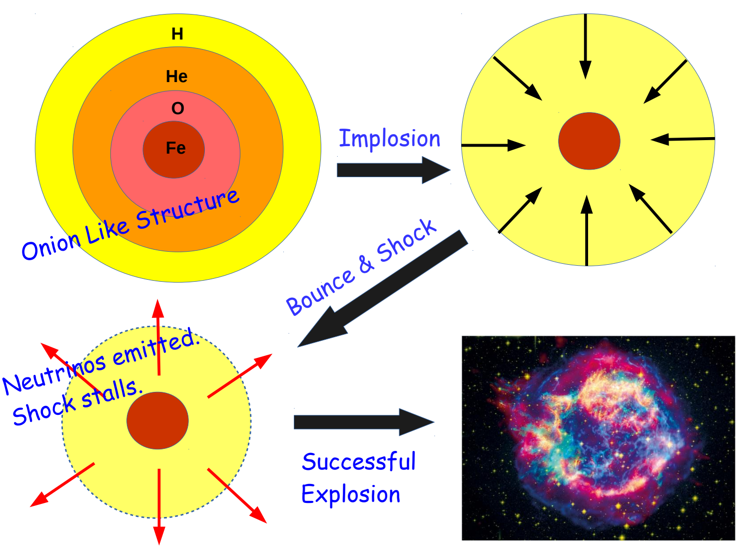
A few moments with supernova neutrinos

Manibrata Sen

Network in Neutrinos, Nuclear Astrophysics and Symmetries (N3AS)
University of California, Berkeley.
Northwestern University, Evanston.

May 16, 2019

SN neutrinos: an odyssey



Neutrino transport: (1+3+3)D problem

- The matrix of neutrino densities $\varrho(t, \mathbf{r}, \mathbf{p})$.

$$\varrho = \begin{bmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_x \rangle \\ \langle \nu_e | \nu_x \rangle & \langle \nu_x | \nu_x \rangle \end{bmatrix}$$

- $\langle \nu_{e,x} | \nu_{e,x} \rangle$ related to total flavor content. $\langle \nu_e | \nu_x \rangle$ related to flavor conversions.
- Equation of motion:

$$\mathrm{d}_t \varrho = -\mathrm{i} \Big[H[\varrho], \varrho \Big] + C[\varrho] \, .$$

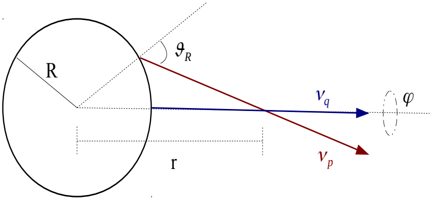
$H[\varrho] \rightarrow$ Hamiltonian containing vacuum mixing and forward scattering.

$C[\varrho] \rightarrow$ Collisions (neglected in this talk!).

Interaction Hamiltonian: 3 scales

- $H[\varrho]_{\mathbf{p}} \supset H^{\text{vac}}_{\mathbf{p}} + H^{\text{MSW}} + H^{\nu\nu}_{\mathbf{p}}$
- Vacuum oscillation: $H^{\text{vac}}_{\mathbf{p}} = \omega = \frac{\Delta m^2}{2E} \simeq 0.3\,\text{km}^{-1}$
- Matter effect : $H^{\text{MSW}} = \lambda = \sqrt{2} G_F n_e \gtrsim 10^5\,\text{km}^{-1}$
- $\nu - \nu$ interaction: $H^{\nu\nu}_{\mathbf{p}} = \mu \int \, d\mathbf{q} \; (1 - \mathbf{v_p}.\mathbf{v_q})\,(\varrho_{\mathbf{q}} - \bar{\varrho_{\mathbf{q}}})$.

  Note $\mu = \sqrt{2} G_F n_\nu \gtrsim 10^5\,\text{km}^{-1}$.

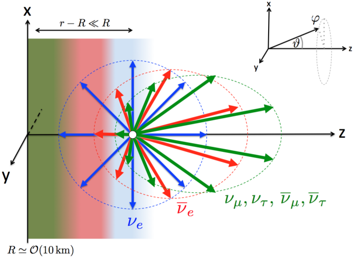


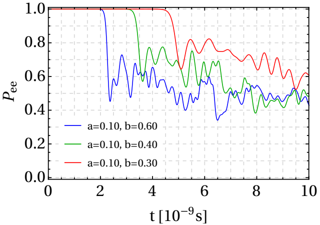
Duan, Carlson, Fuller, and Qian (2006, 2007)

Hannestad, Raffelt, Sigl and Wong(2006)

What are fast flavor oscillations?

- Oscillations growing with a rate $\propto \mu \propto n_\nu$. Rates do not depend on neutrino mass or energy
- New instability, absent for isotropic angular distributions.
- Requires crossing in the neutrino angular distributions. (See Tobias's talk)





Sawyer(2005,2015)
Chakraborty, Izagguire, Hansen and Raffelt (2016)

Dasgupta, Mirizzi, and MS (2017)

Collective Oscillations : Linear stability analysis

- Consider 2 flavors ν_e and ν_x . The flavor density matrices

$$\varrho = \begin{bmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho_{xe} & \varrho_{xx} \end{bmatrix}$$

- Equation of motion (EoM):

$$id_t \varrho_p = i(\partial_t + \mathbf{v} \cdot \nabla)\varrho_p = [H_p, \varrho_p]$$

where,

$$H_p = \omega_p + \lambda + \mu \int d\Gamma \, (1 - \mathbf{v_p}.\mathbf{v_q}) \, (\varrho_q - \bar{\varrho}_q)$$

- Expand :

$$\varrho = \frac{\mathrm{Tr}\varrho}{2} + \frac{g_{\omega v \phi}}{2} \begin{bmatrix} s & S \\ S^* & -s \end{bmatrix}$$

Drop trace.

Banerjee, Dighe and Raffelt (2011)

Linear stability analysis

- Here $s^2 + S^2 = 1$. To begin with, $S << 1$, hence linearise in S. Instability $\Rightarrow$ S blows up.

- Eigenvalue equation.

$$
\begin{aligned}
i\,(\partial_t + \mathbf{v}\cdot\nabla)\,S \quad = \quad & \left(\omega_{\rm vac} + \mu \int d\Gamma'\,(1-\mathbf{v}.\mathbf{v}')\,g'\right) S \\
& -\mu \int d\Gamma'\,(1-\mathbf{v}.\mathbf{v}')\,g'\,S'
\end{aligned}
$$

- $S_{\Omega,\,\mathbf{K}} \propto {\rm Exp}\,(\mathrm{i}\,\mathbf{K}\ \cdot \mathbf{r} - \mathrm{i}\,\Omega\,t)$
- This leads to a dispersion relation for flavor waves: $D(\Omega, \mathbf{K}) = 0$.

- What really matters is the electron lepton number (ELN)

$$
g = \int d\Gamma\, G_{E,\mathbf{v}} = \sqrt{2} G_F \int d\Gamma\ [f_{\nu_e}(E,\mathbf{v}) - f_{\bar{\nu}_e}(E,\mathbf{v})]
$$

The dispersion relation: a covariant form

- Integrate over energy. The ELN angular distribution

$$G_{\mathbf{v}} = \sqrt{2}G_F \int_0^\infty \frac{dE\, E^2}{2\pi^2} \left[f_{\nu_e}(E,\mathbf{v}) - f_{\bar{\nu}_e}(E,\mathbf{v})\right]$$

- The dispersion relation

$$D(\Omega,\mathbf{K}) = \mathrm{Det}\ \left[\Pi^{\mu\nu}(\omega,\mathbf{k})\right] = \mathrm{Det}\ \left[\eta^{\mu\nu} + \int d\mathbf{v}\, G_{\mathbf{v}} \frac{v^\mu v^\nu}{\omega - \mathbf{k}\cdot\mathbf{v}}\right] = 0$$

where

$$\begin{aligned} v^\mu &= (1, \hat{\mathbf{v}}) \\ \omega &= \Omega - \left(\omega_{\mathrm{vac}} + \sqrt{2}G_F\ n_e + \int d\mathbf{v}\ G_{\mathbf{v}}\right) \\ \mathbf{k} &= \mathbf{K} - \int d\mathbf{v}\, \mathbf{v}\, G_{\mathbf{v}} \end{aligned}$$

Izaguirre, Raffelt and Tamborra(2017)

Capozzi, Dasgupta, Lisi and Mirizzi (2018)

Co-rotating frame

- Possible to simplify quantities in a co-rotating frame (similar to a gauge transformation).

$$\omega \quad = \quad \Omega \quad \underbrace{-\left(\omega_{\mathrm{vac}} + \sqrt{2}G_F\, n_e + \int d\mathbf{v}\; G_{\mathbf{v}}\right)}_{\text{rotate away}} \Rightarrow \;\; \omega = \Omega$$

$$\mathbf{k} \quad = \quad \mathbf{K} \quad \underbrace{-\int d\mathbf{v}\,\mathbf{v}\,G_{\mathbf{v}}}_{\text{rotate away}} \Rightarrow \;\; \mathbf{k} = \mathbf{K}$$

- The dispersion relation

$$D(\Omega, \mathbf{K}) = \mathrm{Det}\,\left[\eta^{\mu\nu} + \int d\mathbf{v}\,G_{\mathbf{v}}\frac{v^{\mu}v^{\nu}}{\omega - \mathbf{k}\cdot\mathbf{v}}\right] = 0\,,$$

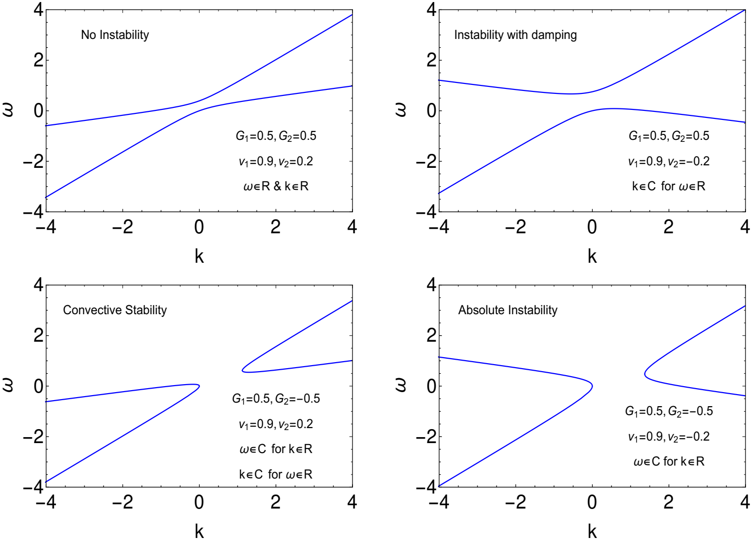
where ω and $\mathbf{k}$ to be Fourier modes in this frame.

Izaguirre, Raffelt and Tamborra(2016)
Capozzi, Dasgupta, Lisi, Marrone and Mirizzi (2017)

Dasgupta, Mirizzi and MS (2018)

Different types of instabilities

Interpret instabilities as gaps in dispersion relations.



Why is this complicated?

- Solving the dispersion relation in general an arduous task. Need to study analytical structures of these dispersion relations, which is almost impossible for multi-D angular distributions.

- Most SN simulations provide us with the moments of the neutrino angular distributions, and not the full distributions.

Moments that matter...

- The dispersion relation

$$\text{Det }[\Pi^{\mu\nu}(\omega,\mathbf{k})] = \text{Det }\left[\eta^{\mu\nu} + \int d\mathbf{v}\, G_{\mathbf{v}} \frac{v^\mu v^\nu}{\omega - \mathbf{k}\cdot\mathbf{v}}\right] = 0$$

- Focus on "zero mode", i.e., $\mathbf{k} = 0$ mode. Simple polynomial in the moments:

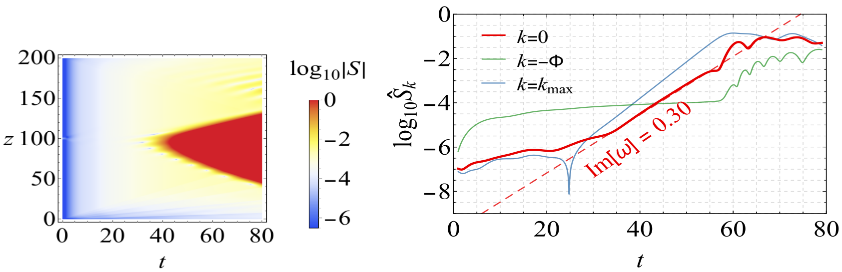
$$\text{Det }[\Pi^{\mu\nu}(\omega,0)] = \text{Det}\left[\eta^{\mu\nu} + \frac{1}{\omega}\int d\mathbf{v}\, G_{\mathbf{v}}\, v^\mu v^\nu\right]$$

- Moments of the neutrino angular distribution $V^{\mu\nu} = \int d\mathbf{v}\, G_{\mathbf{v}}\, v^\mu v^\nu$.
- Growth rate of zero modes in $(1+1)D$.:

$$\omega = \frac{1}{2}\left(V^{11} - V^{00} \pm \sqrt{(V^{00}+V^{11})^2 - (2V^{01})^2}\right)$$

Dasgupta, Mirizzi and MS (2018)

Check for instability: 10 beam model



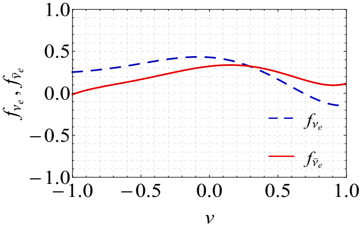
Growth rate : $\mathrm{Im}(\omega) = \frac{1}{2}\left[4\left(V^{01}\right)^2 - \left(V^{00}+V^{11}\right)^2\right]^{1/2}$

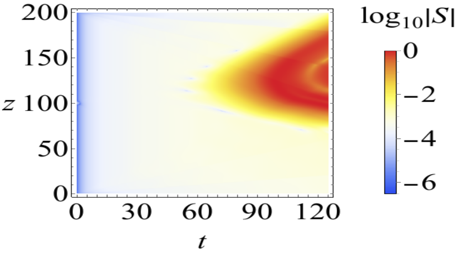
Fourier transform: $\hat{S}_k(t) = \frac{1}{z_{\max}}\int_0^{z_{\max}} dz\, e^{-i\,k\,z}\, S(z,t)$.

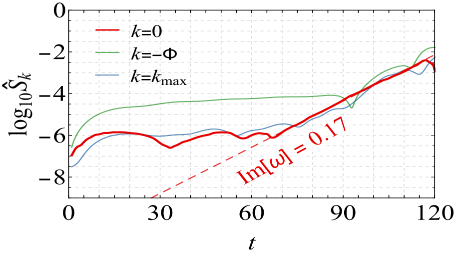
$k = 0$ **mode need not dominate the growth rate.**

Dasgupta, Mirizzi and MS (2018)

Predictions for a realistic spectra







Dasgupta, Mirizzi and MS (2018)

Application to a SN simulation

Three-Dimensional Core-Collapse Supernova Simulations with Multi-Dimensional Neutrino Transport Compared to the Ray-by-Ray-plus Approximation

Robert Glas,[1,2] Oliver Just,[3,1] H.-Thomas Janka,[1] and Martin Obergaulinger[4,5]

[1] *Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Straße 1, D-85748 Garching, Germany*
[2] *Physik Department, Technische Universität München, James-Franck-Straße 1, D-85748 Garching, Germany*
[3] *Astrophysical Big Bang Laboratory, RIKEN Cluster for Pioneering Research, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan*
[4] *Institut für Kernphysik, Technische Universität Darmstadt, Schlossgartenstraße 2, 64289 Darmstadt, Germany.*
[5] *Departament d'Astronomia i Astrofísica, Universitat de València, Edifici d´Investigació Jeroni Muñoz, C/ Dr. Moliner, 50, E-46100 Burjassot (València), Spain*

(Received -; Revised -; Accepted -)

Submitted to ApJ

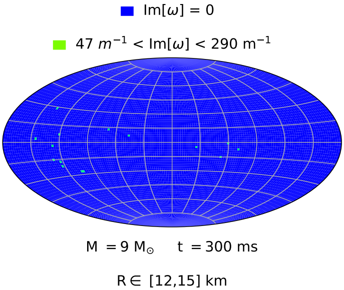
ABSTRACT

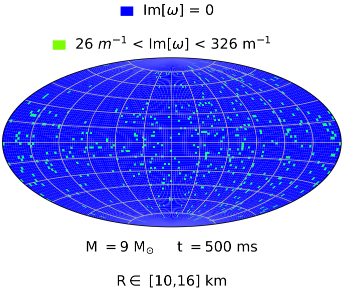
Self-consistent, time-dependent supernova (SN) simulations in three spatial dimensions (3D) are conducted with the Aenus-Alcar code, comparing, for the first time, calculations with fully multi-dimensional (FMD) neutrino transport and the ray-by-ray-plus (RbR+) approximation, both based on a two-moment solver with algebraic M1 closure. We find good agreement between 3D results with FMD and RbR+ transport for both tested grid resolutions in the cases of a 20 $M_\odot$ progenitor, which does not explode with the employed simplified set of neutrino opacities, and of an exploding 9 $M_\odot$ model. This is in stark contrast to corresponding axisymmetric (2D) simulations, which confirm previous claims that the RbR+ approximation can foster explosions in 2D in particular in models with powerful axial sloshing of the stalled shock due to the standing accretion shock instability (SASI). However, while local and instantaneous variations of neutrino fluxes and heating rates can still be considerably higher with RbR+ transport in 3D, the time-averaged quantities are very similar to FMD results because of the absence of a fixed, artificial symmetry axis that channels the flow. Therefore, except for stochastic fluctuations, the neutrino signals and the post-bounce evolution of 3D simulations with FMD and RbR+ transport are also very similar, in particular for our calculations with the better grid resolution. Higher spatial resolution has clearly a more important impact than the differences by the two transport treatments. Our results back up the use of the RbR+ approximation for neutrino transport in 3D SN modeling.

- How does our method apply to a CCSN simulation?
- Consider a fully multi-D simulation of a $9M_\odot$ model. Provide us with the energy dependent angular moments.
  $N_r \times N_\theta \times N_\phi = 640 \times 80 \times 160$

Glas, Just, Janka and Obergaulinger (2018)

Searching for fast conversions





- Scattered points of fast conversions at specific directions.
- Deeper than expected. Instabilities at $R \in (10 - 20)$ km.

Capozzi, Dasgupta, Glas, Janka, Mirizzi, MS and Sigl (in prep)

Understanding the result

- Relating the moments:

$$\begin{aligned} V^{00} &\propto n_{\nu_e} - n_{\bar{\nu}_e} = \Delta n_\nu \\ V^{0r} &\propto F_{\nu_e} - F_{\bar{\nu}_e} = \Delta F_\nu \,, \end{aligned}$$

- In the effective $(1+1)D$ problem,

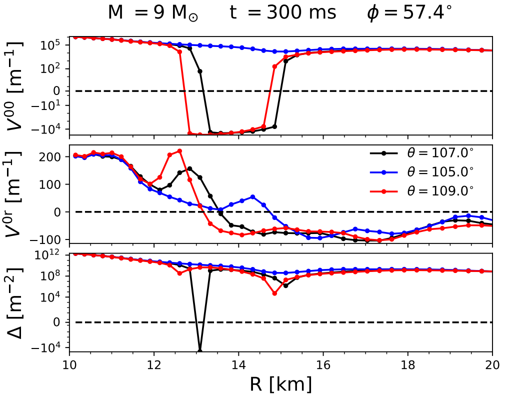
$$\omega = \frac{1}{2}\left(V^{rr} - V^{00} \pm \sqrt{\left(V^{00} + V^{rr}\right)^2 - 4\left(V^{0r}\right)^2}\right).$$

- In optically thick region, $V^{rr} = V^{00}/3$. Condition for complex modes:

$$\Delta = \frac{16}{9}\left(V^{00}\right)^2 - 4\left(V^{0r}\right)^2 < 0\,,$$

Capozzi, Dasgupta, Glas, Janka, Mirizzi, MS and Sigl (in prep)

Can resolution play a role?

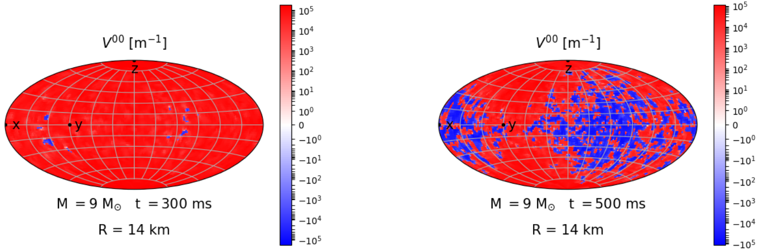


$$\Delta = \frac{16}{9}\left(V^{00}\right)^2 - 4\left(V^{0r}\right)^2 < 0\,,$$

Typically $V^{00} \gg V^{0r}$. Δ can be negative if $V^{00} \simeq 0$.

Modified criteria for instability

- To avoid resolution issues, look for **change in sign of** $V^{00} \propto (n_{\nu_e} - n_{\bar{\nu}_e})$.



- Concentrated away from LESA direction, which points in $+y$ direction. Possible correlation with LESA.

Capozzi, Dasgupta, Glas, Janka, Mirizzi, MS and Sigl (in prep)

Summary

- Fast conversions are associated with a crossing of the ELN. But simulations do not always provide us with ELNs. Easier to get the moments.
- Use a LSA to obtain regions of instability. Focus on the **zero mode**. Allows us to predict instabilities with moments of neutrino angular distributions.
- Applied it to SN simulations of the Garching group. We find fast instabilities deeper inside the “neutrinosphere” ($R \in (10-20)\,$km). Requires $n_{\overline{\nu}_e} > n_{\nu_e}$.
- Possibility of fast conversions deeper? Need more detailed studies.

Thank You