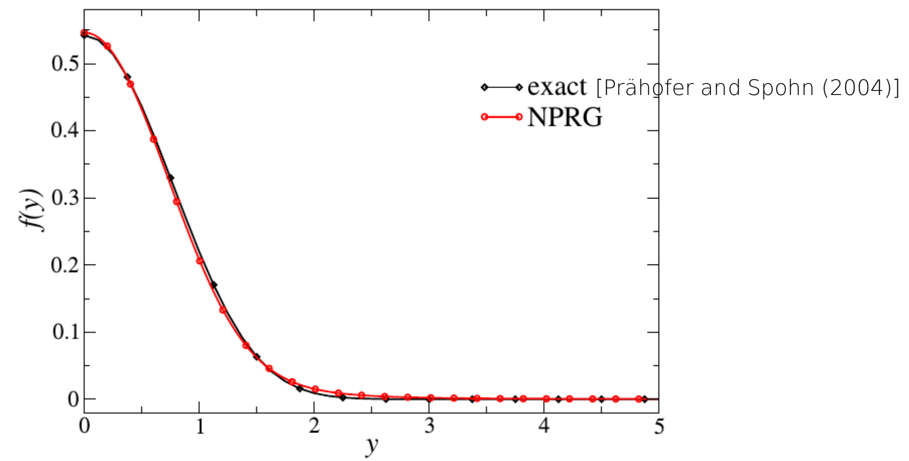
$$\boxed{\Gamma_\kappa[\varphi,\tilde\varphi] = \int_{\mathbf{x}} \left\{ \tilde\varphi f_\kappa^\lambda D_t\varphi - \tilde\varphi f_\kappa^D \tilde\varphi - \frac{1}{2}\Big[\nabla^2\varphi f_\kappa^\nu \tilde\varphi + \tilde\varphi f_\kappa^\nu \nabla^2\varphi\Big]\right\}}$$

with $f_\kappa^X \equiv f_\kappa^X(-\tilde D_t^2, -\nabla^2)\,,\quad \tilde D_t = \partial_t - \lambda\vec\nabla\varphi\cdot\vec\nabla$

and $\eta_\kappa^X = -\frac{1}{X_\kappa}\kappa\partial_\kappa X_\kappa\,,\quad X_\kappa = f_\kappa^X(\omega,q)\big|_{NP}$ $\Longrightarrow$ $\boxed{\begin{array}{c} z = 2-\eta_*^\nu \\ \chi = (2-d+\eta_*^D-\eta_*^\nu)/2 \end{array}}$

$\boxed{\text{Quadratic in } \tilde\varphi \;+\; \text{Linear in } D_t\varphi,\ \nabla^2\varphi \;+\; \text{Functional in } p,\ \omega,\ \varphi}$

Very good results in d=1:



[*]L.Canet *et al.*, PRE **84** (2011)

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Time-Correlation in the KPZ Equation

Real life spoils delta correlation in space and time:

$$\langle \eta(t,\vec{x})\eta(t',\vec{x}')\rangle = 2D(t-t',|\vec{x}-\vec{x}'|)$$

If $D(t-t',|\vec{x}-\vec{x}'|) \neq G(|\vec{x}-\vec{x}'|)\delta(t'-t)$ $\Rightarrow$

Galilean Invariance is broken

$\boxed{\lambda \text{ renormalizes and } z+\chi \neq 2}$

Fluctuation-Dissipation is violated in d=1

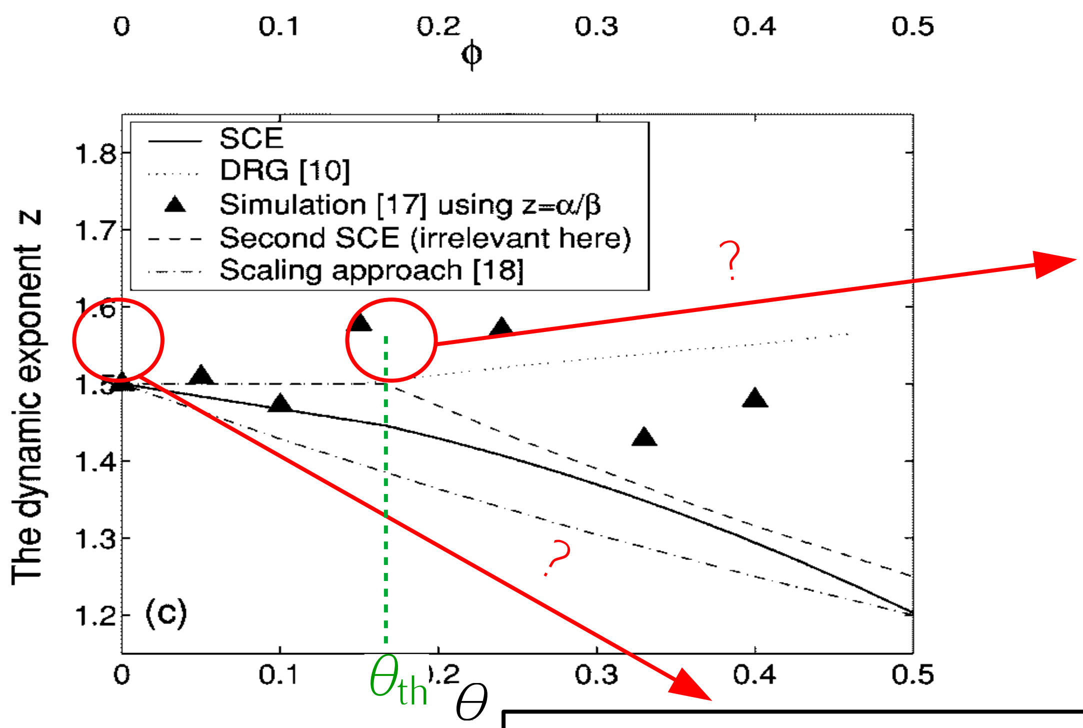
$\boxed{f^D_{\kappa=\Lambda} \neq f^\nu_{\kappa=\Lambda}, \quad \eta^D_{\kappa=\Lambda} \neq \eta^\nu_{\kappa=\Lambda}}$

Does KPZ universality survive to an infinitesimal correlation in time?

Time-Correlation in the KPZ Equation: State of the Art in d=1

Controversial results in the literature for long-range(LR) correlations:

$$D(\omega, q) = D_\theta \omega^{-2\theta}$$



Persistence of pure-KPZ up to $\theta = \theta_{\text{th}}$

Medina *et al.* (1989), Dynamical RG
Fedorenko (2008), Perturbative FRG

Line of LR fixed-points for any $\theta \neq 0$

Ma and Ma (1993), Flory scaling
Katzav and Schwartz (2004), Self-consistent Expansion
Lam *et al.* (1992),Song and Xia (2016), Ales *et al.* (2019), Numerics

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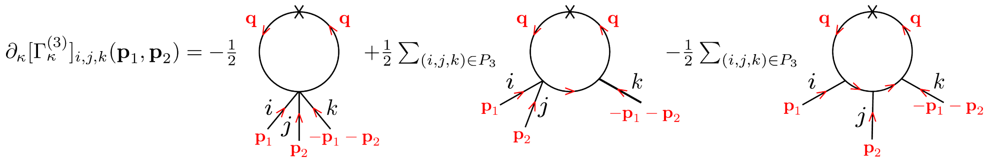
Breaking GI in the NPRG ansatz for KPZ

$$\Gamma_\kappa[\varphi,\tilde\varphi] = \int_{\mathbf{x}} \left\{ \tilde\varphi f_\kappa^\lambda \, D_t\varphi - \tilde\varphi f_\kappa^D \, \tilde\varphi - \frac{1}{2}\Big[\nabla^2\varphi f_\kappa^\nu \, \tilde\varphi + \tilde\varphi f_\kappa^\nu \, \nabla^2\varphi\Big]\right\}$$

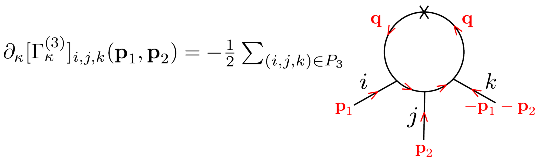
$$\langle \eta(\mathbf{q})\eta(-\mathbf{q})\rangle = G(\omega) \quad \Longrightarrow \quad f^D_{\kappa=\Lambda} = G(\omega)$$

Need for time-dependent f_κ^X … …but is too heavy!

$$\boxed{f_\kappa^X \equiv f_\kappa^X(-\tilde D_t^2, -\nabla^2) \quad \Longrightarrow \quad f_\kappa^X \equiv f_\kappa^X(-\partial_t^2, -\nabla^2)}$$



$\Longrightarrow$

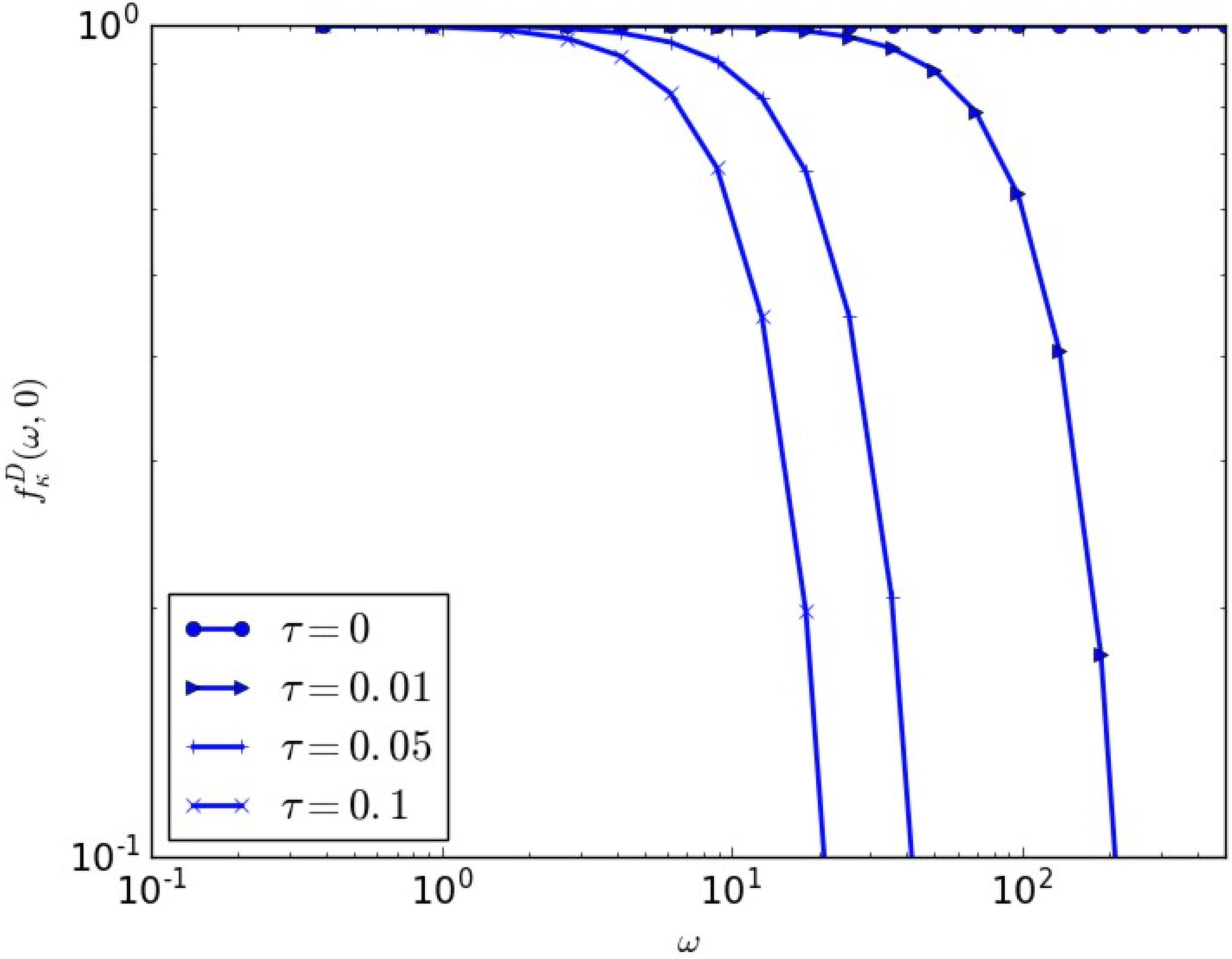


The price to pay is an artificial breaking of Galilean Invariance

$$\partial_\kappa\lambda_k \neq 0 \text{ even if } \theta = 0$$

Short Range (SR) Correlation: $f^D_{\kappa=\Lambda}(\omega, q) = e^{-\omega^2 \tau^2}$

$\boxed{d = 1}$



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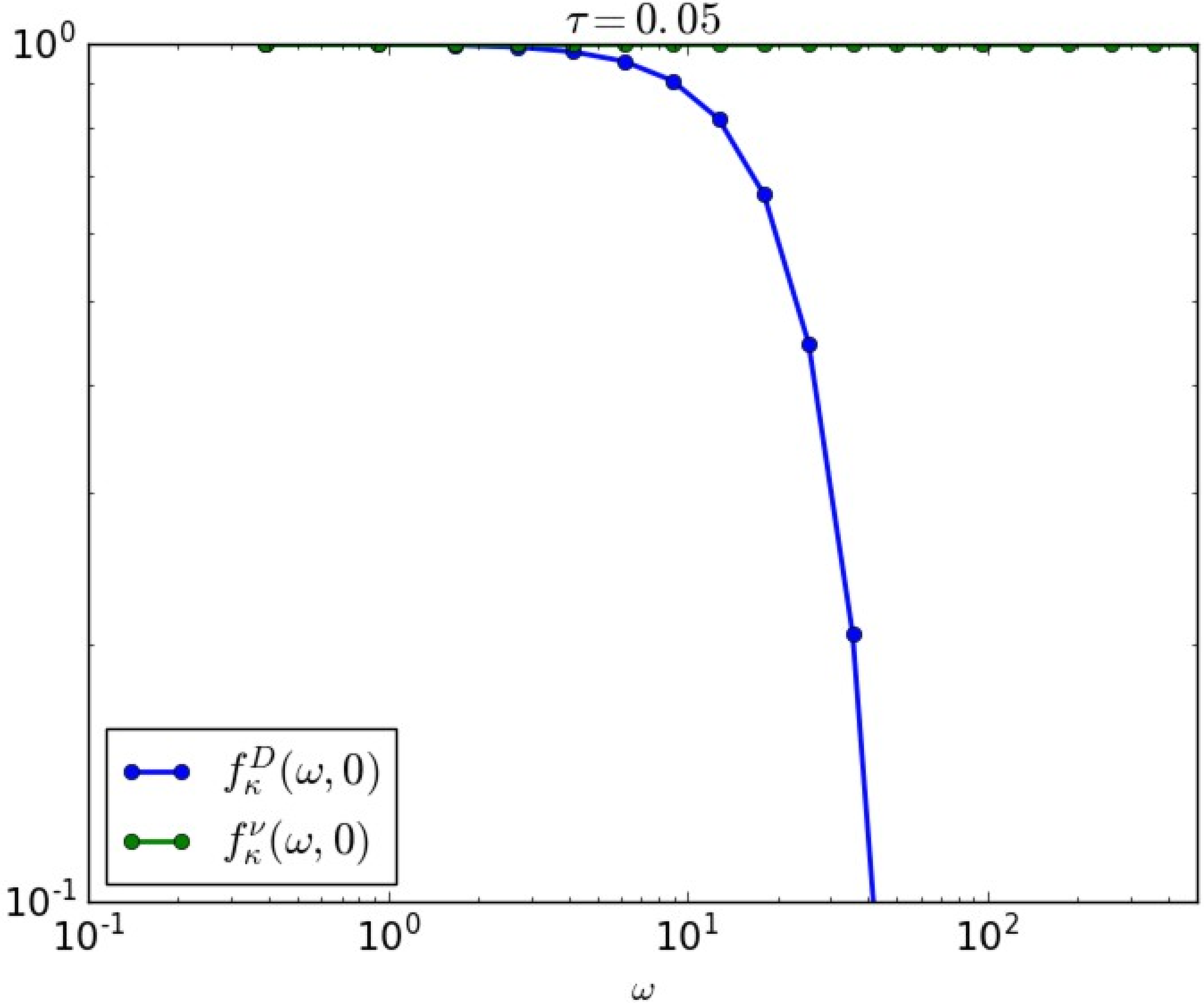
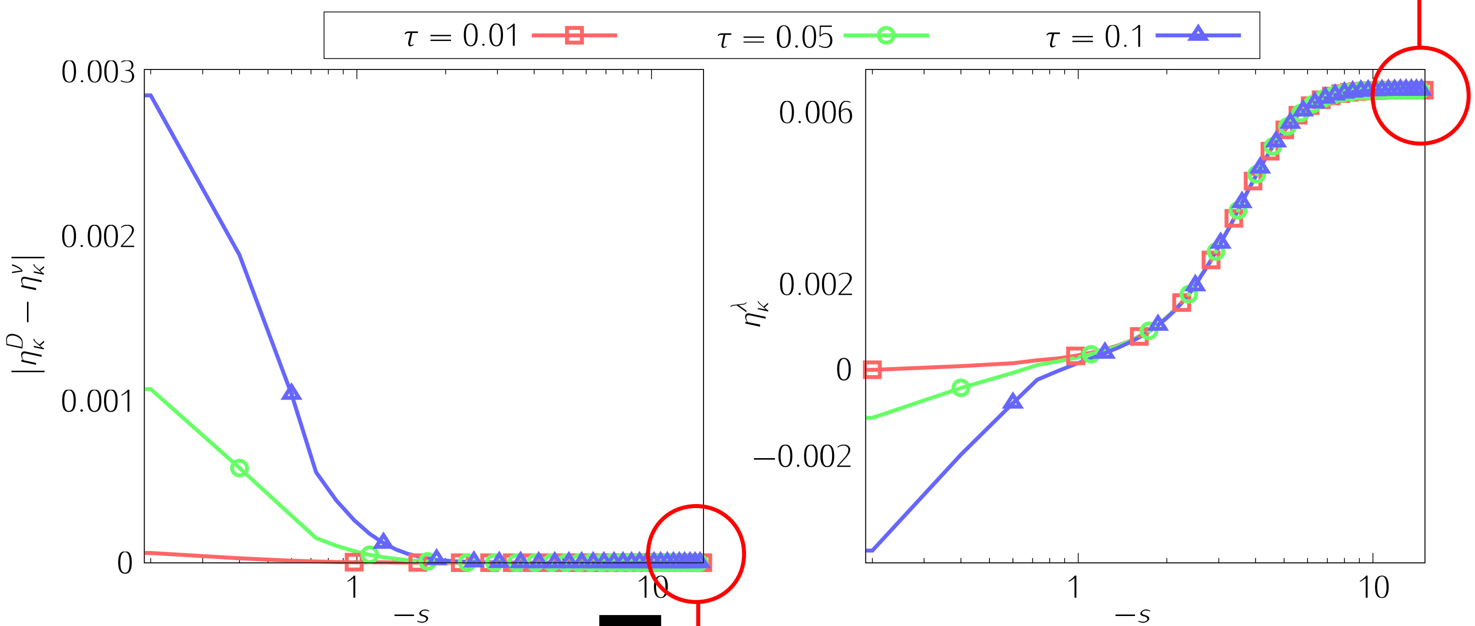


Short Range (SR) Correlation: $f^{D}_{\kappa=\Lambda}(\omega, q) = e^{-\omega^2 \tau^2}$

$$\boxed{X_\kappa \sim \kappa^{-\eta^X_*}}$$

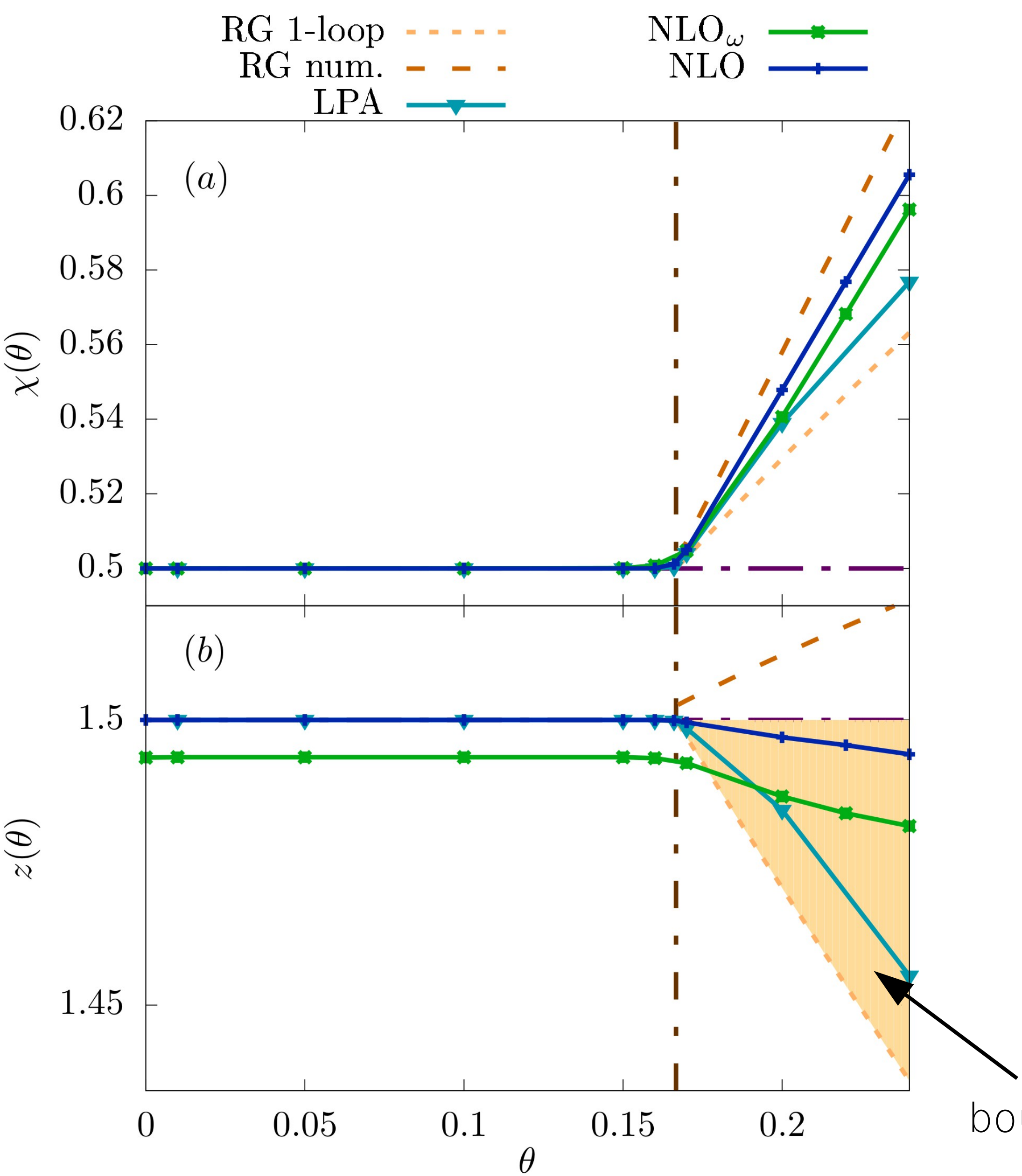
Coming from $\tilde{D}_t \to \partial_t$ in Γ_κ



$|\eta^D_\kappa - \eta^\nu_\kappa| \xrightarrow{FP} 0$

Time reversal symmetry and Galilean invariance are restored for all τ

Long-Range Correlation (LR): $f^{D}_{\kappa=\Lambda}(\omega, q) = D_0 + D_\theta \omega^{-2\theta}$



bounds predicted by Fedorenko (2008)

$d = 1$

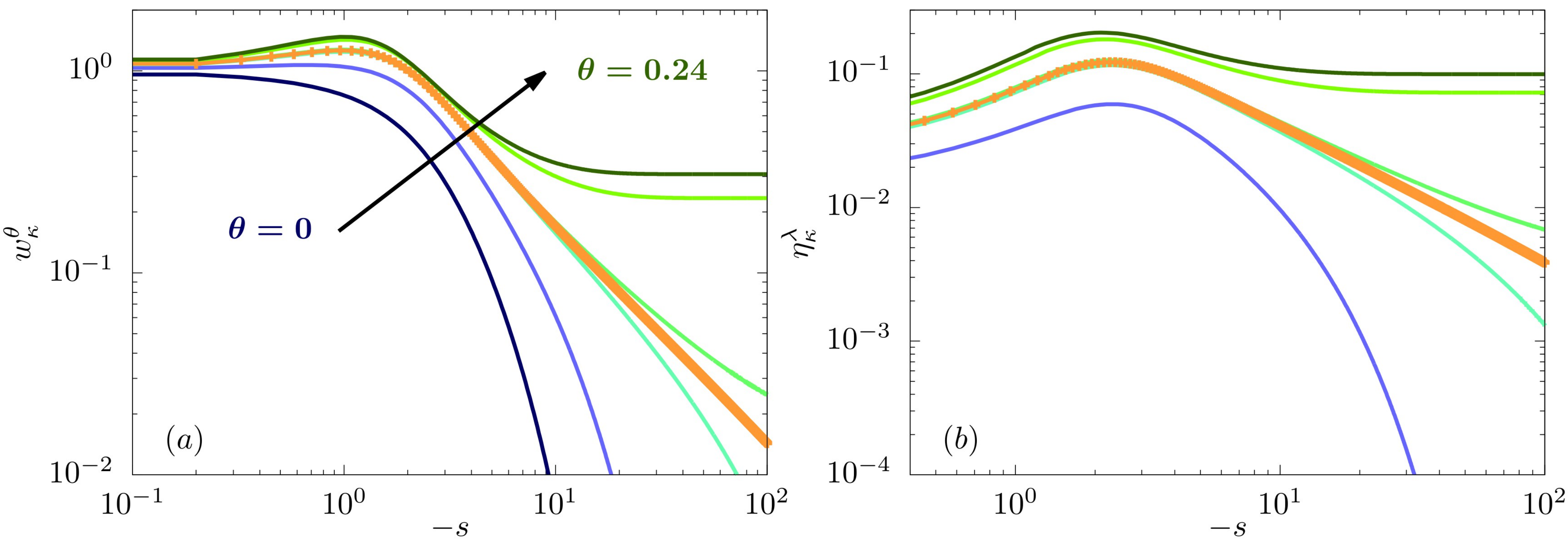
The pure KPZ regime is robust to long-range correlations in time, up to $\theta = \theta_{\rm th}$

$$\theta_{\rm th}(d) = \frac{1}{2(\eta_*^\nu - 2)}\left(2 - d - 3\eta_*^\nu\right)$$

$$\Downarrow$$

$$\theta_{\rm th}(d = 1) = 1/6$$

Long-Range Correlation (LR): $f^{D}_{\kappa=\Lambda}(\omega, q) = D_0 + D_\theta \omega^{-2\theta}$



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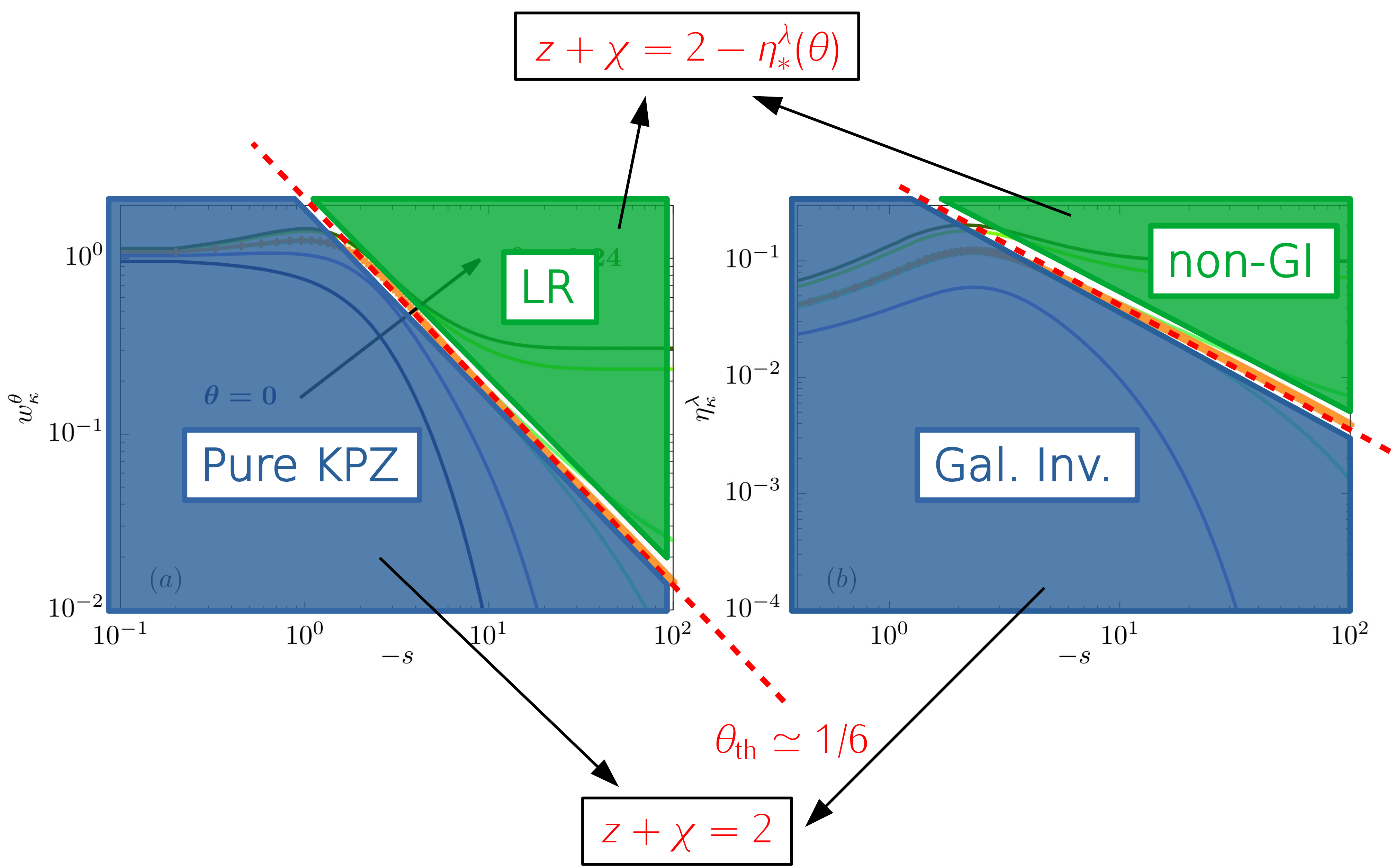

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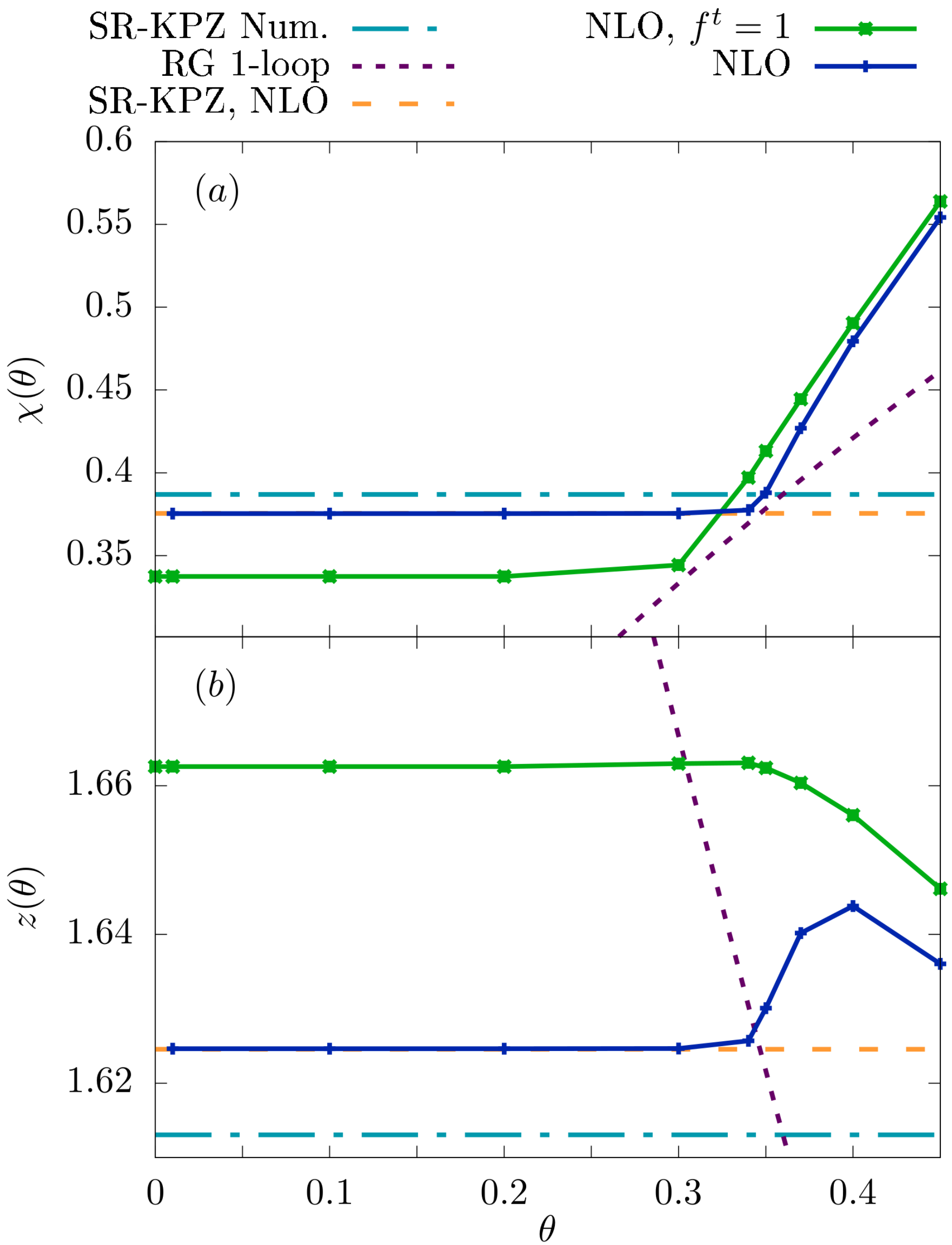
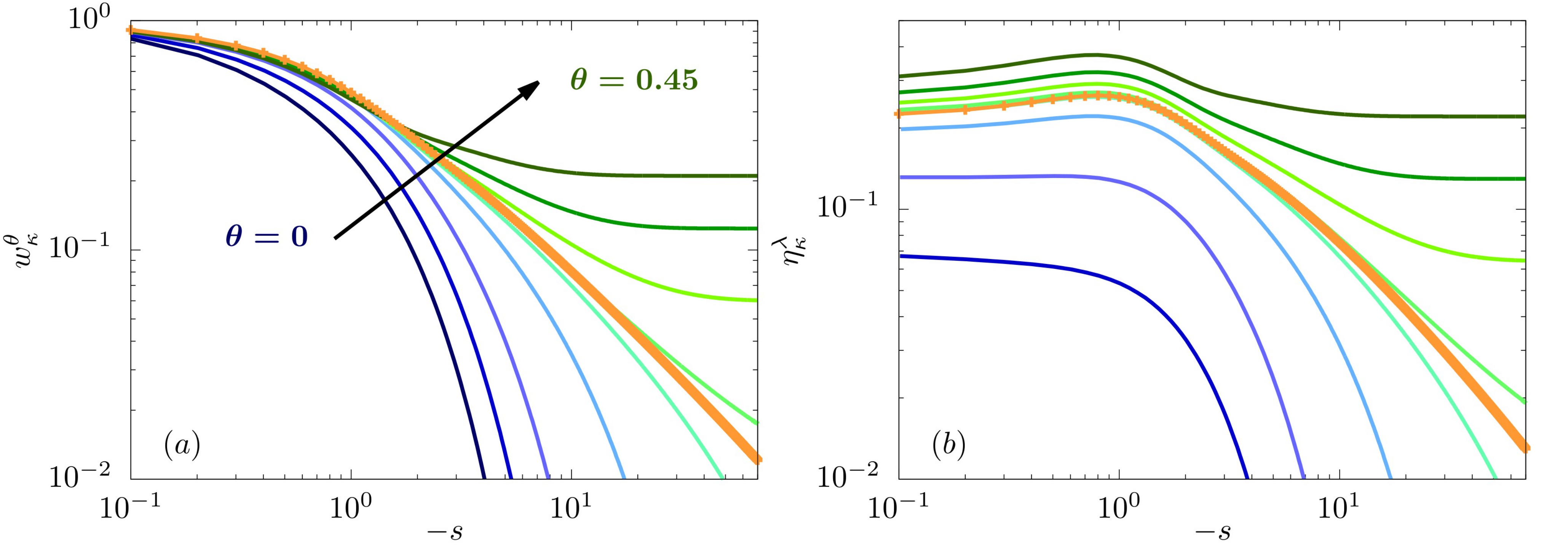


$d = 2$

The pure KPZ regime is robust to long-range correlations in time, up to $\theta = \theta_{\text{th}}$

Long-Range Correlation (LR): $f^{D}_{\kappa=\Lambda}(\omega, q) = D_0 + D_\theta \omega^{-2\theta}$



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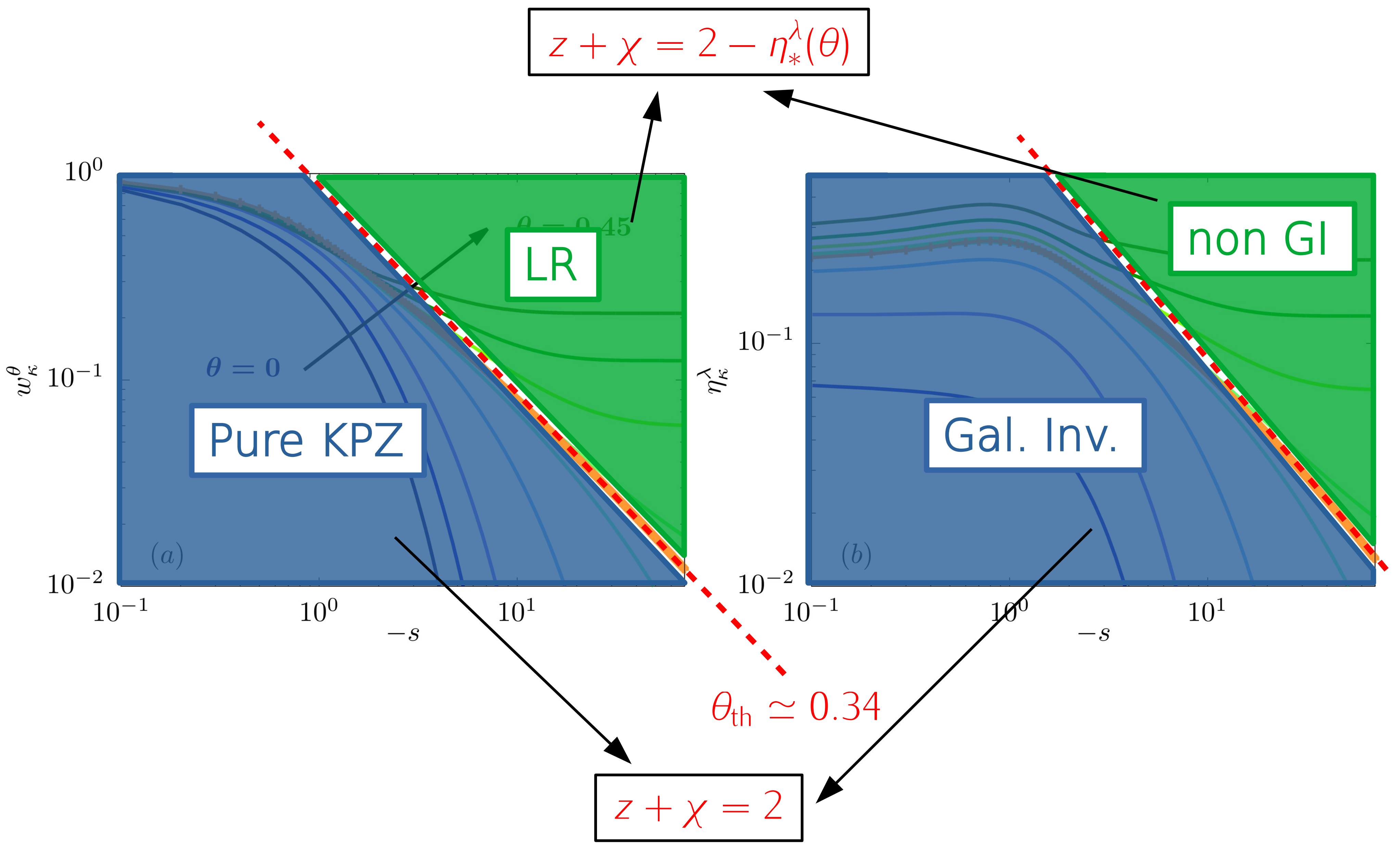


Decoupling between high- and low- energy sectors

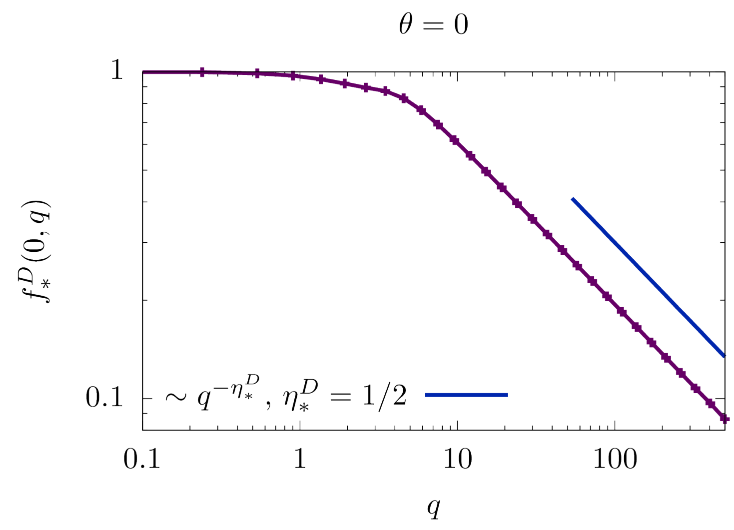
In general, $\partial_s \hat{f}^X_\kappa(\hat{\omega},\hat{q}) = \left(\eta^X_\kappa + (2-\eta^\nu_\kappa)\hat{\omega}\partial_{\hat{\omega}} + \hat{q}\,\partial_{\hat{q}} \right) \hat{f}^X_\kappa(\hat{\omega},\hat{q}) + \hat{I}^X_\kappa(\hat{\omega},\hat{q})$

In pure KPZ $\hat{I}^X_*(\hat{\omega},\hat{q}) \to 0$ for big enough frequency and/or momentum

$\Downarrow$

Decoupling between high- and low-energy sectors

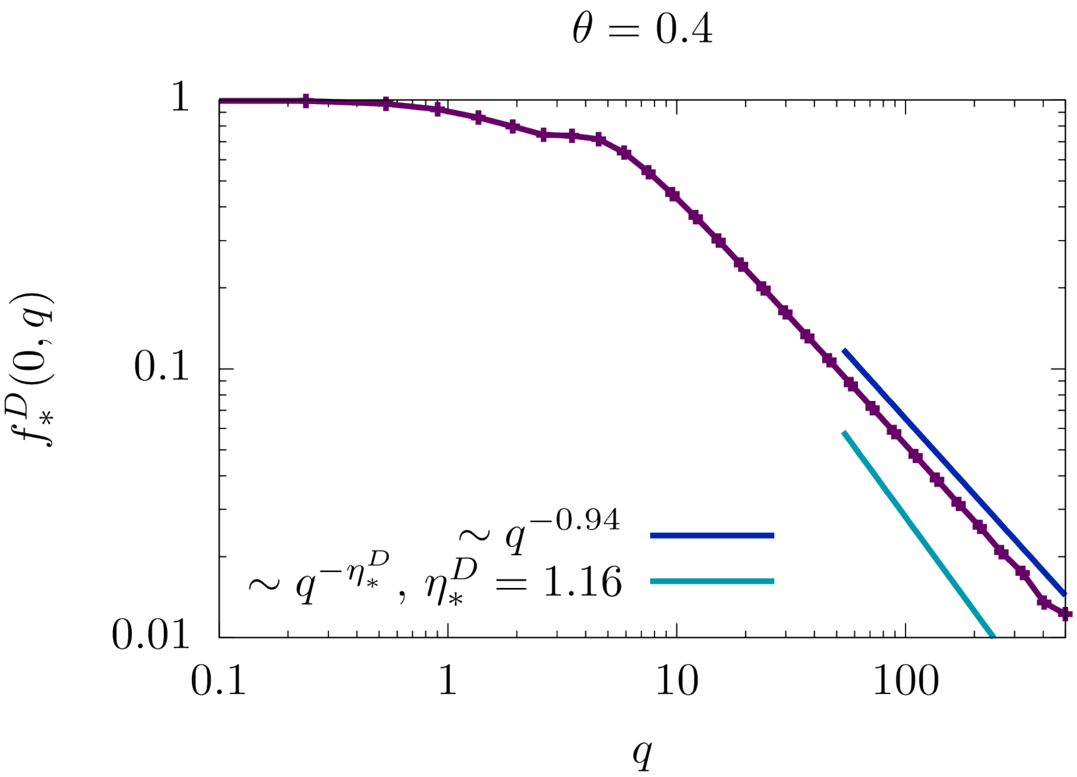
$$\eta^X_* f^X_*(\hat{\omega},\hat{q}) + z\,\hat{\omega}\partial_{\hat{\omega}} f^X_*(\hat{\omega},\hat{q}) + \hat{q}\partial_{\hat{q}} f^X_*(\hat{\omega},\hat{q}) = 0 \Rightarrow \boxed{f^X_*(\hat{\omega},\hat{q}) = q^{-\eta^X_*}\zeta^X\left(\frac{\hat{\omega}}{\hat{q}^z}\right)}$$



$$\boxed{X_\kappa \sim \kappa^{-\eta^X_*}}$$

Intermittency?

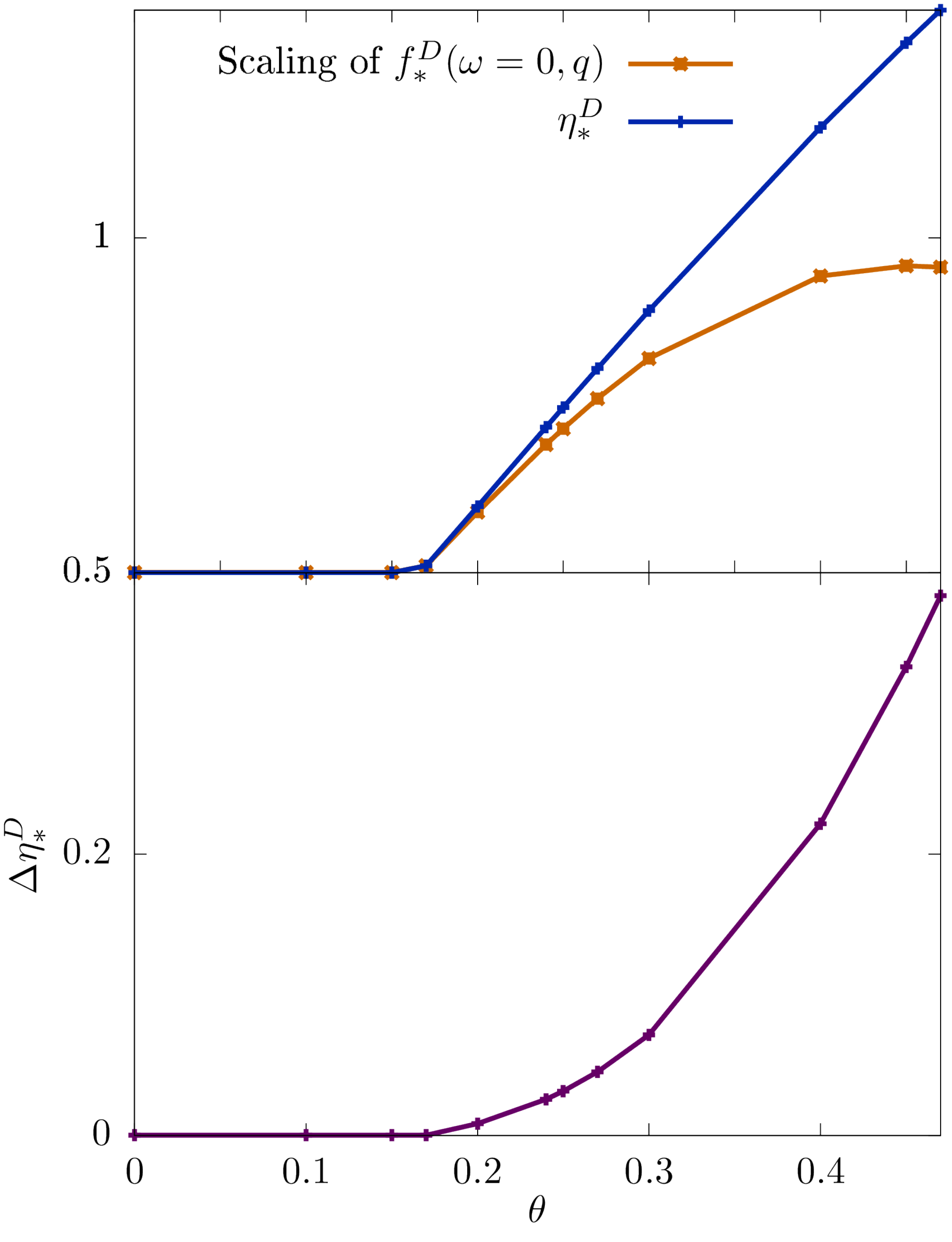
If $\hat{I}_*^X(\hat{\omega}^2, \hat{p}^2) \nrightarrow 0$, η_*^X is not enough to predict the scaling of $f_*^X(\omega, q)$



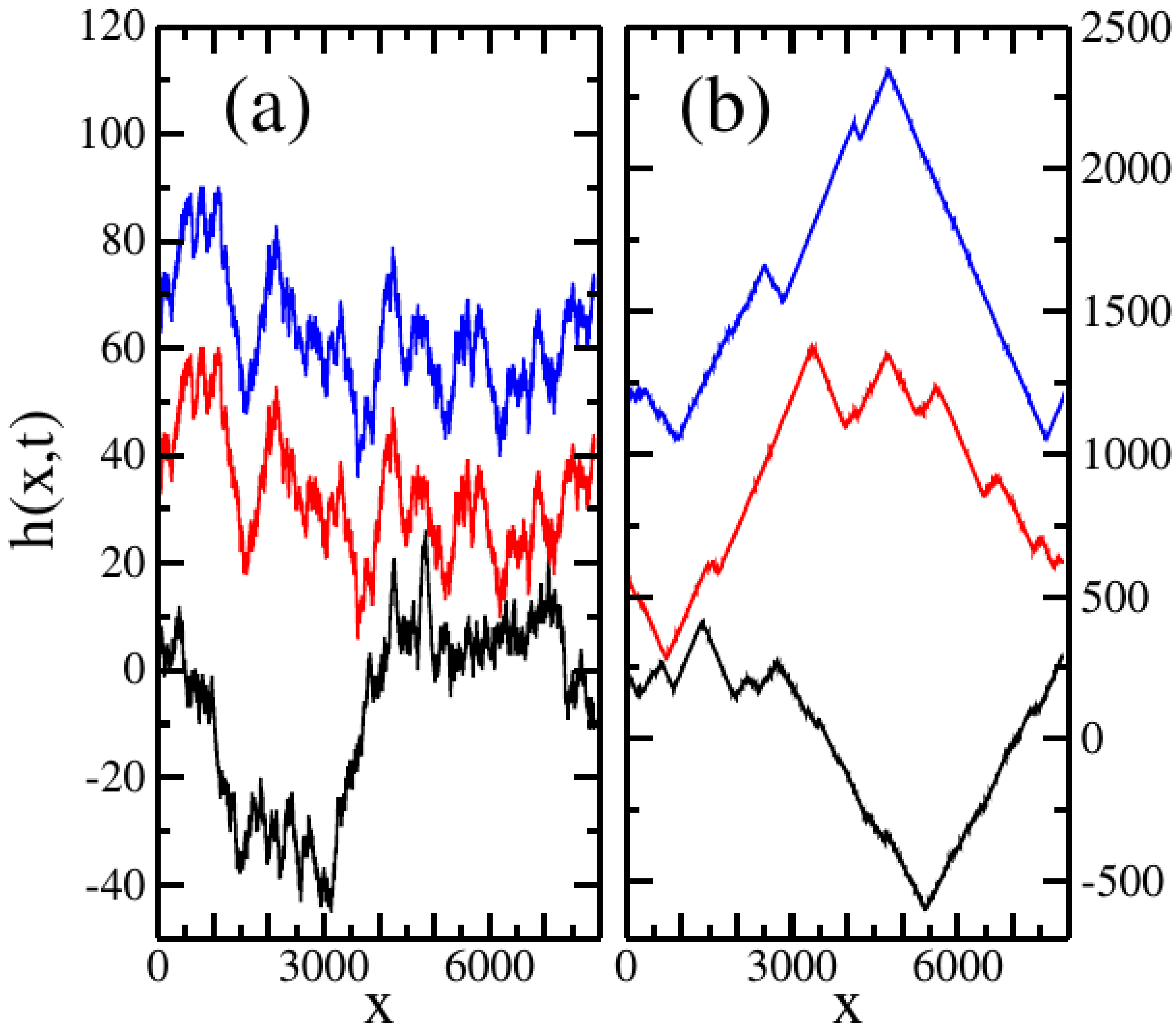
Non-decoupling has been addressed as a key feature of intermittent systems

Canet *et al.* (2016), Tarpin *et al.* (2018)

Intermittency?



Intermittency?



Alés and López (2019)

Faceting, new critical exponent inaccessible from scaling argument

Conclusions

- NPRG allows to study both short- and long-time correlations in the KPZ equation
- KPZ universality is not affected by a short-range correlated noise in d=1.
- In the case of long-range correlations, the pure KPZ universality is recovered up to a critical value of the power-law exponent, in both d=1 and d=2.
- For strong enough correlations, the decoupling of high- and low-momentum sectors seems to cease

Perspectives

- Study the full quadratic ansatz in order to properly tackle short-range correlations in d>1.
- Understand the mechanism leading to the non-decoupling of momenta sectors for strong long-range correlations