RG treatment of topological transitions in 2 dimensions

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Outline

- Motivations
- Pure Phase Model
- O(2) model
- Traditional FRG picture
- Amplitude and phase representation
- Universality of 2d Bose gas
- XY critical temperatures
- Bilayer Models
- 2D BEC-BCS Crossover

Motivations

- Topological phase transitions are widely present in low dimensions
- Realization of 2d BEC-BCS crossover with BKT physics (Heidelberg)
- Disputed role of amplitude fluctuations
- Vortex core energy effects in superconductors

Pure Phase model

$$S[\theta] = \int d^d x (\nabla\theta)^2$$

Single field action

Scale independent Field

Periodic Field

Vortex unbinding

Conjecture

2D Models can be described effectively by villain model with

$$K_v = f(K)$$

Spin waves vs vortexes

$$\nabla\theta = \mathbf{j} = \mathbf{j}_\perp + \mathbf{j}_{||}$$

- Spin waves

$$\nabla \times \mathbf{j}_{||} = 0$$

$$\int \mathbf{j}_\perp \cdot \mathbf{j}_{||} d\mathbf{r} = 0$$

- Vortexes

$$\nabla \cdot \mathbf{j}_\perp = 0$$

$$\oint \mathbf{j}_\perp dl = 2\pi \sum_i q_i$$

$$\beta H = \beta H_{sw} + \beta H_v$$

Coulomb gas Hamiltonian

Sine-Gordon Model

XY Model

$$H = -J \sum_{\langle ij \rangle} \cos\left(\theta_i - \theta_j\right)$$

Coulomb Gas

$$H = -\sum_{i \neq j} q_i q_j \log \left| \frac{r_j - r_i}{a} \right|$$

sine-Gordon

$$S = \int d^d x \left\{ \partial_\mu \varphi \partial_\mu \varphi + u(1 - \cos(\beta\varphi)) \right\}$$

U(1) field theory

$$S = \int d^d x \left\{ \partial_\mu \varphi^* \partial_\mu \varphi + U(\varphi^* \varphi) \right\}$$

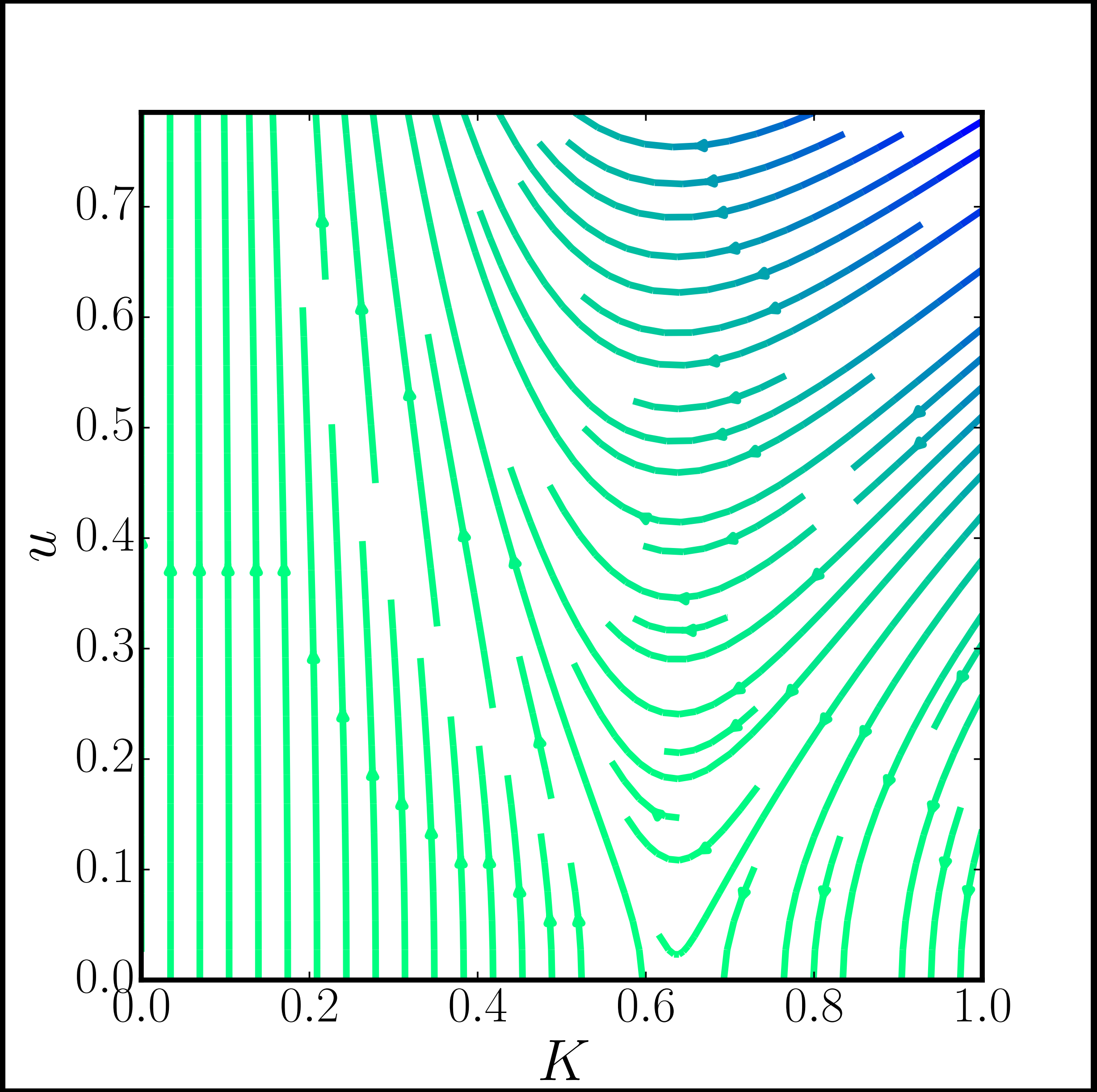
Unit charge approximation: sine-Gordon model

$$S[\theta] = \int d^2x \left\{ \frac{1}{2}\partial_\mu\tilde{\theta}\partial_\mu\tilde{\theta} + u\cos(\beta\tilde{\theta}) \right\}$$

$$\beta = 2\pi\sqrt{K}$$

$$\partial_t K_k = -\pi g_k^2 K_k^2,$$

$$\partial_t g_k = \pi\left(\frac{2}{\pi} - K_k\right) g_k$$



Continuous field theory: O(2) model

$$S[\varphi, \varphi^*] = \int d^2x \left\{ \frac{1}{2}\partial_\mu \varphi \partial_\mu \varphi^* - \mu|\varphi|^2 + \frac{U}{2}|\varphi|^4 \right\}$$

Madelung representation

$$\varphi = \sqrt{\rho} e^{i\theta}$$

$$S[\rho, \theta] = \int d^2x \left\{ \frac{1}{8\rho}\partial_\mu \rho \partial_\mu \rho + \frac{\rho}{2}\partial_\mu \theta \partial_\mu \theta - \mu\rho + \frac{U}{2}\rho^2 \right\}$$

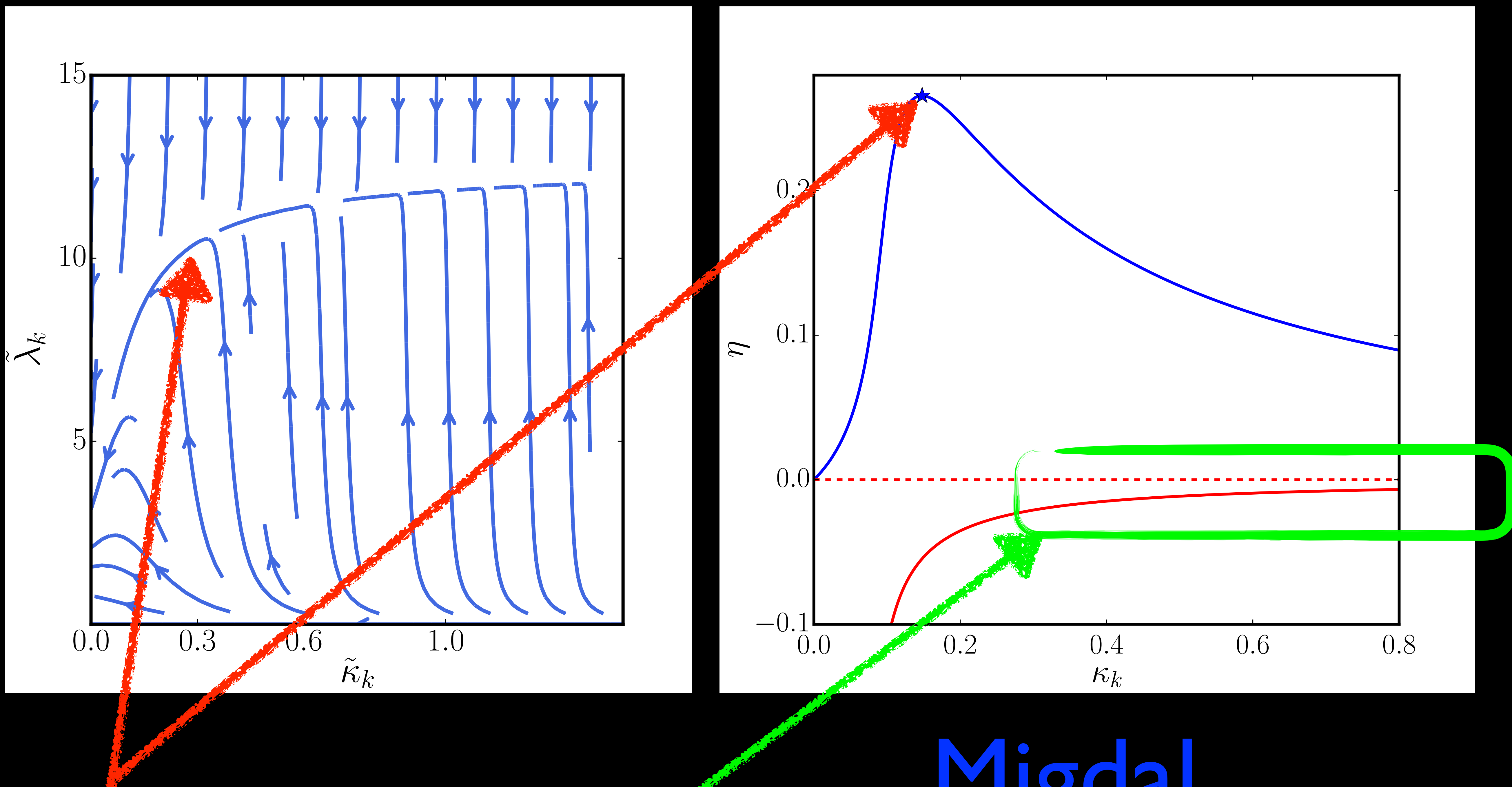
Frozen amplitude fluctuations

$$\rho = \rho_0 + \delta\rho \quad \delta\rho \ll \rho_0$$

$$S[\theta] = \int d^2x \left\{ \frac{\rho_0}{2}\partial_\mu \theta \partial_\mu \theta \right\}$$

FRG Picture

$$U(\rho) = \frac{\lambda_k}{2}(\rho - \kappa_k)^2$$



Smooth Crossover

Vanishing flow

Migdal Approximation

PHYSICAL REVIEW B VOLUME 16, NUMBER 3 1 AUGUST 1977

Renormalization, vortices, and symmetry-breaking perturbations in the two-dimensional planar model

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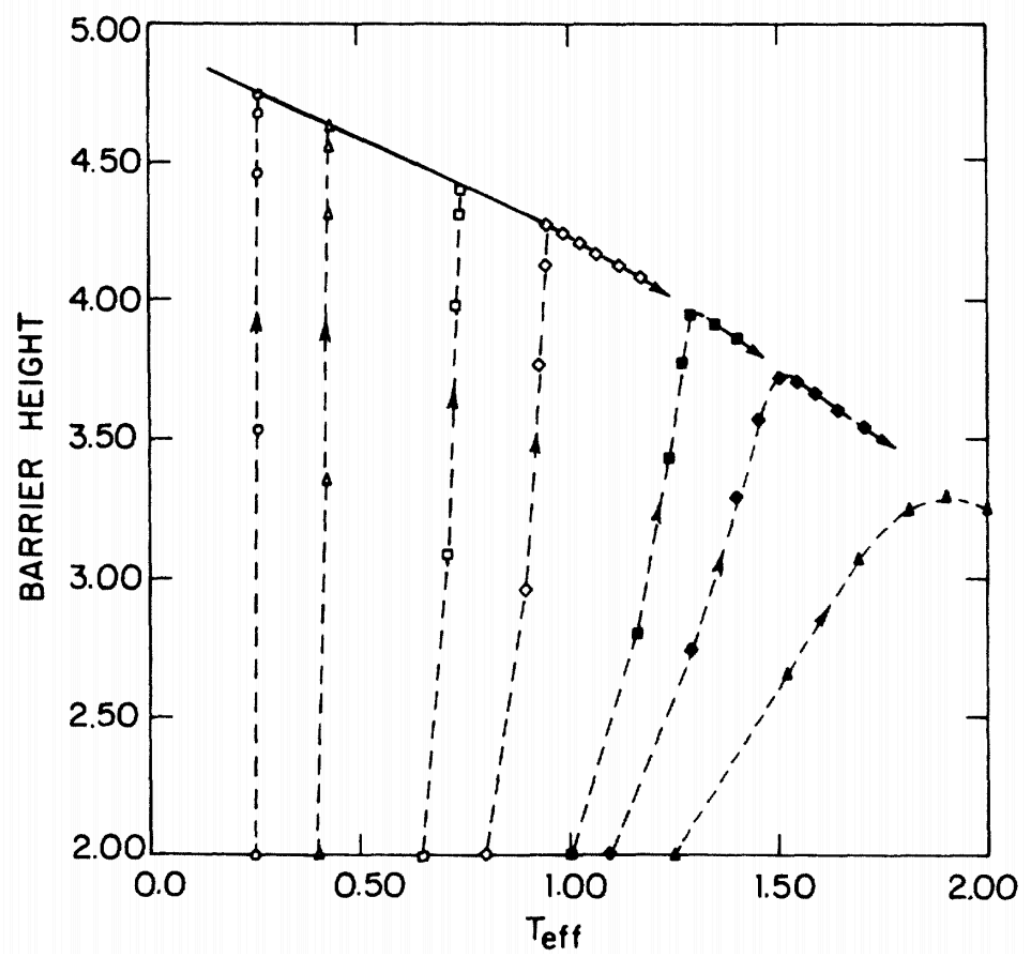
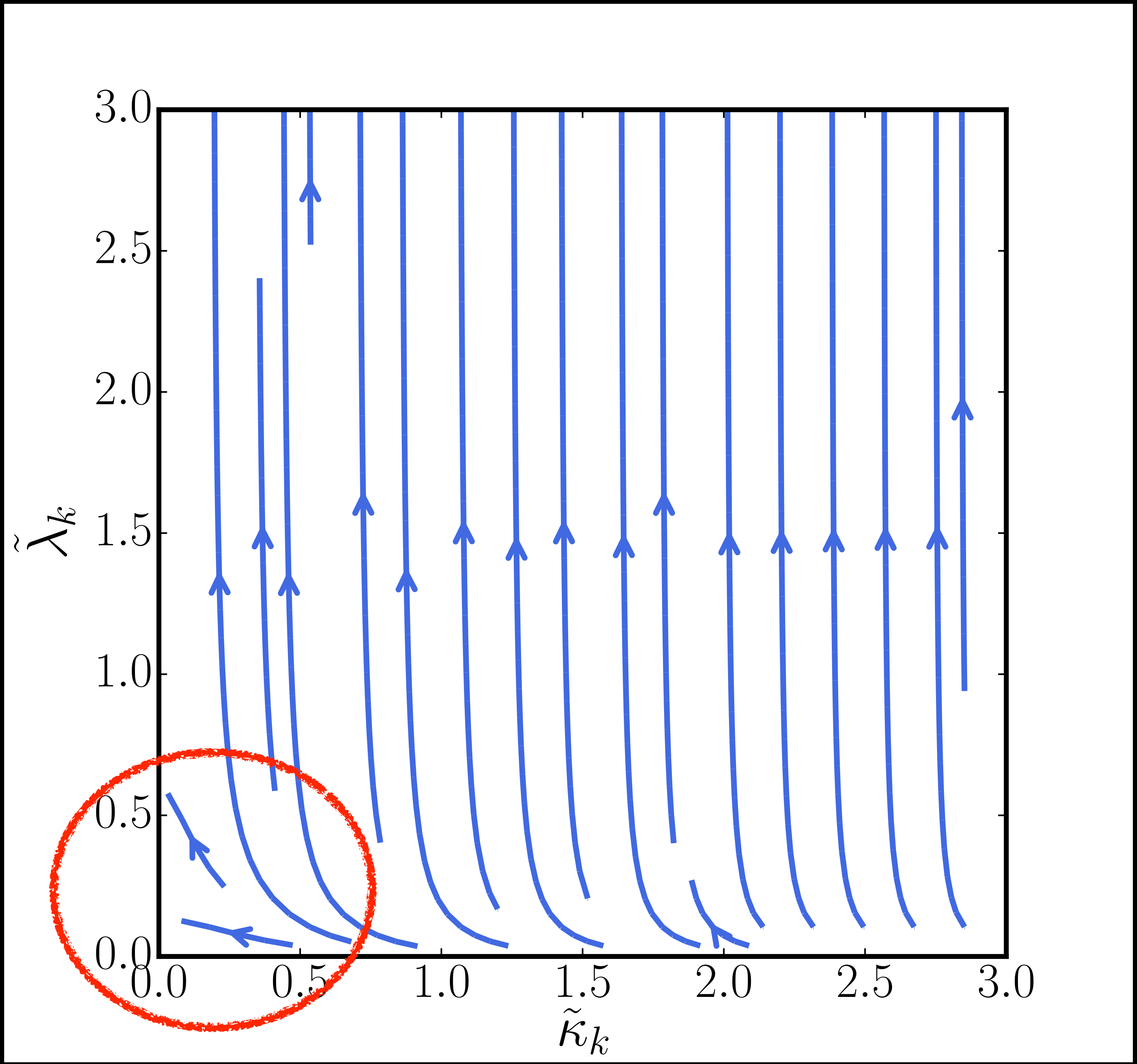


FIG. 7. The Migdal transformation parametrized by an effective temperature and barrier height. The circles, triangles, etc., indicate iterations of the $b=2$ transformation from the line of barrier height =2.0.

Amplitude only flow

Minimum Depletion

Symmetric phase



Always relevant interaction

2d Bose-Gas

$$H_{\rm bg} = \int \left\{ \psi^\dagger(x) \left(-\frac{\nabla^2}{2m} - \mu \right) \psi(x) + \frac{U}{2} \psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x) \right\} d^2x$$

Quasi-condensation

Non degenerate gas $\mu \gg T \rightarrow n_k \propto e^{-\beta\varepsilon_k}$

Degenerate gas $\mu \ll T \rightarrow n_k \propto \dfrac{T}{\varepsilon_k + |\mu|}$

Low energy action

$$S[\varphi, \varphi^*] = \int d^2x \left\{ \frac{1}{2m} \partial_\mu \varphi \partial_\mu \varphi^* - \mu' |\varphi|^2 + \frac{U}{2} |\varphi|^4 \right\}$$

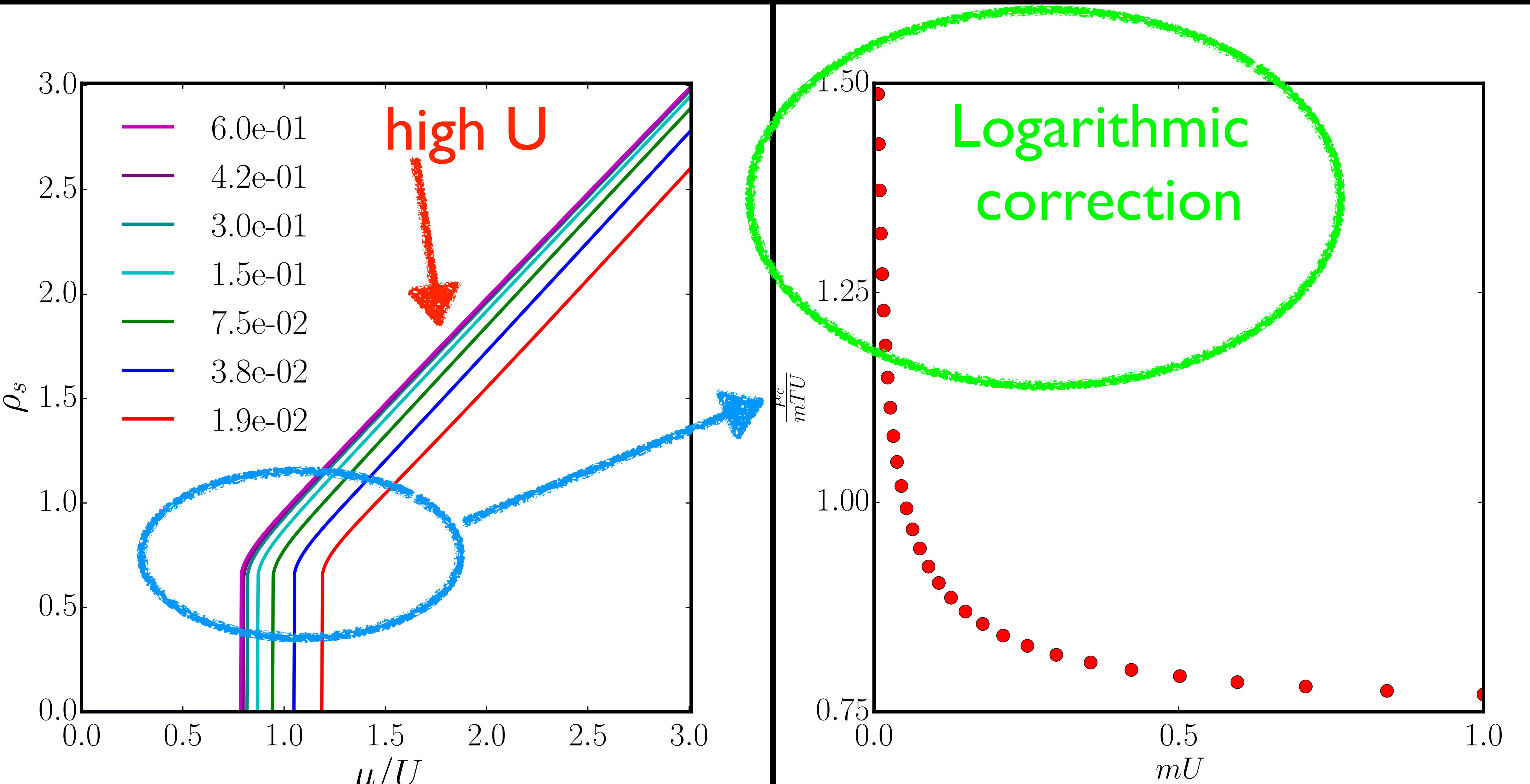
Universality at "weak" coupling

- Small quantum renormalization of U can be neglected
- Critical chemical potential $$\mu_c = \frac{mTU}{\pi} \ln \frac{\xi_\mu}{mU}$$
- Universal variable $$X = \frac{\mu - \mu_c}{mTU}$$
- All models share same behavior for $X \ll 1/mU$
- Superfluid density $$\rho_s = \frac{2mT}{\pi} f(X)$$

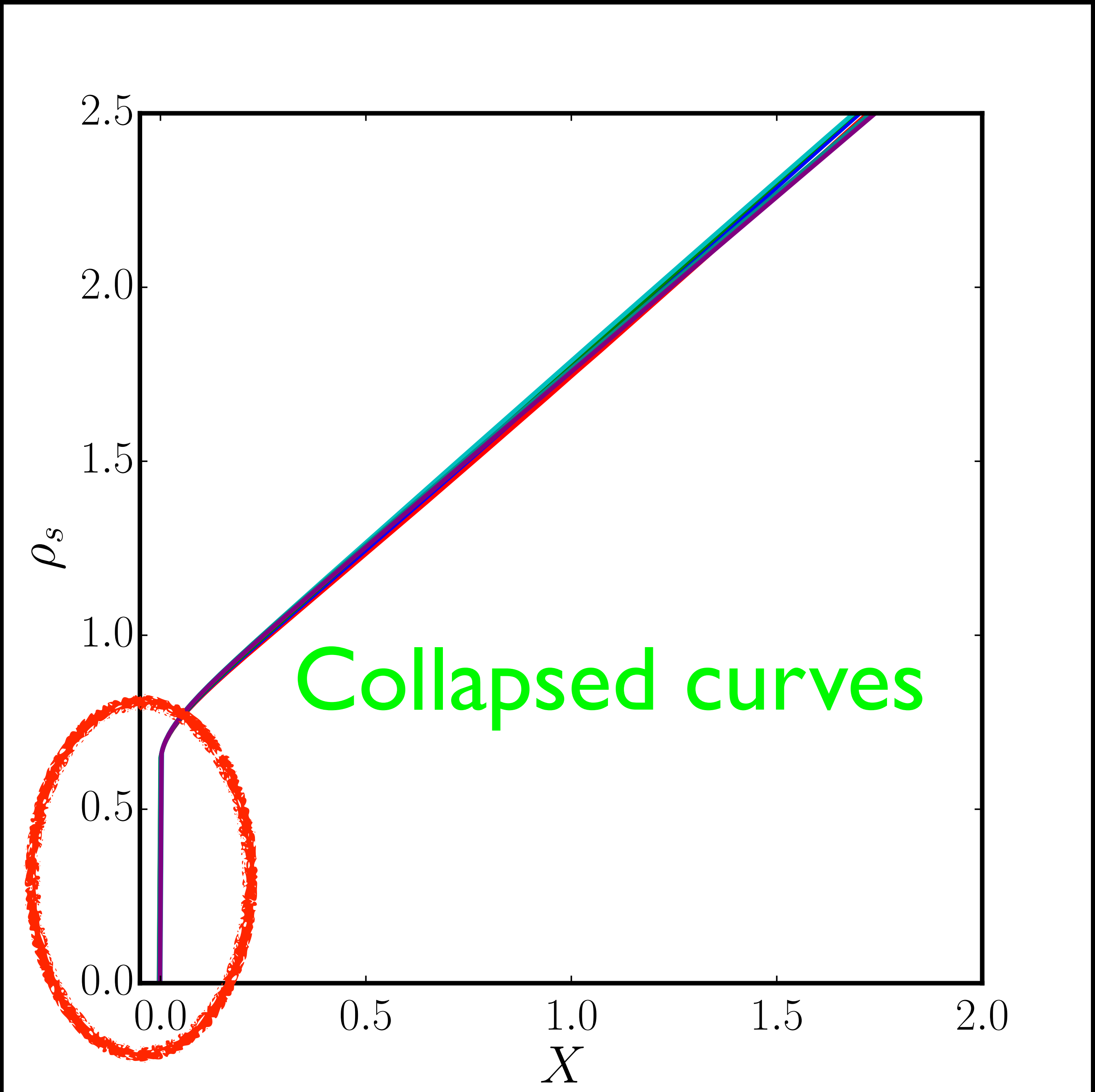
FRG routine

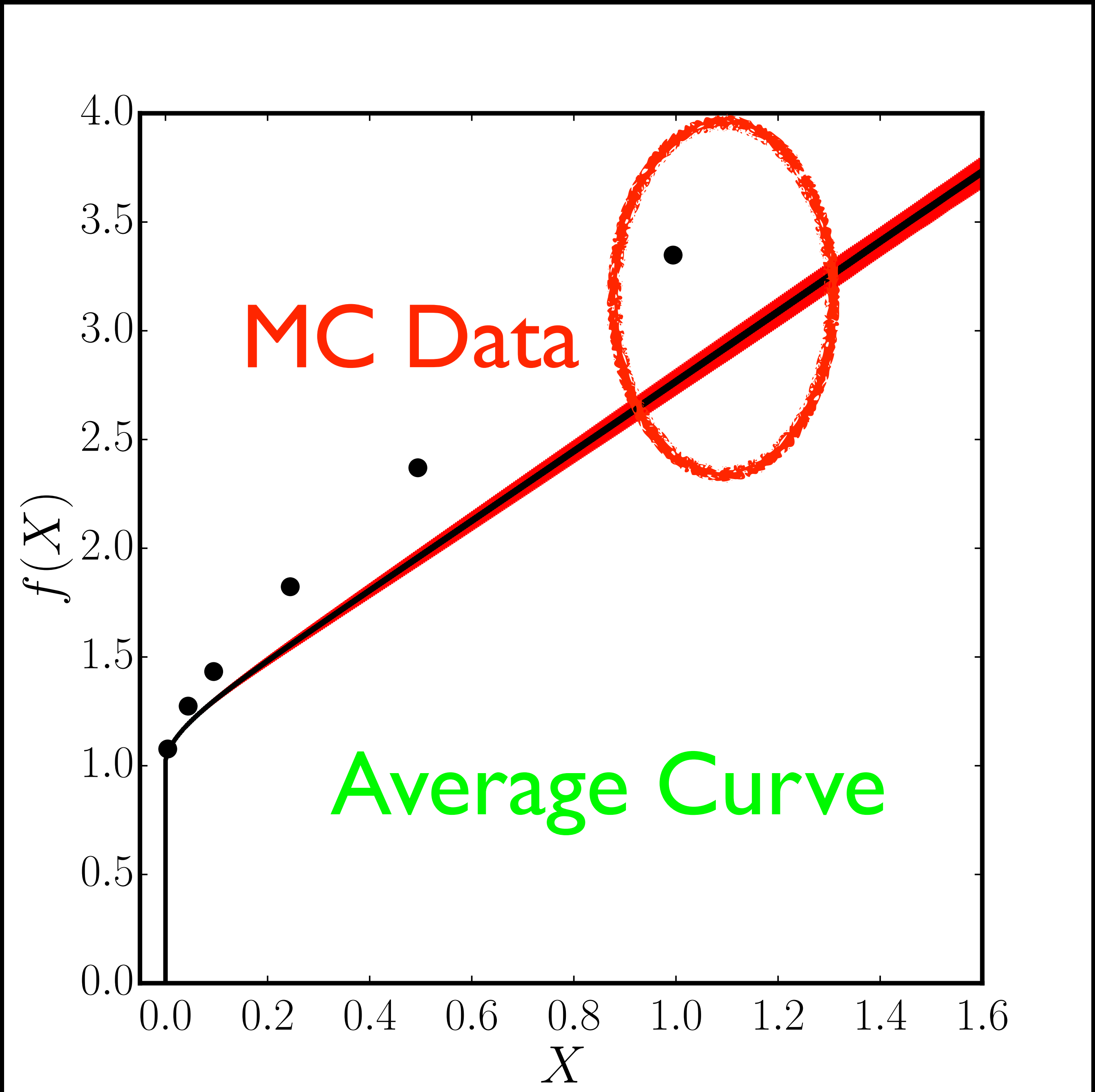
- Run the amplitude flow with $\kappa_\Lambda = \mu/U$
- Extract the expectation value for the field $U'(\rho)\Big|_{\kappa_r} = 0$
- Initiate the SG flow $K = \kappa_r$
- Extract the renormalized superfluid stiffness $K_* = \rho_s$

Looking for μ_c



Universality recovered





The universality in the X variable is confirmed.
Incorrect large X behavior! Higher derivative of the phase?

XY Model

$$H_{XY} = -J \sum_{\langle ij \rangle} \left(s_{x,i} s_{x,j} + s_{y,i} s_{y,j} \right),$$

Hubbard-Stratonovich Transformation

$$S[\varphi] = S_{\mathrm{kin}}[\varphi] + S_{\mathrm{pot}}[\varphi]$$

$$S_{\mathrm{kin}}[\varphi] = \frac{1}{2} \sum_q \varphi_q \varepsilon(q) \varphi_{-q}.$$

$$S_{\mathrm{pot}}[\varphi] = \int d^d x \left[-U \left(2 \sqrt{\frac{\beta}{J}} (Jd + \mu) \varphi \right) + \frac{Jd + \mu}{J} |\varphi|^2 \right]$$

$$\varepsilon(q) = 2(Jd + \mu) \frac{d - \varepsilon_0(q)}{J\varepsilon_0(q) + \mu}.$$

XY Model

Mean field initial condition:

$$U_\Lambda(\rho) = \left[-\log\left(\pi I_0\left(2\sqrt{\frac{\beta}{J}}(Jd+\mu)\varphi\right)\right) + \frac{Jd+\mu}{J}|\varphi|^2 \right]$$

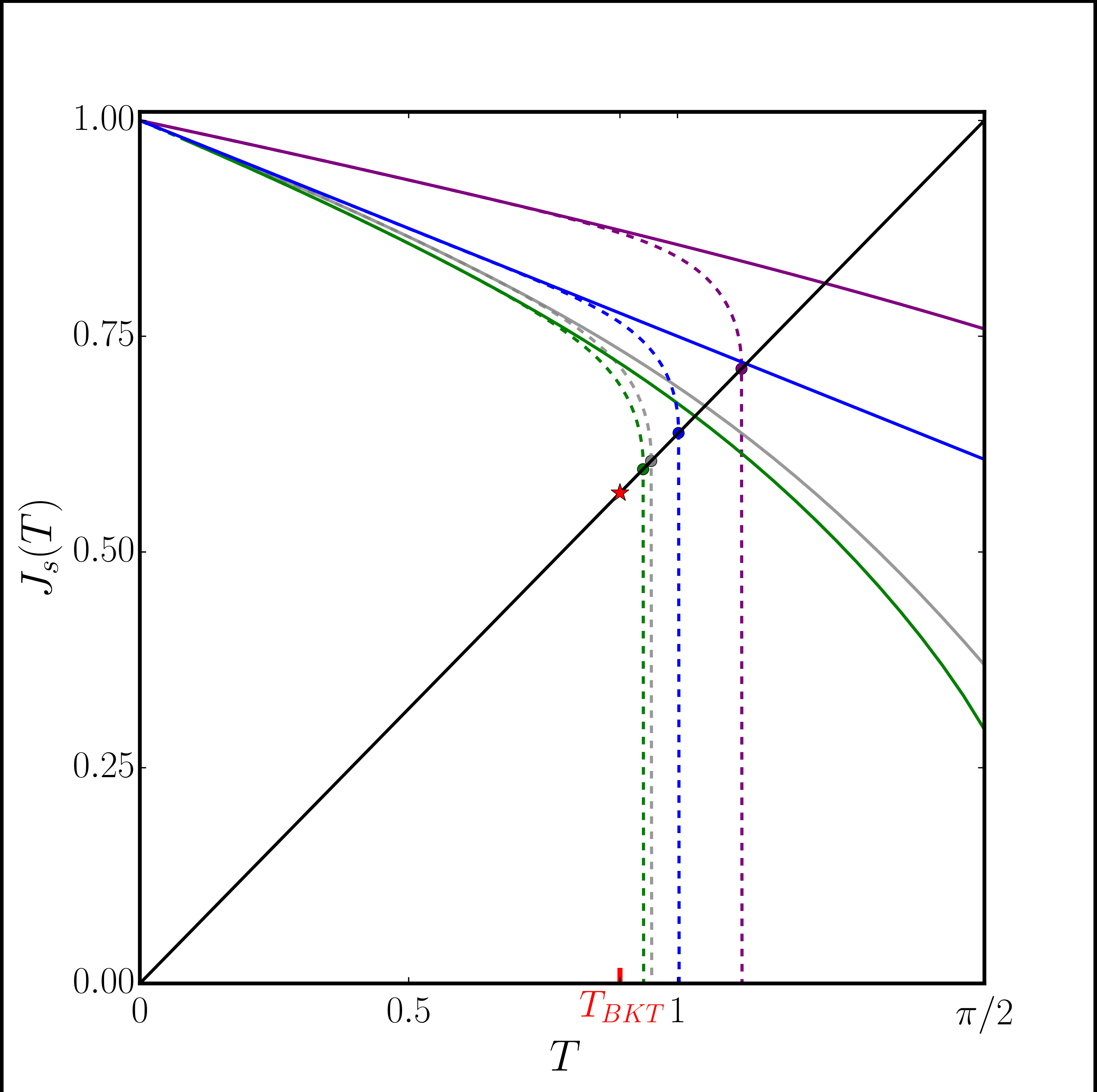
Approximate dispersion: $\varepsilon(q) = 2(Jd+\mu)\dfrac{d-\varepsilon_0(q)}{J\varepsilon_0(q)+\mu} \simeq q^2$

μ dependence recovered

Optimal choice: $\mu = 0$

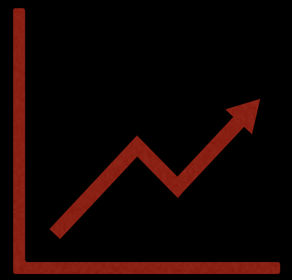
Recovers low temperature expansion

Spin stiffness



Results of first part

- Amplitude fluctuations irrelevant in FRG
- Non universal corrections
- Exact BKT features
- “Weak” Universality in 2d Bose gas
- Good estimation for XY critical temperature



Nonperturbative renormalization group treatment of amplitude fluctuations for $|\varphi|^4$ topological phase transitions

Nicolò Defenu, Andrea Trombettoni, István Nándori, and Tilman Enss
Phys. Rev. B **96**, 174505 – Published 13 November 2017

Bilayer XY Model

$$H = -J \sum_{\langle ij \rangle} \cos\left(\phi_i - \phi_j\right) - J \sum_{\langle ij \rangle} \cos\left(\psi_i - \psi_j\right) - K \sum_{i} \cos\left(\phi_i - \psi_i\right)$$

$$S[\varphi] = \frac{1}{2} \frac{\varphi_+(q)\varphi_+(q)}{K^+(q)} + \frac{1}{2} \frac{\varphi_+(q)\varphi_+(q)}{K^+(q)} + \text{local terms...}$$

$$K^+(q) = 2J\varepsilon_0(q) + 2\mu + 2K \qquad K^-(q) = 2J\varepsilon_0(q) + 2\mu - 2K$$

$$S[\theta] = \int d^2x \left\{ \frac{\rho_+}{2m_+} \partial_\mu \theta_+ \partial_\mu \theta_+ + \frac{\rho_-}{2m_-} \partial_\mu \theta_- \partial_\mu \theta_- \right\}$$

Berezinskii-Kosterlitz-Thouless Paired Phase in Coupled XY Models

Giacomo Bighin, Nicolò Defenu, István Nándori, Luca Salasnich, and Andrea Trombettoni
Phys. Rev. Lett. **123**, 100601 – Published 3 September 2019

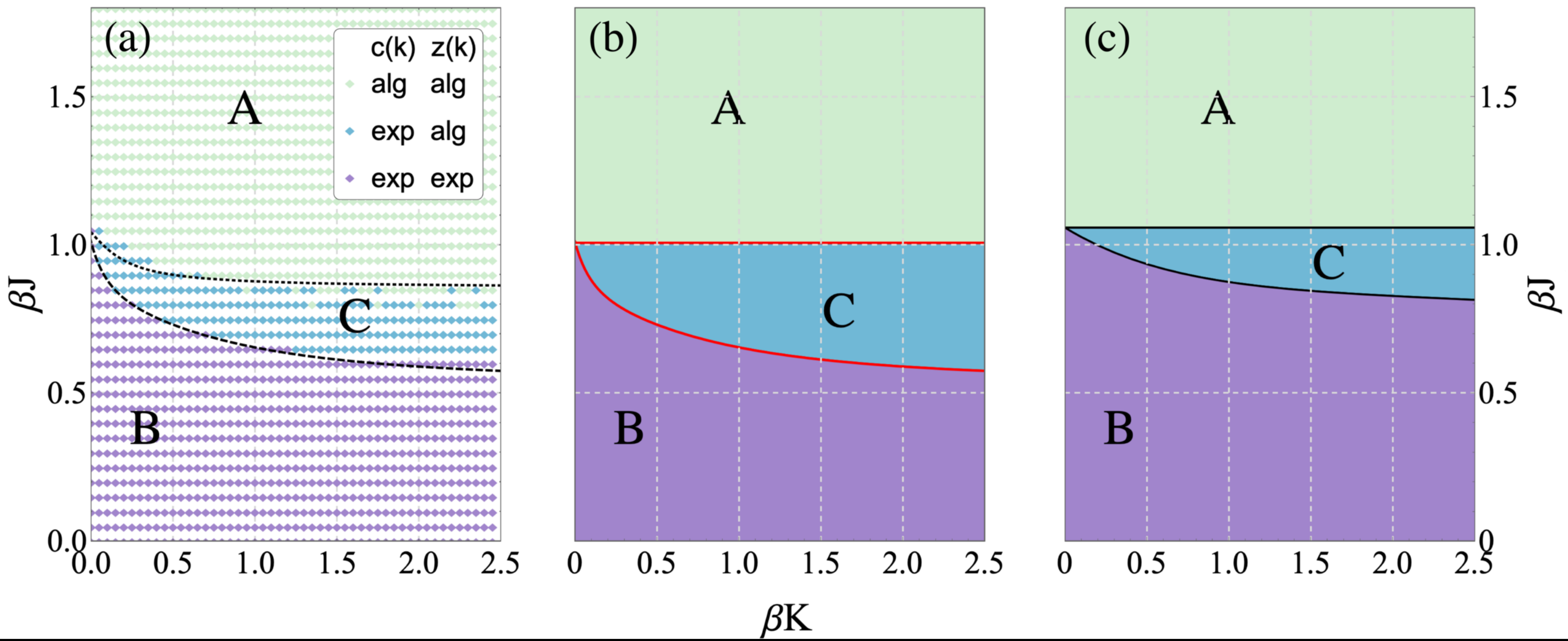
Bilayer XY Model

Numerical Simulations

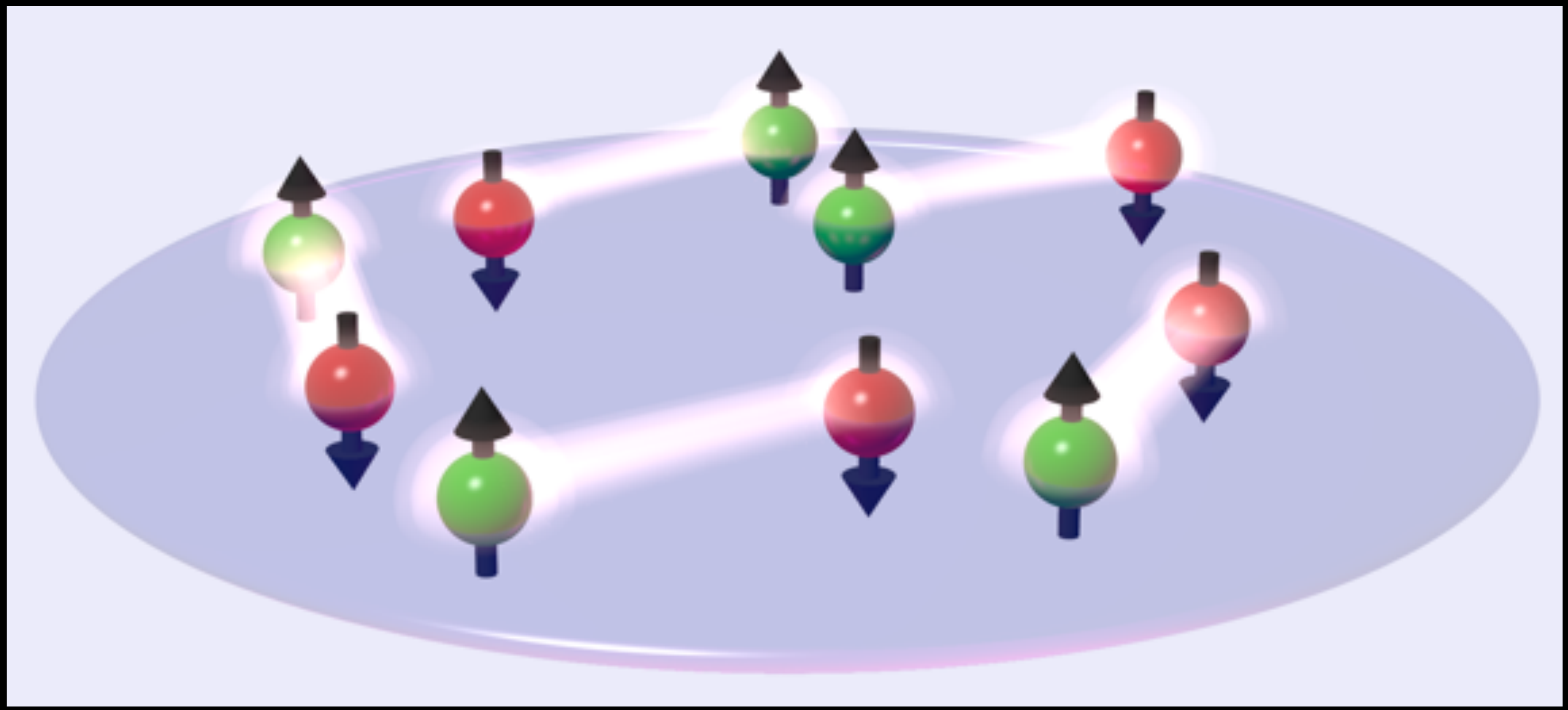
$$c_{\uparrow}(k) = \sum_{|i-j|=k} \exp(\mathrm{i}\phi_i - \mathrm{i}\phi_j)$$

$$c_{\downarrow}(k) = \sum_{|i-j|=k} \exp(\mathrm{i}\psi_i - \mathrm{i}\psi_j)$$

$$z(k) = \sum_{|i-j|=k} \exp(\mathrm{i}\phi_i + \mathrm{i}\psi_i - \mathrm{i}\phi_j - \mathrm{i}\psi_j)$$



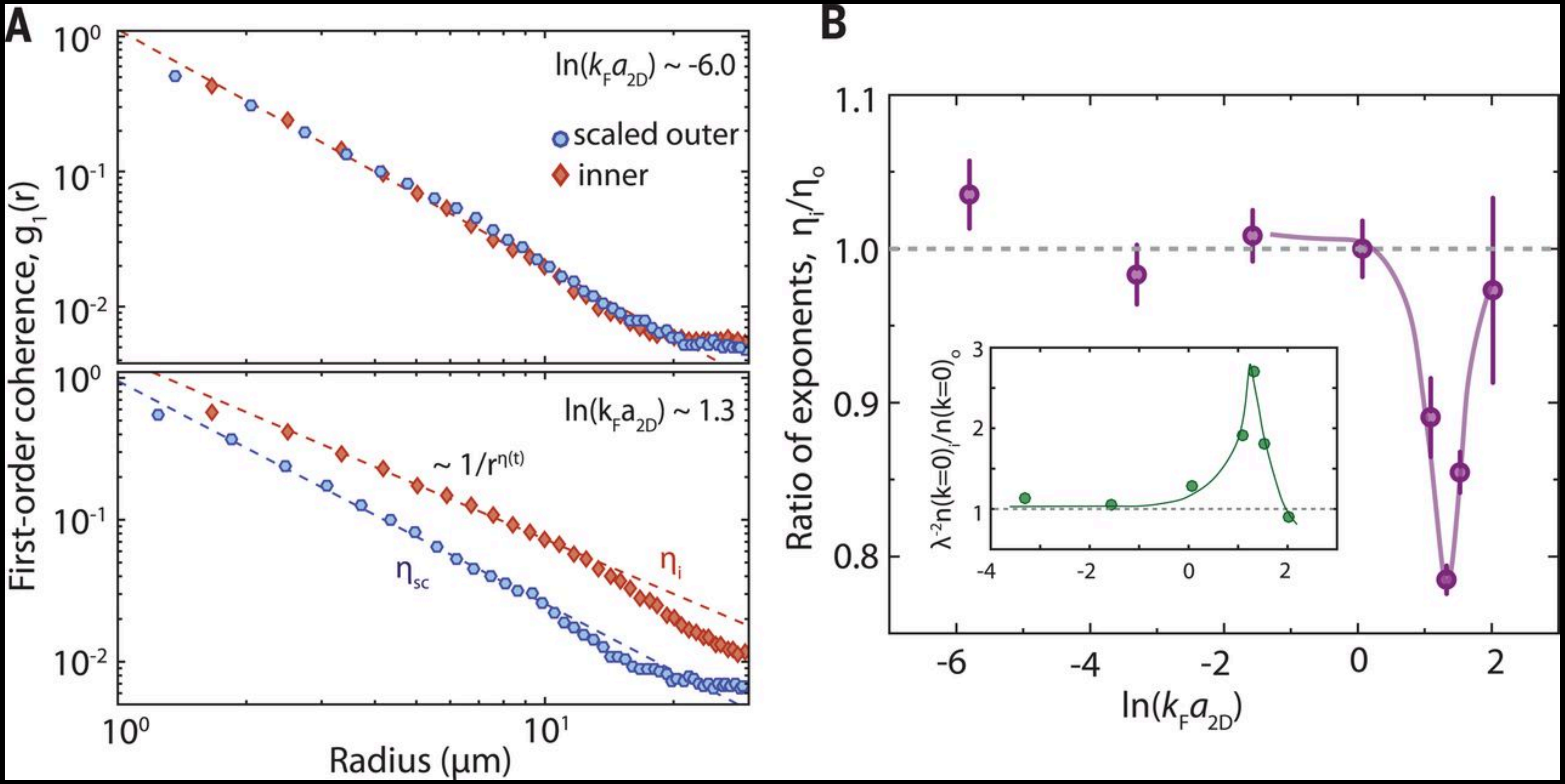
2D BEC-BCS Crossover



dilute gas of ↑ and ↓ fermions with contact interaction:

$$H = \int d^d x \sum_{\sigma=\pm 1} \psi_\sigma^\dagger \left(-\frac{\nabla^2}{2m} - \mu_\sigma \right) \psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

Phase correlations



- density scale invariant but **superfluid density** n_{SF} anomalously enhanced: **scale dependence (scaling violation) of critical exponent**

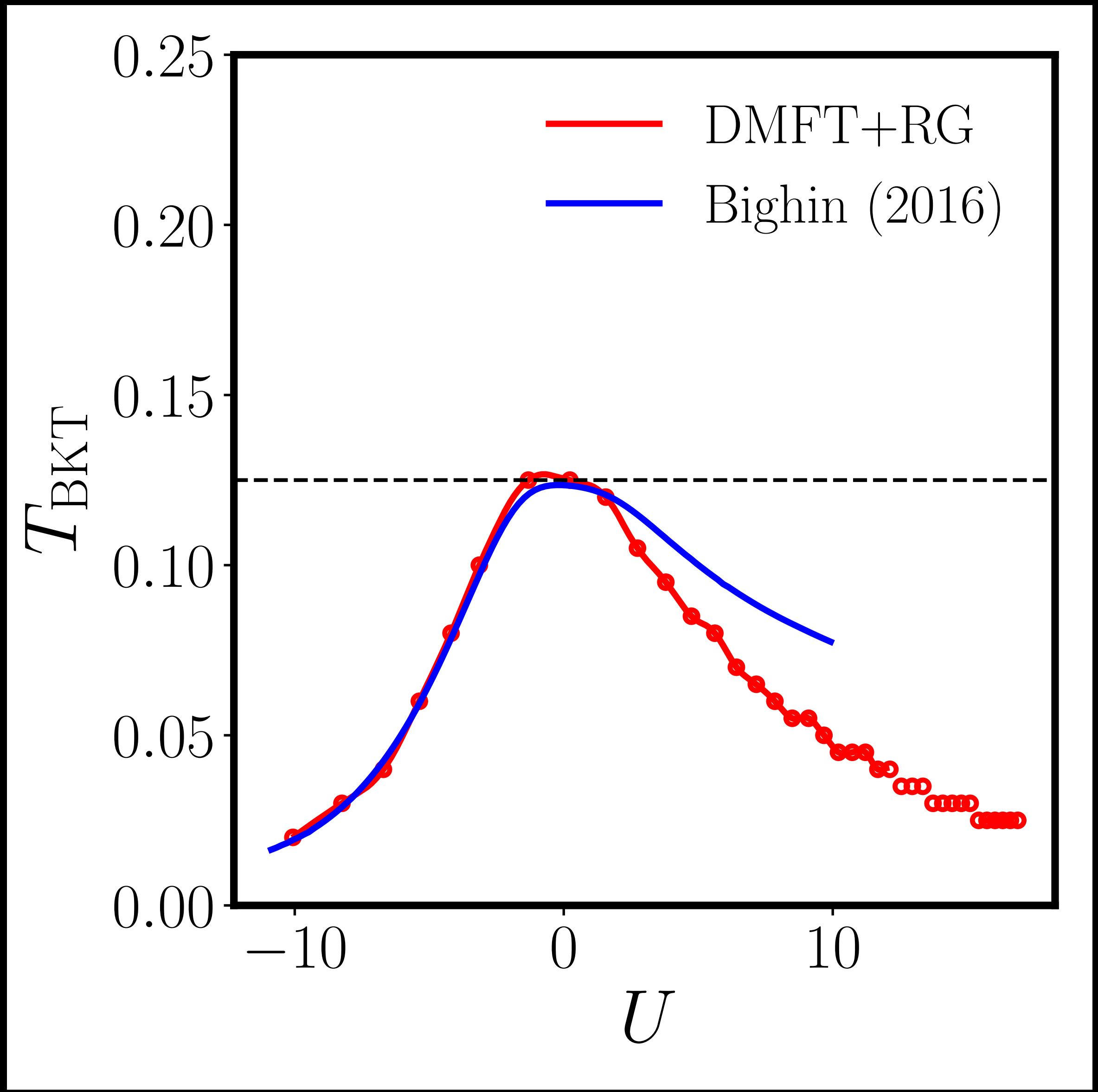
P. A. Murthy, N. Defenu, L. Bayha, M. Holten, P. M. Preiss, T. Enss, S. Jochim, *Science 365, 268 (2019)*

Preliminar RG application

$$H^{cl}_{\text{phase}} = -\sum_{ij} E_J(\Delta)\cos(\theta_i - \theta_j)$$

Δ : superconducting order parameter

$E_J(\Delta)$: effective phase stiffness



Future perspectives

- Investigate the assumption of decoupled spin wave and vortexes.
- Inclusion of lattice dispersion relation in the classical XY model.
- FRG treatment of 2d Fermi gas.

Thank You