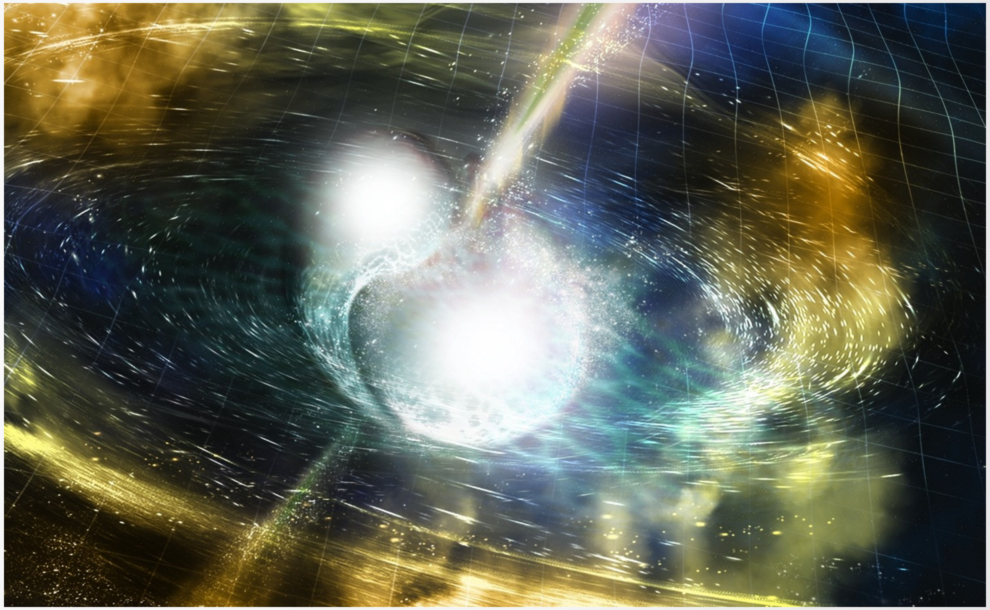
Stellar Modeling for R-Process Nucleosynthesis

-Amber Lauer: Triangle Universities Nuclear Laboratory, Duke University, Durham, NC 27708

Abstract: One of the primary mechanisms for guiding experimental research in nuclear astrophysics is the sensitivity study. In a broad sense, these studies involve a stellar model in which quantities related to nuclear reactions are varied to examine the effects. These variables can include a single, or multiple, reaction rates, Q-values, nuclear masses, and others. Subsequently, various aspects of the model are examined, such as elemental abundances, thermodynamics, and astronomical observables, and compared with theory and observations. This talk will examine the underlying stellar models, their strengths and limitations both broadly and as it applies to R-process, and discuss the findings of the most recent works, as well as improvements that must be made in the future in order to improve the accuracy of the results of the sensitivity study.



TUNL

TRIANGLE UNIVERSITIES NUCLEAR LABORATORY

Stellar Modeling for R-Process Nucleosynthesis: 2010- present

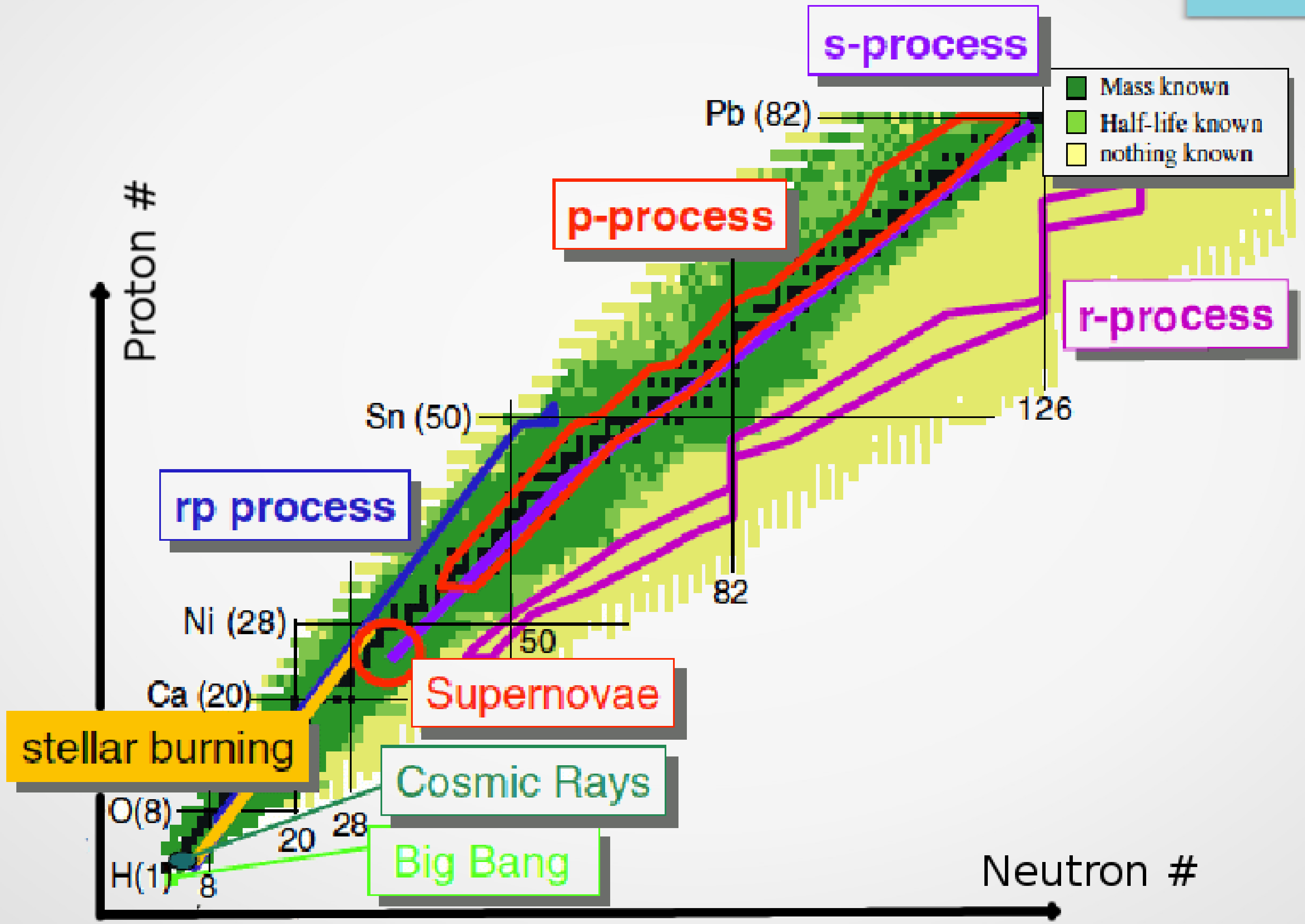
1) Why do we need stellar models?

2) Networks for Nuclear-astrophysics (nucastro):

3) Stellar Models: Theory and limitations

4) Stellar Models for R-process nucleosynthesis

Stellar Modeling for R-Process Nucleosynthesis Sites



Stellar Modeling for R-Process Nucleosynthesis Sites

Why do we need stellar modeling?

Stellar Modelling for R-Process Nucleosynthesis Sites

Why do we need stellar modeling?

- Create and experiment with environments and time scales we will never visit or recreate otherwise*

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*probably

Stellar Modeling for R-Process Nucleosynthesis Sites

Why do we need stellar modeling?

- Create and experiment with environments and time scales we will never visit or recreate otherwise*

- Test theories for physics and the relationships between quantities

*probably

Stellar Modeling for R-Process Nucleosynthesis Sites

Why do we need stellar modeling?

Create and experiment with environments and time scales we will never visit or recreate otherwise*

Test theories for physics and the relationships between quantities

How do test stellar models?

*probably

Stellar Modeling for R-Process Nucleosynthesis Sites

Why do we need stellar modeling?

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- Test theories for physics and the relationships between quantities

How do test stellar models?

- Comparison with theory

- Comparison with observables

*probably

Networks for Nuclear Astrophysics

- Realistic nucleosynthesis 3 levels of complexity:

  1) Nuclear Network Only

  2) Network & Stellar Model- post-processed

  3) Network & Stellar Model (3D)- co-processed

Networks for Nuclear Astrophysics

- Nuclear network only

Networks for Nuclear Astrophysics

- Nuclear network only
  - Solve: diff eq's, NSE
  - Use: Linear algebra, matrix & stiff ODE solvers
  - Input
    - Initial X_i's
    - Rxn rates
    - T & ρ (constant or T(t)/ρ(t)) , from sim. or obs.***

Networks for Nuclear Astrophysics

- Nuclear network only
- Network & stellar model- post-processed net
  - 1st model: 3D Hydro or 1D SE w/Hydro
  - Small (or no), de-coupled Net (5-20 ^{N}Z)
  - More physical (coupled hydro, mixing, EOS, κ)
  - 2nd model: Full/Large net
  - 1 or multi-zone/multi-particle
  - Input
    - T & ρ from 1st, sometimes mixing

Networks for Nuclear Astrophysics

- Nuclear network only
- Network & stellar model- post-processed net
- Network & stellar model (3D)- co-processed net
  - All physics fully coupled in single time step (hydro, SE, & Net)
  - Large (but not necessarily complete) net: 20-500 species

Networks for Nuclear Astrophysics

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Stellar Models: Theory and limitations

"All models are wrong, some are useful"
-G. E. P. Box

- Full 3D coupled physics not yet possible
- Why?
  - C. A. Meakin[1]
    - Need: $\Theta(10^{22})$ computational cells
    - Moore's law: $\log_2(flops/\$) = time/(18\,months)$
    - Horizon Run 2/3 on Tachyon ii[2]
      - 2011
      - 3.74×10^{11} particles

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1. https://arxiv.org/pdf/0806.4542.pdf - (2008)

2. http://sdss.kias.re.kr/astro/Horizon-Runs/Horizon-Run.php

Stellar Models: Theory and limitations

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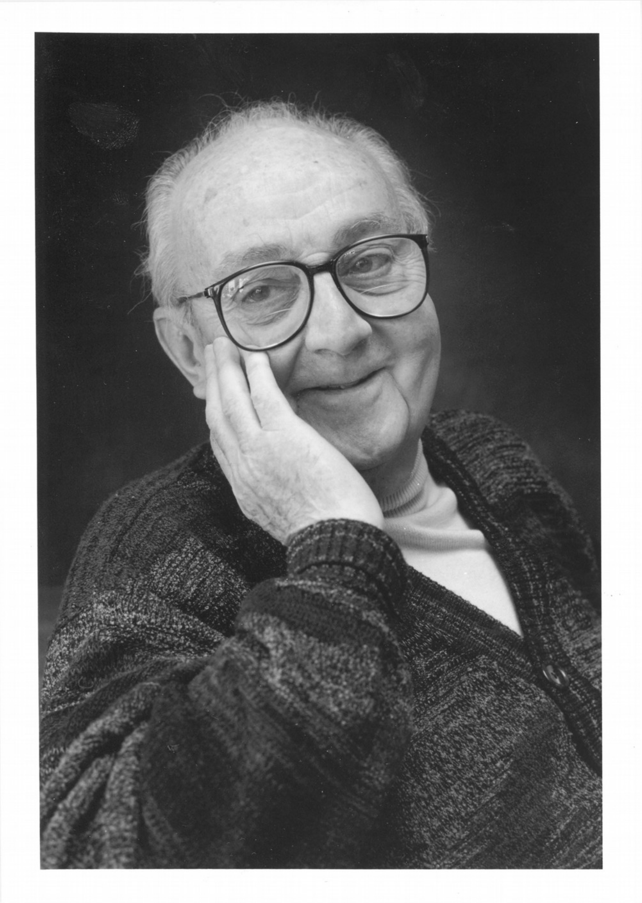

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Stellar models characterized by how they make use of resources: Resolution or evolution?

1. https://arxiv.org/pdf/0806.4542.pdf - (2008)
2. http://sdss.kias.re.kr/astro/Horizon-Runs/Horizon-Run.php

Stellar Models: Theory and limitations

- Stellar Evolution (SE)

Conservation of momentum

$$\frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2},$$

Conservation of mass

$$\frac{\partial r}{\partial M_r} = \frac{1}{4\pi r^2 \rho},$$

Energy Transport

$$\frac{\partial T}{\partial M_r} = -\frac{3\kappa L_r}{64\pi^2 ac T^3 r^4},$$

Conservation of energy

$$\frac{\partial L_r}{\partial M_r} = \epsilon - T\frac{\partial S}{\partial t}.$$

- Fluid/Hydro Dynamics

Conservation of momentum

$$\frac{d\,\rho}{dt} = -\rho\nabla\cdot\vec{v}$$

Conservation of mass

$$\frac{d\,\vec{v}}{dt} = -\frac{\nabla P}{\rho} + \vec{g}$$

Conservation of energy
(heating-Γ , cooling-Λ)

$$\frac{d\,u}{dt} = -\frac{P}{\rho}\nabla\cdot\vec{v} + \Gamma - \Lambda,$$

EOS (eg)

$$P = (\gamma - 1)\rho u$$

[4],[5]

Stellar Evolution (SE) & Hydrodynamics

Hydrodynamic:

Eulerian: M(r)- AMR (FLASH)

3D

LaGrangian: R(m)- SPH (PHANTOM)

Images courtesy Jordi Jose: http://www.fen.upc.edu/users/jjose/images/CRC/

Stellar Models: Theory and limitations

Hydrodynamic:

3D

Eulerian: M(r)- AMR (FLASH)

LaGrangian: R(m)- SPH (PHANTOM)

Common issues:

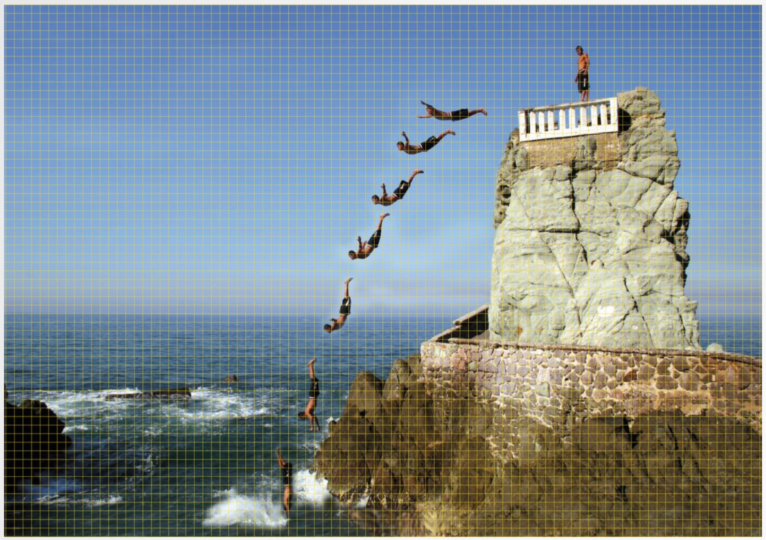
Advection errors
Excessive Diffusion
Angular Momentum errors
Containment of experimental volume

Viscosity issues
Mesh Deformation

Images courtesy Jordi Jose: http://www.fen.upc.edu/users/jjose/images/CRC/

Stellar Models: Theory and limitations

Hydrodynamic:

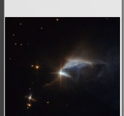


Eulerian: M(r)- AMR

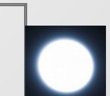
3D

LaGrangian: R(m)- SPH (PHANTOM)

Hydrodynamics + SE: MESA, KEPLER, GENEC, GARSTEC:



1D



Images courtesy Jordi Jose: http://www.fen.upc.edu/users/jjose/images/CRC/, ESO/L. Calcada., NASA/STScI

Stellar Models: Theory and limitations

Hydrodynamic:

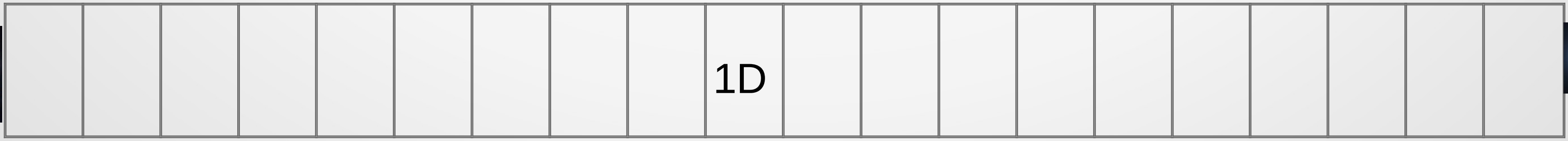
3D

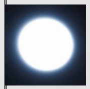
Eulerian: M(r)- AMR

LaGrangian: R(m)- SPH (PHANTOM)

Hydrodynamics + SE: MESA, KEPLER, GENEC, GARSTEC:

Pre-MS-→WD & other long term evol





EOS, Mixing (simple 1D), Large nets (co-processed), mass loss, opacity, diffusion

Images courtesy Jordi Jose: http://www.fen.upc.edu/users/jjose/images/CRC/, ESO/L. Calcada., NASA/STScI

Stellar Models: Theory and limitations

Hydrodynamic:

3D

short timescales

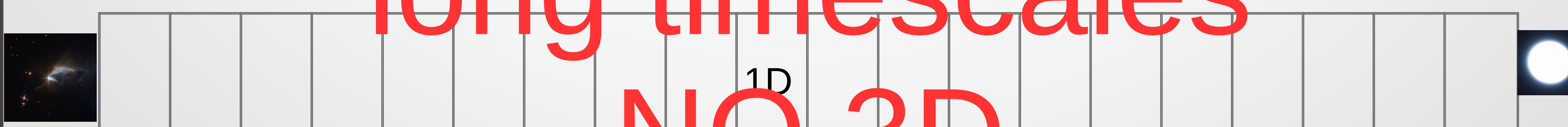
high spatial resolution

Eulerian: M(r)- AMR (FLASH)

LaGrangian: R(m)- SPH

Hydrodynamics + SE: MESA, KEPLER, GENEC, GARSTEC:

long timescales



NO 3D

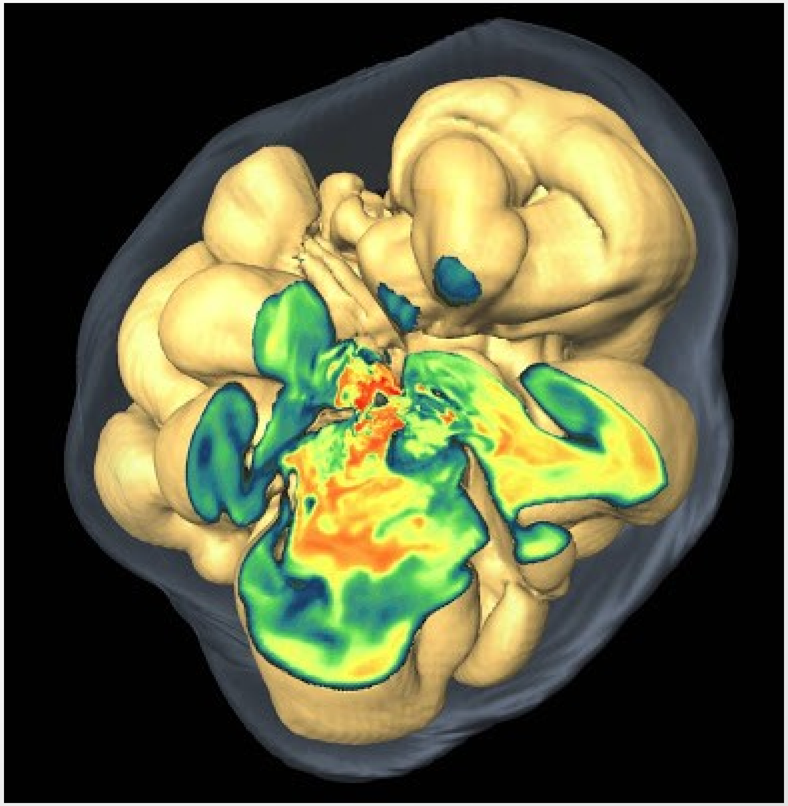
Images courtesy Jordi Jose: http://www.fen.upc.edu/users/jjose/images/CRC/, ESO/L. Calcada., NASA/STScI

Stellar Models for R-process nucleosynthesis

R-Process: *n* rich, high T, seed X_i

$\rightarrow t_{n\_cap} < t_{\beta^-} <\sim 1$ s

- Compact object merger
  - Ejecta- LIGO 2017 [1] ✓
  - Gamma Ray Burst [2]
- Supernovae, CCSN [2]
  - Neutrino-driven wind [2]
  - Relativistic/MHD Jets [2]



[3]

Stellar Models for R-process nucleosynthesis

R-Process: *n* rich, high T, seed X_i

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3D

- Compact object merger
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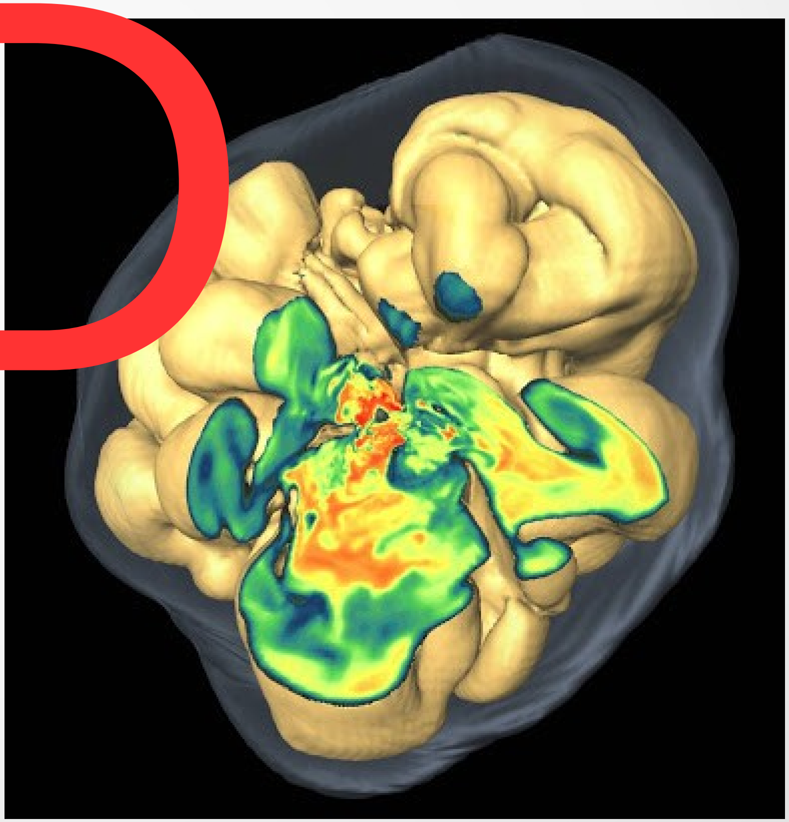

[3]

Stellar Models for R-process nucleosynthesis

- Nuclear network only
  - Solve: diff eq's, NSE
  - Use: Linear algebra, matrix & stiff ODE solvers
  - Input
    - Initial X_i's
    - Rxn rates
    - *T & ρ* (constant or T(t)/*ρ*(t)) , from sim. or obs.***

R-process: S, $Y_e = \frac{n_p}{n_p + n_n}$,τ_{dyn} in order to calculate *T & ρ* profile & set X_n

Physics: MHD, Neutrino transport, GR

Authors	Scenario	Simulation	Tested quantity	year
Beun, Blackmon, Hix McLaughlin, Smith, Surman[6]	η wind	param	*n* capture	2008
Surman,Beun, McLaughlin, Hix[7]	η wind	param	*n* capture	2009
Arcones, Martinez-Pinedo[8]	η wind	PPM	m, S_n	2011
Mumpower, McLaughlin, Surman[9]	Hot, cold	param	m	2012
Brett, Bentley, Paul, Aprahamian, Surman[10]	η winds	param	m, S_n	2012
Surman, Mumpower, Cass, Aprahamian[11]	Early universe?	param	m, S_n	2013
Surman, Mumpower, Cass, Bentley, Aprahamiian, McLaughlin[12]	η winds, NSM,	param, PPM, R-SPH	m,*n* capture, β^- rates	2013
Surman, Mumpower, Sinclair, Jones, Hix, McLaughlin[13]	CSSN	param	*n* capture	2014
Aprahmian, Bentley, Mumpower, Surman[14]	CCSN	param	m, S_n	2014
Mumpower, Cass, Passuci, Surman, Aprahamian[15]	η winds, NSM, GRB	Param, PPM	β^- *rates*	2014
Mumpower, Surman,Fang, Beard, Moller, Kawano, Aprahamian [16]	η winds, NSM	param, PPM, R-SPH	m→ *n* capture, (γ,p), β^-	2015

Abbreviations:
PPM: Piecewise Parabolic Method (Eulerian)
R-SPH: Relativistic Smoothed Particle Hydrodynamics (LaGrangian)
Param: Parameterized: Equation or constant

Stellar Models for R-process nucleosynthesis

Is 3D necessary for r-process nucleosynthesis?

- Ideally, yes.
- But 2D will probably suffice
- Problems with level 2:
    - 1D uses approximations for 3D effects
    - Hydro timescales (dynamical) vs Myrs for SE
        - (either-or)
    - Hydro → some physics issues due to mesh or smoothing
    - Post-processed → Harder to test astron observables
    - Little/No feed back with net in 3D

Stellar modelling for nuc-astro

- What can we improve:
  - Physics: eg EOS, Neutrino treatment
  - Extending Models:
    - SE equations to 2D-3D hydro & vice versa*
    - Adaptive Hydro 1D, 2D → 3D*
    - 2D Nuclear nets w/ mixing &/or Monte Carlo X_i
    - Eulerian smoothing
    - Hybrid Eulerian & Lagrangian*
  - Computing power:
    - This will motivate advancement
  - Algorithms & Solvers:
    - Probably have to rely on CS
  - Access model:

*already under way

Pros:
Deriving from first principles is useful exercise
Encourages variation of solutions

Cons:
Closed source contrary to spirit of scientific method
”black box”, irreproducable
Encourages replication & abandonware
Slows advancement

Take cues from FOSS: “All bugs are shallow” or “Sunlight is the best disinfectant”

Thank You- Collaborators

- Manos Chatzopoulos- LSU
- Zach Meisel- Ohio U
- Carl Fields- MSU
- Art Champagne- TUNL/ UNC
- Modules for Experiments With Astrophysics (MESA)

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When did you become an expert in thermonuclear astrophysics?
Last night.

Finite Difference & Related Methods

Finite differences are simple and effective, although their applicability is restricted to structured meshes. In this technique, variables are assigned to individual grid points (either located at the edges, corners, or centers of each computational cell), and the corresponding derivatives are evaluated as differences of variables in neighboring cells, in a Taylor series expansion.

Finite elements and finite volumes are particularly suited for arbitrary meshes (unstructured grids, in particular).

Finite elements operate by splitting the domain of the problem into a number of subdomains, called finite elements, each represented by a set of equations. But in contrast to the direct (strong-form) approach adopted in finite difference techniques (also in SPH), by which the partial differential equations are directly discretized and solved, finite element methods rely on an indirect or weak-form approach, in which the original equations are transformed, often in integral form, on the basis of trial solutions and weighting functions. The collection of local equations is then reassembled into a global system of equations through a transformation of coordinates.

Finite-volume methods [507, 1855] are gaining popularity as a suitable form of discretization in multidimensional codes. They are somewhat similar to finite-difference and finite-element methods, since they rely on the discretization of a system of partial differential equations onto a discrete set of points of a grid-based geometry. To this end, the computational domain is divided into a set of discrete, nonoverlapping discretization cells or control volumes, with physical variables assigned to the centroid of each control volume. The set of partial differential equations is then integrated over each control volume, resulting in balance equations that are subsequently discretized. The cornerstone of finite-volume methods is the discretization of the fluxes at the boundaries of each control volume. This discretization technique guarantees conservation of physical magnitudes for any control volume as well as for the overall computational domain. Note that discretization is again performed on local balance equations rather than on the original partial differential equations (as in finite-differences). For more information on these and other methods used in computational fluid dynamics, the reader is referred to LeVeque et al. [1083] and Toro [1812].

Nuclear Net Theory

Differential equation of species change:

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_1 u}{\partial x} = -n_1 n_2 \langle \sigma \nu \rangle \qquad n_i = \rho N_a Y_i$$

nuc ↑ term mix ↑ term number density ↑ avg cross section

- Reactions

$$\frac{dY_i}{dt} = -\lambda_i Y_i$$

$$\frac{dY_i}{dt} = -\rho N_A Y_i Y_j \langle \sigma \nu \rangle \qquad \frac{dY_i}{dt} = -\rho^2 N_A^2 Y_i Y_j Y_l \langle \sigma \nu \rangle_{j,k,l}$$

- Total differential equation

$$\dot{Y}_i = \sum_j N_j^i \lambda_j Y_j + \sum_{j,k} N_{j,k}^i \rho \mathrm{N}_A \langle \sigma \nu \rangle_{j,k} Y_j Y_k + \sum_{j,k,l} N_{j,k,l}^i \rho^2 N_A^2 \langle \sigma \nu \rangle_{j,k,l} Y_j Y_k Y_l$$

- Euler's method

$$\frac{\vec{Y}(t+\Delta t) - \vec{Y}(t)}{\Delta t} = (1-\Theta)\dot{\vec{Y}}(t+\Delta t) + \Theta \dot{\vec{Y}}(t)$$

Stellar Evolution & Hydrodynamics

- SE-

Conservation of momentum	$$\frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2},$$	Hydrostatic equilibrium
Conservation of mass	$$\frac{\partial r}{\partial M_r} = \frac{1}{4\pi r^2 \rho},$$	
Energy Transport	$$\frac{\partial T}{\partial M_r} = -\frac{3\kappa L_r}{64\pi^2 ac T^3 r^4},$$	
Conservation of energy	$$\frac{\partial L_r}{\partial M_r} = \epsilon - T\frac{\partial S}{\partial t}.$$	

Stellar Evolution & Hydrodynamics

- SE-

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Conservation of energy
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Thermal Equilibrium

Stellar Evolution & Hydrodynamics

- SE-
  - Conservation of momentum
    $$\frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2},$$
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  - Conservation of energy
    $$\frac{\partial L_r}{\partial M_r} = \epsilon - T\frac{\partial S}{\partial t}.$$
    Thermal Equilibrium

- Fluid dynamics (Eulerian)-

R-process Reaction Rates:

- Rate Types:
  - Fast Neutron Captures
  - Photidisintegration
  - β^- decay
- What do we know about these rates:
  - For R-process: Nothing*.

WE RELY ON THEORETICAL RATE CALCULATIONS
Mass models

*Almost

Nuclear Net Theory

Differential equation of species change:

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_1 u}{\partial x} = -n_1 n_2 \langle \sigma \nu \rangle \qquad n_{\underleftarrow{i}} = \rho N_a Y_i$$

nuc ↑ term   mix ↑ term   no. density & avg ↑ cross section

- Mixing term handled by hydrodynamic code
- Reactions

$$\frac{dY_i}{dt} = -\lambda_i Y_i \quad \frac{dY_i}{dt} = -\rho N_A Y_i Y_j \langle \sigma \nu \rangle \quad \frac{dY_i}{dt} = -\rho^2 N_A^2 Y_i Y_j Y_l \langle \sigma \nu \rangle_{j,k,l}$$

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- Euler's method

$$\frac{\vec{Y}(t+\Delta t) - \vec{Y}(t)}{\Delta t} = (1-\Theta)\dot{\vec{Y}}(t+\Delta t) + \Theta \dot{\vec{Y}}(t)$$