Flavor evolution of dense neutrinos as an emergent phenomenon

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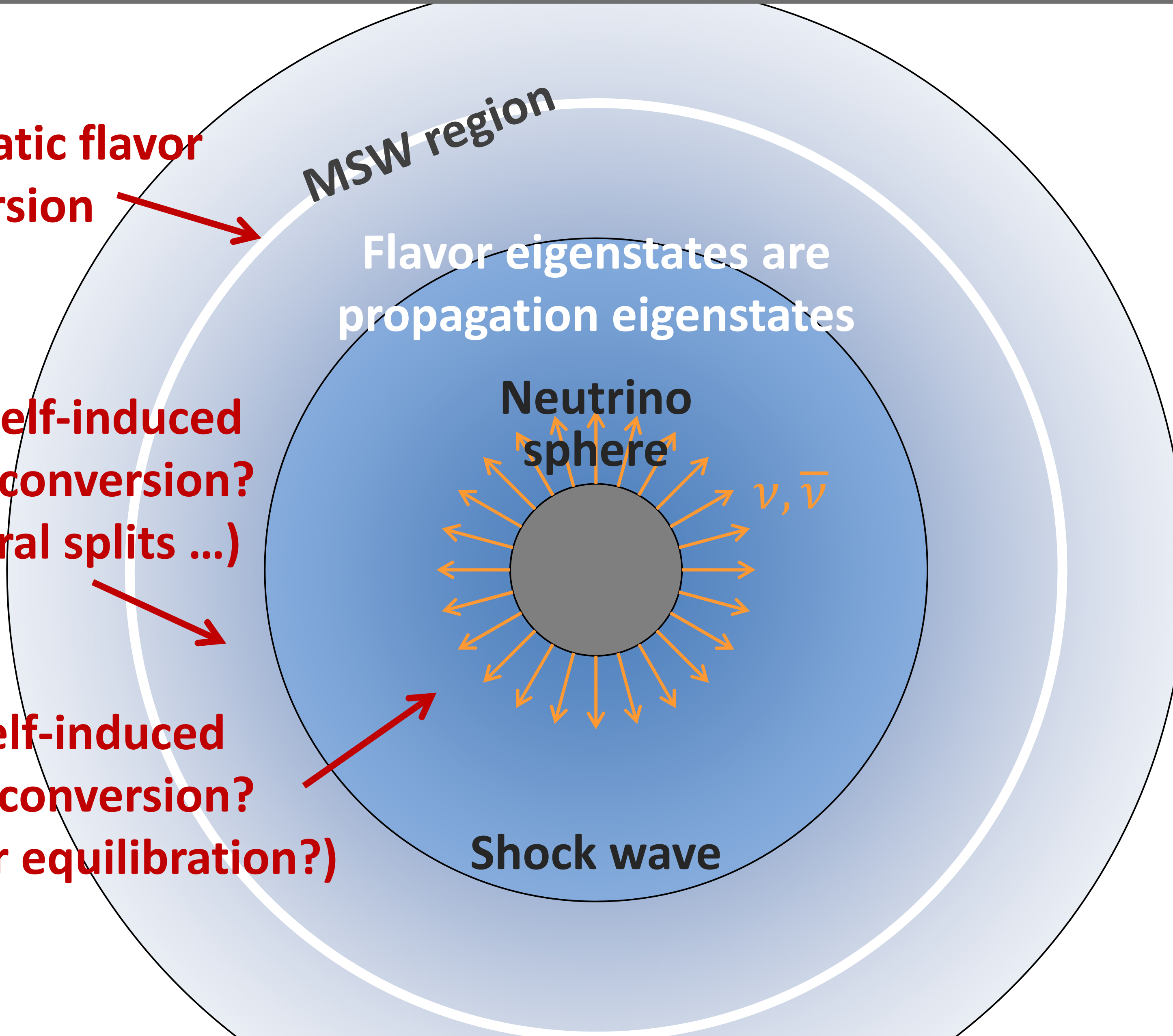
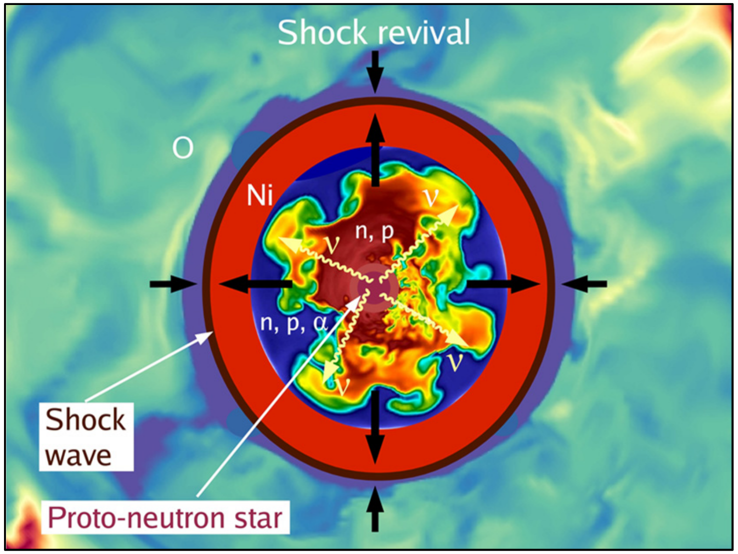
Flavor Conversion in Core-Collapse Supernovae

- **Adiabatic flavor conversion**
- **Slow self-induced flavor conversion? (Spectral splits …)**
- **Fast self-induced flavor conversion? (Flavor equilibration?)**



Why worry about detailed neutrino transport?



- Explosion mechanism:
  Shock-wave revival by nu energy deposition
- Nucleosynthesis in neutrino irradiated outflows in SNe and NS-mergers depends on flavor (beta reactions!)
- Signal interpretation of DSNB and next nearby SN
- **Collective flavor conversion: interesting theoretical problem in its own right**

Kinetic Equation for Neutrino Transport

Flavor-dependent phase-space densities (occupation number matrices)

$$\varrho = \begin{pmatrix} f_{\nu_e} & f_{\langle\nu_e|\nu_\mu\rangle} & f_{\langle\nu_e|\nu_\tau\rangle} \\ f_{\langle\nu_\mu|\nu_e\rangle} & f_{\nu_\mu} & f_{\langle\nu_\mu|\nu_\tau\rangle} \\ f_{\langle\nu_\tau|\nu_e\rangle} & f_{\langle\nu_\tau|\nu_\mu\rangle} & f_{\nu_\tau} \end{pmatrix}$$

Diagonal: Usual occupation numbers
Off-diag: Flavor coherence information

and similar for $\overline{\nu}$

Transport equation

$$\Big(\underbrace{\partial_t + \vec{v}\cdot\vec{\nabla}_x}_{\text{Streaming}} - \underbrace{\vec{F}\cdot\vec{\nabla}_p}_{\substack{\text{Gravitational forces}\\ \text{(redshift, deflection)}}}\Big)\,\varrho(t,\vec{x},\vec{p}) = \underbrace{-\mathrm{i}\,[\mathcal{H}(t,\vec{x},\vec{p}),\varrho(t,\vec{x},\vec{p})]}_{\substack{\text{Flavor oscillations}\\ (\text{vacuum, matter, } \nu\nu)}} + \underbrace{\mathcal{C}[\varrho(t,\vec{x},\vec{p})]}_{\text{Collisions}}$$

- $e^- + p \rightleftharpoons n + \nu_e$
- $e^+ + n \rightleftharpoons p + \bar{\nu}_e$
- $e^- + A \rightleftharpoons \nu_e + A^*$

β

- $\nu + n, p \rightleftharpoons \nu + n, p$
- $\nu + A \rightleftharpoons \nu + A$
- $\nu + e^\pm \rightleftharpoons \nu + e^\pm$
- $N + N \rightleftharpoons N + N + \nu + \bar{\nu}$
- $e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$
- $\nu_x + \nu_e, \bar{\nu}_e \rightleftharpoons \nu_x + \nu_e, \bar{\nu}_e$
  $(\nu_x = \nu_\mu,\ \bar{\nu}_\mu, \nu_\tau, \text{ or } \bar{\nu}_\tau)$
- $\nu_e + \bar{\nu}_e \rightleftharpoons \nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau}$

Typical approximations in numerical simulations:

- **Reducing 6+1 dimensions**
  (Angular moments, ray-by-ray, …)
- **No gravitational deflection**
- **No flavor conversion (large matter effect!)**
- **No muons**
- **3-species transport: $\nu_e, \overline{\nu}_e, \nu_x$**

Kinetic Equation for Neutrino Transport

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Flavor evolution governed by "Hamiltonian matrix" (here for 2 flavors)

$$\mathcal{H} = \underbrace{\frac{\Delta m^2}{4E}\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}}_{\text{Vacuum oscillations}} + \underbrace{\sqrt{2}G_\mathrm{F}\begin{pmatrix} n_e & 0 \\ 0 & 0 \end{pmatrix}}_{\text{MSW effect}} + \underbrace{\sqrt{2}G_\mathrm{F}\int\frac{d^3\vec{p}}{(2\pi)^3}(\varrho+\overline{\varrho})}_{\substack{\text{Nu-nu interactions,} \\ \text{nus feed back on each other}}}$$

- **Flavor evolution is caused by off-diagonal $\mathcal{H}$ elements (vacuum or nu-nu term)**
- **For $\Delta m^2 = 0$, nu-nu term can still cause run-away modes!**

Self-Induced Flavor Conversion

Flavor conversion (vacuum or MSW) for a neutrino of given momentum p

- Requires lepton flavor violation by masses and mixing

$$\nu_e(p) \to \nu_\mu(p)$$

$$\boxed{\frac{\Delta m^2_{\rm atm}}{2E} = 10^{-10}\,\mathrm{eV} = 0.5\ \mathrm{km}^{-1}}$$

Pair-wise flavor exchange by ν–ν refraction (forward scattering)

- No net flavor change of pair
- Requires dense neutrino medium (collective effect of interacting neutrinos)
- **Can occur without masses/mixing (and then does not depend on $\Delta m^2/2E$)**
- Familiar as neutrino pair process $\mathcal{O}(G_{\rm F}^2)$ Here as coherent refractive effect $\mathcal{O}(G_{\rm F})$

$$\nu_e(p) + \overline{\nu}_e(k) \to \nu_\mu(p) + \overline{\nu}_\mu(k)$$

$$\nu_e(p) + \nu_\mu(k) \to \nu_\mu(p) + \nu_e(k)$$

$$\boxed{\sqrt{2}G_{\rm F}n_\nu = 10^{-5}\,\mathrm{eV} = 0.5\ \mathrm{cm}^{-1}}$$

$$E = 12.5\ \mathrm{MeV}$$
$$R = 80\ \mathrm{km}$$
$$L_\nu = 40 \times 10^{51}\,\mathrm{erg/s}$$

Transport of Particles and Flavor Coherence

How to interpret the Liouville operator (advection term)?

↓

$$\mathcal{D}\,\varrho(t,\vec{x},\vec{p}) = -\mathrm{i}\,[\mathcal{H}(t,\vec{x},\vec{p}),\varrho(t,\vec{x},\vec{p})] + \mathcal{C}[\varrho(t,\vec{x},\vec{p})]$$

- Flavor evolution along trajectories (of individual neutrinos or wave packets)

$$\frac{d}{ds}\varrho(\underbrace{t,\vec{x}}_{\downarrow},\vec{p}) = -\mathrm{i}\,[\mathcal{H}(t,\vec{x},\vec{p}),\varrho(t,\vec{x},\vec{p})] + \mathcal{C}[\varrho(t,\vec{x},\vec{p})]$$

Time and space coordinates related by neutrino velocity along trajectory

- Flavor evolution in space and time (of neutrino radiation field)

$$\partial_t\varrho + \frac{1}{2}\{\underbrace{\vec{\nabla}_p\mathcal{H}}_{\downarrow},\vec{\nabla}_x\varrho\} - \frac{1}{2}\{\vec{\nabla}_x\mathcal{H},\vec{\nabla}_p\varrho\} = -\mathrm{i}\,[\mathcal{H},\varrho] + \mathcal{C}[\varrho]$$

Velocity operator (matrix in flavor space)

Neutrino quantum field obeys the Dirac equation

Kinetic equation for ϱ is always an approximation / simplification / expansion

Different intuitive pictures should give the same answer for observables

Velocity in Advection Term

- Advection term after "separation of scales" between gradients and neutrino $\vec{p}$
- Wigner function $\varrho(t,\vec{x},\vec{p})$ makes sense as a phase-space density

$$\partial_t\varrho+\frac{1}{2}\{\underbrace{\vec{\nabla}_p\mathcal{H}}_{\downarrow},\vec{\nabla}_r\varrho\}-\frac{1}{2}\{\vec{\nabla}_x\mathcal{H},\vec{\nabla}_p\varrho\}=-\mathrm{i}\,[\mathcal{H},\varrho]+\mathcal{C}[\varrho]$$

Velocity operator (matrix in flavor space) $\quad \vec{\nabla}_p E_{\mathrm{kin}}=\vec{\nabla}_p\sqrt{\vec{p}^2+m^2}=\frac{\vec{p}}{E}=\vec{v}$

Ultrarelativistic limit and ignore gravity or refractive deflection

$$\partial_t\varrho+\underset{\downarrow}{\vec{v}}\cdot\vec{\nabla}_r\varrho=-\mathrm{i}\left[\frac{\mathcal{M}^2}{2\underset{\downarrow}{E}}+\mathcal{H}_{\mathrm{matter}}+\mathcal{H}_{\nu\nu},\varrho\right]+\mathcal{C}[\varrho]$$

Unit vector $\qquad E=|\vec{p}|$

Leading-order effect of ν masses on the rhs in the usual oscillation term

Stirner, Sigl & Raffelt: ***Liouville term for neutrinos: Flavor structure and wave interpretation,*** JCAP 1805 (2018) 016 [arXiv:1803.04693]

Kinetic equations for flavor transport

Rudszky: ***Kinetic equations for neutrino spin- and type-oscillations in a medium***,
Astrophys. Space Sci. 165 (1990) 65

Raffelt & Sigl: ***General kinetic description of relativistic mixed neutrinos***,
NPB 406 (1993) 423

Sirera & Perez: ***Relativistic Wigner function approach to neutrino propagation in matter,***
PRD 59 (1999) 125011 [hep-ph/9810347]

Yamada: ***Boltzmann equations for neutrinos with flavor mixings***,
PRD 62 (2000) 093026 [astro-ph/0002502]

Cardall: ***Liouville equations for neutrino distribution matrices,***
PRD 78 (2008) 085017 [arXiv:0712.1188]

Vlasenko, Fuller & Cirigliano: ***Neutrino quantum kinetics***,
PRD 89 (2014) 105004 [arXiv:1309.2628]

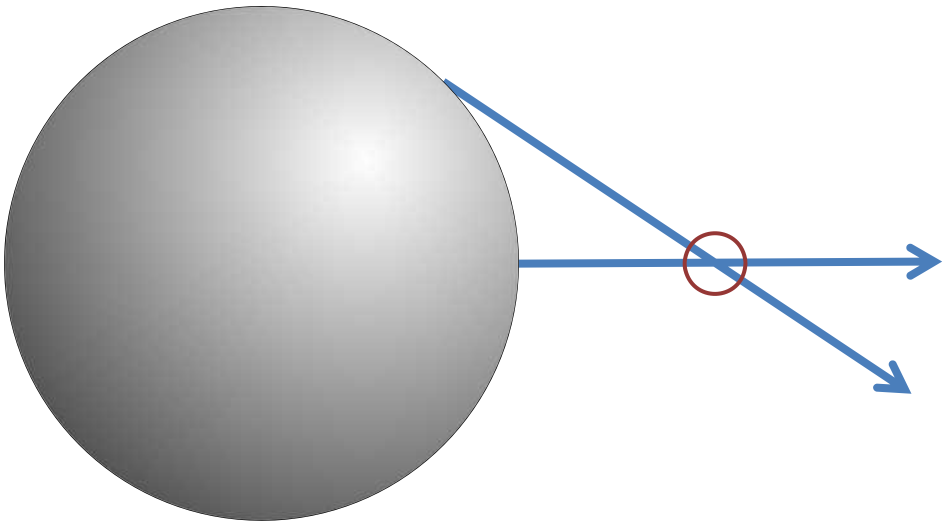
Serreau & Volpe: ***Neutrino-antineutrino correlations in dense anisotropic media,***
PRD 90 (2014) 125040 [arXiv:1409.359]

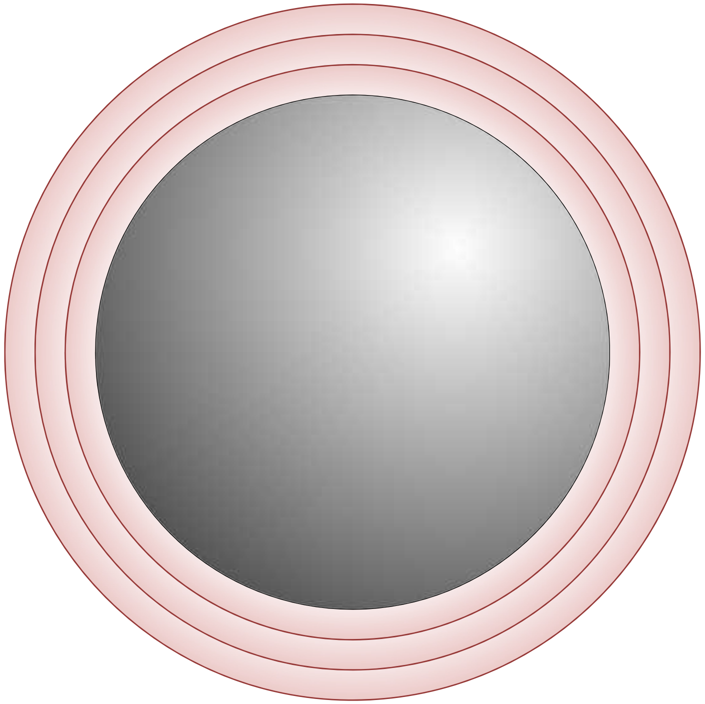
Hansen & Smirnov: ***The Liouville equation for flavour evolution of neutrinos and neutrino wave packets,*** JCAP 1612 (2016) 019 [arXiv:1610.00910]

Stirner, Sigl & Raffelt*:* ***Liouville term for neutrinos: Flavor structure and wave interpretation,***
JCAP 1805 (2018) 016 [arXiv:1803.04693]

Correlated Trajectories vs. Field of Flavor Coherence

Assume globally spherically symmetric neutrino emission from SN core





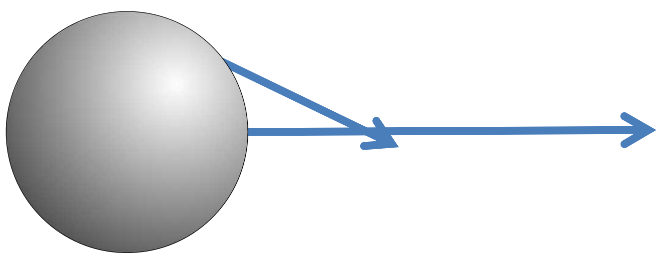
- Every ν meets every other ν at most once
- Nonlinear feedback on flavor evolution?

- Oscillating (or unstable) field $\varrho(r)$ of flavor coherence, acting back upon itself
- Do not worry about individual neutrinos

Evolution of the Questions

Bulb model of neutrino emission:

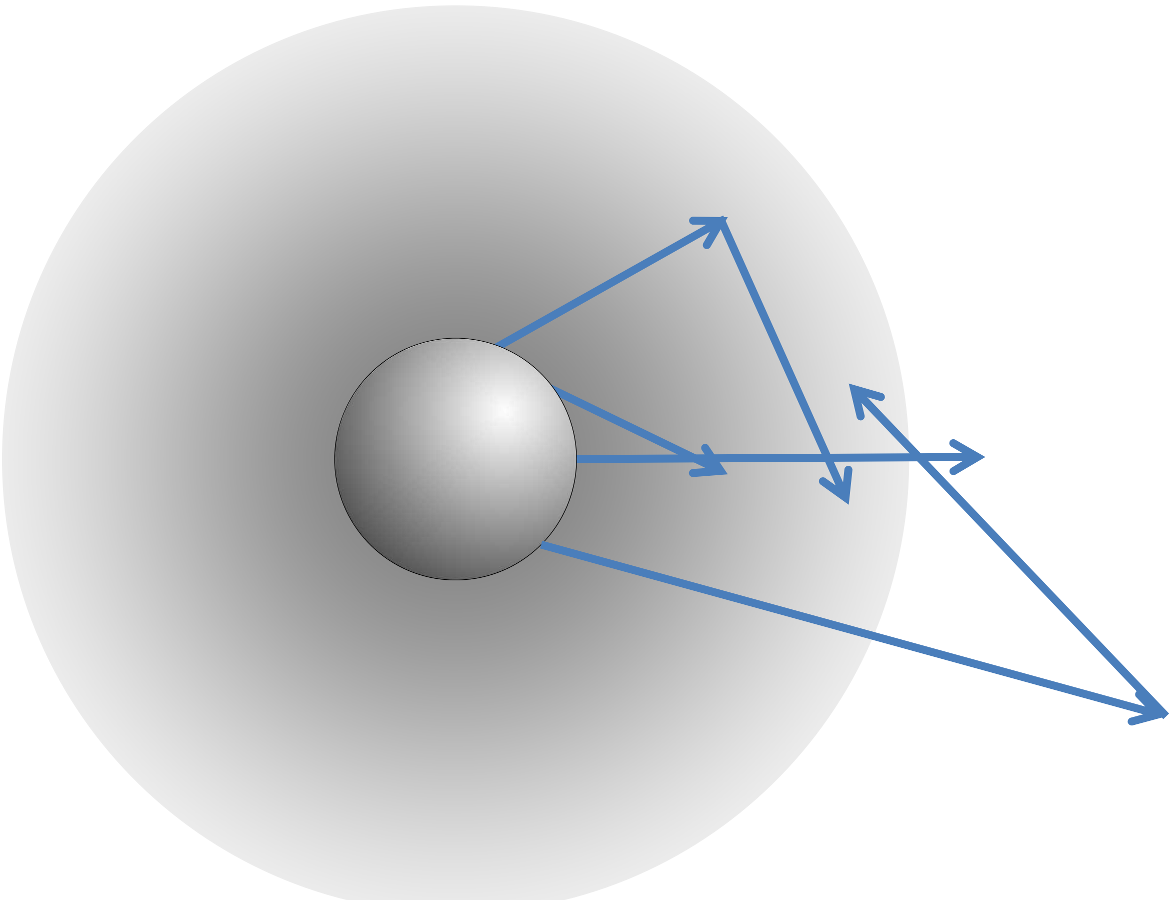
- Nu-nu interaction determined by aspect ratio of emission surface
- Instability as a function of radius – adiabatic conversion possible
- Flavor pendulum, spectral splits, multi-angle matter effect, three-flavor effects, …
- Spurious instabilities (need many angle bins in numerical studies)
- Instability in the transverse direction: Spontaneous symmetry breaking



Evolution of the Questions

Halo effect:

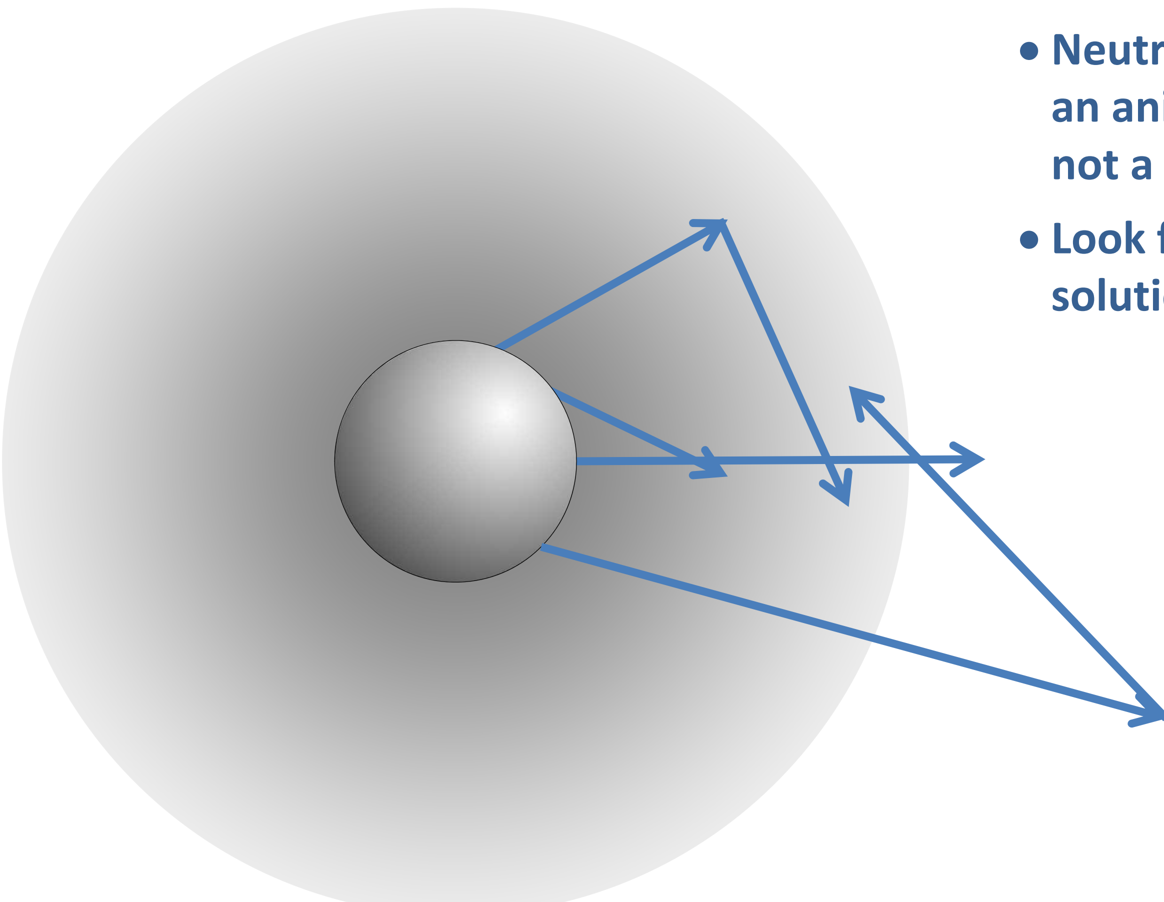
- Small re-scattered flux, much larger angular leverage
- Impact on collective effects?
- How to deal with backward flux?



Evolution of the Questions

Non-stationarity:

- Time-variation (of SN emission) in source region?
- Self-induced time variation, “pulsating modes” more unstable than stationary ones?



- **Neutrino field begins to look like an anisotropic “gas” or “medium”, not a “beam”**
- **Look for full space-time dependent solution $\varrho(t,\vec{x})$**

Fast Flavor Conversion

Flavor evolution governed by "Hamiltonian matrix" (here for 2 flavors)

$$\mathcal{H}_{\vec{p}} = \frac{\Delta m^2}{4E}\begin{pmatrix}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{pmatrix} + \mathcal{H}_{\rm mat} + \sqrt{2}G_{\rm F}\int \frac{d^3\vec{p}'}{(2\pi)^3}(1-\vec{v}\cdot\vec{v}')\begin{pmatrix} f_{\nu_e} & f_{\langle\nu_e|\nu_x\rangle} \\ f_{\langle\nu_x|\nu_e\rangle} & f_{\nu_x}\end{pmatrix}$$

Flavor conversion caused by off diagonals

Energy scales of the problem:

$\mu = \sqrt{2}\,G_{\rm F}\,n_{\nu\bar\nu}$ *Required for any collective effects*

$\omega_E = \Delta m^2/2E$ *Vacuum oscillation frequency*

$\lambda = \sqrt{2}\,G_{\rm F}\,n_{\rm e}$ *Matter effect*

Slow modes:

Require $\omega_E \neq 0$

Possible growth rate: $\kappa \sim \sqrt{\mu\omega_E}$, requires "crossing" of ω_E distribution

Fast modes:

Dynamical even for $\omega_E = 0$

Growth rate: $\kappa \sim \sqrt{\mu\omega_E}$ slow growth

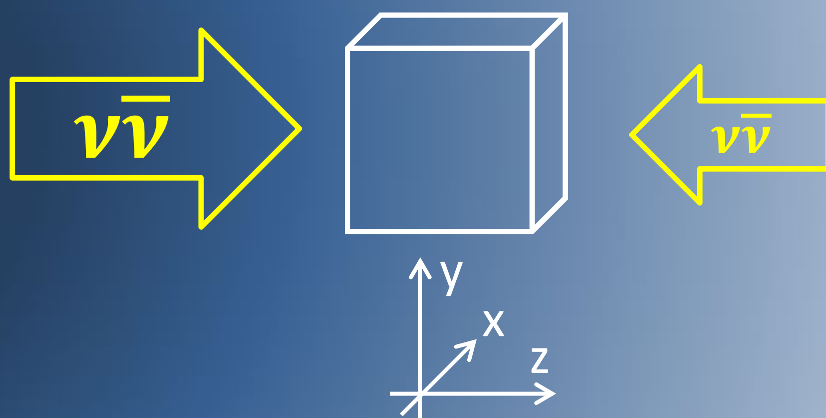
$\kappa \sim \mu$ fast growth, requires "crossing" of angle distribution

Neutrino gas in the near-free streaming regime

Small test volume for fast modes

- Homogeneous conditions
- Need $f_\nu(E,\theta,\varphi)$ for all species
- Large mean free path
- What is flavor evolution (t,x,y,z) ?

PNS



Shock

Linearised problem:
Study dispersion relation of
“flavor waves”

Linearisation for Fast Flavor Modes

Evolution equation: $iv^\alpha\partial_\alpha\varrho = [\mathcal{H},\varrho]$ with $v^\alpha = (1,\vec{v})$

Linearisation to find (propagating or unstable) collective modes:

$$\varrho = \begin{pmatrix} f_{\nu_e} & f_{\langle\nu_e|\nu_x\rangle} \\ f_{\langle\nu_x|\nu_e\rangle} & f_{\nu_x} \end{pmatrix} = \frac{f_{\nu_e}+f_{\nu_x}}{2} + \frac{f_{\nu_e}-f_{\nu_x}}{2}\begin{pmatrix} s & S \\ S^* & -s \end{pmatrix}$$

Field of flavor coherence

Linearised EOM for field of flavor coherence

$$iv^\alpha\partial_\alpha S_{\vec{p}} = \Bigg(\underbrace{\frac{\Delta m^2}{2E}}_{\text{Ignore for fast modes}} + v^\alpha\Lambda_\alpha\Bigg)S_{\vec{p}} - \mu v^\alpha \underbrace{\int\frac{d^3\vec{p}\,'}{(2\pi)^3}v'_\alpha\left(g_{\vec{p}\,'}S_{\vec{p}\,'} - \overline{g}_{\vec{p}\,'}\overline{S}_{\vec{p}\,'}\right)}_{\text{Same for all } E \text{ and } \nu \text{ and } \overline{\nu}}$$

Angle distribution of electron lepton number (ELN) carried by neutrinos

$$G_{\vec{v}} = \int\frac{dE\,E^2}{2\pi^2}\,\frac{f_{\nu_e,\vec{p}} - f_{\overline{\nu}_e,\vec{p}} - f_{\nu_x,\vec{p}} + f_{\overline{\nu}_x,\vec{p}}}{2}$$

Linearised EOM for field of flavor coherence

$$iv^\alpha(\partial_\alpha + i\Lambda_\alpha)S_{\vec{v}} = -\mu v^\alpha\int\frac{d\vec{v}\,'}{4\pi}v'_\alpha G_{\vec{v}\,'}S_{\vec{v}\,'}$$

Matter effect, “rotate away” by including it in derivative
if medium is homogeneous and stationary

Dispersion Relation for Fast Flavor Modes

Linearised EOM for field of flavor coherence – a wave equation

$$iv^\alpha \partial_\alpha S_{\vec{v}} = -\mu v^\alpha \int \frac{d\vec{v}'}{4\pi} v'_\alpha G_{\vec{v}'} S_{\vec{v}'}$$

Plane-wave ansatz

$$S_{\vec{v}}(t,\vec{r}) = Q_{\vec{v}}\big(\Omega,\vec{K}\big)\, e^{-i(\Omega t - \vec{K}\cdot\vec{r})}$$

EOM in Fourier space

$$\big(\Omega - \vec{v}\cdot\vec{K}\big) Q_{\vec{v}} = -\mu \int \frac{d\vec{v}'}{4\pi} (1 - \vec{v}\cdot\vec{v}')\, G_{\vec{v}'} Q_{\vec{v}'}$$

Non-collective solutions:

$(\Omega,\vec{\mathrm{K}})$ real and “below the light cone”

$\big(\Omega - \vec{v}\cdot\vec{K}\big) = 0$ for some mode $\vec{v}$

Dispersion relation:

$\Omega = \vec{v}\cdot\vec{K}$

for every $\vec{K}$ continuos infinity of frequencies

Collective solutions:

$\big(\Omega - \vec{v}\cdot\vec{K}\big) \neq 0$ for all $\vec{v}$ modes

$(\Omega,\vec{\mathrm{K}})$ real and “outside the light cone” or imaginary part

Eigenfunctions $Q_{\vec{v}} \propto \frac{\vec{a}\cdot\vec{v}+b}{\Omega-\vec{v}\cdot\vec{K}}$

Dispersion relation: $\det \Pi = 0$

$$\Pi^{\mu\nu} = \eta^{\mu\nu} + \int \frac{d\vec{v}}{4\pi} G_{\vec{v}} \frac{v^\mu v^\nu}{\Omega - \vec{v}\cdot\vec{K}}$$

PRL **118,** 021101 (2017) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2017

Fast Pairwise Conversion of Supernova Neutrinos: A Dispersion Relation Approach

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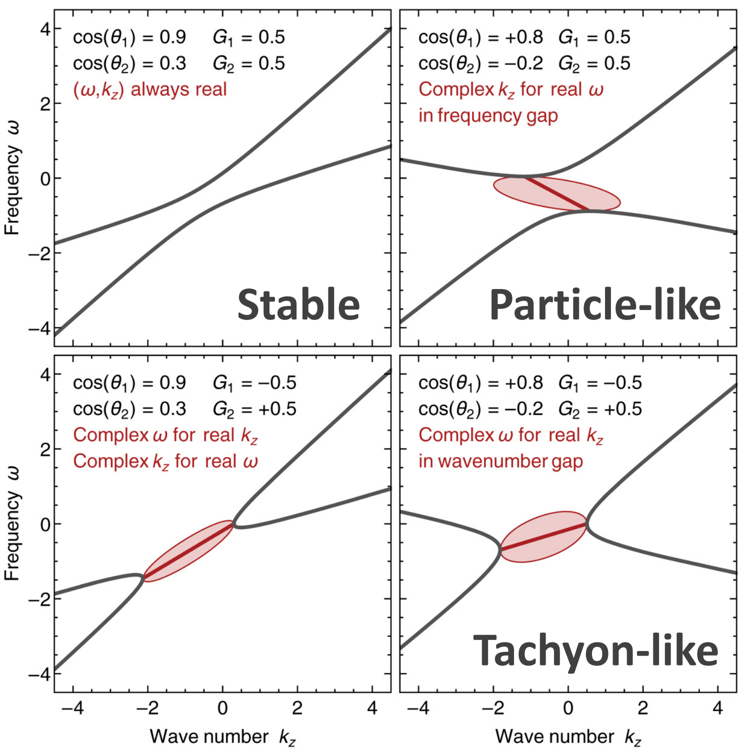
(Received 10 October 2016; published 10 January 2017)

Classification of instabilities of “flavor waves” (Two-beam model)



Classification of instabilities of plasma waves (Two-beam model)

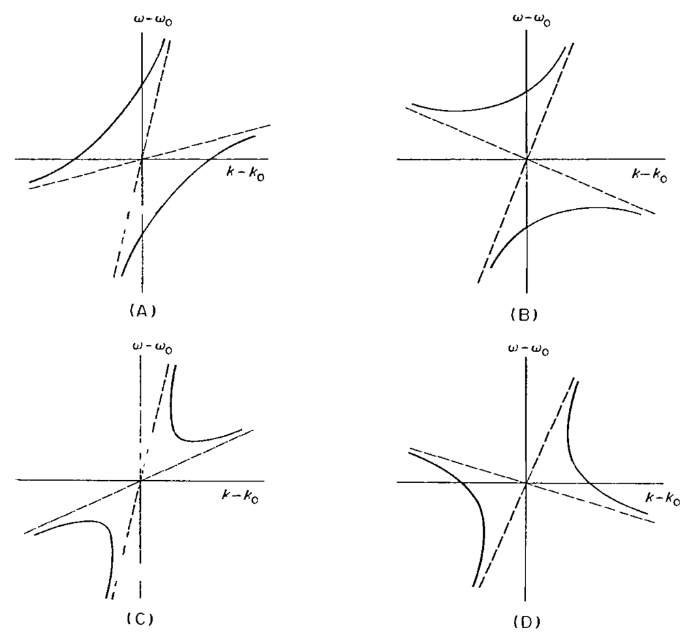
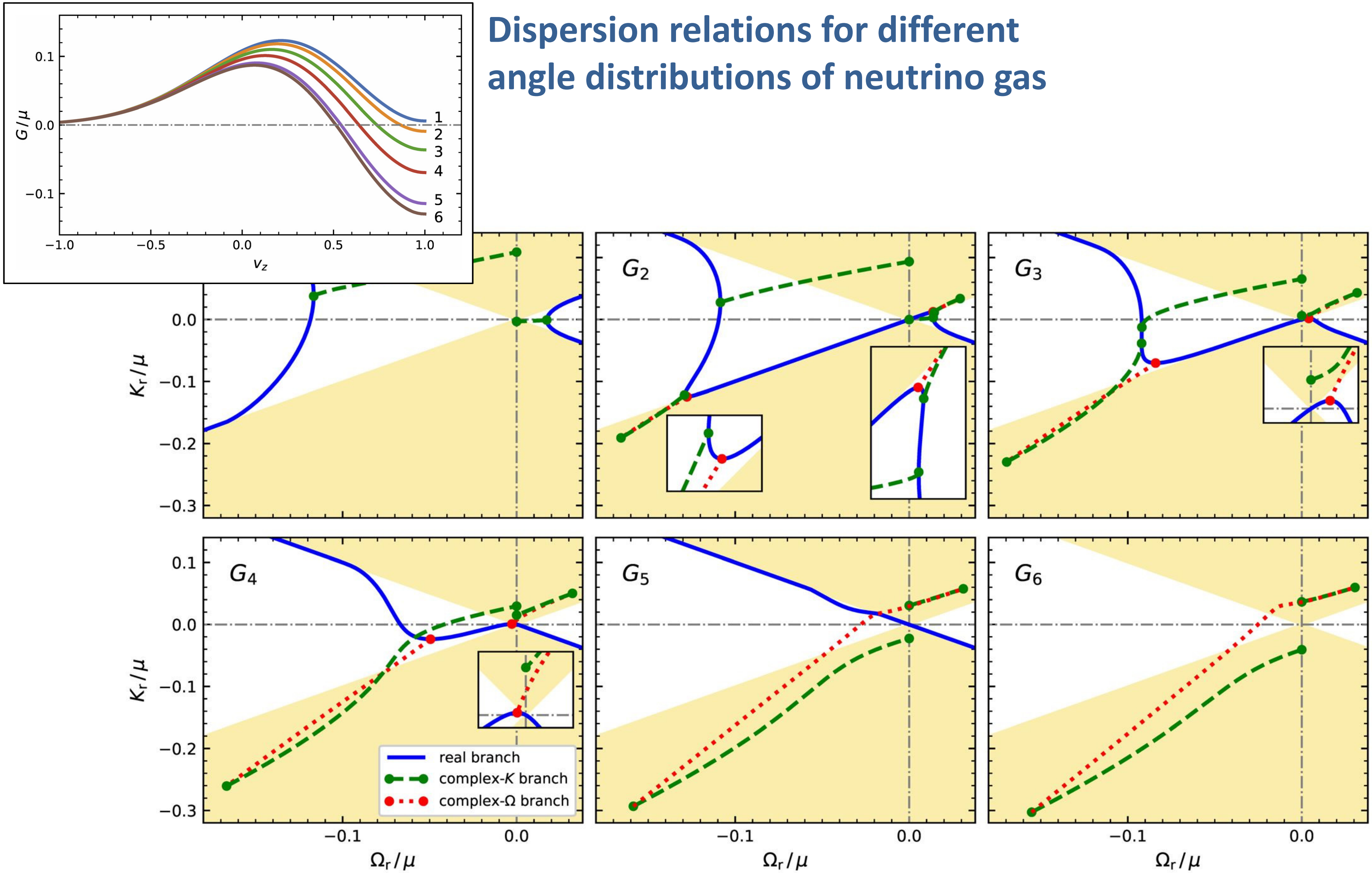


FIG. 23.

Landau & Lifshitz, Vol.10, Physical Kinetics
Chapter VI, Instability Theory

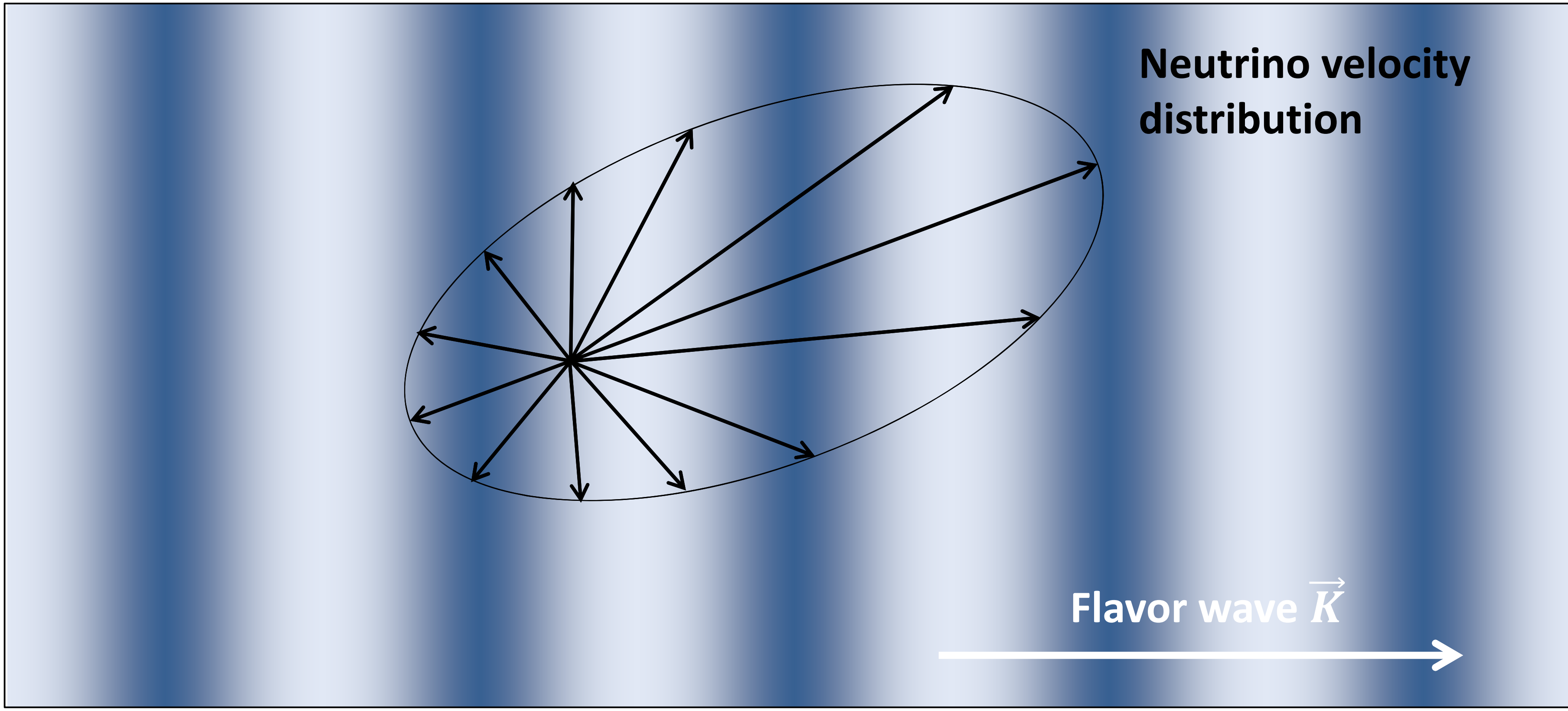
Dispersion Relation for Neutrino Flavor Waves

Dispersion relations for different angle distributions of neutrino gas



Yi, Ma, Martin & Duan, PRD 99 (2019) 063005 [arXiv:1901.01546]

Fast Flavor Waves



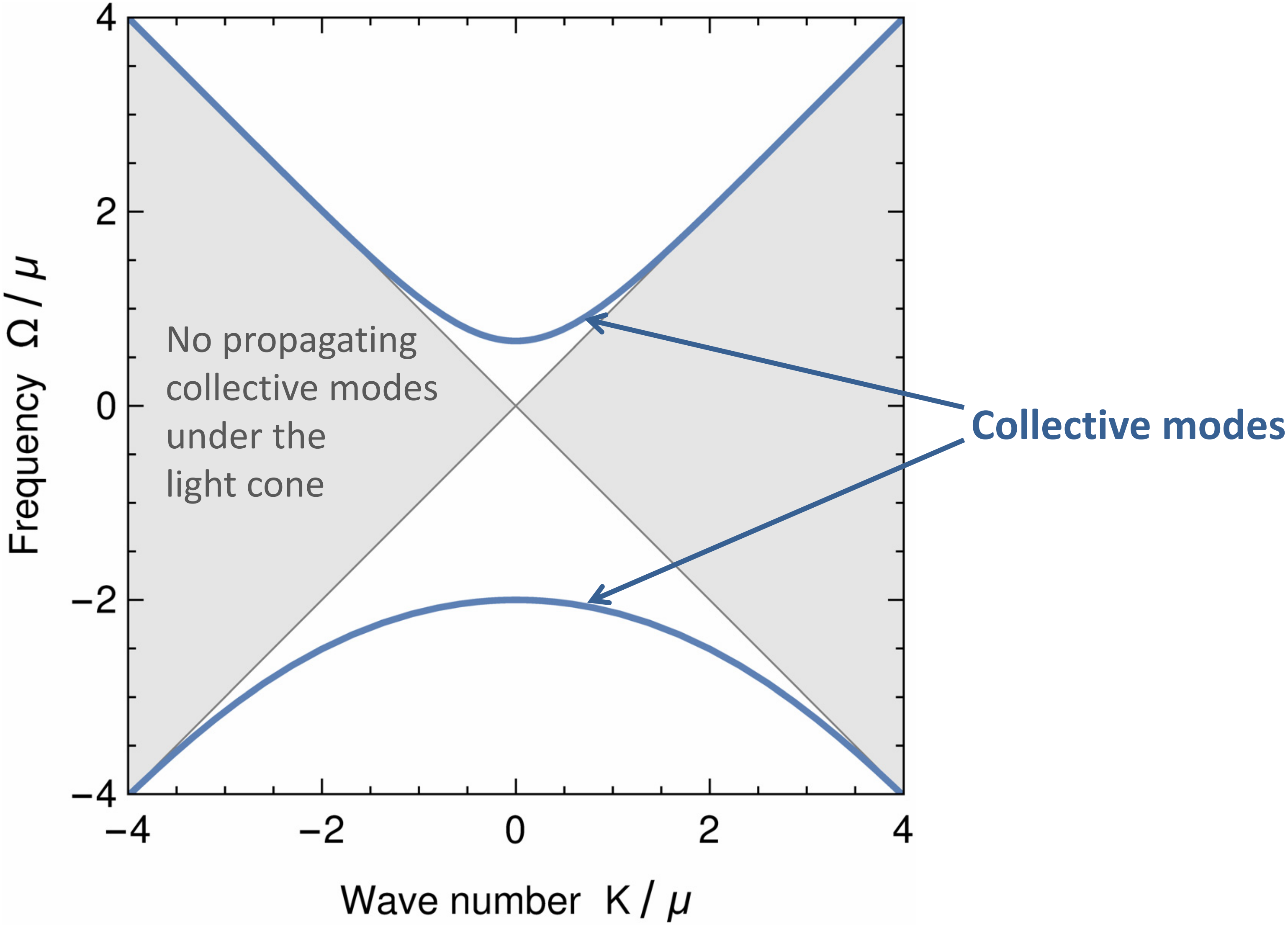
Non-collective modes: **Infinitely many $\Omega = \vec{\mathbf{v}} \cdot \vec{K}$**

- Flavor coherence carried by every neutrino mode separately
- Quick kinematical decoherence

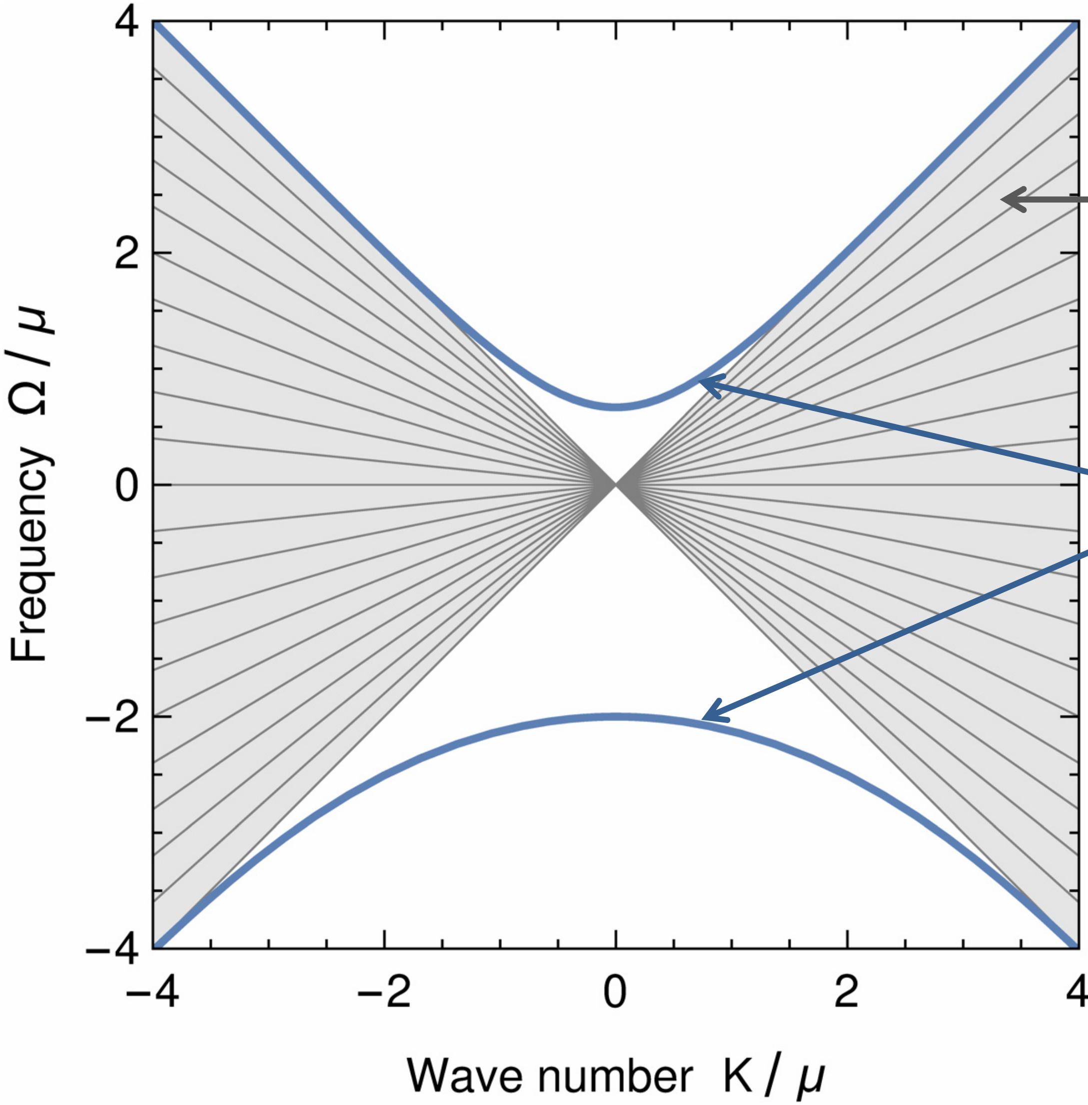
Collective modes: $\boldsymbol{\Omega}(\vec{K})$ according to collective dispersion relation

- Flavor wave (or wave packet) propagates and/or grows

Dispersion Relation for Isotropic Case



Dispersion Relation for Isotropic Case



Non-collective modes with

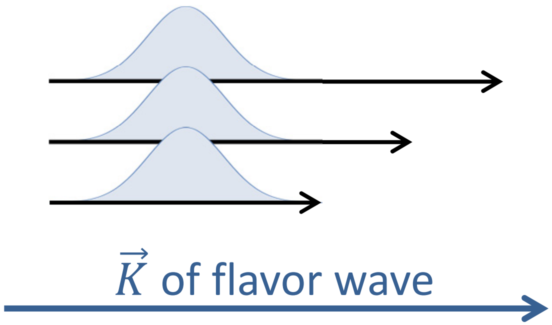
$\Omega = \vec{v} \cdot \vec{K}$

carried by neutrino "beams"

Collective modes

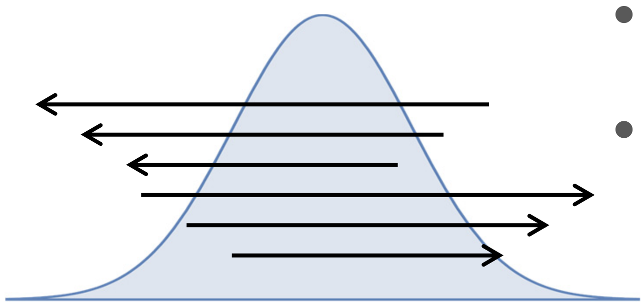
Fast Flavor Waves

Non-collective modes:



- Infinitely many neutrino velocity projections on $\vec{K}$
- Each carries along its initial flavor coherence
- Kinematical decoherence of initial wave packet (Does not happen in two-beam model)
- Fast dissipation of any initial wave packet

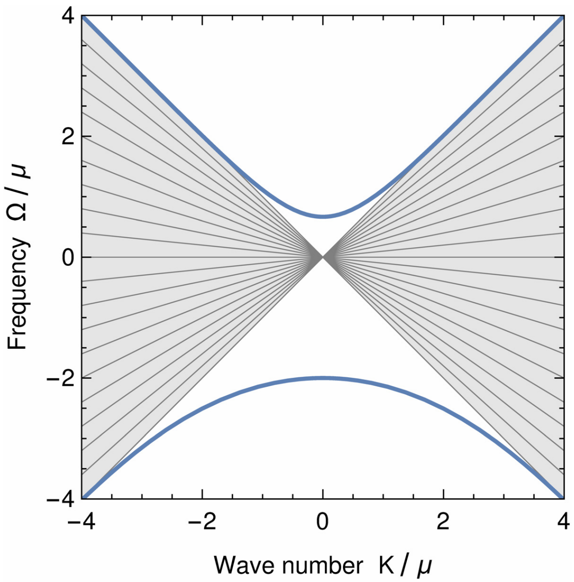
Collective modes



Wave packet of flavor coherence

- Infinitely many neutrino velocity projections on $\vec{K}$
- Move through wave packet (here taken with vanishing central wave number)
- Wave packet moves in neutrino gas, independently of velocities of neutrino "beams"

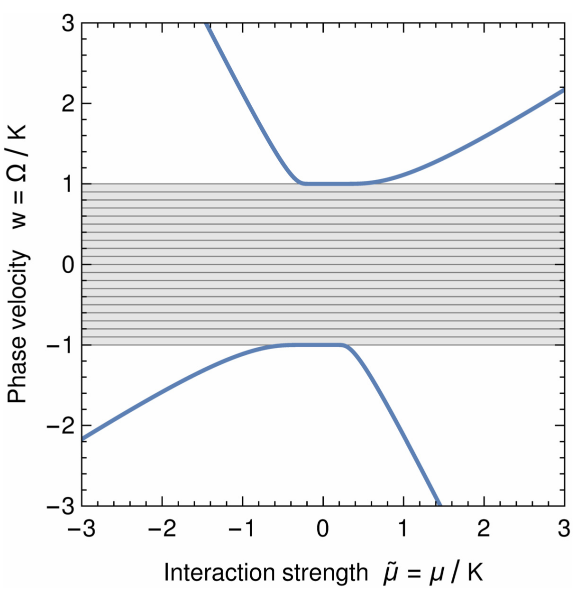
Dispersion Relation vs. Eigenvalues of Hamiltonian



Dispersion relation:

For fixed μ find $\Omega(K)$ from

$$\left(\Omega - \vec{v}\cdot\vec{K}\right)Q_{\vec{v}} = -\mu\int\frac{d\vec{v}'}{4\pi}\left(1-\vec{v}\cdot\vec{v}'\right)G_{\vec{v}'}Q_{\vec{v}'}$$



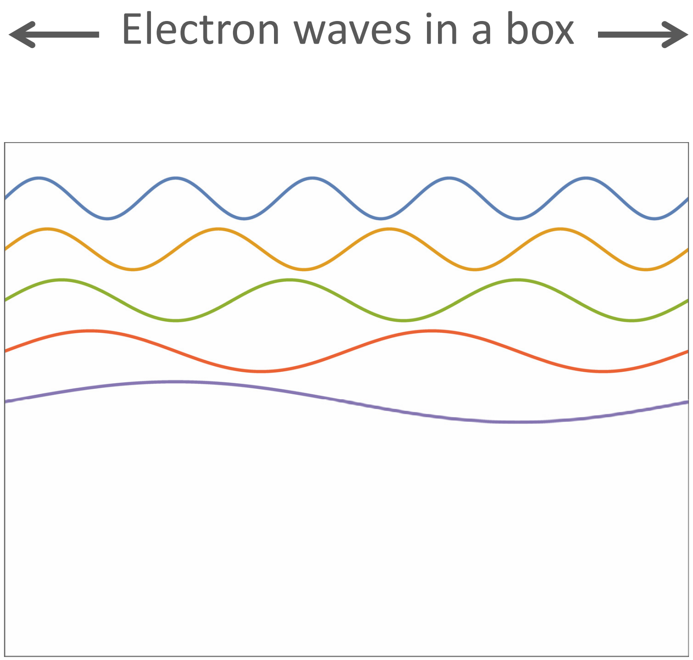
Eigenvalues of Hamiltonian:

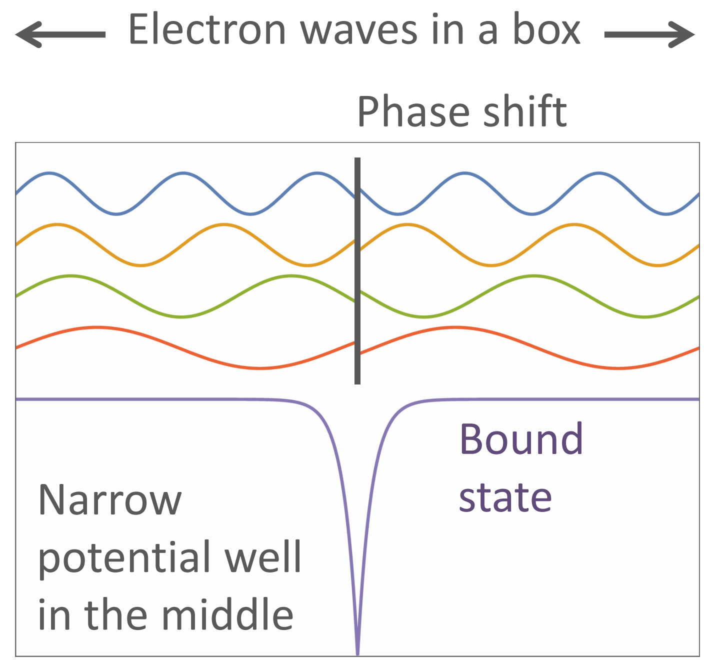
For fixed K find eigenvalues $w = \Omega(\mu)/K$ of $\mathcal{H}$

$$i\partial_t S_{\vec{v}}\left(t,\vec{K}\right) = \mathcal{H}\left(S_{\vec{v}}\right)$$

$$\mathcal{H}\left(S_{\vec{v}}\right) = \vec{v}\cdot\vec{K}\,S_{\vec{v}} - \mu\int\frac{d\vec{v}'}{4\pi}\left(1-\vec{v}\cdot\vec{v}'\right)G_{\vec{v}'}S_{\vec{v}'}$$

Bound vs Scattering States





Continuum limit: Box size $\to \infty$

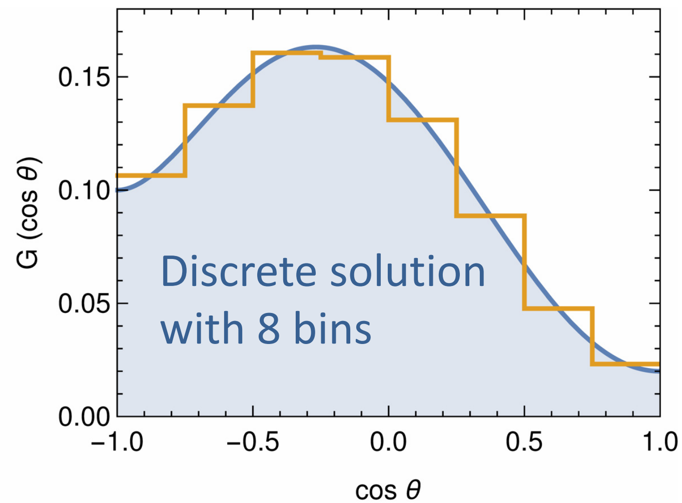
Continuum of scattering states

Continuum of scattering states (with phase shifts) + Bound state

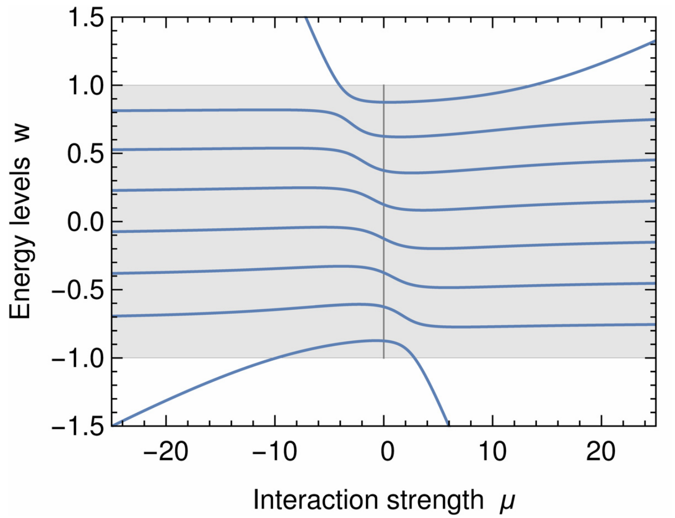
Non-collective modes ~ Scattering states
Collective modes ~ Bound states

No spectral crossing

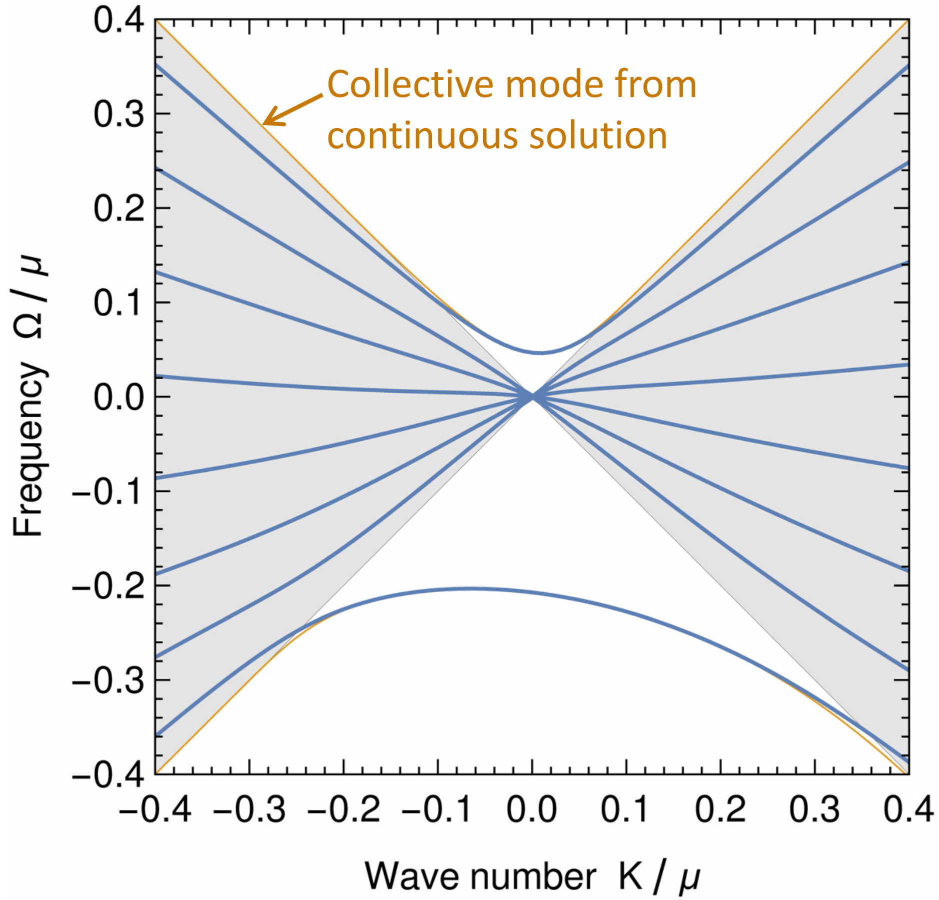
ELN angle distribution



Energy levels

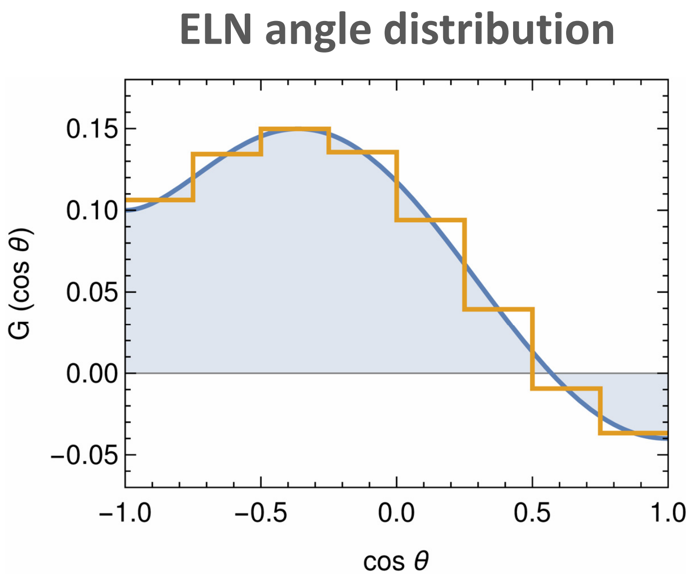


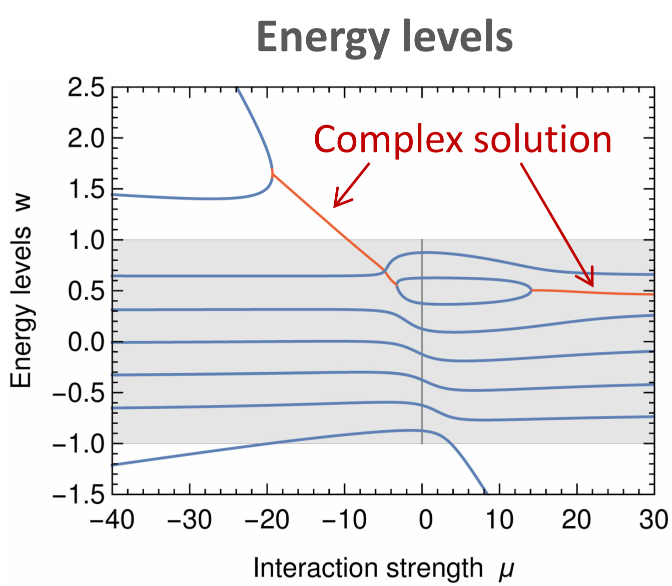
Dispersion relation

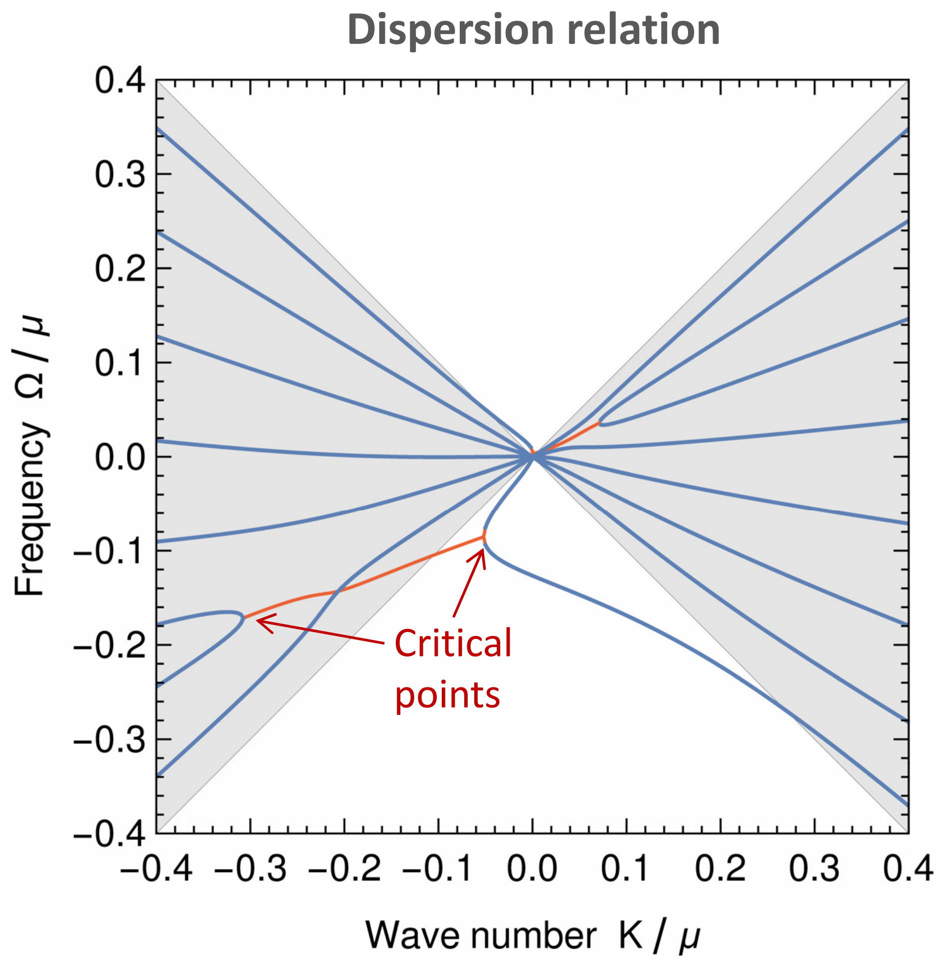


- **Two propagating collective modes**
- **"Peel off" from non-collective modes**

"Weak" spectral crossing



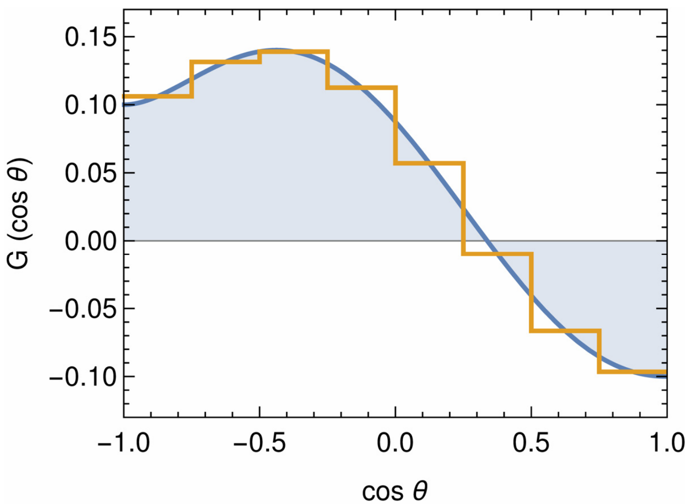




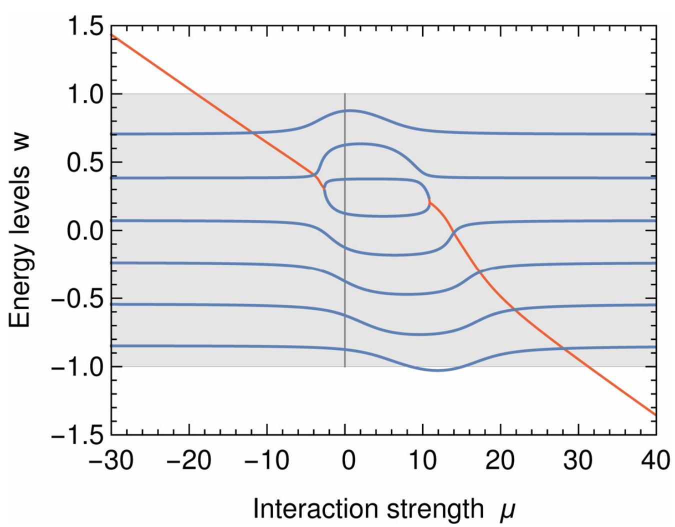
- **Propagating and unstable collective modes**
- **Unstable modes begin under the light cone**
- **No spurious instabilities in discrete case**

“Strong” spectral crossing

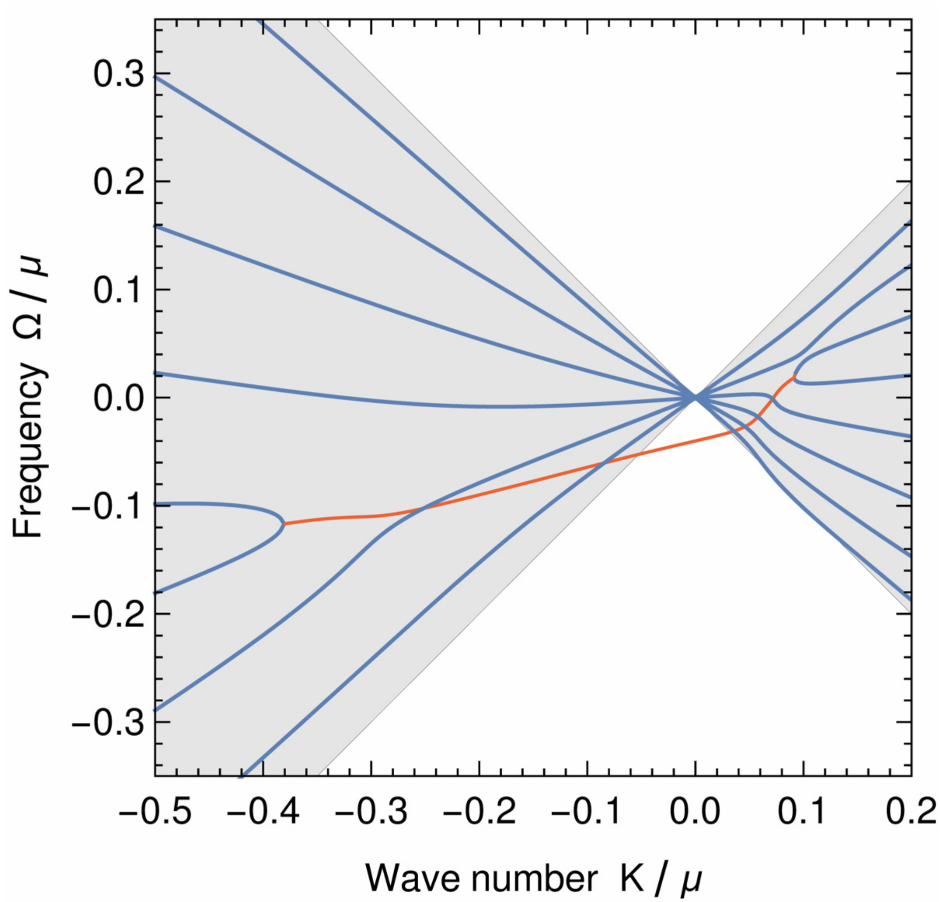
ELN angle distribution



Energy levels

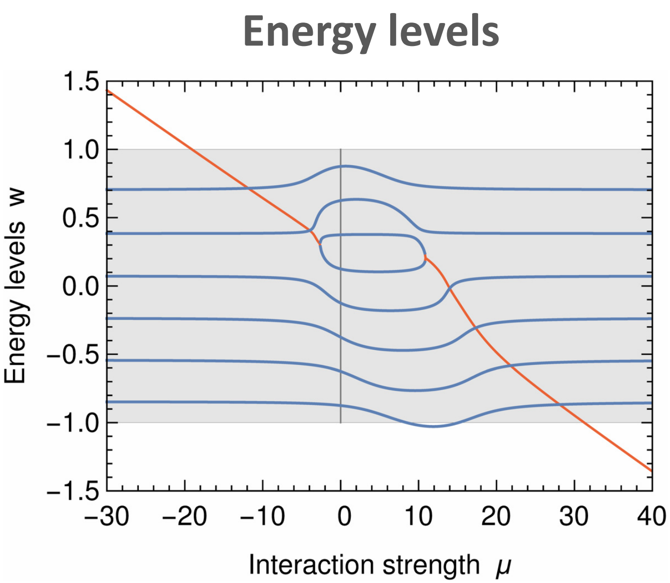


Dispersion relation

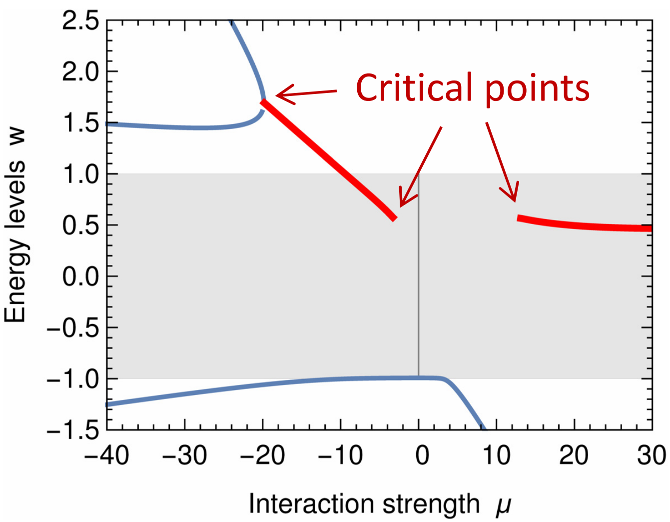


- **Only unstable collective modes**
- **Begin and end under the light cone**

Spectral crossing – Continuous Limit



- Solutions with complex eigenvalues appear as merging of two real eigenvalues
- With increasing μ must emerge below the light cone



Continuous limit of vanishing mode spacing:

- Critical points at $w=\cos(\theta_0)$, i.e. at crossing where $G(\cos\theta_0)=0$
- Interaction strength $\mu_{1,2}$ of critical points follow easily from $G(\cos\theta)$
- Single crossing: Complex solution guaranteed
- Several crossings: Not guaranteed

→ *See Tobias Stirner's talk*

Summary

- Neutrino-neutrino interactions lead to emergence of collective modes of flavor coherence (propagating or unstable)
- Need not exist for every $\vec{K}/\mu$ (dispersion relations can end)
- Co-exist with non-collective modes
- "Wave packet of flavor coherence" dissipates by kinematical decoherence between non-collective modes
- Contains non-dissipating (propagating or growing) projection for sufficiently strong nu-nu interaction effect
- Explicit formulation of eigenfunctions for non-collective modes leads to simple identification of critical points → Tobias Stirner's talk
- Stable collective modes "peel off" from the light cone and exist only outside
- Unstable collective modes begin/end under the light cone from coalescence of non-collective modes

Capozzi, Raffelt & Stirner, work in progress (2019)

Many Open Questions

Flavor evolution in dense neutrino flows still on the level of simplified toy models and parametric studies

- **Realistic normal-mode analysis without symmetry assumptions?**
- **Realistic triggering of stable or unstable flavor waves?**
- **Do tachyonic modes really lead to flavor equilibration? (Going beyond linearised stability analysis)**
- **Realistic impact on SN explosion and nucleosynthesis?**

It is only the beginning. A lot more work ahead of us …