Nuclear and astrophysics aspects for the rapid neutron capture process in the era of multimessenger observations, Trento, 1-5.7.2019.

Improving the nuclear energy density functionals: constraints from the ground state and collective excitations

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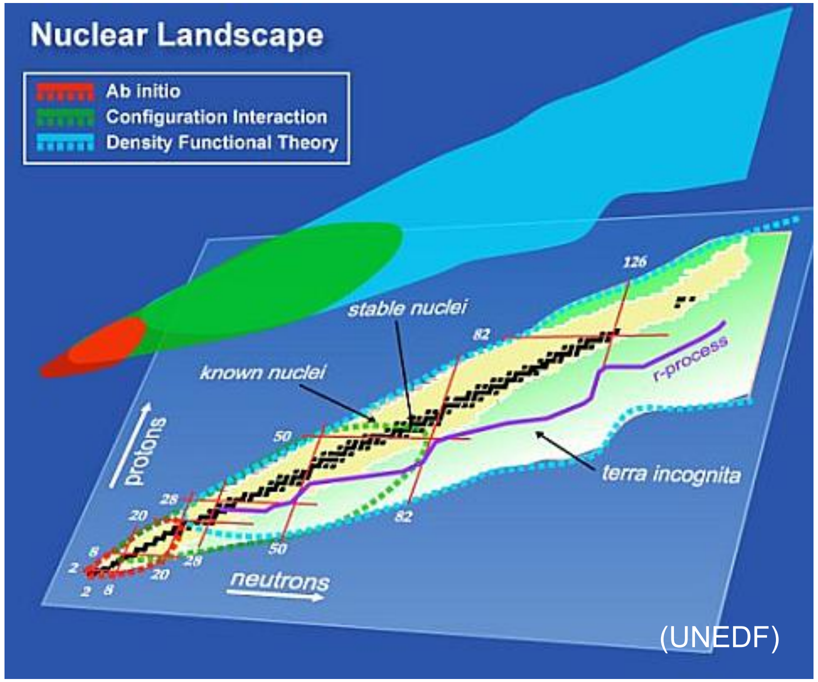
NUCLEAR ENERGY DENSITY FUNCTIONALS

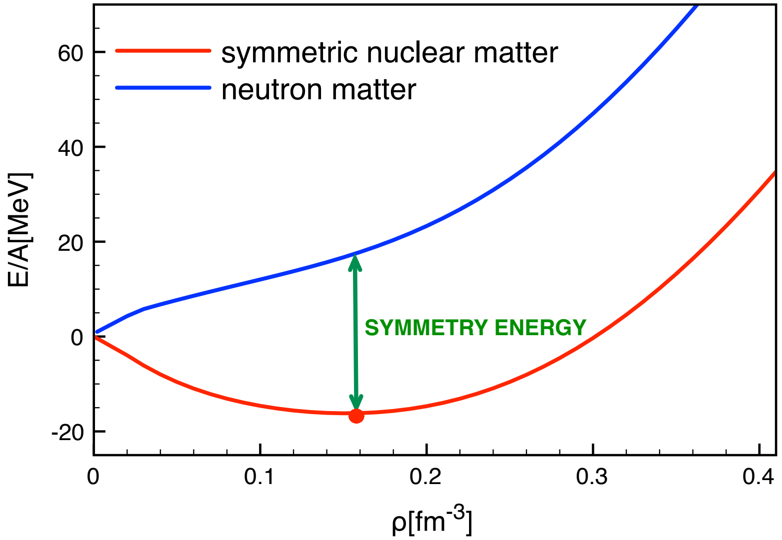
- **nuclear energy density functional (EDF)**

unified approach to describe at the quantitative level nuclear properties across the nuclide map including exotic nuclei − astrophysically relevant properties

  - ☐ static and dynamic properties of finite nuclei across the nuclide map (nuclear masses, excitations,...)
  - ☐ nuclear processes and reactions (beta decays, beta delayed neutron emission, neutron capture, neutrino-induced reactions, nuclear fission ...)
    - ➢ talk by **Ante Ravlić** on electron capture in presupernova collapse
  - ☐ equation of state of nuclear matter



- **Nonrelativistic**
  - ☐ Skyrme functionals
  - ☐ Gogny functionals
  - ☐ ...
- **Relativistic**
  - ☐ finite range meson exchange functionals
  - ☐ point coupling functionals
  - ☐ density dependent functionals
  - ☐ ...

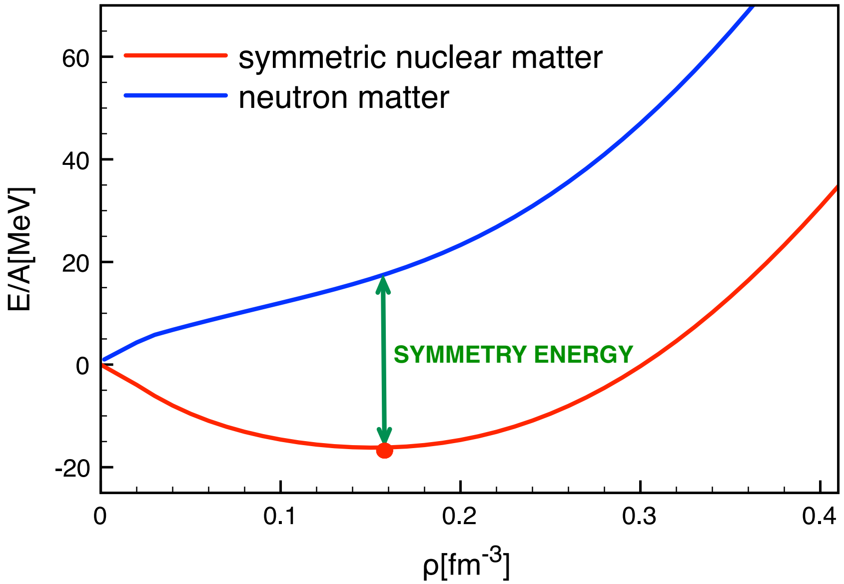


INTRODUCTION

- Nuclear matter equation of state (EOS) describes the energy per nucleon as a function of the nucleon densities of an uniform and infinite system at zero temperature

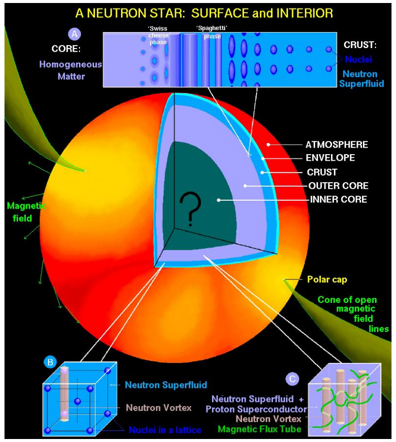
The "Holy Grail" of nuclear physics

(J. Piekarewicz, Florida St. Univ.)



- Symmetry energy $S(\rho)$ describes the increase in the energy of the N≠Z system as protons are turned into neutrons;
- It is important for understanding the properties of neutron-rich matter and neutron rich nuclei
- There are various theoretical approaches to the EOS (ab initio, energy density functional theory,...)

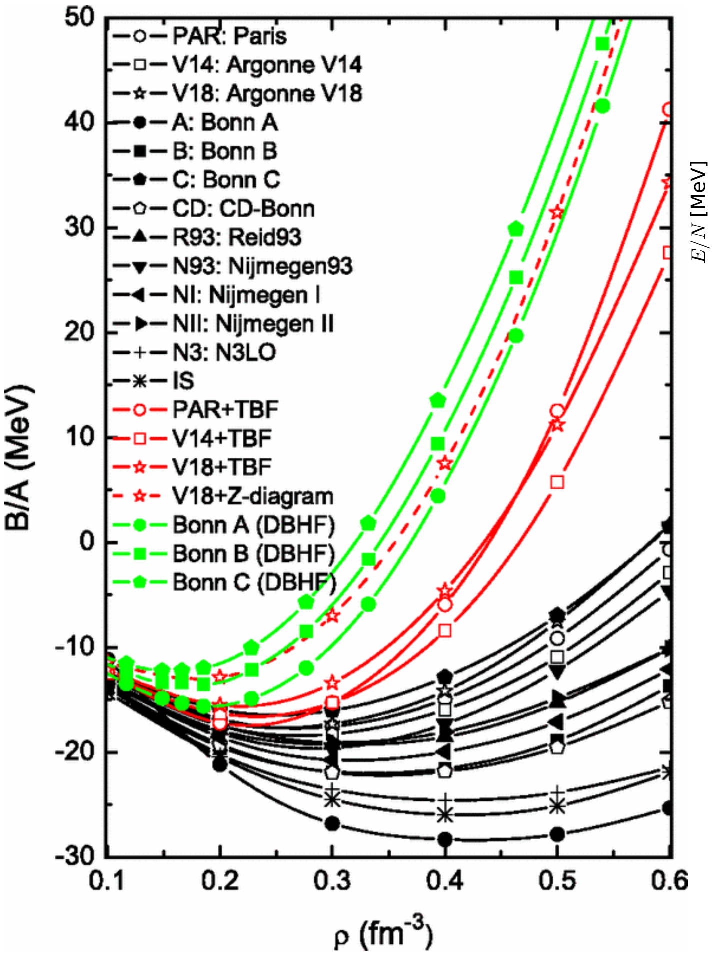
- The knowledge on the EOS is essential in nuclear physics and astrophysics



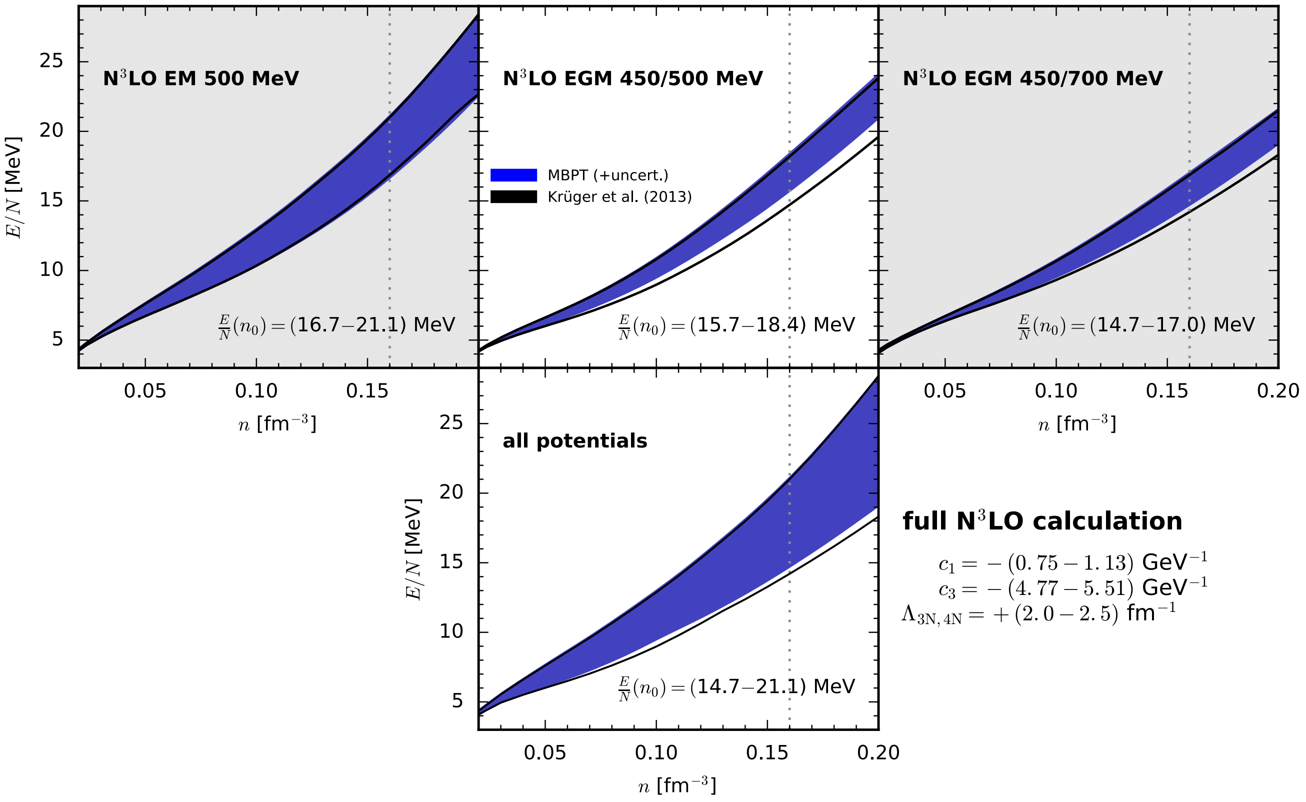
- Neutron star properties
- Core collapse supernovae
- Nucleosynthesis

NUCLEAR EQUATION OF STATE (EOS) – "ab initio"

- the EOS from the modern nucleon-nucleon interactions (*"ab initio"* models)
- differences in the results for various NN interactions and approaches (with 3N forces, relativistic/ nonrelativistic, chiral NN+3N calculations at N^{3}LO order, etc.)



Z. H. Li et al., PRC 74, 047304 (2006).



C. Drischler et al., PRC 94, 054307 (2016).

NUCLEAR EQUATION OF STATE (EOS) FROM THE EDFs

- **the EOS from the modern nuclear energy density functionals**
- supported by the experiments that probe the nuclear properties at or around saturation density

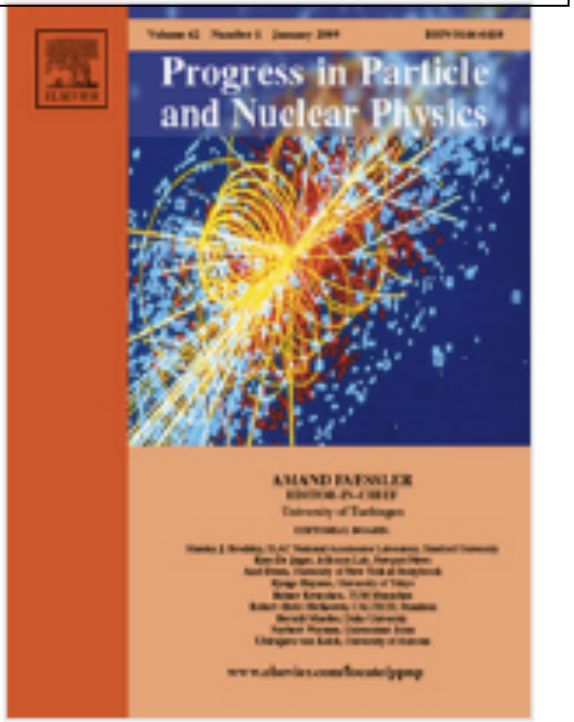
X. Roca Maza, N. Paar,
Nuclear equation of state from ground and collective excited state properties of nuclei
Progress in Particle and Nuclear Physics 101, 96-176 (2018).

6 Excitations in nuclei and EOS
6.1 Isoscalar modes
6.1.1 Isoscalar giant monopole resonance
6.1.2 Isoscalar giant dipole resonance
6.1.3 Isoscalar giant quadrupole resonance
6.2 Isovector modes

CONSTRAINTS
FOR THE ISOSCALAR
PROPERTIES

6.2.1 Isovector giant dipole resonance
6.2.2 Dipole polarizability
6.2.3 Pygmy dipole strength
6.2.4 Isovector giant quadrupole resonance
6.3 Charge-exchange modes
6.3.1 Isobaric analog resonance
6.3.2 Gamow-Teller resonance
6.3.3 Spin-dipole resonance
6.3.4 Anti-analogue giant dipole resonance

CONSTRAINTS
FOR THE ISOVECTOR
PROPERTIES


Progress in Particle and Nuclear Physics
AMAND FAESSLER

Experimental and theoretical efforts in finding and measuring observables specially sensitive to the EoS properties are of paramount importance, not only for low-energy nuclear physics but also for nuclear astrophysics applications.

CONSTRAINING THE EDFs

- The EDFs have been parametrized with the experimental data on the ground state properties of finite nuclei

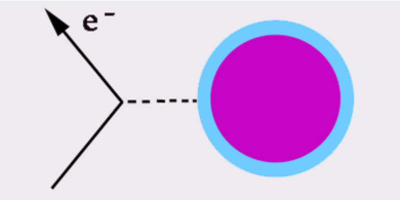
$$\chi^2(\boldsymbol{p}) = \sum_{\imath=1}^{m} \left(\frac{\mathcal{O}_\imath^{\mathrm{theo.}}(\boldsymbol{p}) - \mathcal{O}_\imath^{\mathrm{ref.}}}{\Delta\mathcal{O}_\imath^{\mathrm{ref.}}} \right)^2$$

- Nuclear ground state properties are often not enough to constrain the effective interaction completely, especially its isovector channel (that is especially relevant for the neutron-rich nuclei, neutron skins, symmetry energy, etc.).

- The protocols to determine the EDF's often included additional constraints on the *pseudo-observables* on the *nuclear matter properties* (often they are arbitrary).

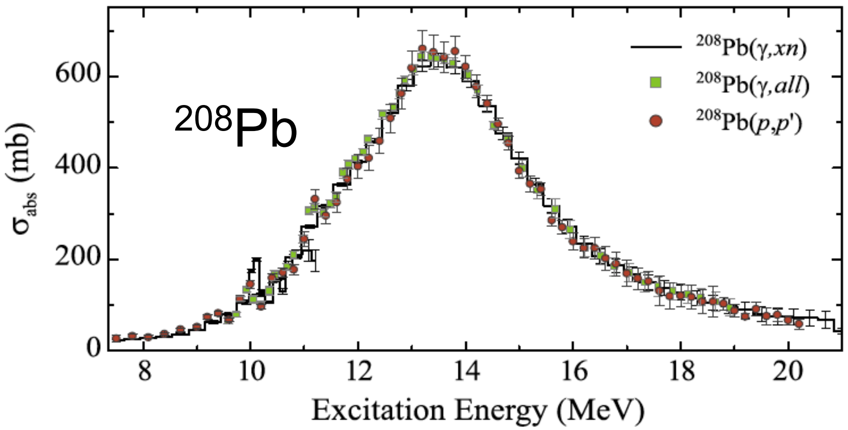
- The neutron skin thickness r_{np} may be useful probe for the isovector channel of the EDFs. However, the data on r_{np} are often model dependent.

- Results from parity violating electron scattering experiment (PREX) on ^{208}Pb have large uncertainties: $R_n - R_p = 0.33^{+0.16}_{-0.18}$ fm Abrahamyan et al. PRL 108, 112502 (2012).

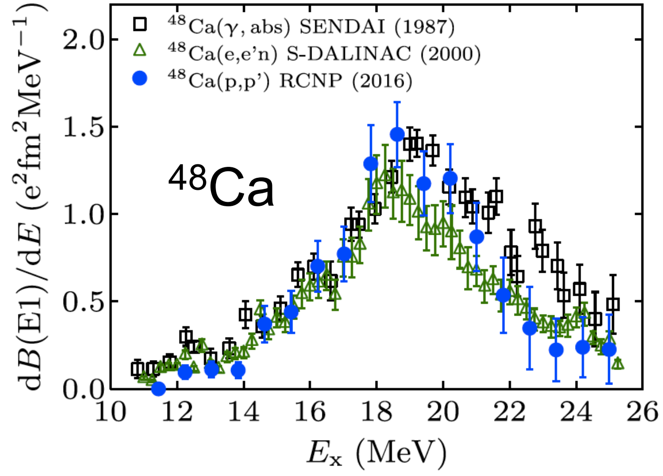


DIPOLE EXCITATIONS FROM THE EXPERIMENT

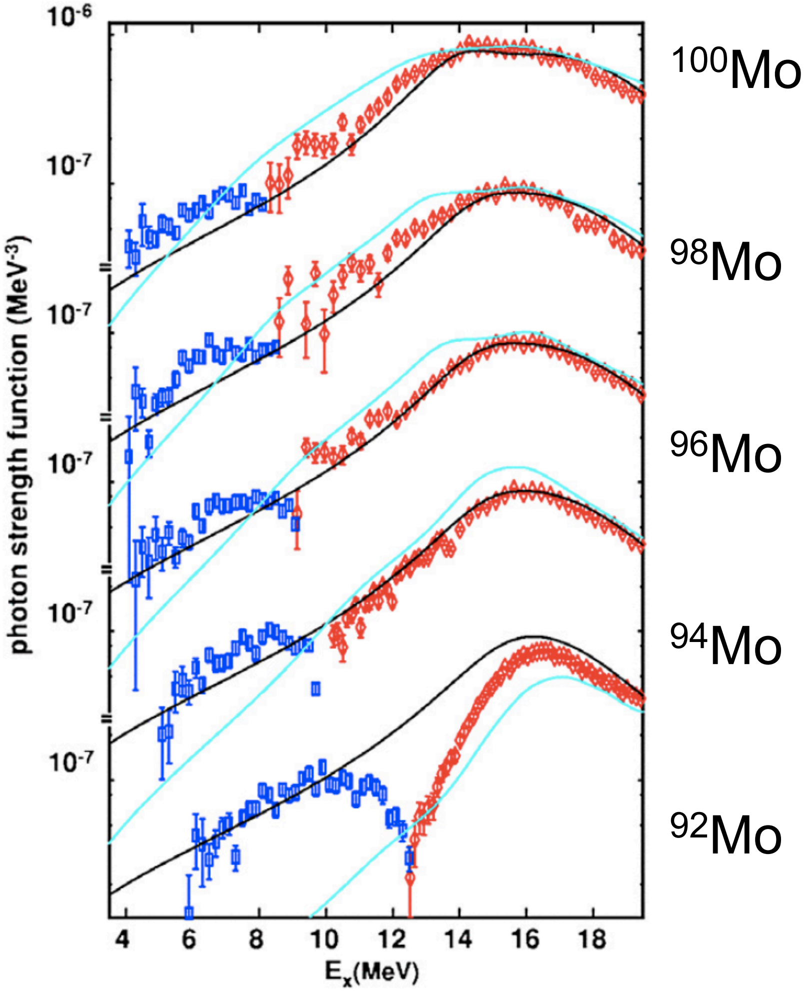
- Isovector dipole transitions in nuclei may provide valuable constraints for the EDFs
- A variety of experimental data from different approaches, e.g.:



A. Tamii, P. von Neumann-Cosel, I. Poltoratska, EPJ A 50 (2), 28 (2014)



J. Birkhan et al., PRL 118, 252501 (2017)



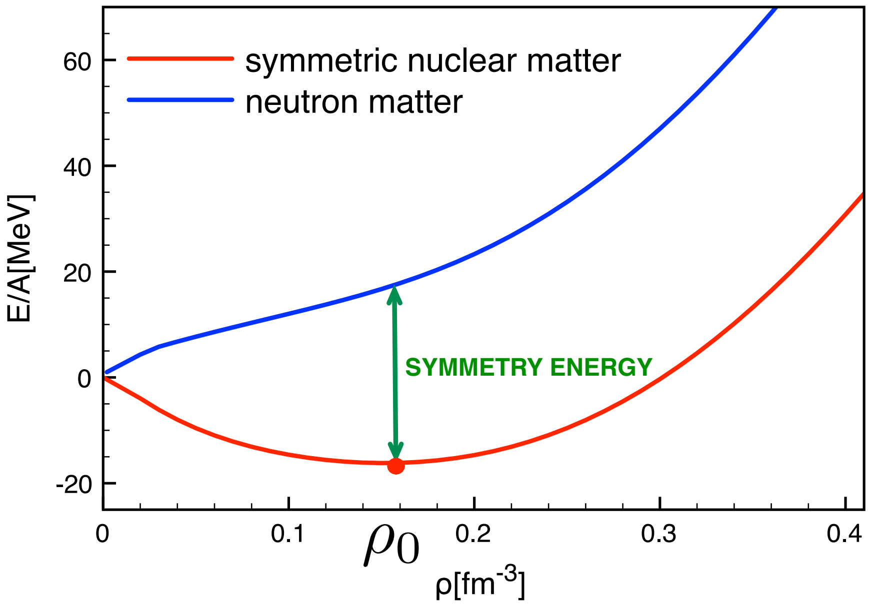
M. Erhard, A. R. Junghans, C. Nair, R. Schwengner, et al. Phys. Rev. C 81, 034319 (2010)

NUCLEAR MATTER EQUATION OF STATE

- Nuclear matter equation of state (for the uniform and infinite system)

$$E(\rho,\delta) = E_{SNM}(\rho) + E_{sym}(\rho)\delta^2 + \dots$$

$$\rho = \rho_n + \rho_p \qquad \delta = \frac{\rho_n - \rho_p}{\rho}$$



- Symmetry energy *$S_2(\rho)$* describes the increase in the energy of the *N≠Z* system as protons are turned into neutrons

$$E_{sym}(\rho) \equiv S_2(\rho) = J - L\epsilon + \dots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

$$L = 3\rho_0 \frac{dS_2(\rho)}{d\rho}|_{\rho_0}$$

$$K_0 \equiv 9\rho_0^2 \frac{\partial^2 E(\rho,0)}{\partial\rho^2}|_{\rho=\rho_0}$$

J – symmetry energy at saturation density

L – slope of the symmetry energy (related to the pressure of neutron matter)

K_0 – incompressibility of symmetric nuclear matter

RELATIVISTIC NUCLEAR ENERGY DENSITY FUNCTIONAL

- Relativistic point coupling model

- The basis is an effective Lagrangian with four-fermion (contact) interaction terms; isoscalar-scalar, isoscalar-vector, isovector-vector, derivative term

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}(i\gamma\cdot\partial - m)\psi \\
&- \frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^{\mu}\psi)(\bar{\psi}\gamma_{\mu}\psi) \\
&- \frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^{\mu}\psi)(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi) \\
&- \frac{1}{2}\delta_S(\partial_{\nu}\bar{\psi}\psi)(\partial^{\nu}\bar{\psi}\psi) - e\bar{\psi}\gamma\cdot A\frac{(1-\tau_3)}{2}\psi
\end{aligned}
$$

- many-body correlations encoded in density-dependent coupling functions that are motivated by microscopic calculations but parameterized in a phenomenological way − 10 model parameters

- Extensions: pairing correlations in finite nuclei
  - Relativistic Hartree-Bogoliubov model   T. Niksic, et al., Comp. Phys. Comm. 185, 1808 (2014).

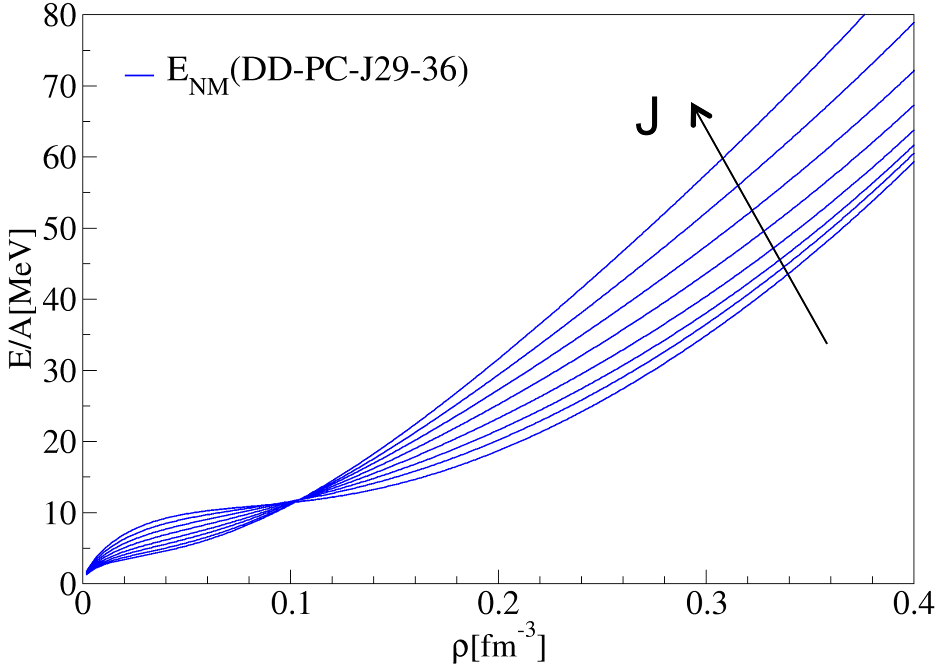
- In the small amplitude limit, self-consistent quasiparticle random phase approximation (QRPA) is used to compute nuclear excitations

- supplemented by the co-variance analysis to determine statistical uncertainties of calculated quantities

VARIATION OF THE SYMMETRY ENERGY IN CONSTRAINING THE EDF

- Establish a set of 8 relativistic point coupling interactions that span the range of values of the symmetry energy at saturation density: J=29,30,…36 MeV
- Adjust the properties of 72 spherical nuclei to exp. data (binding energies, charge radii, diffraction radii, surface thickness, pairing gaps)
- Each interaction is determined independently using the same dataset supplemented with an additional constraint on J

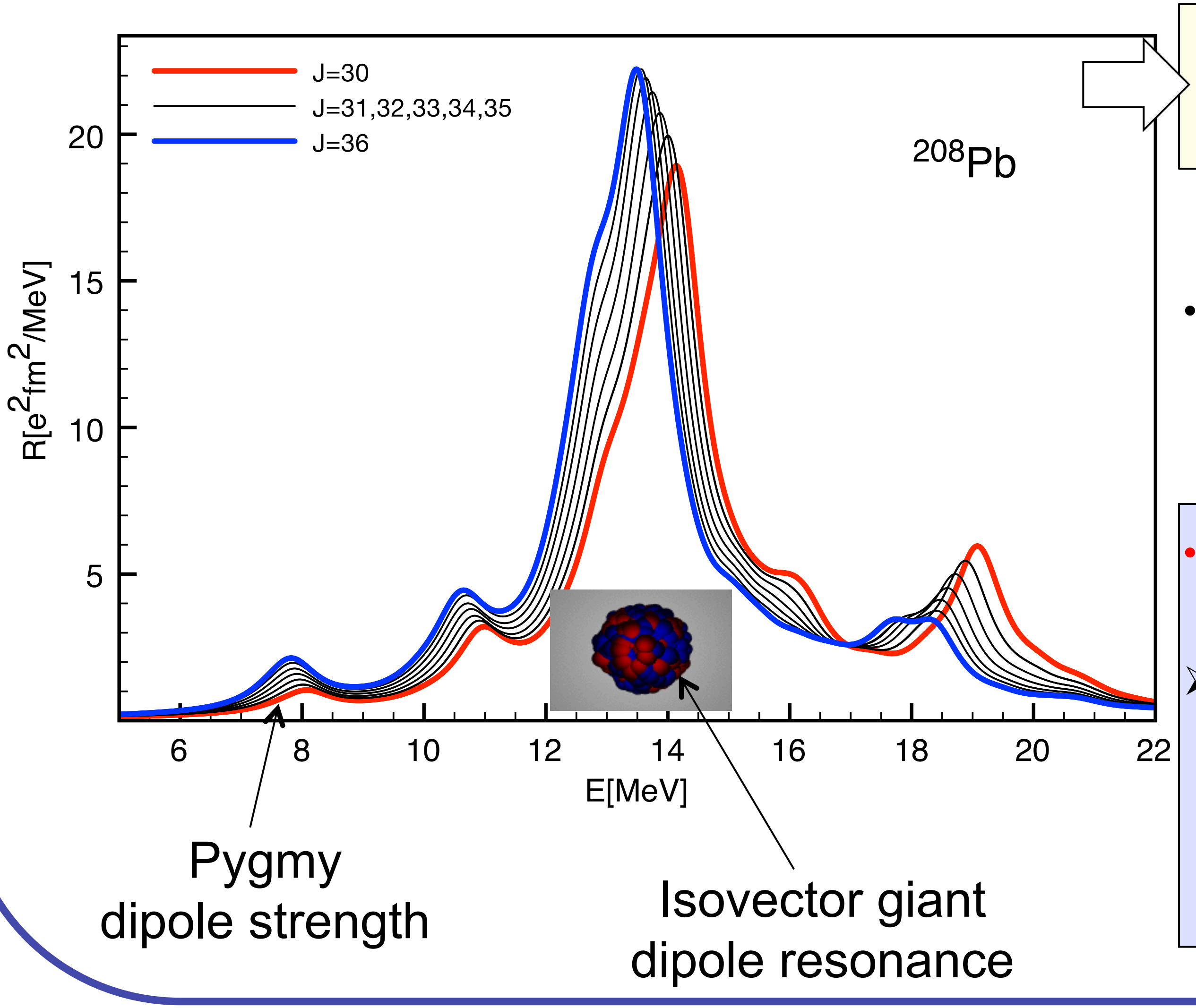
NEUTRON MATTER ENERGY p.p.



J[MeV]	L[MeV]
29	31.9
30	37.0
31	44.1
32	52.5
33	62.2
34	72.3
35	83.4
36	94.3

CONSTRAINING THE SYMMETRY ENERGY

- Isovector dipole transition strength for ^{208}Pb using a set of relativistic point coupling interactions which vary the symmetry energy properties (J=30,31,…,36 MeV)



- Isovector giant dipole resonance
- Pygmy dipole strengths
- Dipole polarizability ($\alpha_D \sim m_{-1}$)

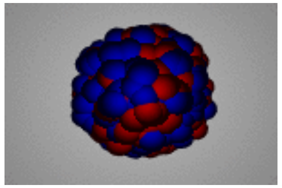
- The transition strength is sensitive on the properties of symmetry energy - (J,L)

- **Strategy to determine symmetry energy parameters:**
  - build the EDF which accurately reproduces experimental data on the dipole transitions (e.g., dipole polarizability, pygmy strength, etc.)

CONSTRAINING THE NUCLEAR MATTER INCOMPRESSIBILITY

- Nuclear matter incompressibility $K_{nm} = 9\rho_0^2 \frac{d^2}{d\rho^2}\frac{E}{A}|_{\rho=\rho_0}$

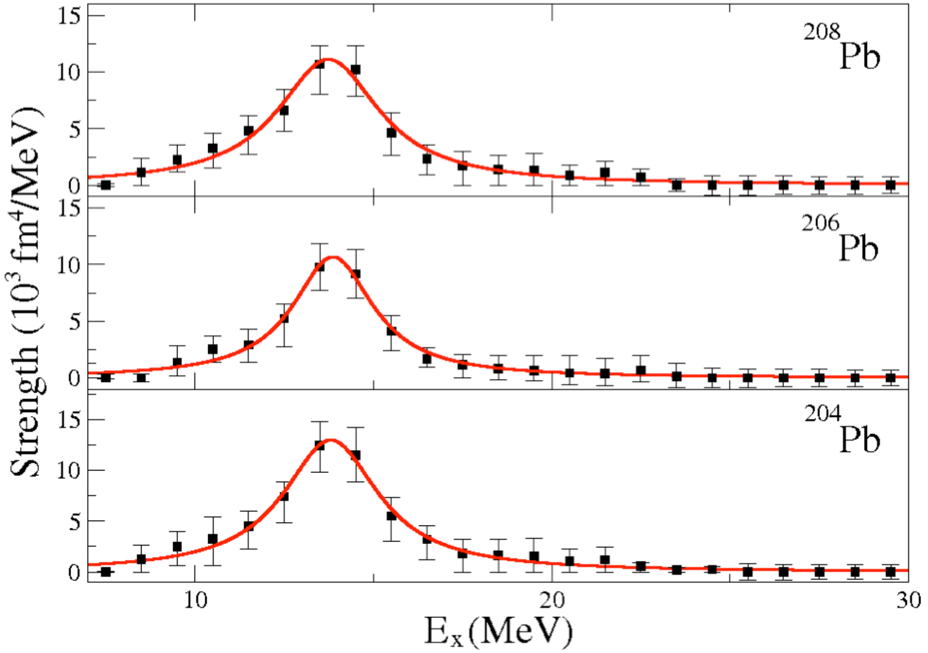
ISGMR



- It can be determined from the energies of compression mode in nuclei: Isoscalar Giant Monopole Resonance (ISGMR)

- ISGMR energies from inelastic scattering of α-particles, e.g. D. Patel et al., PLB 726, 178 (2013)

$$E_{ISGMR} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}$$ (for nuclei)



Strategy to determine K_{nm} (for nuclear matter)

- ❑ Build the EDF that accurately reproduces (e.g. by using the RPA) the experimental data on the ISGMR excitation energy
- ➢ The K_{nm} value associated with the EDF that best describes the experimental ISGMR energy is considered as the *"correct"* one.

CONSTRAINING THE ENERGY DENSITY FUNCTIONAL

- The new relativistic point coupling EDF constrained by the nuclear ground state properties and collective excitation properties in finite nuclei − **DD-PCX**
  - ➢ E. Yuksel, T. Marketin, N.P., PRC 99, 034318 (2019)

- The observables used directly in χ^2 minimization to constrain the functional:
  - **standard** – *nuclear masses* (34), *charge radii* (26), *pairing gaps* (15)
  - **additional** – *dipole polarizability* (^{208}*Pb)* to constrain the symmetry energy

Exp. data: $\alpha_D = 19.6 \pm 0.6$ fm^3 A. Tamii et al. PRL 107, 062502 (2011)
A. Tamii (private communication (2015)

– *isoscalar giant monopole resonance* (^{208}*Pb)* to constrain the nuclear matter incompressibility

Exp. data: $E_{ISGMR} = 13.5 \pm 0.1$ MeV D. Patel et al., PLB 726, 178 (2013)

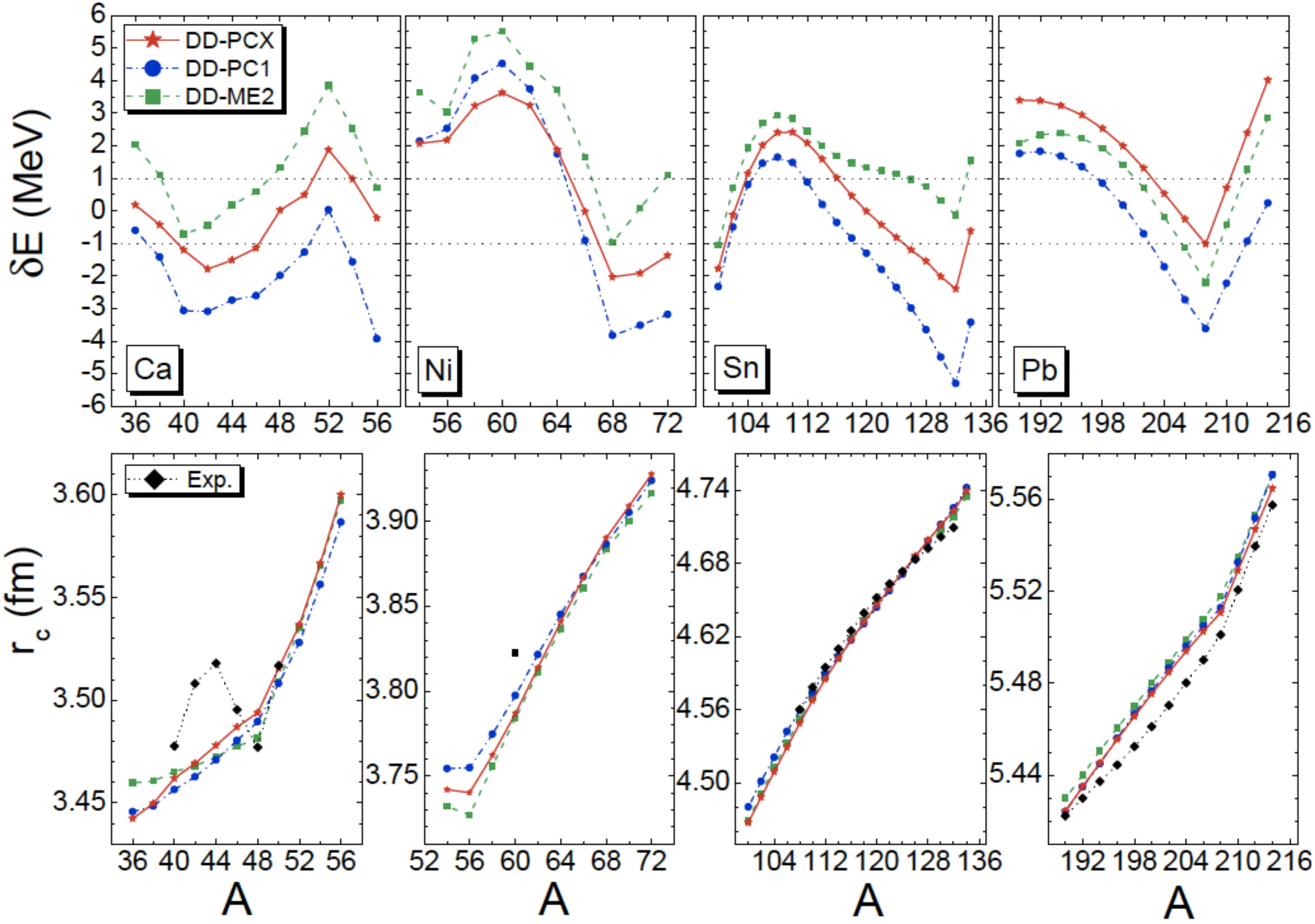
✓ **DD-PCX** ➔ the first EDF constrained by nuclear ground state properties, dipole polarizability and ISGMR excitation energy

➢ The EDF constrained by this procedure will have the *correct symmetry energy* – the one that is necessary to reproduce the experimental data on dipole polarizability

➢ It will also have the *correct nuclear matter incompressibility* that is compatible with the measured ISGMR excitation energy

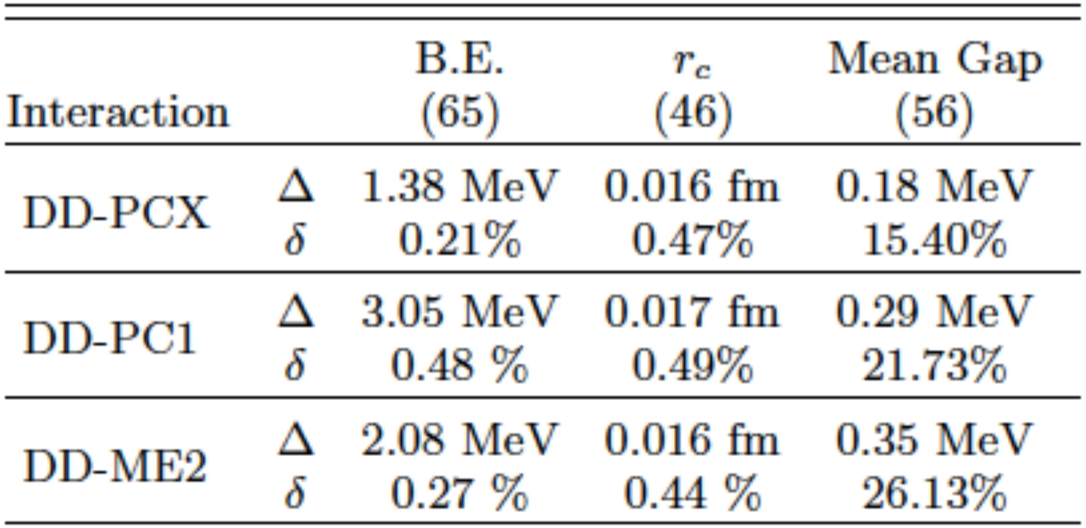
NUCLEAR BINDING ENERGIES AND CHARGE RADII

DD-PCX: new relativistic density dependent point coupling interaction



THE PROPERTIES OF FINITE NUCLEI AND NUCLEAR MATTER

DD-PCX: RMS errors for binding energies, charge radii, and mean gap values for a selection of nuclei

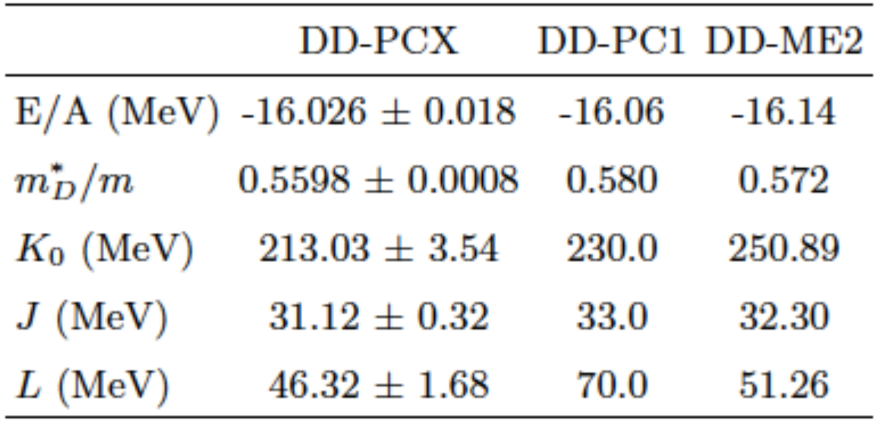


Interaction		B.E. (65)	r_c (46)	Mean Gap (56)
DD-PCX	Δ	1.38 MeV	0.016 fm	0.18 MeV
	δ	0.21%	0.47%	15.40%
DD-PC1	Δ	3.05 MeV	0.017 fm	0.29 MeV
	δ	0.48 %	0.49%	21.73%
DD-ME2	Δ	2.08 MeV	0.016 fm	0.35 MeV
	δ	0.27 %	0.44 %	26.13%

$$\Delta = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i^{exp} - y_i^{th})^2}$$

$$\delta = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(y_i^{exp} - y_i^{th})^2}{(y_i^{exp})^2}}$$

saturation properties of nuclear matter

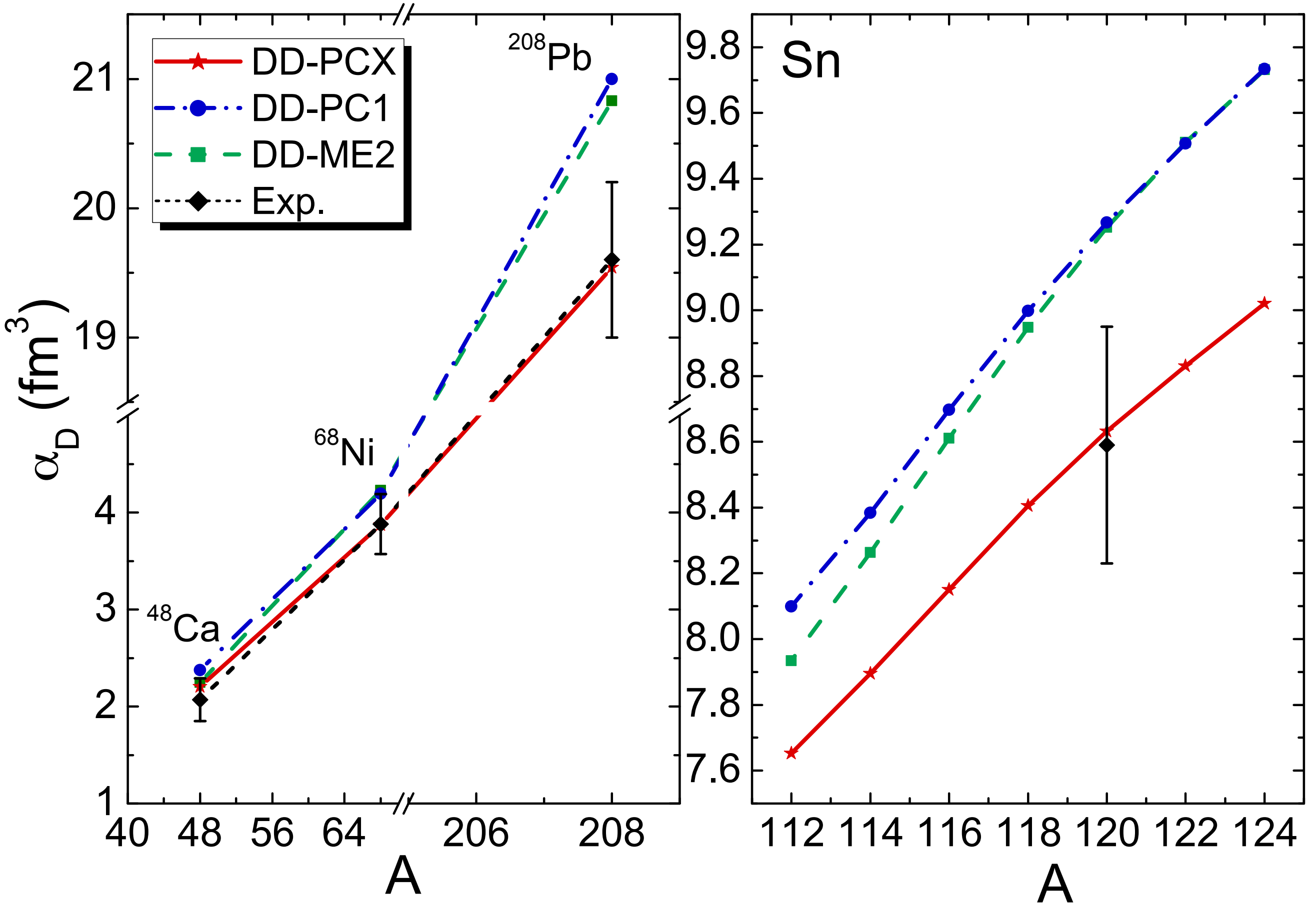


	DD-PCX	DD-PC1	DD-ME2
E/A (MeV)	-16.026 ± 0.018	-16.06	-16.14
m_D^*/m	0.5598 ± 0.0008	0.580	0.572
K_0 (MeV)	213.03 ± 3.54	230.0	250.89
J (MeV)	31.12 ± 0.32	33.0	32.30
L (MeV)	46.32 ± 1.68	70.0	51.26

E. Yuksel, T. Marketin, N.P., PRC 99, 034318 (2019).

DIPOLE POLARIZABILITY

- dipole polarizability (α_D) in several nuclei – only the **DD-PCX** interaction reproduces all the experimental data E. Yuksel, T. Marketin, N.P., PRC 99, 034318 (2019).



- Exp. data on α_D:

^{48}Ca - J. Birkhan et al., PRL 118, 252501 (2017).
^{68}Ni - D. M. Rossi et al., PRL 111, 242503 (2013).
^{120}Sn - T. Hashimoto et al., PRC 92, 031305 (2015).
^{208}Pb – A. Tamii et al., PRL 107, 062502 (2011); A. Tamii (priv. comm.) (2015).

DIPOLE POLARIZABILITY IN LIGHT NUCLEI

DD-PCX:

	^{16}O	^{40}Ca
r_{ch} [fm]	2.76	3.46
α_D [fm^3]	0.57	1.90

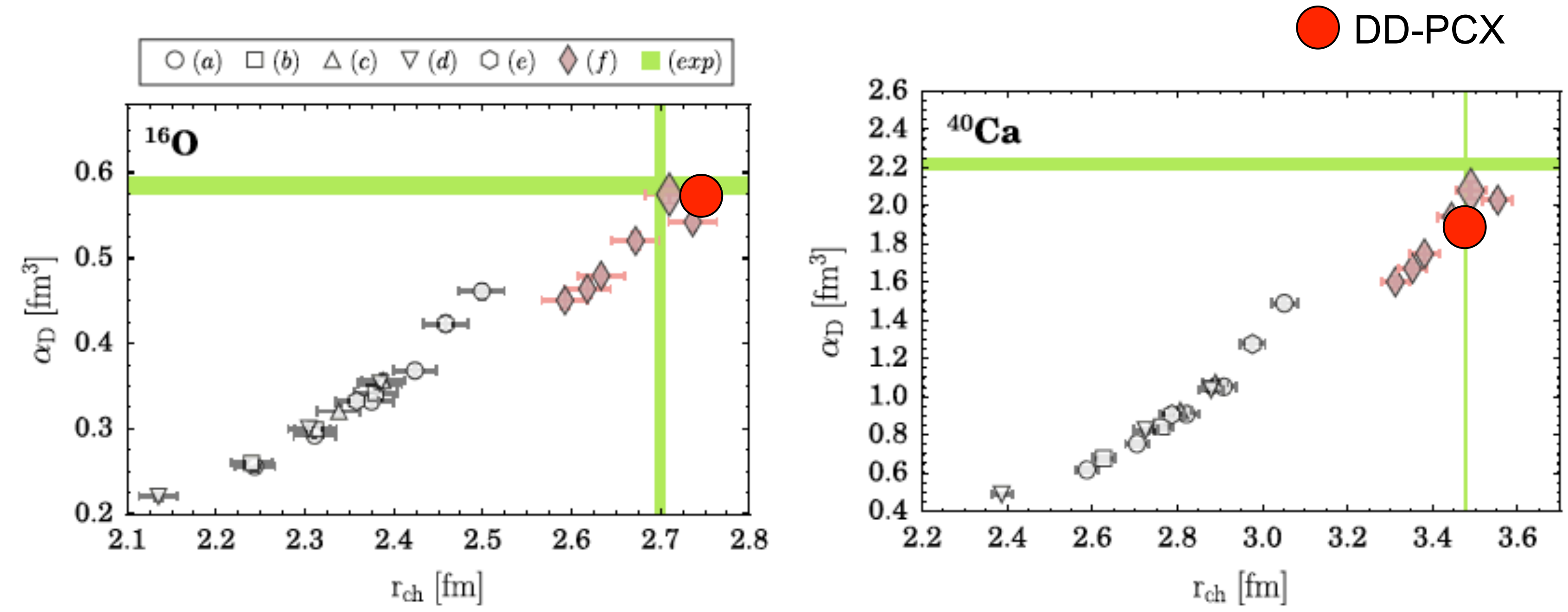
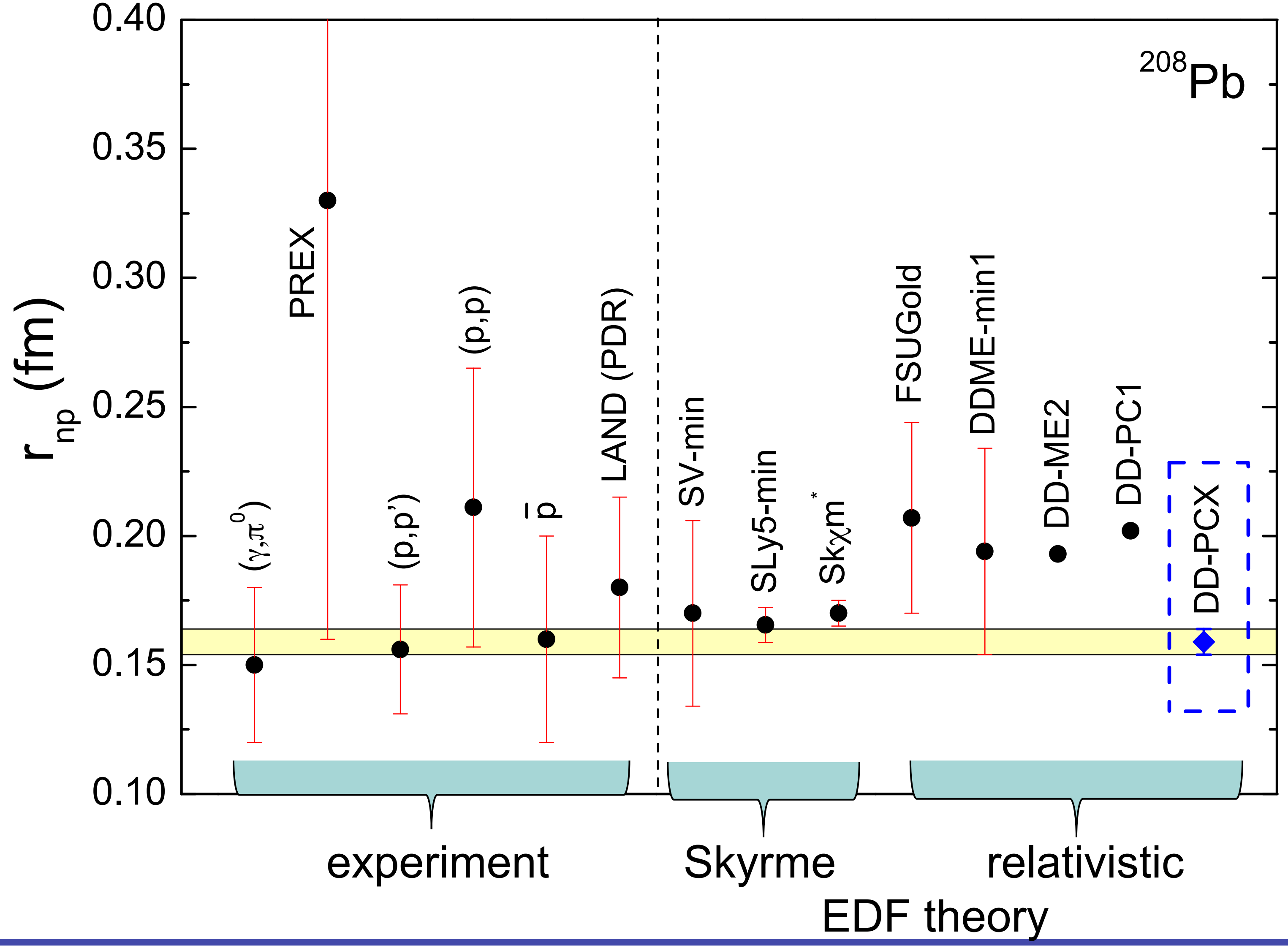


Fig. 43. Relationship α_D versus r_{ch} in ^{16}O and ^{40}Ca. Results for *NN* interactions include (empty symbols) (*a*) SRG evolved interaction [337], $\Lambda = 500$ MeV/c and $\lambda = \infty$, 3.5, 3.0, 2.5 and 2.0 fm^{-1}, (*b*) SRG evolved interaction [337] with $\Lambda = 600$ MeV/c and $\lambda = 3.5$, 3.0 and 2.5 fm^{-1}, (*c*) SRG evolved CD-BONN [338] with $\lambda = 4.0$ and 3.5 fm^{-1}, (*d*) V_{low-k} evolved CD-BONN with $\lambda = 3.0$, 2.5 and 2.0 fm^{-1} and (*e*) V_{low-k}-evolved AV18 [339] interaction and $\lambda = 3.0$ and 2.5 fm^{-1}. The red diamonds (*f*) denote results including 3NF: the large one is from NNLO$_{sat}$ [340] and the others from chiral interactions as in Ref. [341]. Experimental data are denoted by green bands [342,18].
Source: Figure taken from Ref. [334].

Figure taken from M. Miorelli, S. Bacca, N. Barnea, et al., Phys. Rev. C 94 (2016) 034317.

NEUTRON SKIN THICKNESS

- neutron skin thickness in neutron-rich nucleus ^{208}Pb
- **DD-PCX**: r_{np} (^{208}Pb) = 0.159 ± 0.005 fm



CONCLUDING REMARKS

- **DD-PCX relativistic density dependent point coupling interaction**
  - the first EDF constrained by nuclear ground state properties (E_B,r_c,$\Delta_{n,p}$) and collective excitation properties (dipole polarizability and the ISGMR excitation energy in ^{208}Pb)
  - supplemented by the co-variance analysis to determine theoretical statistical uncertainties

- **The synergy of the EDF theory with the experiments** on ground state and excitations allow to improve the EDFs
  - isovector properties – neutron-rich nuclei, neutron-skin thickness, symmetry energy of the EOS
  - isoscalar properties – incompressibility of nuclear matter
  - nuclear ground state properties

Outlook

- more systematic calculations – astrophysically relevant quantities
- introducing additional constraints on excitation properties to improve the EDFs
- identifying key observables from the experimental studies

In collaboration with E. Yuksel (Istanbul), T. Marketin (Zagreb)

E. Yuksel, T. Marketin, N.P., PRC 99, 034318 (2019).

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