MAX-PLANCK-INSTITUT FÜR KERNPHYSIK HEIDELBERG

Stability of three neutrino flavor conversion in supernovae

Christian Döring

based on: arXiv:1905.03647

Together with:
Rasmus S. L. Hansen and
Manfred Lindner

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SN neutrinos at the crossroads

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- Motivation
- Neutrinos in dense media
- Three flavor stability analysis
- Application to toy-systems
- Analysis in basis of propagting states
- Conclusion

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- Extend analysis to three flavors and search for instabilities

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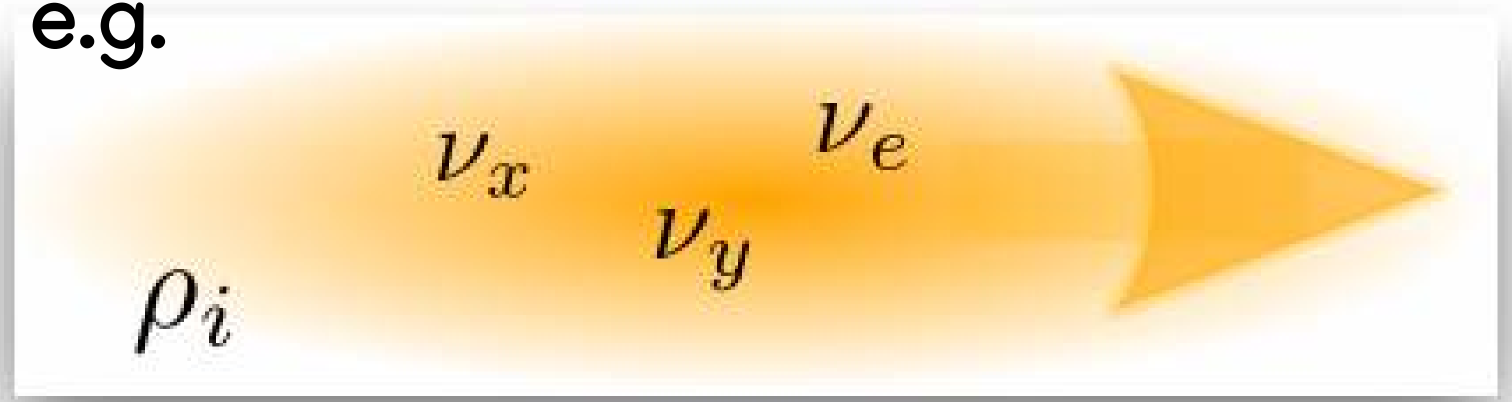

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$$i\,\partial_t \rho_i(t, \mathbf{v}) = [H_i(t, \mathbf{v}), \rho_i(t, \mathbf{v})]$$

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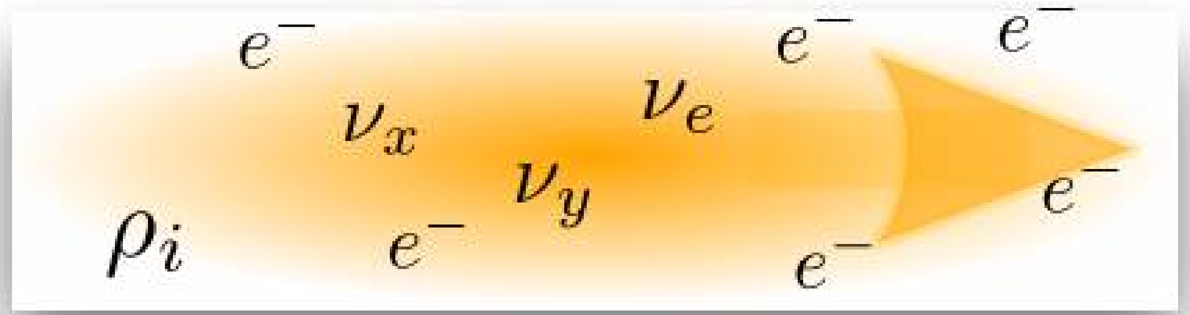

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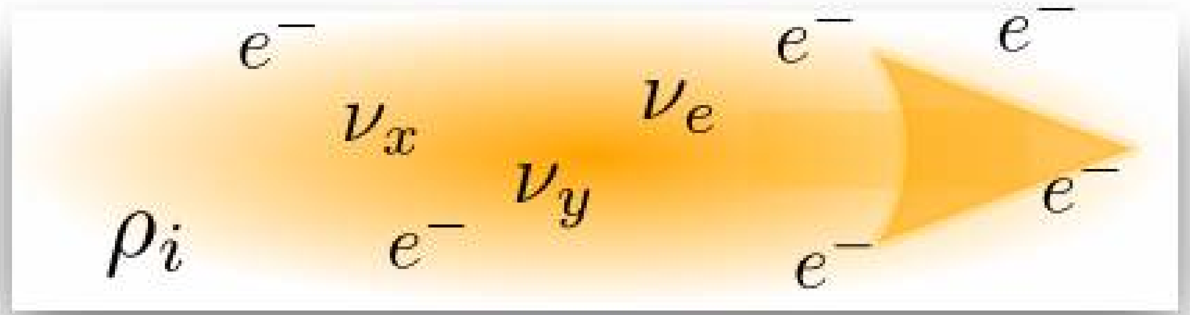

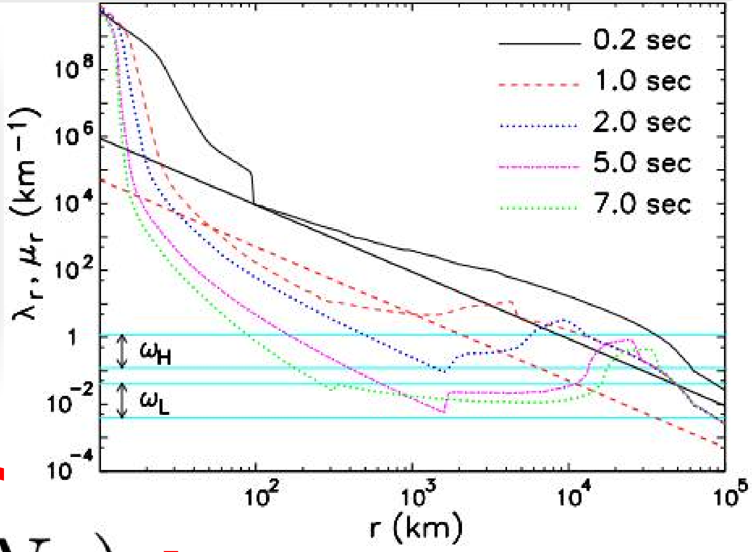

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Mirizzi et al: arXiv:1508.00785]

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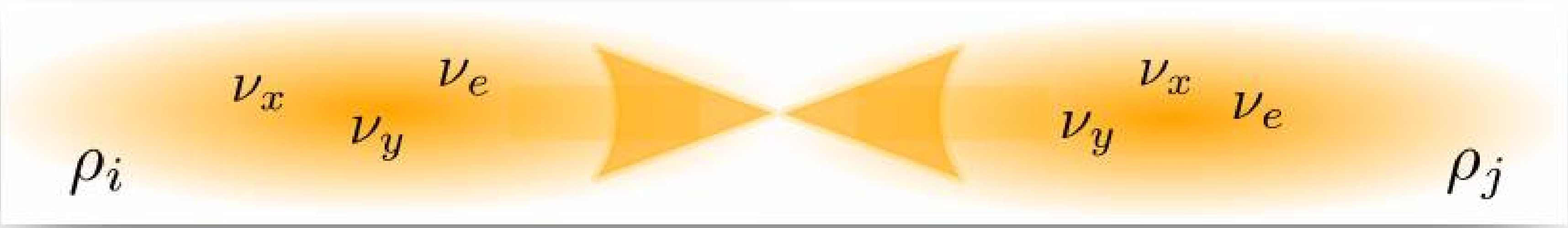

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- Aim: Solving the eigenvalue equation and looking for imaginary omega

Two-Beam-System

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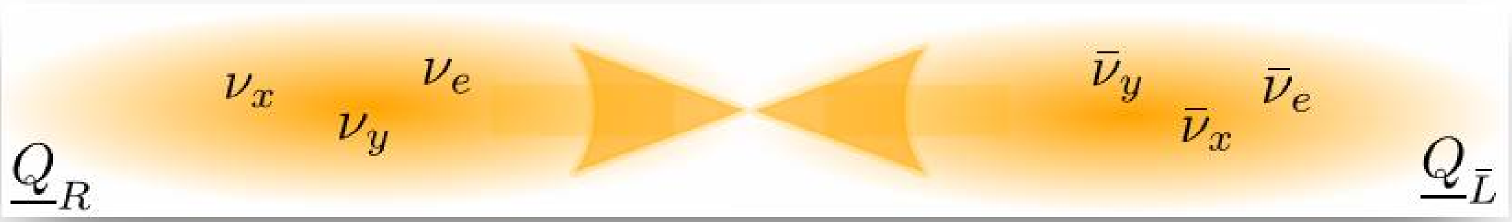

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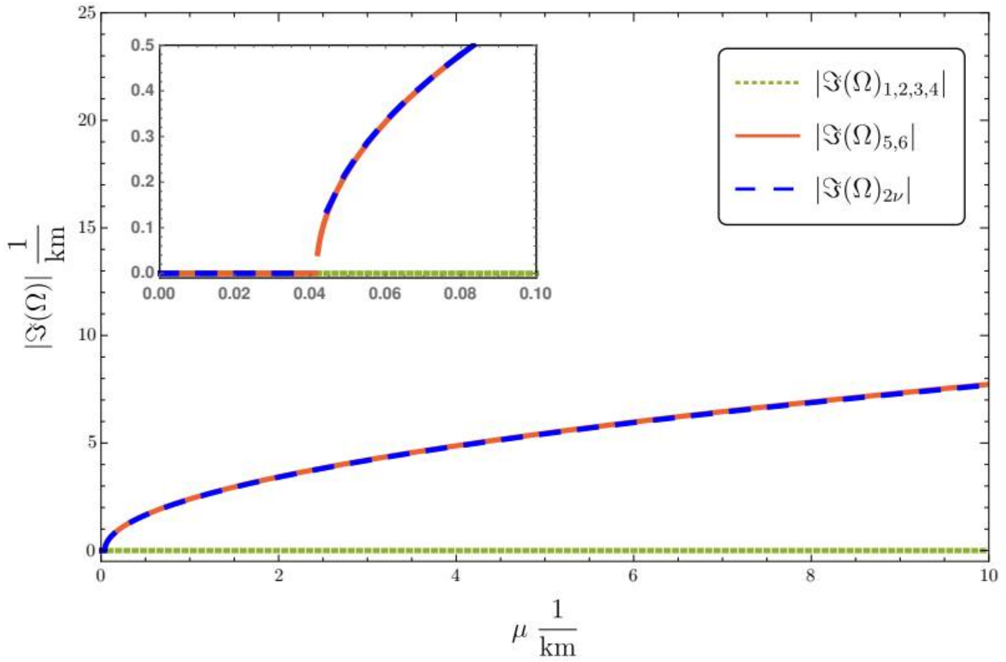

Inverted mass ordering
Two-neutrino limit

[Chakraborty et al: arXiv:1602.00698]

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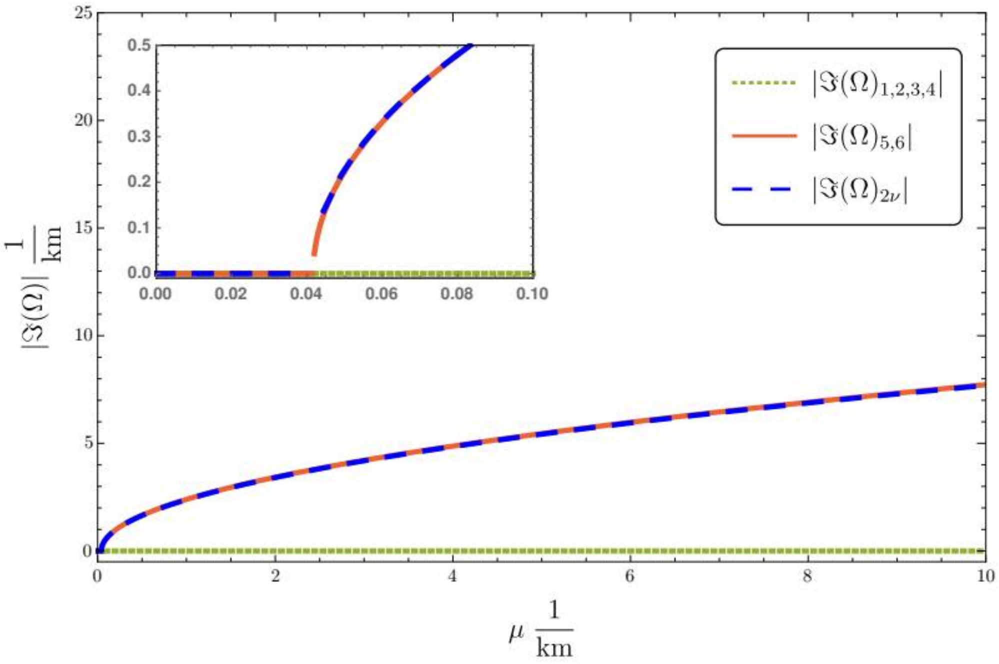

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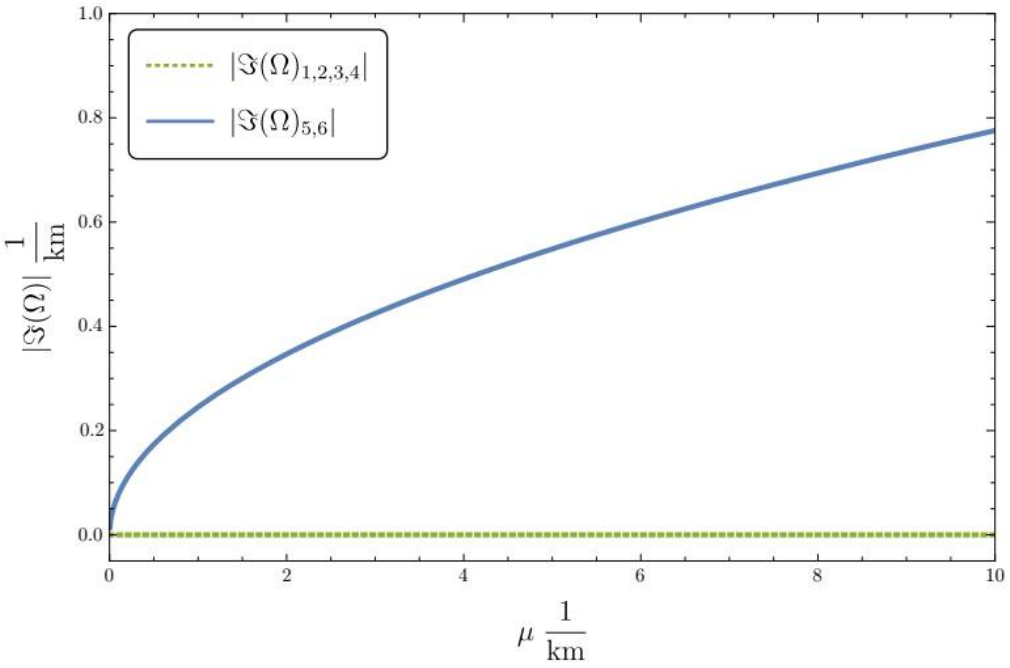

Normal mass ordering

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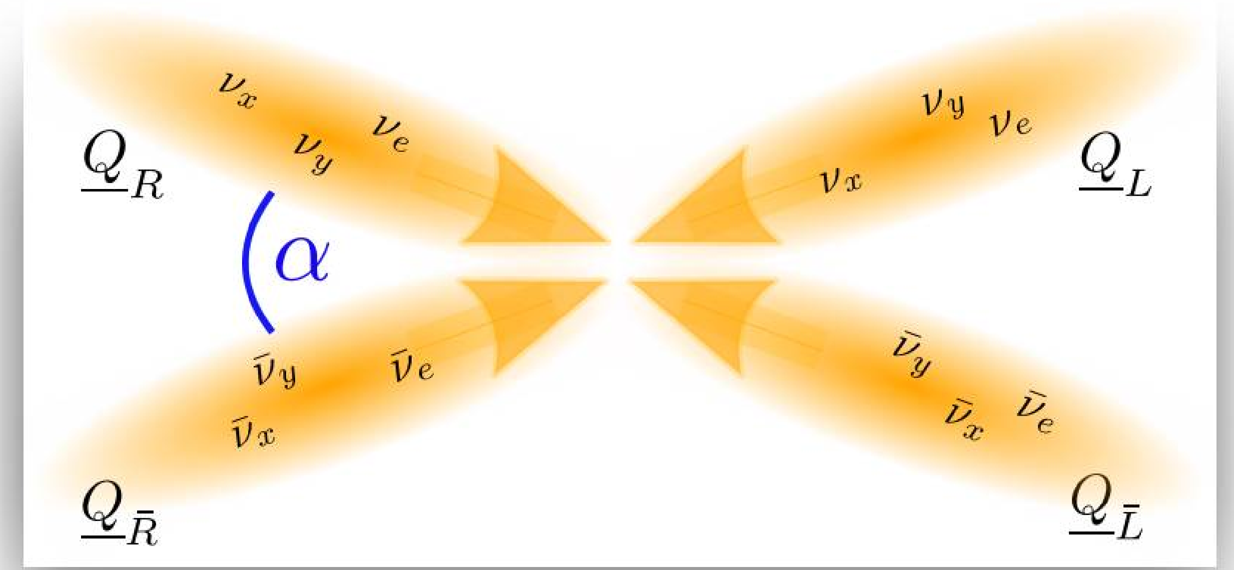

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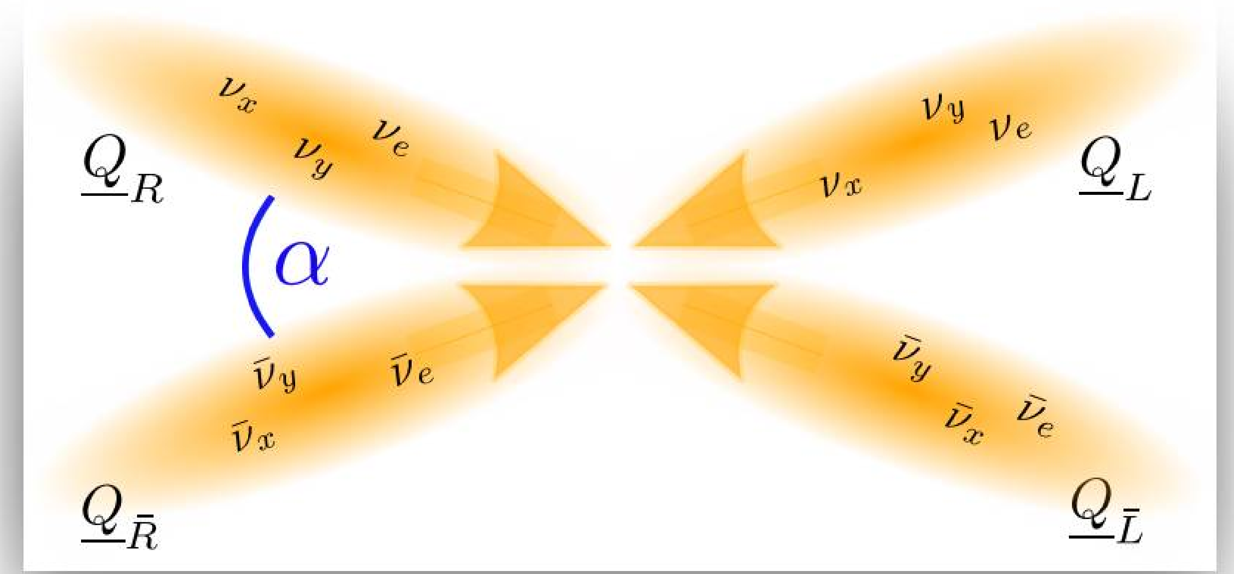

$$A_\pm := \frac{1}{2}(L \pm R) \qquad \bar{A}_\pm := \frac{1}{2}(\bar{L} \pm \bar{R})$$

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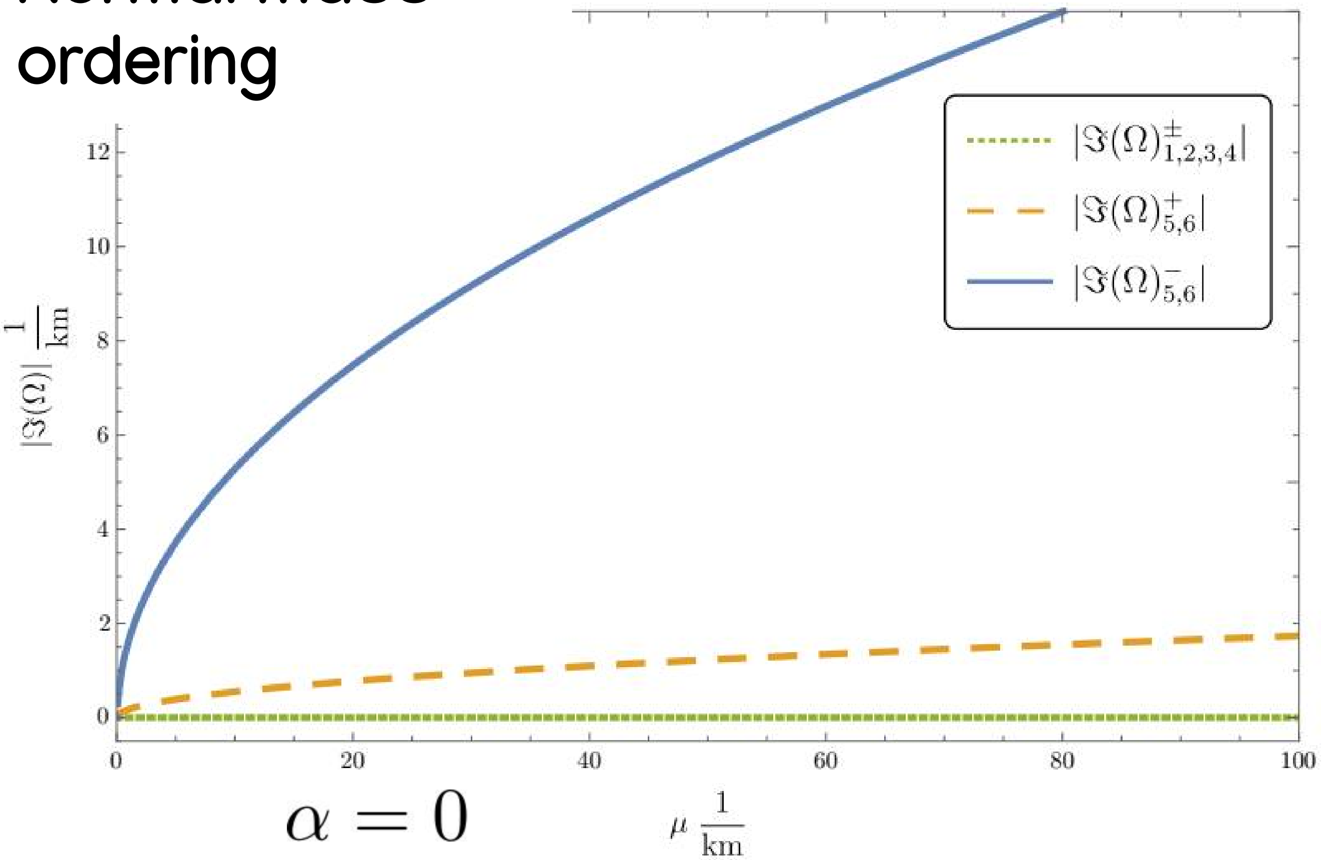
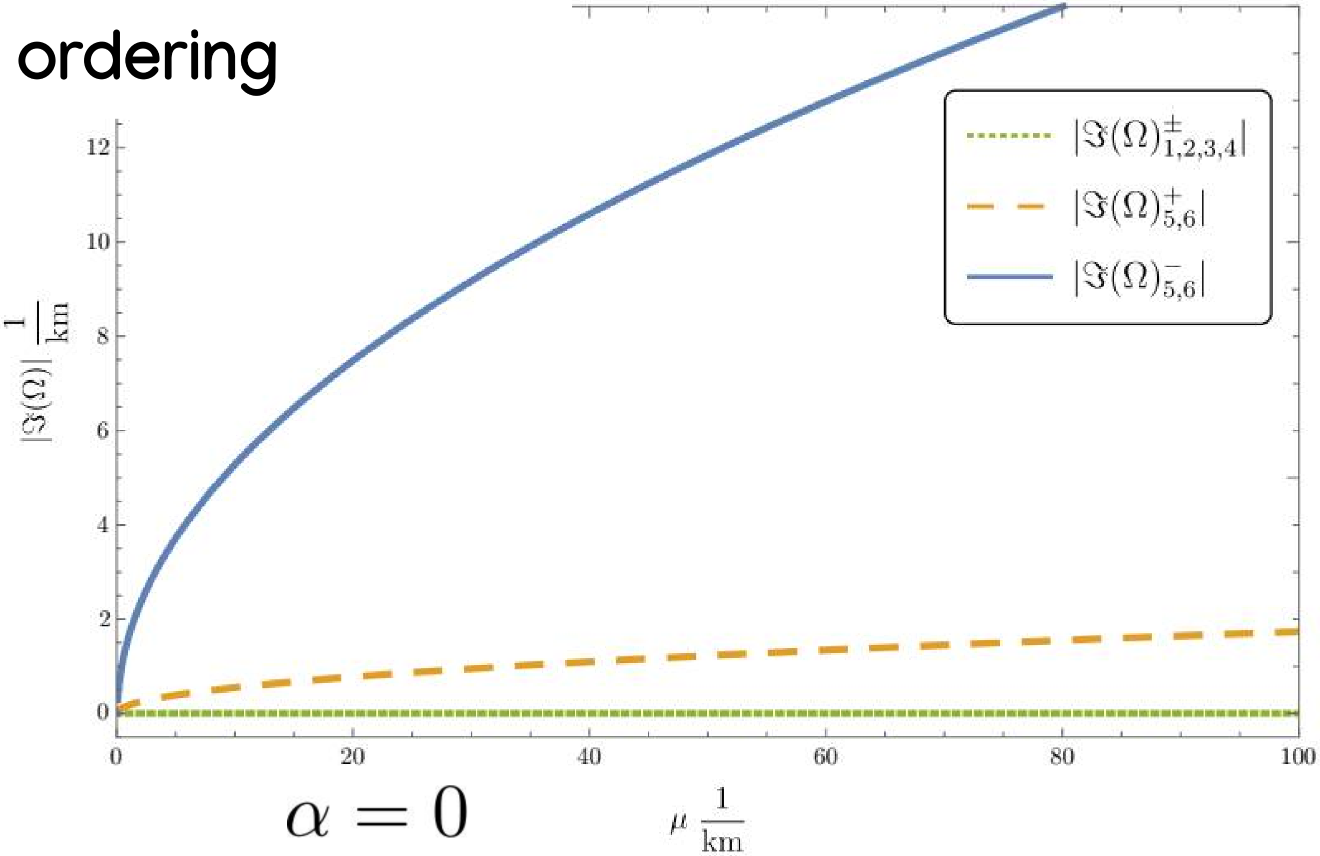


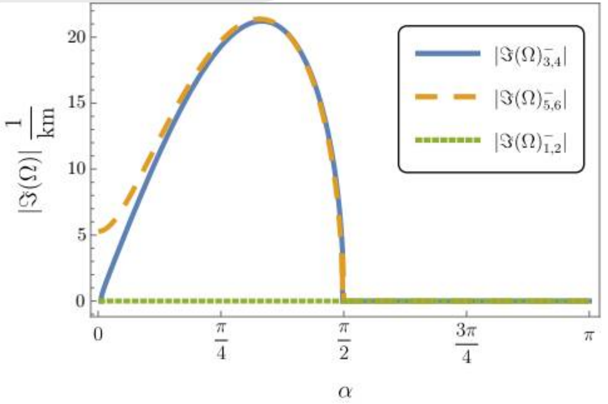
$\alpha = 0$

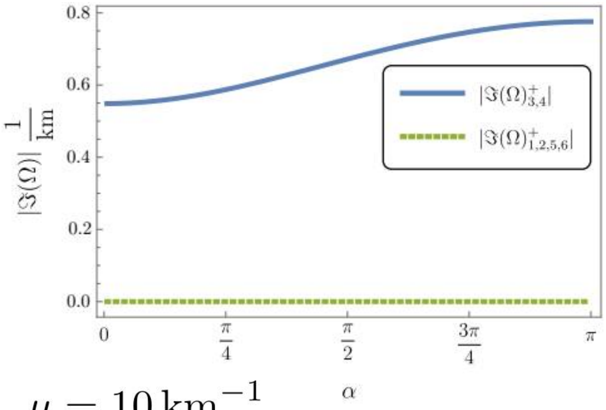
Four-Beam-System

Normal mass ordering



$\alpha = 0$





$\mu = 10\,\mathrm{km}^{-1}$

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$$H'_{\rm diag} \approx \begin{pmatrix} \lambda + \omega_i(s_{12}^2 c_{13}^2 + \eta s_{13}^2) & 0 & 0 \\ 0 & \omega_i c_{12}^2 & 0 \\ 0 & 0 & \omega_i(s_{12}^2 s_{13}^2 + \eta c_{13}^2) \end{pmatrix}$$

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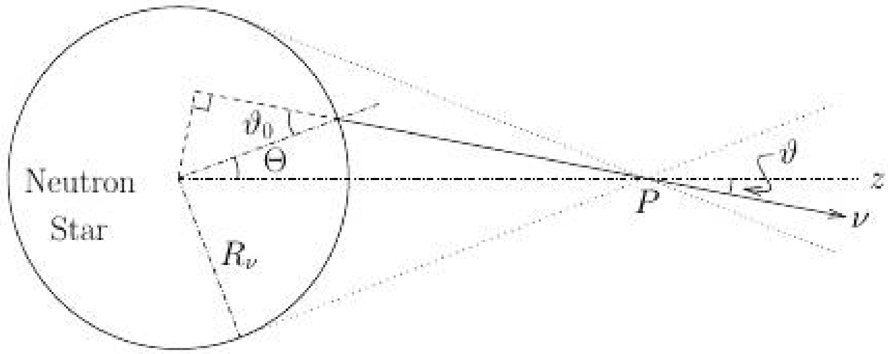
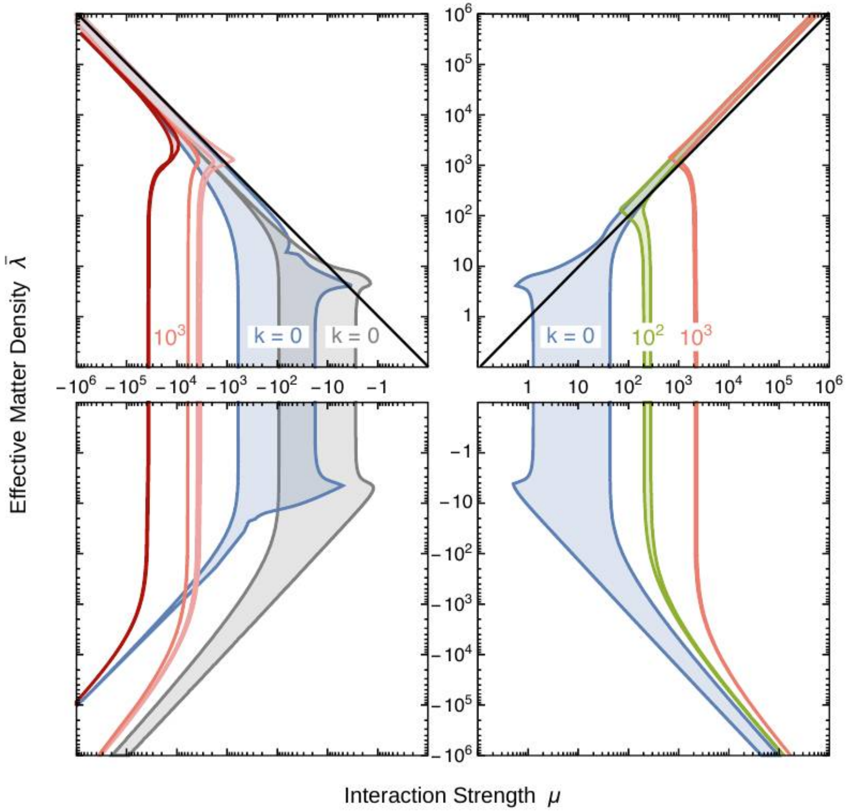
$$\omega_{\text{eff}} = \omega(-c_{12}^2 + c_{13}^2 s_{12}^2 + \eta s_{13}^2)$$

Bulb model



NO

IO



[Duan et al:arXiv:astro-ph/0606616]

[Chakraborty et al: arXiv:1507.07569]

Summary

- For high neutrino densities as in a CcSN neutrino-neutrino interactions must be considered
- We studied collective oscillations in a large matter background including all three flavors and search for flavor instabilities
- Besides known flavor instabilities for IO, we find instabilities also for NO in simple toy models
- These new instabilities can be understood as originating from an effective two-neutrino system with IO