Slow and fast flavor oscillations: 3 flavor picture

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SNnu Workshop, ICT
Trento, 14^{th} May, 2019

Neutrino Transport & Flavor Oscillations:

Liouville like equations

$$v^\beta \partial_\beta \rho = -i[H_p, \rho_p]$$

$$v^\beta = (1, \mathbf{v})$$
$$\beta = 0, \ldots, 3$$

Matrix of densities & Hamiltonian

$$\rho_\mathbf{p} = \begin{pmatrix} \rho_\mathbf{p}^{ee} & \rho_\mathbf{p}^{e\mu} & \rho_\mathbf{p}^{e\tau} \\ \rho_\mathbf{p}^{\mu e} & \rho_\mathbf{p}^{\mu\mu} & \rho_\mathbf{p}^{\mu\tau} \\ \rho_\mathbf{p}^{\tau e} & \rho_\mathbf{p}^{\tau\mu} & \rho_\mathbf{p}^{\tau\tau} \end{pmatrix} \quad \& \quad H_\mathbf{p} = \frac{M^2}{2E} + H^{matter}$$

$$H^{matter} = v_\beta \lambda^\beta$$

$$\lambda_l^\beta = \sqrt{2}\, G_F \int d\mathbf{p} \left[2\, v_l^\beta \, (f_{l,\mathbf{p}} - \bar{f}_{l,\mathbf{p}}\,) + v^\beta \, (\rho_\mathbf{p}^{ll} - \bar{\rho}_\mathbf{p}^{ll}\,) \right]$$

are the diagonal parts of $\sqrt{2}\, G_F \, (F_l^\beta + F_\nu^\beta)$

S. Airen, F. Capozzi, S.C, B. Dasgupta, G. Raelt & T. Stirner, JCAP 2018

Neutrino Transport & Flavor Oscillations:

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$$v^\beta = (1, \mathbf{v})$$
$$\beta = 0, \ldots, 3$$

$$\rho_\mathbf{p} = \begin{pmatrix} \rho_\mathbf{p}^{ee} & \rho_\mathbf{p}^{e\mu} & \rho_\mathbf{p}^{e\tau} \\ \rho_\mathbf{p}^{\mu e} & \rho_\mathbf{p}^{\mu\mu} & \rho_\mathbf{p}^{\mu\tau} \\ \rho_\mathbf{p}^{\tau e} & \rho_\mathbf{p}^{\tau\mu} & \rho_\mathbf{p}^{\tau\tau} \end{pmatrix}$$

Taking into account the off-diagonals of ρ upto linear order,

$$i\, v^\beta \partial_\beta \rho_\mathbf{p}^{e\mu} \;=\; \left[\frac{m_1^2 - m_2^2}{2E} + v_\beta(\lambda_e^\beta - \lambda_\mu^\beta)\right]\rho_\mathbf{p}^{e\mu} \;-\sqrt{2}\, G_F\,(\rho_\mathbf{p}^{ee} - \rho_\mathbf{p}^{\mu\mu})\, v^\beta \int d\mathbf{p}' v'_\beta\,(\rho_{\mathbf{p}'}^{e\mu} - \bar{\rho}_{\mathbf{p}'}^{e\mu})$$

Similarly for ‘e-τ’ and ‘μ-τ’

2 Flavor System

Only for 'e-μ'

Evolution of the off diagonal 'S_p' holds all flavor coherence information,

$$\varrho_{\mathbf{p}} = \frac{f_{\nu_e,\mathbf{p}} + f_{\nu_\mu,\mathbf{p}}}{2} \mathbb{1} + \frac{f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}}}{2} \begin{pmatrix} s_{\mathbf{p}} & S_{\mathbf{p}} \\ S^*_{\mathbf{p}} & -s_{\mathbf{p}} \end{pmatrix}$$

In the flavor Iso-spin picture,

To linear order $s_p \sim 1$

$$g_{E,\mathbf{v}} = \sqrt{2} G_{\mathrm{F}} \begin{cases} f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} & \text{for } E > 0, \\ f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} & \text{for } E < 0. \end{cases}$$

$$\int d\Gamma = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} \int \frac{d\mathbf{v}}{4\pi},$$

$$i\, v^\alpha \partial_\alpha S_{E,\mathbf{v}} = \left(\omega_E + v^\alpha \Lambda_\alpha\right) S_{E,\mathbf{v}} - v^\alpha \int d\Gamma'\, v'_\alpha\, g_{E',\mathbf{v}'} S_{E',\mathbf{v}'}$$

Normal Mode analysis to stability or dispersion picture

Linearized Stability Analysis:

$$i(\partial_t + \vec{v}\cdot\vec{\nabla}_x)\, S_{t,\vec{x},\omega,\vec{v}} = \left[\omega + \frac{\lambda + \epsilon\mu}{2} v^2\right] S_{t,\vec{x},\omega,\vec{v}} - \mu \int d\Gamma'\, g_{\omega',\vec{v}'} \frac{(\vec{v}-\vec{v}')^2}{2} S_{t,\vec{x},\omega',\vec{v}'}$$

nu-nu interaction energy
$\sqrt{2} G_F n_\nu R^2/r^2$

Matter effect
$\sqrt{2} G_F n_e R^2/r^2$

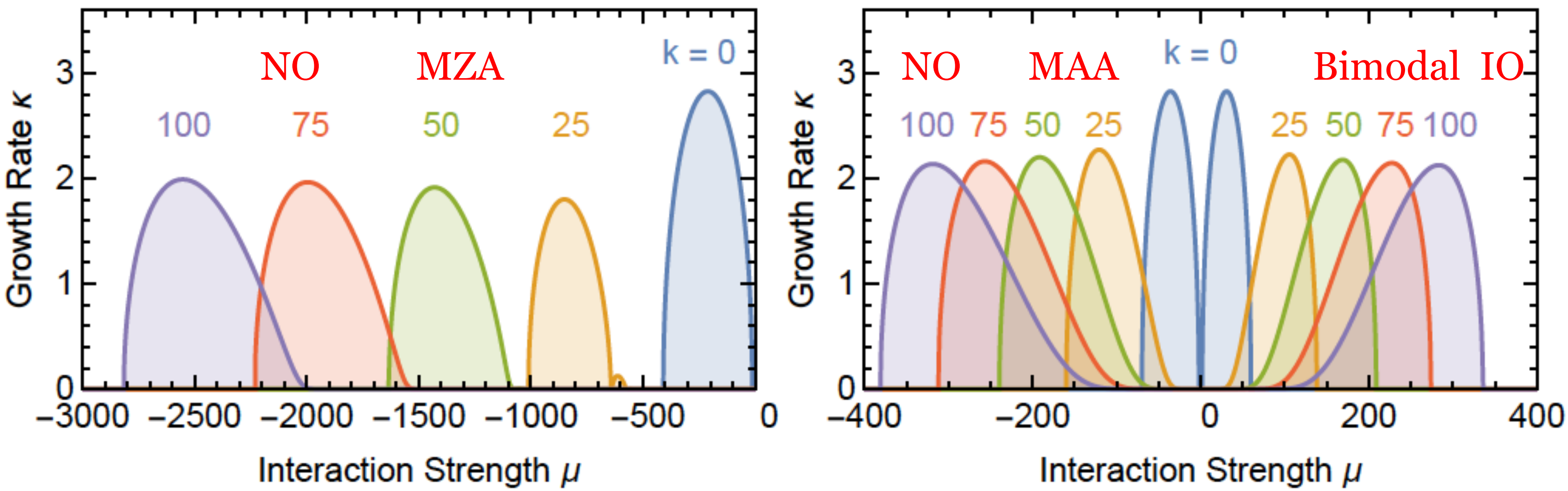
Spatial Fourier transform $\vec{v}\cdot\vec{\nabla}_x \to i\vec{k}\cdot\vec{v}$

eigenmodes $S_{t,\vec{k},\omega,\vec{v}} = Q_{\Omega,\vec{k},\omega,\vec{v}}\, e^{-i\Omega t}$

$$\left[\frac{\lambda + \epsilon\mu}{2} v^2 + \vec{k}\cdot\vec{v} + \omega - \Omega\right] Q_{\Omega,\vec{k},\omega,\vec{v}} = \mu \int d\Gamma'\, g_{\omega',\vec{v}'} \frac{(\vec{v}-\vec{v}')^2}{2} Q_{\Omega,\vec{k},\omega',\vec{v}'}$$

S.C, Hansen, Izaguirre & Raffelt, JCAP 2015

2 Flavor system: Stability picture



S.C, Hansen, Izaguirre & Raffelt, JCAP 2015

2 Flavor system: Dispersion picture

$$i\,v^\alpha\partial_\alpha S_{E,\mathbf{v}} = \left(\omega_E + v^\alpha\Lambda_\alpha\right)S_{E,\mathbf{v}} - v^\alpha\int d\Gamma'\,v'_\alpha\,g_{E',\mathbf{v}'}S_{E',\mathbf{v}'}$$

$$S_{\Gamma,r} = Q_{\Gamma,K}\,e^{-i(K_0t-\mathbf{K}\cdot\mathbf{r})}\,,\ \text{where }\Gamma = \{E,\mathbf{v}\},\ r = (t,\mathbf{r})\text{ and }K = (K_0,\mathbf{K}).$$

$$\left(v_\alpha k^\alpha - \omega_E\right)Q_{\Gamma,k} = v_\alpha A_k^\alpha \qquad \text{with} \qquad A_k^\alpha = -\int d\Gamma\,v^\alpha\,g_\Gamma Q_{\Gamma,k}$$

$$k^\alpha = K^\alpha - \Lambda^\alpha$$

$$\Pi_k^{\alpha\beta} = h^{\alpha\beta} + \int\frac{d\mathbf{v}}{4\pi}\,G_\mathbf{v}\,\frac{v^\alpha v^\beta}{v_\gamma k^\gamma} \qquad \det\Pi_k^{\alpha\beta} = 0$$

This equation can be schematically written as a dispersion relation

$$D\,(K_0, \mathbf{K}) = 0$$

Lepton number angle distribution

$$G_\mathbf{v} = \int_{-\infty}^{+\infty}\frac{E^2dE}{2\pi^2}\,g_{E,\mathbf{v}} = \sqrt{2}G_\mathrm{F}\int_0^\infty\frac{E^2dE}{2\pi^2}\left(f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} + f_{\bar{\nu}_\mu,\mathbf{p}}\right)$$

2 Flavor system: Dispersion picture

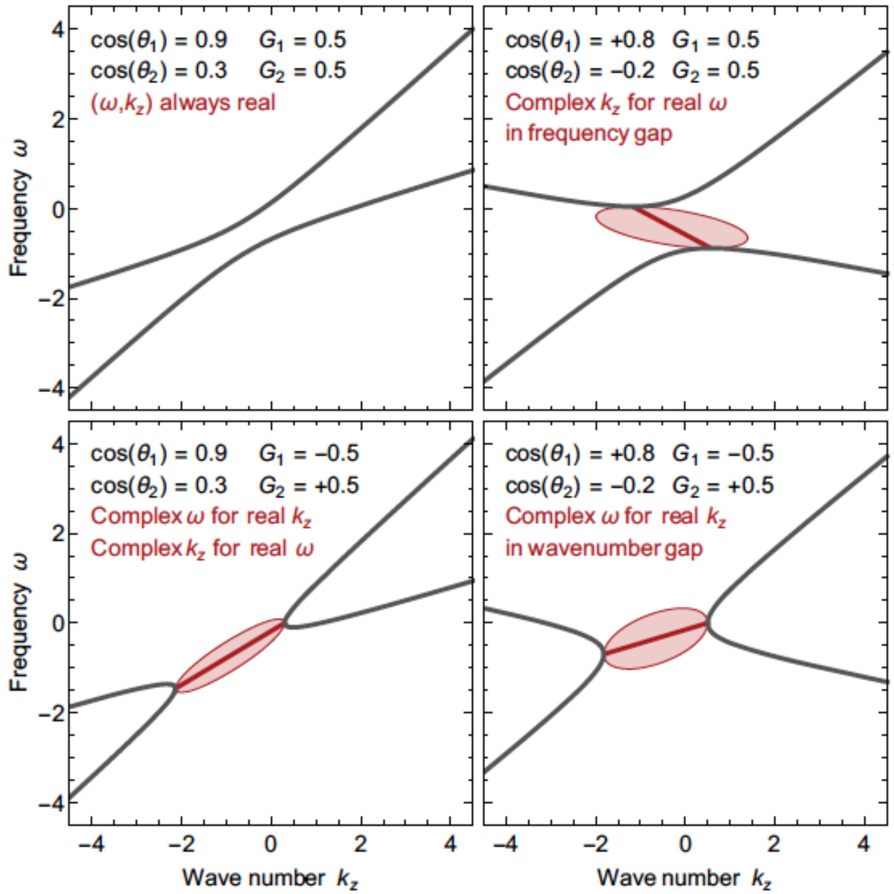


FIG. 1. Dispersion relations (black lines) for two θ modes. The thick red line is $\mathrm{Re}(\omega)$ for real k_z or $\mathrm{Re}(k_z)$ for real ω. The width of the blob is $\pm\mathrm{Im}(\omega)$ or $\pm\mathrm{Im}(k_z)$. *Left:* Only outward modes. *Right:* One outward and one backward mode. *Top:* Both ν_e excess. *Bottom:* Forward mode $\bar{\nu}_e$ excess.

Temporal stability analysis

$$\omega \in C, \ \vec{k} \in R$$

Spatial stability analysis

$$\vec{k} \in C, \omega \in R$$

Izaguirre, Raffelt and Tamborra, PRL 2016

3 Flavor System

For 'e-μ' and 'e-τ' and 'μ-τ'

$$i\, v^{\beta}\partial_{\beta}\rho^{e\mu}_{\mathbf{p}} \;=\; \left[\frac{m_1^2-m_2^2}{2E}+v_{\beta}(\lambda^{\beta}_{e}-\lambda^{\beta}_{\mu})\right]\rho^{e\mu}_{\mathbf{p}} \;-\sqrt{2}\,G_F\,(\rho^{ee}_{\mathbf{p}}-\rho^{\mu\mu}_{\mathbf{p}})v^{\beta}\int d\mathbf{p}'v'_{\beta}\,(\rho^{e\mu}_{\mathbf{p}'}-\bar{\rho}^{e\mu}_{\mathbf{p}'})$$

Evolution of the off-diagonal 'S_p' holds all flavor coherence information,

$$\rho_{\mathbf{p}} = \frac{f_{\nu_e,\mathbf{p}}+f_{\nu_\mu,\mathbf{p}}+f_{\nu_\tau,\mathbf{p}}}{3}\,1+\frac{f_{\nu_e,\mathbf{p}}-f_{\nu_\mu,\mathbf{p}}}{3}\begin{pmatrix} s_{\mathbf{p}} & S_{1\mathbf{p}} & 0\\ S^{*}_{1\mathbf{p}} & -s_{\mathbf{p}} & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$+\frac{f_{\nu_e,\mathbf{p}}-f_{\nu_\tau,\mathbf{p}}}{3}\begin{pmatrix} s_{\mathbf{p}} & 0 & S_{2\mathbf{p}}\\ 0 & 0 & 0\\ S^{*}_{2\mathbf{p}} & 0 & -s_{\mathbf{p}}\end{pmatrix}+\frac{f_{\nu_\mu,\mathbf{p}}-f_{\nu_\tau,\mathbf{p}}}{3}\begin{pmatrix} 0 & 0 & 0\\ 0 & s_{\mathbf{p}} & S_{3\mathbf{p}}\\ 0 & S^{*}_{3\mathbf{p}} & -s_{\mathbf{p}}\end{pmatrix}$$

Three Different S_p's

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3 Flavor System

In the flavor Iso-spin picture,

$$g_{1E,\mathbf{v}} = \begin{cases} f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} & \text{for } E > 0, \\ f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} & \text{for } E < 0. \end{cases}$$

$$g_{2E,\mathbf{v}} = \begin{cases} f_{\nu_e,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} & \text{for } E > 0, \\ f_{\bar{\nu}_\tau,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} & \text{for } E < 0. \end{cases}$$

$$g_{3E,\mathbf{v}} = \begin{cases} f_{\nu_\mu,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} & \text{for } E > 0, \\ f_{\bar{\nu}_\tau,\mathbf{p}} - f_{\bar{\nu}_\mu,\mathbf{p}} & \text{for } E < 0. \end{cases}$$

$$i\, v^\beta \partial_\beta S_{1E,\mathbf{v}} = (\omega_{12} + v^\beta \lambda_{1\beta}) S_{1E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{1E',\mathbf{v}'} S_{1E',\mathbf{v}'}$$

$$i\, v^\beta \partial_\beta S_{2E,\mathbf{v}} = (\omega_{13} + v^\beta \lambda_{2\beta}) S_{2E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{2E',\mathbf{v}'} S_{2E',\mathbf{v}'}$$

$$i\, v^\beta \partial_\beta S_{3E,\mathbf{v}} = (\omega_{13} - \omega_{12} + v^\beta \lambda_{3\beta}) S_{3E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{3E',\mathbf{v}'} S_{3E',\mathbf{v}'}$$

$$\int d\Gamma = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} \int \frac{d\mathbf{v}}{4\pi},$$

Normal Mode analysis to stability picture or dispersion

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3 Flavor System

$$i\, v^\beta \partial_\beta S_{1E,\mathbf{v}} = (\omega_{12} + v^\beta \lambda_{1\beta}) S_{1E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{1E',\mathbf{v}'} S_{1E',\mathbf{v}'}$$

$$i\, v^\beta \partial_\beta S_{2E,\mathbf{v}} = (\omega_{13} + v^\beta \lambda_{2\beta}) S_{2E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{2E',\mathbf{v}'} S_{2E',\mathbf{v}'}$$

$$i\, v^\beta \partial_\beta S_{3E,\mathbf{v}} = (\omega_{13} - \omega_{12} + v^\beta \lambda_{3\beta}) S_{3E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{3E',\mathbf{v}'} S_{3E',\mathbf{v}'}$$

- Three off diagonal modes evolve independently!
- Three flavor evolution is equivalent to three two flavor evolution
- $(\omega_{13} - \omega_{12}) \simeq \omega_{13}$

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Slow Oscillations: 3 Flavor Stability Analysis

3 Flavor system: Stability Analysis

Stationary, homogeneous (k=0), isotropic ($\lambda_1, \lambda_2, \lambda_3$ = 0) and only slow modes

$$i\partial_r S_{1r,\omega_1,u,\phi} = \left[\omega_1 + u(\lambda_1 + \epsilon_1 \mu)\right] S_{1r,\omega_1,u,\phi} - \mu \int d\Gamma'_1 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_{1\omega'_1,u',\phi'} S_{1r,\omega'_1,u',\phi'}$$

$$i\partial_r S_{2r,\omega_2,u,\phi} = \left[\omega_2 + u(\lambda_2 + \epsilon_2 \mu)\right] S_{2r,\omega_2,u,\phi} - \mu \int d\Gamma'_2 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_{2\omega'_1,u',\phi'} S_{2r,\omega'_2,u',\phi'}$$

$$i\partial_r S_{3r,\omega_2,u,\phi} = \left[\omega_2 + u(\lambda_3 + \epsilon_3 \mu)\right] S_{3r,\omega_2,u,\phi} - \mu \int d\Gamma'_2 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_{3\omega'_2,u',\phi'} S_{3r,\omega'_2,u',\phi'}$$

Three different λ, ε, & g

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3 Flavor system: Stability Analysis

Stationary, homogeneous (k=0), isotropic ($\lambda_1, \lambda_2, \lambda_3$ = 0) and only Slow modes

$$i\partial_r S_{1r,\omega1,u,\phi} = \Big[\omega_1 + u(\lambda_1 + \epsilon_1 \mu)\Big] S_{1r,\omega1,u,\phi} - \mu \int d\Gamma'_1 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_{1\omega'_1,u',\phi'} S_{1r,\omega'_1,u',\phi'}$$

$$i\partial_r S_{2r,\omega_2,u,\phi} = \Big[\omega_2 + u(\lambda_2 + \epsilon_2 \mu)\Big] S_{2r,\omega_2,u,\phi} - \mu \int d\Gamma'_2 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_{2\omega'_1,u',\phi'} S_{2r,\omega'_2,u',\phi'}$$

$$i\partial_r S_{3r,\omega_2,u,\phi} = \Big[\omega_2 + u(\lambda_3 + \epsilon_3 \mu)\Big] S_{3r,\omega_2,u,\phi} - \mu \int d\Gamma'_2 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_{3\omega'_2,u',\phi'} S_{3r,\omega'_2,u',\phi'}$$

$$S_{1r,\omega1,u,\phi} = Q_{1\omega1,u,\phi} e^{-i\Omega_1 r},\ S_{2r,\omega2,u,\phi} = Q_{2\omega2,u,\phi} e^{-i\Omega_2 r} \text{ and } S_{3r,\omega2,u,\phi} = Q_{3\omega2,u,\phi} e^{-i\Omega_3 r}$$

eigenvalue equations in Q's

$$\Big[\omega_1 + u(\lambda_1 + \epsilon_1 \mu) - \Omega_1\Big] Q_1 = \mu \int d\Gamma'_1 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_1 Q_1$$

$$\Big[\omega_2 + u(\lambda_2 + \epsilon_2 \mu) - \Omega_2\Big] Q_2 = \mu \int d\Gamma'_2 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_2 Q_2$$

$$\Big[\omega_2 + u(\lambda_3 + \epsilon_3 \mu) - \Omega_3\Big] Q_3 = \mu \int d\Gamma'_2 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_3 Q_3$$

3 Flavor system: Stability Analysis

eigenvalue equations in Q's

$$\Big[\omega_1 + u(\lambda_1 + \epsilon_1\mu) - \Omega_1\Big]Q_1 = \mu \int d\Gamma_1' \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_1 Q_1$$

$$\Big[\omega_2 + u(\lambda_2 + \epsilon_2\mu) - \Omega_2\Big]Q_2 = \mu \int d\Gamma_2' \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_2 Q_2$$

$$\Big[\omega_2 + u(\lambda_3 + \epsilon_3\mu) - \Omega_3\Big]Q_3 = \mu \int d\Gamma_2' \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_3 Q_3$$

Normal Stellar collapse like scenario,

$$N_{\nu_\mu} = N_{\bar{\nu}_\mu} \text{ and } N_{\nu_\tau} = N_{\bar{\nu}_\tau} \quad \& \quad n_\mu = n_\tau = 0$$

$$\lambda_1 = \frac{\sqrt{2}G_F R^2}{2r^2}[(n_e - \bar{n_e}) - (n_\mu - \bar{n_\mu})]$$

$$\lambda_2 = \frac{\sqrt{2}G_F R^2}{2r^2}[(n_e - \bar{n_e}) - (n_\tau - \bar{n_\tau})]$$

$$\lambda_3 = \frac{\sqrt{2}G_F R^2}{2r^2}[(n_\mu - \bar{n_\mu}) - (n_\tau - \bar{n_\tau})]$$

$$\epsilon_1 = \frac{\int d\Gamma_1 (f_{\nu_e} - f_{\nu_\mu})}{N} = \frac{(N_{\nu_e} - N_{\bar{\nu}_e}) - (N_{\nu_\mu} - N_{\bar{\nu}_\mu})}{N}$$

$$\epsilon_2 = \frac{\int d\Gamma_2 (f_{\nu_e} - f_{\nu_\tau})}{N} = \frac{(N_{\nu_e} - N_{\bar{\nu}_e}) - (N_{\nu_\tau} - N_{\bar{\nu}_\tau})}{N}$$

$$\epsilon_3 = \frac{\int d\Gamma_3 (f_{\nu_\mu} - f_{\nu_\tau})}{N} = \frac{(N_{\nu_\mu} - N_{\bar{\nu}_\mu}) - (N_{\nu_\tau} - N_{\bar{\nu}_\tau})}{N}$$

$$\mu = \sqrt{2}G_F \frac{\left[2(N_{\nu_e} + N_{\bar{\nu}_e}) - (N_{\nu_\tau} + N_{\bar{\nu}_\tau} + N_{\nu_\mu} + N_{\bar{\nu}_\mu})\right]}{4\pi r^2} \frac{R^2}{2r^2}$$

$$\lambda_1 = \lambda_2 = \lambda, \lambda_3 = 0 \quad \& \quad \epsilon_1 = \epsilon_2 = \epsilon, \epsilon_3 = 0.$$

3 Flavor system: Stability Analysis

Eigenvalue equations in Q's

$$\lambda_1 = \lambda_2 = \lambda, \lambda_3 = 0 \quad \& \quad \epsilon_1 = \epsilon_2 = \epsilon, \epsilon_3 = 0.$$

$$\Big[\omega_1 + u(\lambda + \epsilon\mu) - \Omega_1\Big] Q_1 = \mu \int d\Gamma_1' \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_1 Q_1$$

$$\Big[\omega_2 + u(\lambda + \epsilon\mu) - \Omega_2\Big] Q_2 = \mu \int d\Gamma_2' \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_2 Q_2$$

$$\Big[\omega_2 - \Omega_3\Big] Q_3 = \mu \int d\Gamma_2' \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_3 Q_3$$

- Evolution of first two off diagonal differ by ω
- Third one is independent of matter effect!
- Decoupled solar & atmospheric evolution for $N_{\nu_\mu} = N_{\bar{\nu}_\mu} = N_{\nu_\tau} = N_{\bar{\nu}_\tau}$ as g_3 vanishes

$$g_{3E,\mathbf{v}} = \begin{cases} f_{\nu_\mu,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} & \text{for } E > 0, \\ f_{\bar{\nu}_\tau,\mathbf{p}} - f_{\bar{\nu}_\mu,\mathbf{p}} & \text{for } E < 0. \end{cases}$$

In preparation, M. Chakraborty & **S.C**

3 Flavor system: Stability Analysis

Eigenvalue equations in Q's

$$\lambda_1 = \lambda_2 = \lambda, \lambda_3 = 0 \quad \& \quad \epsilon_1 = \epsilon_2 = \epsilon, \epsilon_3 = 0.$$

$$\Big[\omega_1 + u(\lambda + \epsilon\mu) - \Omega_1\Big] Q_1 = \mu \int d\Gamma'_1 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_1 Q_1$$

$$\Big[\omega_2 + u(\lambda + \epsilon\mu) - \Omega_2\Big] Q_2 = \mu \int d\Gamma'_2 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_2 Q_2$$

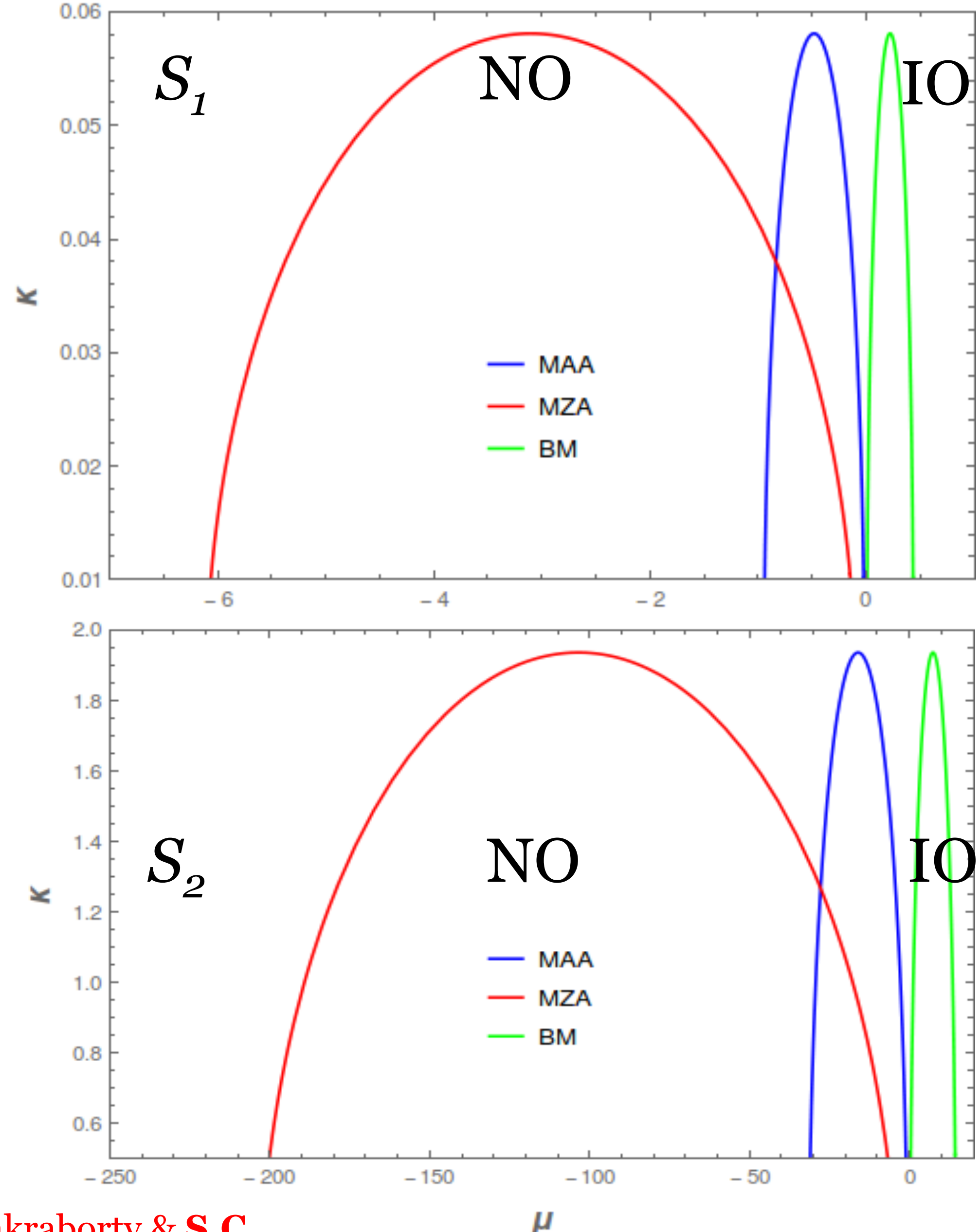
$$\Big[\omega_2 - \Omega_3\Big] Q_3 = \mu \int d\Gamma'_2 \left[u + u' - 2\sqrt{uu'}\cos(\phi - \phi')\right] g_3 Q_3$$

$$g_j = h_j(\omega)\, f_j(u) \qquad j = 1,2,3$$

$$h_j(\omega) = \left(1 + \frac{\epsilon_j}{2}\right)\delta(\omega_j - \omega'_j) - \left(1 - \frac{\epsilon_j}{2}\right)\delta(\omega_j + \omega'_j)$$

3 Flavor system: Stability Analysis

$\overline{\lambda} = 0, K = 0$



Solar

$\varepsilon_1 = 0.5,$
$\omega_1 = 0.015$

Atmospheric

$\varepsilon_2 = 0.5,$
$\omega_2 = 0.5$

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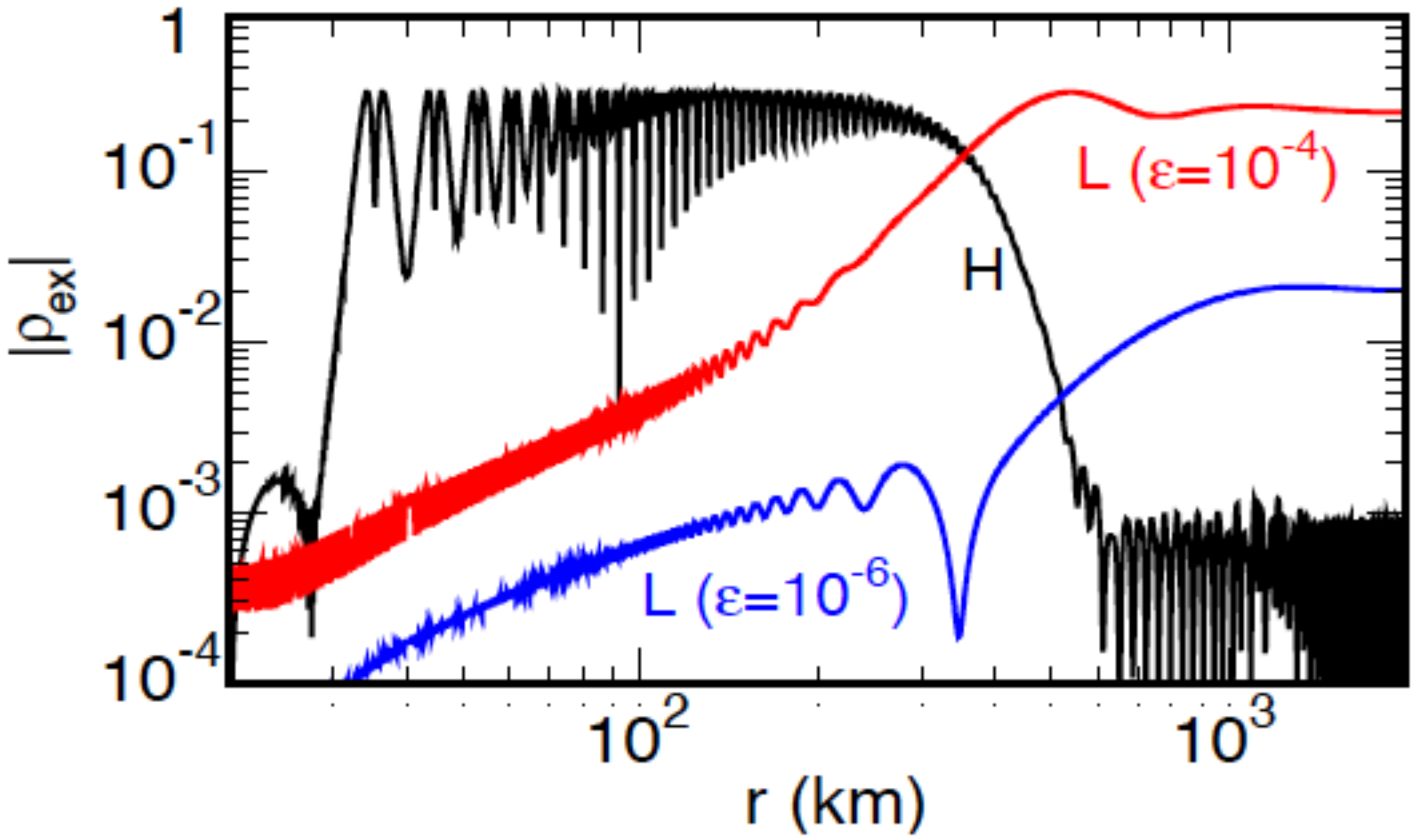
3 Flavor system: Stability Analysis

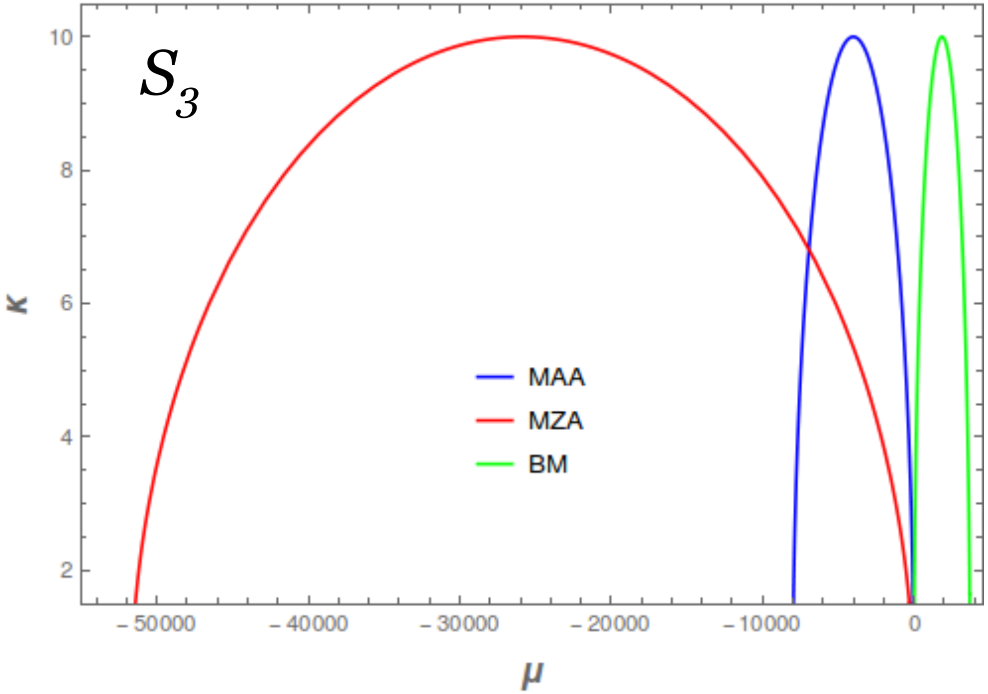


Decoupled solar & atmospheric evolution!

B. Dasgupta, A. Mirizzi, I. Tamborra & R. Tomàs, PRD 2010

3 Flavor system: Stability Analysis

$\overline{\lambda} = 0, K = 0$



$\varepsilon_3 = 10^{-2}$,
$\omega_{12} = 0.05$

Coupling the
solar and atmospheric
sector

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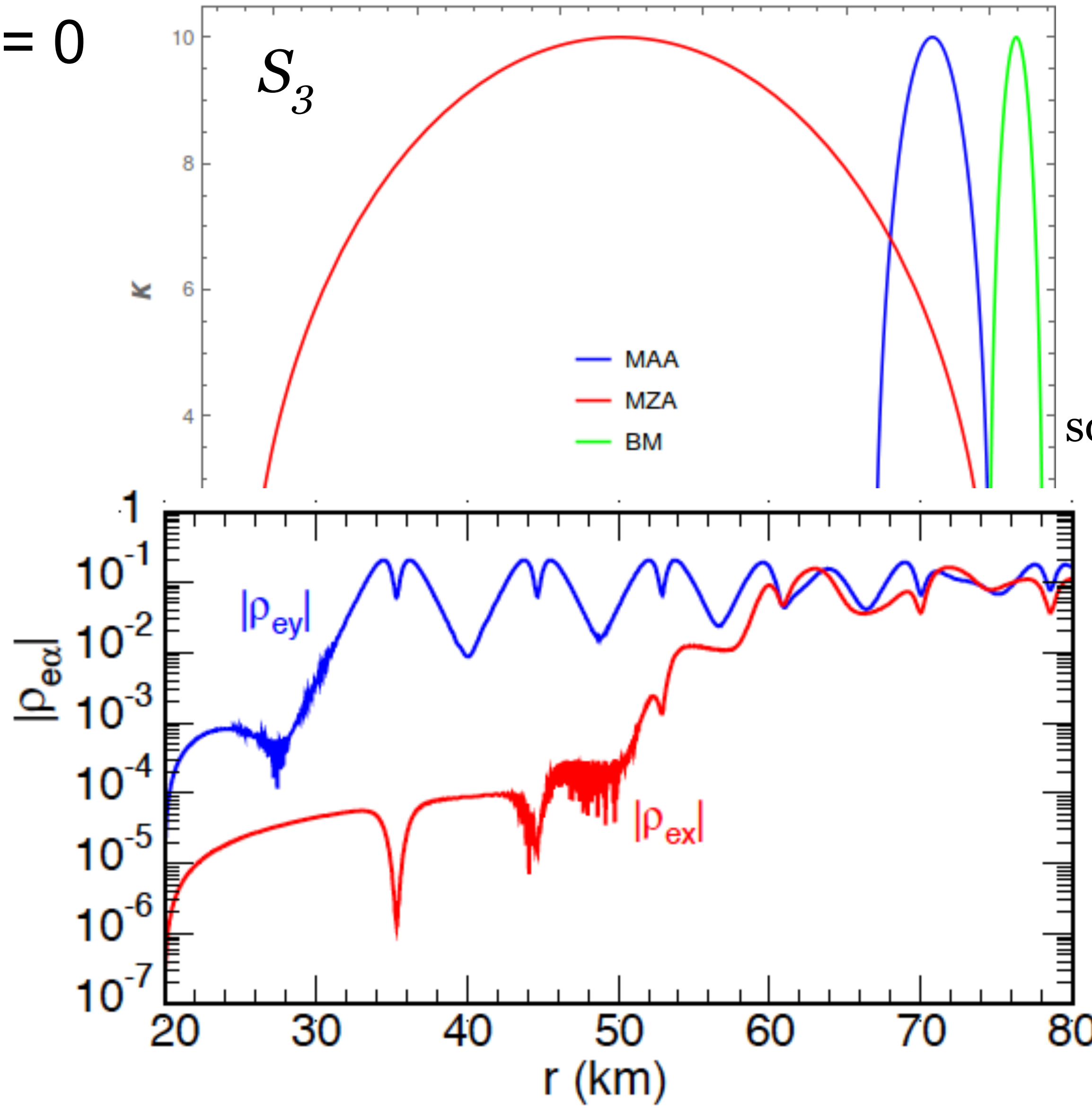
3 Flavor system: Stability Analysis

$\overline{\lambda} = 0$, $K = 0$



$\varepsilon_3 = 10^{-2}$,
$\omega_{12} = 0.05$

Coupling the
solar and atmospheric
sector

B. Dasgupta, A. Mirizzi, I. Tamborra & R. Tomàs, PRD 2010

Fast oscillations: 3 Flavor Dispersion picture

Fast oscillations: 3 Flavor Dispersion picture

Go back to the linearized equations

$$i\,v^\beta \partial_\beta S_{1E,\mathbf{v}} = (\omega_{12} + v^\beta \lambda_{1\beta}) S_{1E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{1E',\mathbf{v}'} S_{1E',\mathbf{v}'}$$

$$i\,v^\beta \partial_\beta S_{2E,\mathbf{v}} = (\omega_{13} + v^\beta \lambda_{2\beta}) S_{2E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{2E',\mathbf{v}'} S_{2E',\mathbf{v}'}$$

$$i\,v^\beta \partial_\beta S_{3E,\mathbf{v}} = (\omega_{13} - \omega_{12} + v^\beta \lambda_{3\beta}) S_{3E,\mathbf{v}} - \sqrt{2}\, G_F\, v^\beta \int d\Gamma'\, v'_\beta\, g_{3E',\mathbf{v}'} S_{3E',\mathbf{v}'}$$

$$S_{1\Gamma,r} = Q_{1\Gamma,K}\, e^{-i(K_1^0 t - \mathbf{K}_1\cdot\mathbf{r})}\,,\; S_{2\Gamma,r} = Q_{2\Gamma,K}\, e^{-i(K_2^0 t - \mathbf{K}_2\cdot\mathbf{r})}\,,\; S_{3\Gamma,r} = Q_{3\Gamma,K}\, e^{-i(K_3^0 t - \mathbf{K}_3\cdot\mathbf{r})}$$

Fast oscillation limit ω ➔ 0

$$\Pi^{\alpha\beta}_{1K,\lambda} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{1\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_1^\gamma - \lambda_1^\gamma)}$$

$$\Pi^{\alpha\beta}_{2K,\lambda} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{2\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_2^\gamma - \lambda_2^\gamma)}$$

$$\Pi^{\alpha\beta}_{3K,\lambda} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{3\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_3^\gamma - \lambda_3^\gamma)}$$

$$\eta^{\alpha\beta} = \mathrm{diag}(+,-,-,-)$$

$$\det \Pi^{\alpha\beta}_{jK} = 0$$

➔ Dispersion relations: Connecting K_i^0 & $\mathbf{K_i}$

Fast oscillations: 3 Flavor Dispersion picture

Fast oscillation limit ω ➔ 0

$$\Pi^{\alpha\beta}_{1K,\lambda} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{1\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_1^\gamma - \lambda_1^\gamma)}$$

$$\Pi^{\alpha\beta}_{2K,\lambda} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{2\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_2^\gamma - \lambda_2^\gamma)}$$

$$\Pi^{\alpha\beta}_{3K,\lambda} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{3\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_3^\gamma - \lambda_3^\gamma)}$$

$$G_{1\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{1E,\mathbf{v}} = \sqrt{2} G_F \int_0^\infty \frac{E^2 dE}{2\pi^2} (f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} + f_{\bar{\nu}_\mu,\mathbf{p}})$$

$$G_{2\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{2E,\mathbf{v}} = \sqrt{2} G_F \int_0^\infty \frac{E^2 dE}{2\pi^2} (f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}})$$

$$G_{3\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{3E,\mathbf{v}} = \sqrt{2} G_F \int_0^\infty \frac{E^2 dE}{2\pi^2} (f_{\nu_\mu,\mathbf{p}} - f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}})$$

$$\lambda_{1\beta} = (\lambda_e^\beta - \lambda_\mu^\beta) = \sqrt{2} G_F \int d\mathrm{p} \left[2\left(v_e^\beta \left(f_{e,\mathrm{p}} - \bar{f}_{e,\mathrm{p}}\right) - v_e^\beta \left(f_{\mu,\mathrm{p}} - \bar{f}_{\mu,\mathrm{p}}\right) \right) + v^\beta \left(\left(f_{\nu_e,\mathrm{p}} - \bar{f}_{\nu_e,\mathrm{p}} - f_{\nu_\mu,\mathrm{p}} + \bar{f}_{\nu_\mu,\mathrm{p}}\right) \right) \right]$$

$$\lambda_{2\beta} = (\lambda_e^\beta - \lambda_\tau^\beta) \text{ and } \lambda_{3\beta} = (\lambda_\mu^\beta - \lambda_\tau^\beta)$$

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Fast oscillations: 3 Flavor Dispersion picture

$$G_{1\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2}\, g_{1E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left(f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} + f_{\bar{\nu}_\mu,\mathbf{p}}\right)$$

$$G_{2\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2}\, g_{2E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left(f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}}\right)$$

$$G_{3\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2}\, g_{3E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left(f_{\nu_\mu,\mathbf{p}} - f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}}\right)$$

In a stellar collapse scenario spectra of muon and tau neutrinos spectra is same as there antineutrinos,

$$G_{3\mathbf{v}} = 0 \text{ and } G_{1\mathbf{v}} = G_{2\mathbf{v}}$$

Fast oscillations: 3 Flavor Dispersion picture

$$\lambda_{1\beta} = (\lambda_e^\beta - \lambda_\mu^\beta) = \sqrt{2}G_F \int d\mathrm{p} \Big[2 \Big(v_e^\beta (f_{e,\mathrm{p}} - \bar{f}_{e,\mathrm{p}}) - v_e^\beta (f_{\mu,\mathrm{p}} - \bar{f}_{\mu,\mathrm{p}}) \Big) + v^\beta \Big((f_{\nu_e,\mathrm{p}} - \bar{f}_{\nu_e,\mathrm{p}} - f_{\nu_\mu,\mathrm{p}} + \bar{f}_{\nu_\mu,\mathrm{p}}) \Big) \Big]$$

$$\lambda_{2\beta} = (\lambda_e^\beta - \lambda_\tau^\beta) \text{ and } \lambda_{3\beta} = (\lambda_\mu^\beta - \lambda_\tau^\beta)$$

In a stellar collapse scenario spectra of mu and tau neutrinos (and leptons) spectra is same as there anti particles

$\lambda_{3\beta} = 0$ and $\lambda_{1\beta} = \lambda_{2\beta}$

Fast oscillations: 3 Flavor Dispersion picture

$$\Pi^{\alpha\beta}_{1K,\lambda} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{1\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_1^\gamma - \lambda_1^\gamma)}$$

$$\Pi^{\alpha\beta}_{2K,\lambda} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{2\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_2^\gamma - \lambda_2^\gamma)}$$

$$\Pi^{\alpha\beta}_{3K,\lambda} = \eta^{\alpha\beta} + \int \frac{d\mathbf{v}}{4\pi} G_{3\mathbf{v}} \frac{v^\alpha v^\beta}{v_\gamma (K_3^\gamma - \lambda_3^\gamma)}$$

In a stellar collapse scenario, $\lambda_{3\beta} = 0$ and $\lambda_{1\beta} = \lambda_{2\beta}$

&

$$G_{3\mathbf{v}} = 0 \text{ and } G_{1\mathbf{v}} = G_{2\mathbf{v}}$$

- Only one dispersion relation.
- Consistent with 2 Flavor scenario.

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Fast oscillations: 3 Flavor Dispersion picture

However, λ's and G's may not be exactly equal or vanish,
Say some muon-antimuon flux

$$\lambda_{1\beta} = (\lambda_e^\beta - \lambda_\mu^\beta) = \sqrt{2}G_F \int d\mathbf{p} \left[2 \left(v_e^\beta \left(f_{e,\mathbf{p}} - \bar{f}_{e,\mathbf{p}} \right) - v_e^\beta \left(f_{\mu,\mathbf{p}} - \bar{f}_{\mu,\mathbf{p}} \right) \right) + v^\beta \left(\left(f_{\nu_e,\mathbf{p}} - \bar{f}_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} + \bar{f}_{\nu_\mu,\mathbf{p}} \right) \right) \right]$$

$$\lambda_{2\beta} = (\lambda_e^\beta - \lambda_\tau^\beta) \text{ and } \lambda_{3\beta} = (\lambda_\mu^\beta - \lambda_\tau^\beta)$$

$$\text{we consider } \lambda_1^\gamma = 0.9\lambda_2^\gamma \quad \& \quad \lambda_3^\gamma = 0.1\lambda_2^\gamma.$$

$$\boxed{\lambda_2^0 = 10, \lambda_2 = 10,\ \lambda_1^0 = 9, \lambda_1 = 9,\ \lambda_3^0 = 1, \lambda_3 = 1}$$

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Fast oscillations: 3 Flavor Dispersion picture

However, λ's and G's may not be exactly equal or vanish,
Say some muon-antimuon flux

$$G_{1\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{1E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left(f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} + f_{\bar{\nu}_\mu,\mathbf{p}} \right)$$

$$G_{2\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{2E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left(f_{\nu_e,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}} \right)$$

$$G_{3\mathbf{v}} = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} g_{3E,\mathbf{v}} = \sqrt{2}\, G_F \int_0^{\infty} \frac{E^2 dE}{2\pi^2} \left(f_{\nu_\mu,\mathbf{p}} - f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\nu_\tau,\mathbf{p}} + f_{\bar{\nu}_\tau,\mathbf{p}} \right)$$

$G_{1v} = 0.9\, G_{2v}$ and $G_{3v} = 0.1\, G_{2v}$

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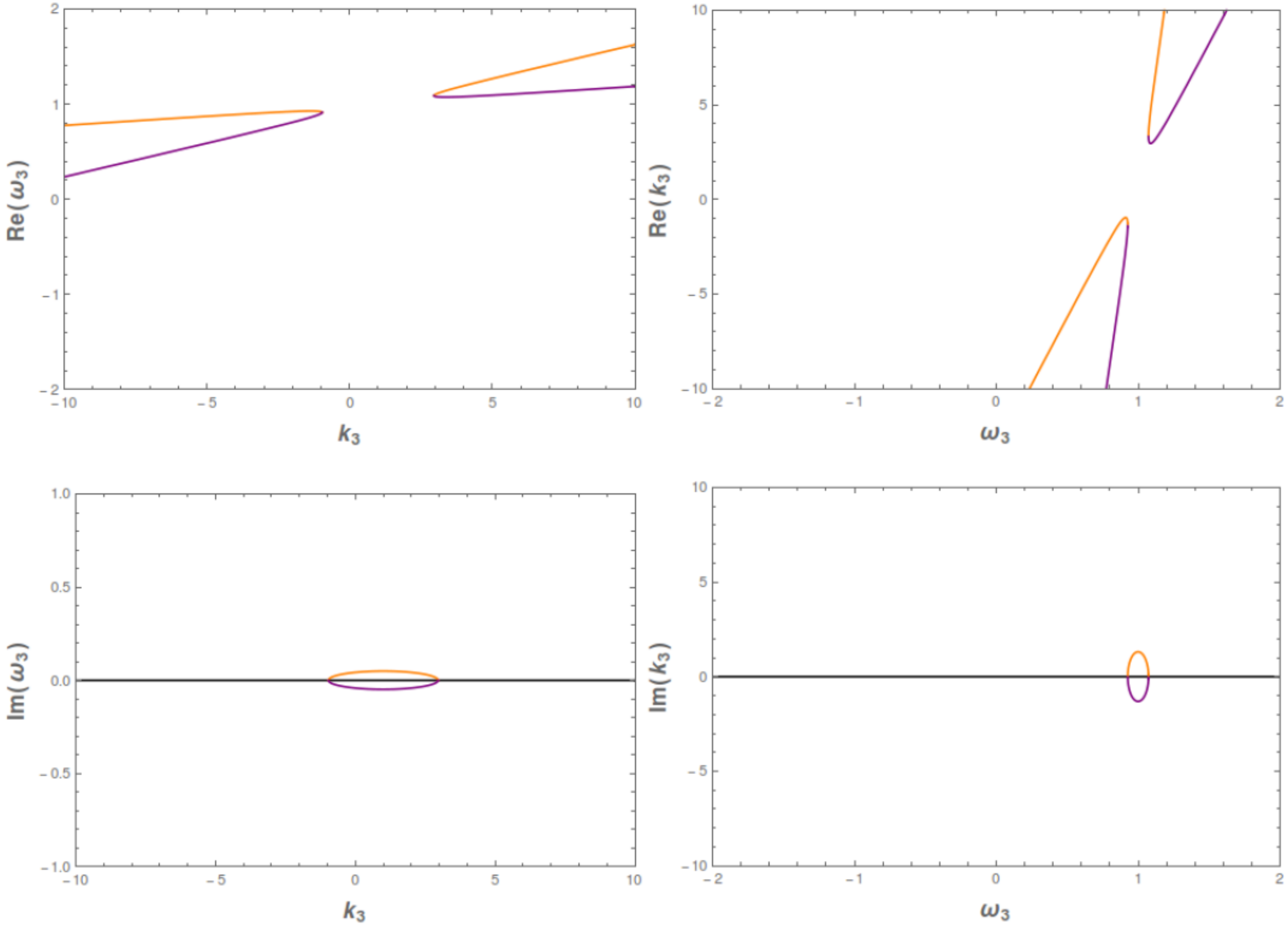
Fast oscillations: 3 Flavor Dispersion picture

Two beam neutrino ELN spectra, $G_{\mathbf{v}} = 4\pi\left[G_1\delta(\mathbf{v}-\mathbf{v}_1) + G_2\delta(\mathbf{v}-\mathbf{v}_2)\right]$



$g_3' = -0.04, g_3'' = 0.06, v_1 = 0.07, v_2 = 0.02.$

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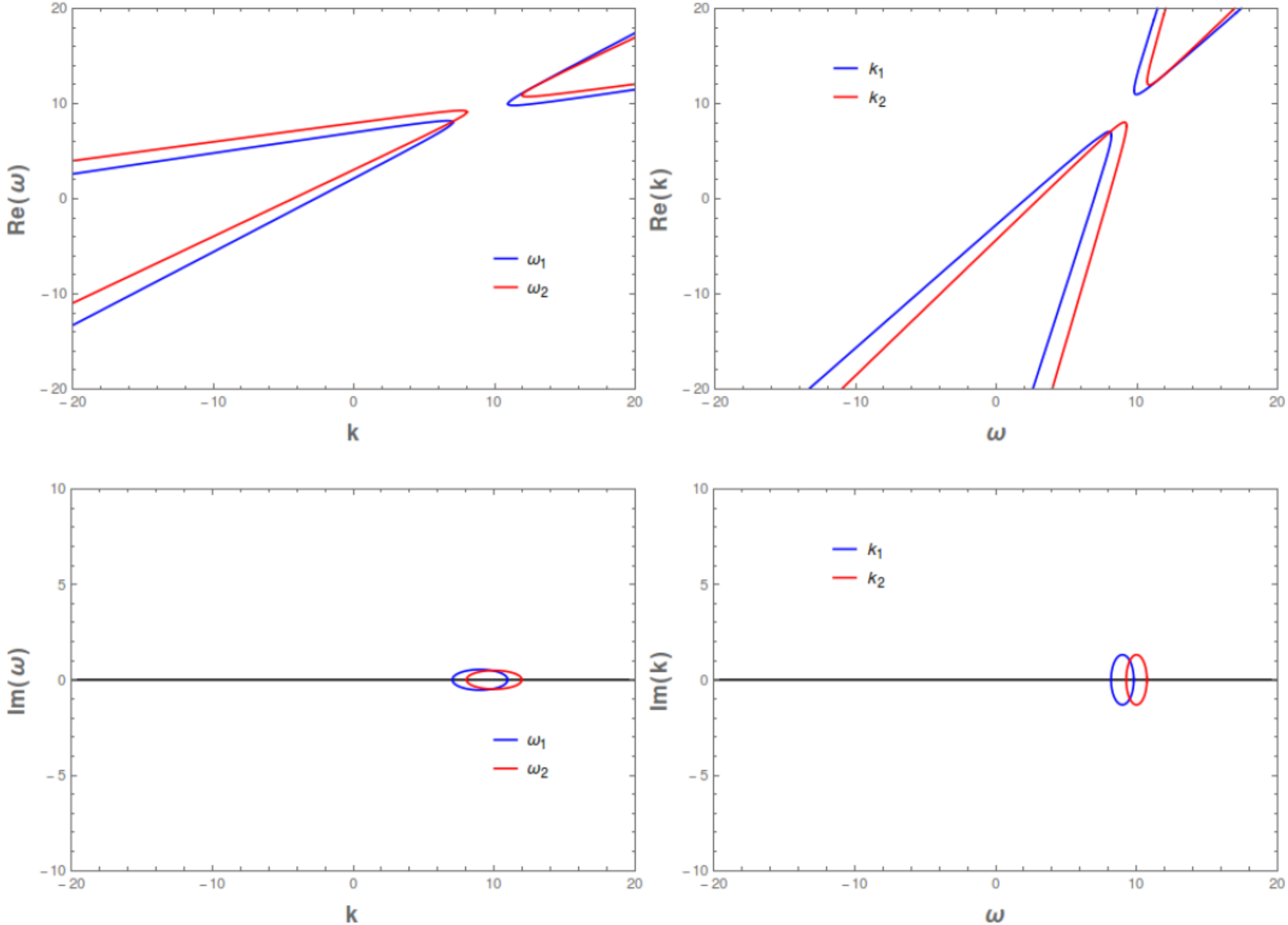
Fast oscillations: 3 Flavor Dispersion picture

Two beam neutrino ELN spectra, $G_{\mathbf{v}} = 4\pi \left[G_1 \delta(\mathbf{v} - \mathbf{v}_1) + G_2 \delta(\mathbf{v} - \mathbf{v}_2) \right]$



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$g_1' = -0.44, g_1'' = 0.66, v_1 = 0.77, v_2 = 0.22$ and $g_2' = -0.4, g_2'' = 0.6, v_1 = 0.7, v_2 = 0.2.$

Conclsuion:

- 3 flavor picture consistent with the 2 flavor results in both slow and fast
- $N_{\nu_\mu} = N_{\bar{\nu}_\mu}$ and $N_{\nu_\tau} = N_{\bar{\nu}_\tau}$ gives effective two flavor solutions!
- Slow oscillations: 3 flavor effect can speed up effects of solar sector
- Fast oscillations: In principle three different dispersion relation ??
- Does it help overall instability or are the triggered modes are far apart?
- Open questions:
    - Mixing of slow and fast modes
    - Triggering of the modes & matter inhomogeneity
    - Collisons

Thank you!

NEUTRINO TRANSPORT & FLAVOR OSCILLATIONS:

Liouville like equations

$$v^\beta \partial_\beta \rho = -i[H_p, \rho_p]$$

$$v^\beta = (1, \mathbf{v})$$
$$\beta = 0, \ldots, 3$$

Matrix of densities & Hamiltonian

$$H_{\mathbf{p}} = \frac{M^2}{2E} + \sqrt{2}\, G_F\, v_\beta (F_l^\beta + F_\nu^\beta)$$

Neutrino Flux,

$$F_\nu^\beta = \int d\mathbf{p}\ v^\beta\ (\rho_{\mathbf{p}} - \bar{\rho}_{\mathbf{p}})$$

Charged lepton Flux,

$$F_l^\beta = \int 2\, d\mathbf{p} \begin{pmatrix} v_e^\beta (f_{e,\mathbf{p}} - \bar{f}_{e,\mathbf{p}}) & 0 & 0 \\ 0 & v_\mu^\beta (f_{\mu,\mathbf{p}} - \bar{f}_{\mu,\mathbf{p}}) & 0 \\ 0 & 0 & v_\tau^\beta (f_{\tau,\mathbf{p}} - \bar{f}_{\tau,\mathbf{p}}) \end{pmatrix}$$

NEUTRINO TRANSPORT & FLAVOR OSCILLATIONS:

Liouville like equations

$$v^\beta \partial_\beta \rho = -i[H_p, \rho_p]$$

$$v^\beta = (1, \mathbf{v})$$
$$\beta = 0, \ldots, 3$$

Define overall matter effect,

$$H^{matter} = v_\beta \lambda^\beta$$

$$\lambda_l^\beta = \sqrt{2}\, G_F \int d\mathbf{p} \left[2\, v_l^\beta\, (f_{l,\mathbf{p}} - \bar{f}_{l,\mathbf{p}}) + v^\beta\, (\rho_{\mathbf{p}}^{ll} - \bar{\rho}_{\mathbf{p}}^{ll}) \right]$$

are the diagonal parts of $\sqrt{2}\, G_F\, (F_l^\beta + F_\nu^\beta)$

2 Flavor system

Only for ‘e-μ’

Evolution of the off diagonal ‘S_p’ holds all flavor coherence information,

$$\varrho_{\mathbf{p}} = \frac{f_{\nu_e,\mathbf{p}} + f_{\nu_\mu,\mathbf{p}}}{2} \mathbb{1} + \frac{f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}}}{2} \begin{pmatrix} s_{\mathbf{p}} & S_{\mathbf{p}} \\ S^*_{\mathbf{p}} & -s_{\mathbf{p}} \end{pmatrix}$$

$$i\, v^\alpha \partial_\alpha S_{\mathbf{p}} = (\omega_E + v^\alpha \Lambda_\alpha) S_{\mathbf{p}} - v^\alpha \int d\mathbf{p}' v'_\alpha \left(S_{\mathbf{p}'} g_{\mathbf{p}'} - \bar{S}_{\mathbf{p}'} \bar{g}_{\mathbf{p}'}\right)$$

In the flavor Iso-spin picture,

$$g_{E,\mathrm{v}} = \sqrt{2} G_{\mathrm{F}} \begin{cases} f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} & \text{for } E > 0, \\ f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} & \text{for } E < 0. \end{cases}$$

$$i\, v^\alpha \partial_\alpha S_{E,\mathbf{v}} = (\omega_E + v^\alpha \Lambda_\alpha) S_{E,\mathbf{v}} - v^\alpha \int d\Gamma'\, v'_\alpha\, g_{E',\mathbf{v}'} S_{E',\mathbf{v}'}$$

$$\int d\Gamma = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} \int \frac{d\mathbf{v}}{4\pi},$$

Normal Mode analysis to stability picture or dispersion