Functional renormalization group approach to 2D turbulence

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24/04/2019

Functional and Renormalization Group Methods

ECT* – European Centre for Theoretical studies in nuclear physics and related areas, Trento

Based on 1809.00909 with Tarpin, Canet, and Wschebor.

Outline

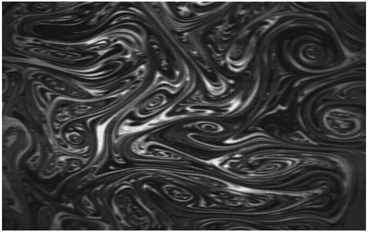
Brief reminder of 2D turbulence

Turbulent 2D phenomena describe (a first approximation of):

- large scale motion of the atmosphere and oceans;
- geostrophic turbulence;
- stratified atmosphere;

and are applied in many context (e.g. MHD turbulence).

Typical experimental setup via soap films.



(From Belmonte et al., Physics of fluids (1999).)

2D dynamics

We consider incompressible flows:

$$\partial_i v^i = 0\,.$$

From 3D Navier-Stokes equation

$$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial^2 v^i + f^i\,.$$

Taking $v^z \approx 0$ and $v^{x,y} \sim z^2$

$$\Rightarrow \nu \partial_z^2 v^{x,y} \approx -\alpha v^{x,y}\,,$$

one obtains the 2D Ekman-Navier-Stokes equation:

$$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial^2 v^i - \alpha v^i + f^i\,,$$

where α is the so called Ekman friction.

Basics of 2D turbulence

Turbulence in 2D is characterized by a completely different phenomenology from the 3D case. In particular there are *two cascades*

- a direct (enstrophy) cascade (towards smaller scales),
- an inverse (energy) cascade (towards larger scales).

A crucial role is play by the *vorticity* ω:

$$\omega \equiv \epsilon^{ij}\partial_i v_j \, .$$

Considering $\alpha \approx 0$ the vorticity dynamics is given by

$$\partial_t \omega + u^j \partial_j \omega - \nu \partial^2 \omega = f_\omega \, .$$

(Similar to the equation for transport of scalar quantities.)

The two cascades

Consider the enstrophy $\zeta \equiv \int_x \omega^2/2$ for zero forcing. We have

$$\partial_t \int_{\vec{p}} \frac{\omega(p)\,\omega(-p)}{2} = -\nu \underbrace{\int_{\vec{p}} p^2 \omega(p)\,\omega(-p)}_{>0} < 0\,,$$

implying that the enstrophy ζ decreases with time.

Moreover

$$\partial_t \int_{\vec{p}} \frac{v_i(p)\,v^i(-p)}{2} = -\nu \underbrace{\int_{\vec{p}} \omega(p)\,\omega(-p)}_{\leq \#} \geq -\nu \#\,.$$

Therefore in the inviscid limit $(\nu \to 0)$

$$\lim_{\nu\to 0} \partial_t \int v^2 \geq \lim_{\nu \to 0} -\nu \# = 0\,.$$

$\Rightarrow$ no energy dissipative anomaly is possible in 2D!

The two cascades

However,

- enstrophy *has* a dissipative anomaly ($\lim_{\nu\to 0}\zeta \neq 0$):

$$\partial_t \langle \zeta \rangle = -\nu \langle \partial_i \omega \partial_i \omega \rangle + \langle f_\omega \omega \rangle \,,$$

  like energy for the 3D Navier-Stokes case. (*Direct enstrophy cascade*.)
- energy is dynamically transferred to larger scales. (*Inverse energy cascade*.)

$\Rightarrow$ Kolmogorov-type of arguments leads to two different spectra in the two cascades.

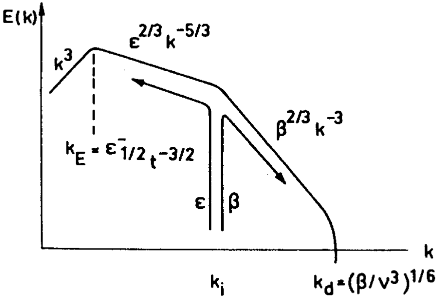
The two cascades

Summarizing

- enstrophy *has* a dissipative anomaly,
- energy is dynamically transferred to larger scales.

(Possible to give a "stochastic calculus based" derivation.)

Let $E(p) = \pi p \langle v^i(p) v^i(-p) \rangle$ (taken from Lesieur (1983)):



(Possible log-corrections (Kraichnan 1971).)

Functional formulation of 2D turbulence

In order to keep the fluid in a steady state we need to consider the forced equation of motion:

$$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial^2 v^i - \alpha v^i + f^i \,.$$

We construct the partion function via (Martin-Siggia-Rose PRA 1973, Janssen Z.Phys. 1976, deDominicis J.Phys. 1976)

$$\begin{aligned} Z &= \int \mathcal{D}v \mathcal{D}f \, P_{ff} \, \delta\left(v_a - v_a^{(\mathrm{sol})}\right) \\ &= \int \mathcal{D}v \mathcal{D}p \mathcal{D}f \, P_{ff} \, \delta\left(\partial_t v^i + v^j \partial_j v^i + \partial^i p - \nu \partial^2 v^i + \alpha v^i - f^i\right) \,, \end{aligned}$$

where P_{ff} implements a stochastic forcing with Gaussian distribution of variance

$$\left\langle f_\alpha^{\mathrm{inj}}(t,\vec{x}) f_\beta^{\mathrm{inj}}(t',\vec{x}\,') \right\rangle = 2\, \delta_{\alpha\beta}\, \delta(t-t') N_{L^{-1}}(|\vec{x}-\vec{x}\,'|) \,.$$

Functional formulation of 2D turbulence

One can exponentiate the Dirac delta and integrate out the Gaussian forcing:

$$S = \int \left\{ \bar{v}_\alpha \left[\partial_t v_\alpha - \nu \nabla^2 v_\alpha + v_\beta \partial_\beta v_\alpha + \frac{1}{\rho} \partial_\alpha p \right] + \bar{p}\, \partial_\alpha v_\alpha \right\}$$

$$\Delta S = \int \left\{ \bar{v}_\alpha R_{L_0^{-1}} v_\alpha - \bar{v}_\alpha N_{L^{-1}} \bar{v}_\alpha \right\},$$

where $R_{L_0^{-1}}$ implements a non-local generalization of the Ekman friction term.

By integrating over the pressure fields we retrieve that the velocity field and the velocity response field are divergenceless:

$$\begin{aligned} \partial_\alpha v_\alpha &= 0 \\ \partial_\alpha \bar{v}_\alpha &= 0\,. \end{aligned}$$

Incompressibility in 2D

We consider incompressible fluids:

$$\partial_i v_i = 0\,.$$

Helmoltz theorem:

$$v_a = \epsilon_{ab}\partial_b\psi - \underbrace{\partial_a\varphi}_{=0}\,,$$

where the scalar ψ is called *stream function*.

Relation to vorticity

$$\omega = \epsilon_{ab}\partial_a v_b = -\partial^2\psi\,.$$

The action reads

$$\mathcal{S}_\psi[\psi,\bar\psi] = \int \partial_\alpha\bar\psi\Big[\partial_t\partial_\alpha\psi - \nu\nabla^2\partial_\alpha\psi + \epsilon_{\beta\gamma}\,\partial_\gamma\psi\,\partial_\beta\partial_\alpha\psi\Big]$$

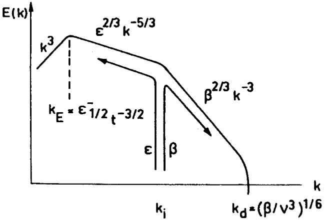
$$\Delta\mathcal{S}_\psi[\psi,\bar\psi] = \int \Big\{\partial_\alpha\bar\psi R_{L_0^{-1}}\partial'_\alpha\psi - \partial_\alpha\bar\psi N_{L^{-1}}\partial'_\alpha\bar\psi\Big\}\,,$$

FRG approach to 2D turbulence

The action

$$\Delta\mathcal{S}_\psi[\psi,\bar\psi] = \int_{t,\vec{x},\vec{x}'} \Big\{ \partial_\alpha\bar\psi(t,\vec{x}) R_{L_0^{-1}}(|\vec{x}-\vec{x}'|)\partial'_\alpha\psi(t,\vec{x}') - \partial_\alpha\bar\psi(t,\vec{x}) N_{L^{-1}}(|\vec{x}-\vec{x}'|)\partial'_\alpha\bar\psi(t,\vec{x}') \Big\}\,,$$

is characterised by Fourier transforms that are smooth functions, which vanish exponentially for wave-numbers large compared to L_0^{-1} or L^{-1}, and which regularize the fluctuating fields for small wave-numbers.



FRG approach to the direct cascade

We take $L_0^{-1} = L^{-1} = \kappa$ and study the momenta $p \gg \kappa$, i.e.

- the inertial range, between the energy injection scale and the dissipation scale;
- the direct enstrophy cascade.

The ideal limit $(L_0, L) \to \infty$ corresponds to $\kappa \to 0$. To reconstruct this limit we consider the EAA dependence on κ (Wetterich 1993):

$$\partial_\kappa \Gamma_\kappa = \frac{1}{2} \int_{\mathbf{x},\mathbf{y}} \partial_\kappa [\mathcal{R}_\kappa]_{ij}(|\mathbf{x} - \mathbf{y}|) \Big[\Gamma_\kappa^{(2)} + \mathcal{R}_\kappa\Big]^{-1}_{ji}(\mathbf{y}, \mathbf{x})\,.$$

$\Rightarrow$ look for IR fixed point corresponding to $\kappa \to 0$.

(Extended) symmetries

Extended symmetries $\equiv$ transformation that leaves the action invariant up to terms linear in the field.

- time-gauged shift symmetries ($v \sim \partial\psi$ and $\bar{v} \sim \partial\bar{\psi}$)

$$a) \quad \delta\psi = \eta(t)\,, \qquad \bar{a}) \quad \delta\bar{\psi} = \bar{\eta}(t)$$

- time-gauged shift of the (velocity) response field (present also in the velocity formulation)

$$b) \quad \delta\psi = 0\,, \qquad \delta\bar{\psi} = x_\alpha \bar{\eta}_\alpha(t)$$

- new time-gauged shift of the response field

$$c) \quad \delta\psi = 0\,, \qquad \delta\bar{\psi} = \frac{x^2}{2}\bar{\eta}(t)$$

(Extended) symmetries

Extended symmetries $\equiv$ transformation that leaves the action invariant up to terms linear in the field.

- time-gauged Galilean symmetry

$$d) \quad \delta\psi = \epsilon_{\alpha\beta} x_\alpha \dot{\eta}_\beta(t) + \eta_\alpha(t)\partial_\alpha\psi\,, \qquad \delta\bar{\psi} = \eta_\alpha(t)\partial_\alpha\bar{\psi}$$

- time-gauged rotation

$$e) \quad \delta\psi = -\dot{\eta}(t)\frac{x^2}{2} + \eta(t)\epsilon_{\alpha\beta} x_\beta \partial_\alpha\psi\,, \qquad \delta\bar{\psi} = \eta(t)\epsilon_{\alpha\beta} x_\beta \partial_\alpha\bar{\psi}$$

(Present in the velocity formulation at the price of considering non-local shift in the pressure fields.)

Symmetries and Ward identities

The extended symmetries a),...,e) encode exact, non-perturbative information regarding the RG of the theory.

$$a) \quad \int_{\vec{x}} \frac{\delta\Gamma_\kappa}{\delta\Psi(\mathbf{x})} = 0, \qquad \bar{a}) \int_{\vec{x}} \frac{\delta\Gamma_\kappa}{\delta\bar{\Psi}(\mathbf{x})} = 0$$

$$b) \quad \int_{\vec{x}} x_\alpha \frac{\delta\Gamma_\kappa}{\delta\bar{\Psi}(\mathbf{x})} = 0$$

$$c) \quad \int_{\vec{x}} \frac{x^2}{2} \frac{\delta\Gamma_\kappa}{\delta\bar{\Psi}(\mathbf{x})} = -2 \int_{\vec{x}} \partial_t \Psi$$

$$d) \quad \int_{\vec{x}} \Big\{ \big(-\epsilon_{\beta\alpha} x_\beta \partial_t + \partial_\alpha \Psi \big) \frac{\delta\Gamma_\kappa}{\delta\Psi(\mathbf{x})} + \partial_\alpha \bar{\Psi} \frac{\delta\Gamma_\kappa}{\delta\bar{\Psi}(\mathbf{x})} \Big\} = 0$$

$$e) \quad \int_{\vec{x}} \Big\{ \big(\frac{x^2}{2} \partial_t + \epsilon_{\alpha\beta} x_\beta \partial_\alpha \Psi \big) \frac{\delta\Gamma_\kappa}{\delta\Psi(\mathbf{x})} + \epsilon_{\alpha\beta} x_\beta \partial_\alpha \bar{\Psi} \frac{\delta\Gamma_\kappa}{\delta\bar{\Psi}(\mathbf{x})} \Big\} = 2 \int_{\vec{x}} \partial_t^2 \bar{\Psi} \,.$$

Ward identities in momentum space

The momentum space version can be employed in the flow equation. ($\delta^n \Gamma = (2\pi)^d \delta\left(\sum p\right) \bar{\Gamma}^{(n)}$.)

$$a), \bar{a}) \quad \bar{\Gamma}_\kappa^{(m,\bar{m})}(\dots, \varpi, \vec{q}, \dots)\Big|_{\vec{q}=0} = 0$$

$$b) \quad \frac{\partial}{\partial q^i} \bar{\Gamma}_\kappa^{(m,\bar{m}+1)}(\{\mathbf{p}_\ell\}_{1\leq\ell\leq m}, \varpi, \vec{q}, \{\mathbf{p}_\ell\}_{1\leq\ell\leq\bar{m}-1})\Big|_{\vec{q}=0} = 0$$

$$c) \quad \frac{\partial^2}{\partial q^2} \bar{\Gamma}_\kappa^{(m,\bar{m}+1)}(\{\mathbf{p}_\ell\}_{1\leq\ell\leq m}, \varpi, \vec{q}, \{\mathbf{p}_\ell\}_{1\leq\ell\leq\bar{m}-1})\Big|_{\vec{q}=0} = 0$$

$$\text{except} \quad \frac{\partial^2}{\partial q^2} \bar{\Gamma}_\kappa^{(1,1)}(\varpi, \vec{q})\Big|_{\vec{q}=0} = -4i\varpi$$

$$d) \quad \frac{\partial}{\partial q^i} \bar{\Gamma}_\kappa^{(m+1,\bar{m})}(\varpi, \vec{q}, \{\mathbf{p}_\ell\}_{1\leq\ell\leq m+\bar{m}-1})\Big|_{\vec{q}=0} = i\epsilon_{\alpha\beta} \tilde{\mathcal{D}}_\beta(\varpi) \bar{\Gamma}_\kappa^{(m,\bar{m})}(\{\mathbf{p}_\ell\})$$

$$e) \quad \frac{\partial^2}{\partial q^2} \bar{\Gamma}_\kappa^{(m+1,\bar{m})}(\varpi, \vec{q}, \{\mathbf{p}_\ell\}_{1\leq\ell\leq m+\bar{m}-1})\Big|_{\vec{q}=0} = \tilde{\mathcal{R}}(\varpi) \bar{\Gamma}_\kappa^{(m,\bar{m})}(\{\mathbf{p}_\ell\}),$$

where we have introduced the two operators $\tilde{\mathcal{D}}_\alpha(\varpi)$ and $\tilde{\mathcal{R}}(\varpi)$ defined as:

$$\tilde{\mathcal{D}}_\alpha(\varpi)F(\{\mathbf{p}_\ell\}_{1\leq\ell\leq n}) \equiv$$
$$-\sum_{k=1}^{n} p_k^\alpha \left[\frac{F(\{\mathbf{p}_\ell\}_{1\leq\ell\leq k-1}, \omega_k + \varpi, \vec{p}_k, \{\mathbf{p}_\ell\}_{k+1\leq\ell\leq n}) - F(\{\mathbf{p}_\ell\})}{\varpi} \right]$$

$$\tilde{\mathcal{R}}(\varpi)F(\{\mathbf{p}_\ell\}_{1\leq\ell\leq n}) \equiv$$
$$2i\epsilon_{\alpha\beta} \sum_{k=1}^{n} p_k^\alpha \frac{\partial}{\partial p_k^\beta} \left[\frac{F(\{\mathbf{p}_\ell\}_{1\leq\ell\leq k-1}, \omega_k + \varpi, \vec{p}_k, \{\mathbf{p}_\ell\}_{k+1\leq\ell\leq n}) - F(\{\mathbf{p}_\ell\})}{\varpi} \right].$$

Fixed point scaling

We consider the fixed point (scaling) behaviour by introducing

- the anomalous dimensions:

$$\eta_D(\kappa) = -\kappa\partial_\kappa \ln D_\kappa\,, \qquad \eta_\nu(\kappa) = -\kappa\partial_\kappa \ln \nu_\kappa\,,$$

  where $N_\kappa(q) = D_\kappa (q/\kappa)^2\, \hat{n}_\kappa (q/\kappa)^2$;
- the dimensionless momentum and frequency are defined by $\hat{p} = p/\kappa$ and $\hat{\omega} \equiv \omega\kappa^{-2}\nu_\kappa^{-2}$;
- one finds the dynamical scaling exponent $\omega \sim p^z$

$$z = 2 - \eta_\nu^*\,;$$

- fixed point and non-renormalization of the "Galilei covariant derivative":

$$\partial_t + v^i\partial_i \to \partial_t + \lambda_k v^i \partial_i$$

$$\Rightarrow \quad \kappa\partial_\kappa\hat{\lambda}_\kappa = -\frac{1}{2}\hat{\lambda}_\kappa\left(2 + \eta_D^* - 3\eta_\nu^*\right) \stackrel{!}{=} 0\,.$$

Fixed point scaling

We consider the fixed point (scaling) behaviour by introducing

- The fixed point anomalous dimension

$$\eta_\nu^* = \frac{2 + \eta_D^*}{3};$$

- η_D is fixed by requiring the presence of the enstrophy cascade

$$\begin{aligned} \varepsilon_\omega &= \langle f_\omega \omega \rangle = \int_{\omega, q} N(q)\, G^{(\bar{\omega},\omega)}(\omega, q) \\ &= D_\kappa \kappa^4 \int_{\hat{\omega},\hat{q}} \hat{q}^6 \hat{n}(\hat{q})\, \hat{G}_\psi^{(1,1)}(\hat{\omega}, \hat{q}) \,. \end{aligned}$$

Imposing a non-trivial ε_ω in the inertial limit $\kappa \to 0$ implies

$$\Rightarrow \eta_D = 4 \,.$$

Scaling and sub-leading logarithmic corrections

The dynamical scaling exponent reads

$$z = 2 - \eta_\nu^* = 2 - \frac{2 + \eta_D^*}{3} = 0\,.$$

$\Rightarrow$ We look for subleading corrections

$$\nu_\kappa \sim \kappa^{-\eta_\nu^*} \left(\ln(\kappa/\Lambda)\right)^{\gamma_\nu^*}, \qquad D_\kappa \sim \kappa^{-\eta_D^*} \left(\ln(\kappa/\Lambda)\right)^{\gamma_D^*}.$$

This sub-leading behaviour modifies the equation for η_D and η_ν

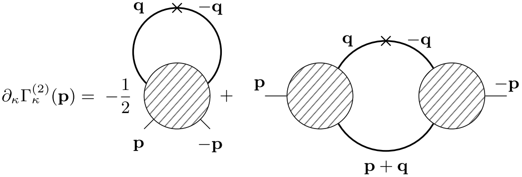
$$\kappa\partial_\kappa \hat{\lambda}_\kappa = -\frac{1}{2}\hat{\lambda}_\kappa \left(2 + \eta_D^* - 3\eta_\nu^*\right) + \frac{1}{2}(\gamma_D^* - 3\gamma_\nu^*)(\ln(\kappa/\Lambda))^{-1}\,\hat{\lambda}_\kappa\,.$$

The new term holds

$$\gamma_D^* = 3\gamma_\nu^* \equiv 3\gamma\,.$$

Closure of the flow equation

Consider the flow equation for the two-point function of the EAA



- approximation in the EAA vertices for soft momenta

$$\bar{\Gamma}_\kappa^{(3)}(q, p) \approx \bar{\Gamma}_\kappa^{(3)}(0, p)$$

- use the Ward identities!

$$\bar{\Gamma}_\kappa^{(3)}(0, p) \sim \bar{\Gamma}_\kappa^{(2)}(p)$$

$\Rightarrow$ closure of the flow equation!

Closure of the flow equation

- After implementing these approximations, the flow equation for the n-point function can be written as

$$\partial_\kappa G^{(n)}_{\psi,i_1\ldots i_n}(\{\mathbf{p}_\ell\}_{1\le\ell\le n}) = \frac{1}{2}\int_{\mathbf{q}_1,\mathbf{q}_2}\tilde{\partial}_\kappa G^{(2)}_{v_\mu v_\nu}(-\mathbf{q}_1,-\mathbf{q}_2) \times\mathcal{D}_\mu(\varpi_1)\mathcal{D}_\nu(\varpi_2)G^{(n)}_{\psi,i_1\ldots i_n}(\{\mathbf{p}_\ell\})\,.$$

- The velocity correlation function are retrieved by using

$$G^{(n)}_{v,k_1\cdots k_n}(\{\omega_k,\vec{p}_k\}) = (i)^n\,\epsilon_{k_1\ell_1}p_1^{\ell_1}\cdots\epsilon_{k_n\ell_n}p_n^{\ell_n}\,G^{(n)}_{\psi,i_1\ldots i_n}(\{\omega_k,\vec{p}_k\})\,.$$

The two-point function in the limit $\kappa \to 0$ (direct cascade)

In the stream function formulation we denote $C_\psi \equiv G_\psi^{(2,0)}$

$$\kappa\partial_\kappa C_\psi(t,\vec{p}) = -\frac{1}{2} p^2 C_\psi(t,\vec{p}) \int_\varpi \frac{\cos(\varpi t) - 1}{\varpi^2} J_\kappa(\varpi)\,,$$

where $J_\kappa(\varpi)$ can be expressed as

$$J_\kappa(\varpi) = -2 \int_{\vec{q}} \left\{ \kappa\partial_\kappa N_\kappa(\vec{q})\, |\bar{G}(\varpi,\vec{q})|^2 - \kappa\partial_\kappa R_\kappa(\vec{q})\, \bar{C}(\varpi,\vec{q}) \mathrm{Re}\big[\bar{G}(\varpi,\vec{q})\big] \right\}.$$

The equation can be simplified in the large and small time limits:

$$\kappa\partial_\kappa C_\psi(t,\vec{p}) = C_\psi(t,\vec{p}) \times \begin{cases} \frac{I_\kappa^0}{4} t^2 p^2 & I_\kappa^0 = \int_\varpi J_\kappa(\varpi) & t \ll 1 \\ \frac{I_\kappa^\infty}{4} |t|\, p^2 & I_\kappa^\infty = J_\kappa(0) & t \gg 1 \end{cases}.$$

The two-point function in the limit $\kappa \to 0$ (direct cascade)

We denote $s \equiv \log \kappa$

$$\left[\partial_s - 6 + \frac{2\gamma}{s} - \hat{p}\partial_{\hat{p}} + \frac{\gamma}{s}\hat{t}\partial_{\hat{t}}\right]\hat{C}_s(\hat{t},\hat{\vec{p}}) = \hat{C}_s(\hat{t},\hat{\vec{p}}) \times \begin{cases} \hat{\alpha}_s^0\,\hat{p}^2\,\hat{t}^2 & t \ll 1 \\ \hat{\alpha}_s^\infty\,\hat{p}^2\,|\hat{t}| & t \gg 1 \end{cases},$$

The (dimensionful) solution is

$$C_\psi(t,\vec{p}) = C_0\frac{\varepsilon_\omega^{2/3}}{p^6}\ln(pL)^{-2\gamma}\hat{\mathcal{F}}_{0,\infty}\left(\bar{\nu}_0\varepsilon_\omega^{1/3}\,t\ln(pL)^{-\gamma}\right) \times \begin{cases} \exp(-\beta_L^0\,t^2\int_0^{pL} x\ln(x)^{2\gamma}dx) & t \ll 1 \\ \exp(-\beta_L^\infty\,|t|\int_0^{pL} x\ln(x)^{\gamma}dx) & t \gg 1 \end{cases}.$$

The parameter γ should be determined by a direct integration of the flow equation. Following a self-consistency argument by Kraichnan (1971) one has $\gamma = -\frac{1}{6}$.

Vorticity two-point function at equal times

From

$$C_\psi(0,\vec{p}) \sim |\vec{p}|^{-6} \left(\ln(|\vec{p}|L)\right)^{2\gamma} .$$

and obtains for the energy spectrum

$$E(p) = 2\pi p^3\, C_\psi(0,\vec{p}) \sim\; p^{-3} \left(\ln(pL)\right)^{-1/3} .$$

(Logarithmic correction are difficult to assess experimentally and numerically Boffetta & Ecke 2012.)

The vorticity two-point function is given by

$$C_\omega(0,\vec{p}) = p^4\, C_\psi(0,\vec{p}) \sim p^{-2} \ln(pL)^{-1/3} .$$

Going to real space

$$C_\omega(0,\vec{r}) = \int_0^\pi d\theta \int dp \frac{1}{p} \ln(pL)^{-1/3} e^{i|\vec{p}||\vec{r}|\cos\theta} \sim \left(\log \frac{\|\vec{r}\|}{L}\right)^{2/3} .$$

Generalization to other correlation functions

At equal time the limit $\kappa \to 0$ gives

$$\left[-4n + 2 + \frac{n}{3s} - \sum_{k=1}^{2n-1} \hat{p}_k \partial_{\hat{p}_k} \right] \hat{G}^{(2n,0)}_{\omega,i_1\dots i_n} = 0 \,.$$

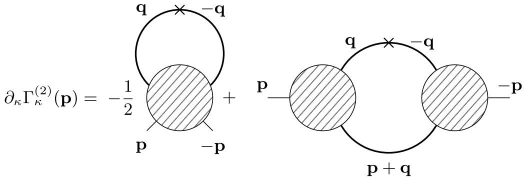
The general solution of this equation reads

$$\begin{aligned} \hat{G}^{(2n,0)}_{\omega,i_1\dots i_{2n}}(0,\hat{\vec{p}}_1,\cdots,0,\hat{\vec{p}}_{2n-1}) &= \left(\prod_{k=1}^{2n-1} \hat{p}_k^{-2} (\ln \hat{p}_k)^{-1/6} \right) \\ &\quad \times \ln |\hat{\vec{p}}_1 + \cdots + \hat{\vec{p}}_{2n-1}|^{-1/6} \hat{F}^{(2n)} \,, \end{aligned}$$

where $\hat{F}^{(2n)}$ is a scaling function.
(Compatible with the logarithmic corrections proposed by Falkovich & Lebedev 1994.)

Used all the symmetries?

Not yet:



- approximation in the EAA vertices for soft momenta

$$\bar{\Gamma}^{(3)}_{\kappa}(q,p) \approx \bar{\Gamma}^{(3)}_{\kappa}(0,p) + q\partial_q \bar{\Gamma}^{(3)}_{\kappa}(0,p)$$

- use the Ward identities from rotations.
- not enough to fully close the equation at equal times in the q-expansion.
- the terms under control vanish at equal times $\Rightarrow$ hint that possible anomalous correction are very weak if any?

Summary

- We studied the direct cascade for 2D turbulent flows via functional methods.
- We studied and unveiled new symmetries of the functional formalism.
- We showed that the FRG framework is a suitable non-perturbative framework to study 2D turbulence in its functional formulation thanks to the symmetries of the formalism.
- We computed the vorticity two-point function and studied its time dependence.

Outlook

- Extension of the present study to the explicit solution of the n-point function.
- Numerical integration of the flow equation
- Extension to other system, e.g. passive scalar (underway).

THANK YOU!!