Experimental Quantum Cosmology: Probing Analogue TransPlanckian Physics in Dipolar Bose-Einstein Condensates

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Probing the Transplanckian World with Analogue Gravity

Analogue “Gravity” \equiv

Fundamental **Kinematical** Effects [Visser, PRL 80 (1998)]

For Quantum Fields

Propagating In Curved Spacetime

(Spacetime Fixed by Experimentalist!)

In Particular:

I. Extreme Spacetime Curvatures
II. Impact of Transplanckian Physics

Can be Investigated in Analogue Experiments
Examples for Kinematical (≡ non-Einstein Equation) Effects in BECs

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- **Cosmological Particle Production, Sakharov Oscillations**
  Jaskula et al, PRL 109 (2012); Hung et al, Science 341 (2013)]
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- **Universe Inflation and Mode Freezing**
  [URF & Schützhold, PRA 70 (2004); Eckel et al, PRX 8 (2018)]
I. Cosmological Particle Production (CPP)

Time Dependent Variation of Spacetime Metric in Expanding Cosmos

OR Time Dependence of Boundary Conditions in Cavity (Dynamical Casimir Effect)

CPP Concept: Schrödinger, Physica 6 (1939)
“The proper vibrations of the expanding universe”

QFT Elaboration of CPP: Parker, PRL 21 (1968)


CPP aka Dynamical Casimir (aka Parametric Downconversion in Quantum Optics)
Dynamical Casimir or CPP in BECs

Simple Recipe: Do Something Explicitly Time-Dependent to System

For Example:

(a) Coupling Constant(s) \( g_c(t) \) [Feshbach Resonance]

(b) Trapping \( V_{\text{trap}} = V_{\text{trap}}(t) \)

\[ \omega_{\text{trap}} = \omega_{\text{trap}}(t) \text{ or Split into Double Well} \]

\[ \rightarrow \] CPP Generic Effect of Mode Mixing

(Conversion of Positive to Negative Frequency Modes)
II. Scale Invariance of Cosmological Power Spectrum

Cosmological Density Perturbations (≡ Inflaton) in

Inflationary Quantum Cosmology


Inflaton Mode Freezing After Crossing of Cosmological Horizon

Standard Quantum Mechanism for Galaxy Formation:
Formation of Inhomogeneities in Initially Homogeneous Universe

Mukhanov, Physical Foundations of Cosmology (CUP 2005)
Chapter 1 of Transplanckian Analogue Simulation:  

Inflation and Scale Invariance of Power Spectrum

Exponential De Sitter Evolution of Cosmological Scale Factor

$$a(\tau) = \exp[H\tau] \quad H \propto \sqrt{\Lambda} \equiv \text{Hubble Constant}$$

Consider Power Spectrum of Phase-Phase (Inflaton) Correlator

$$\langle 0 | \delta \hat{\phi}(x, \tau) \delta \hat{\phi}(y, \tau) | 0 \rangle := \frac{1}{V} \sum_k P(k) e^{ik \cdot (x-y)}$$

Late Cosmological Times $\tau$: $P(k) \rightarrow \frac{\hbar H}{\pi mc_0^2} k^{-2}$

Definition of Scale Invariance:

$$\Delta^2(k) := k^2P(k) \quad \text{Independent of } k$$

Hallmark Signature of Inflationary Quantum Cosmology

Note: Use D=2 Variant Here $[P(k) \propto k^{-D}]$
Trans-Planckian Problem of Inflationary Cosmology

Comoving Wavelengths $\lambda \sim \mathcal{O}(\lambda_{PL})$

Probe Planck Scale During Early Cosmological Stages

Scale Invariance Broken by Trans-Planckian Effects??

“Trans-Planckian Problem of Inflationary Cosmology”

Martin & Brandenberger, PRD 63 (2001)

Problem: Dispersion Relation Not Known for “Real” Gravity

No Quantum Gravity Exists!

$\implies$ Take Analog Model with Well-Known Microscopics:

Condensed Matter Physics Comes to the Rescue...

Volovik, “The Universe in a Helium Droplet”, Oxford UP
Dipolar Bose-Einstein Condensate

\[ V_{dd}(r) = \frac{3g_d}{4\pi} \frac{1 - 3z^2/|r|^2}{|r|^3} \]

\[ g_d \propto \mu_{m,e}^2 \]

Pancake With $\perp$-Polarized Dipoles For Stability

Large Magnetic Dipole Moment $\mu_m$: Cr, Dy, Er

(Tune in Addition Contact Coupling with Feshbach...)

Pfau (Stuttgart), Lev (Stanford), Ferlaino (Innsbruck)

Modugno (Firenze, Pisa) [more to come...]

Electrically Dipolar BEC: Ongoing Work in Progress
(Quasi-2D) Bogoliubov Excitations

\[ \zeta = kd_z \]

\[ \frac{\varepsilon^2}{(\hbar \omega_{z,0})^2} = A\zeta^2 \left( 1 - \frac{3R}{2} \zeta \exp[\zeta^2/2] \text{erfc}[\zeta/\sqrt{2}] \right) + \frac{\zeta^4}{4} \]

\[ A \propto \rho_0 d_z^2 g_d / R \quad \text{Chemical Potential} \]

\[ R = \sqrt{\pi/2}(1 + g_c/2g_d)^{-1} \simeq \sqrt{\pi/2} \quad \text{Coupling Ratio} \]

URF, PRA Rapids 73 (2006)
Definition of “Trans-Planckian” Physics

Beyond Linear (Sound) Part:

Physics not (Pseudo-)Lorentz Invariant

Large Dipolar Coupling \( R \rightarrow \sqrt{\pi/2} \): Curvature Negative

Dispersion Develops “Roton” Minimum

Impact of High-Momentum, Low-Energy Quasi-Particles

On (small-\( k \)) Quantum Fields in Curved Spacetime?

Question Here: Scale Invariance Maintained?
Scaling Expansion of Dipolar BEC

Scaling Ansatz [Kagan et al., Castin & Dum (1996); Gritsev et al (2010)]

\[ x := \frac{r}{b(t)} \quad \tau := \int_0^t \frac{dt'}{b^2(t')} \quad \Psi(r, t) := \exp \left[ i \frac{mr^2}{2\hbar} \frac{\partial_t b}{b} \right] \frac{\psi(x, \tau)}{b} \]

provided

\[ \frac{b^3 \partial_t^2 b + b^4 \omega^2(t)}{\omega_0^2} = \frac{g_c(t)}{g_{c,0} b} = \frac{g_d(t)}{g_{d,0} b} \]

Exact Solution for Constant \( g_c, g_d \):

(Slow) Square Root Expansion

\[ b(t) = \sqrt{4Ht + 1}, \quad \omega^2(t) = \frac{\omega_0^2}{b^5} + \frac{4H^2}{b^4} \]

Robertson-Walker (a) and BEC (b) Universes Have

Scale Factor Relation \( a^2 = b \) \quad (Contact Interaction: \ a = b)
Time-Dependent Bogoliubov Mode Functions

Analytical Solution when \( b(t) = \sqrt{4Ht + 1} \)

\[
\delta \tilde{\phi}_k(s) = s \sqrt{\frac{\pi \hbar VH}{4mc_0^2k^2}} \left[ J_1(s) + iY_1(s) \right] := h_k(s)
\]

Initial Free Oscillation \( \implies \) Overdamped for Large \( s \)

\[
s(t) = \frac{c_0/H}{a(t)/k} \equiv \frac{\text{Hubble radius}}{\text{rescaled wavelength}}
\]

Modes \( h_k(s) \) Correspond to

Instantaneous **Bunch-Davies Vacuum**


“BD Vacuum” \( \equiv \) (Quasi-)Particle-Vacuum Free-Falling Observer

\( \implies \) Yields in (Formally) Infinite Past

BEC Quasi-Particle Vacuum in Minkowski Space
Horizon Crossing and Mode Freezing

In Conformal Time: \( ds^2 = a^2[d\eta^2 - dx^2] \)

\[ -\Im(k\delta\tilde{\phi}_k) \]

relativistic limit

\( \zeta = 0.0005 \)

\( \zeta = 0.04 \)

\( \zeta = 0.1 \)

“Galaxy Formation” in Analogue Inflationary Quantum Cosmology

\( \iff \) Bogoliubov Modes Get “Frozen In”
Violation of Scale Invariance?

Microscopic Excitation Spectrum During Expansion

Expansion: Gas Becomes More Dilute

However: Initial “Roton” Minimum Leaves Imprint On Power Spectrum of Phase-Phase Correlations

⇒ Window on Transplanckian Physics In Early (Analogue) Universe!

Uwe R. Fischer
Experimental Quantum Cosmology
Phase-Phase Correlations in Fourier Space: Power Spectrum

**Significant Violations of Scale Invariance**

For Almost Critical (Initial) Spectrum

**Roton Minimum Imprint!**

Adiabaticity of Mode Propagation

**Violated** for Deep Minimum

Chä & URF, PRL 118, 130404 (2017)
Chapter 2 of Transplanckian Analogue Simulation:

Roton Entanglement and Steering in Quenched Condensates

Dynamical Casimir Effect (aka Cosmological Particle Production)

Here: Created by Quench of Sound Speed

⇒ Pairs of Quasiparticles Created with Momenta $k$ and $-k$

Enhanced Production Near Roton Minimum (Heisenberg Helps...)

Quasiparticle Pairs Entangled/Steered?

Dilute Gas: Tractable with Time-Dependent Bogoliubov Theory
Entanglement and Steering Primer

Entanglement $\equiv$ Nonseparability of Quantum States
Entanglement: Schrödinger, Naturwissenschaften 23 (1935)

Steerable States $\equiv$ Subset of Nonseparable States
Steering: Schrödinger, Math Proc Cambr Phil Soc 31 (1935)

Bipartite Steerable States Have “Half” of Bell-Nonlocality:

Alice Performs Uncharacterized Measurements Not Accessible (Hidden) to Bob

$\implies$ Inferred Standard Dev of Noncommuting $A, B$ Observables Can Violate Standard Heisenberg Uncertainty Relation

Steerable Quantum States
(a) Subset of Entangled States
(b) Superset of Bell States
Entanglement and Steering Criteria for CV Systems

Density Fluctuation Operator

\[ \frac{\delta \hat{\rho}_k(\tau)}{\rho_0} = (u_k + v_k)(\hat{\varphi}_k(\tau) + \hat{\varphi}_k^\dagger(\tau)) \]

\( u_k, v_k \quad \) Bogoliubov Transformation Amplitudes

\( \hat{\varphi}_k \quad \) Quasiparticle Field Operators

Density-Density Correlation Function

\[ G_{2,k}(\tau) = \frac{\langle |\delta \hat{\rho}_k(\tau)|^2 \rangle}{\rho_0^2} = (u_k + v_k)^2(2n_k + 1 + 2\Re[c_k e^{-2i\omega_k \tau}]) \]

\( n_k = \langle \hat{b}_k^\dagger \hat{b}_k \rangle \quad \) Mean Quasiparticle Occupation Number

\( c_k = \langle \hat{b}_k \hat{b}_{-k} \rangle \quad \) Quasiparticle Pair Amplitude
Assessing Entanglement and Steering

Density-Density Correlations:

Experimentally Accessible Measure of Quasiparticle Entanglement

Robertson, Michel, Busch & Parentani
PRA 89 (2014), PRD 95 & 96 (2017)

Finazzi & Carusotto, PRA 90 (2014)

\[ G_{2,k}(\tau) < G_{2,k}^{\text{vac}} = (u_k + v_k)^2 \quad \text{[Nonseparable]} \]

\[ G_{2,k}(\tau) < \frac{1}{2} G_{2,k}^{\text{vac}} \quad \text{[Steerable, More Strict Criterion!]} \]

\[ \Rightarrow \quad \text{Vacuum Amplitude of Density-Density Correlations} \]

Decides On Entanglement and Steering
Stationary Density-Density Correlations

Correlation Peak at Roton Minimum
Contact Interactions: No Peak
Quench of Sound Speed

Scale Factor $b(t)$ of Gas in Lab Time

$\frac{b}{t}$ \(\Rightarrow\) Gas Contraction
Roton-Enhanced Entanglement and Steering

Normalized Correlations $\tilde{G}_{2,k} := G_{2,k}/G_{2,k}^{\text{vac}}$

Increasing Roton Depth $\Rightarrow$ Increased Entanglement Potential
Temperature $T = 0$

Black: Solely Contact Interactions

Purple: Critical Dipole-Dipole Interaction Dominated Gas

$\implies$ Roton Minimum Generally Strongly Enhances Entanglement and Steering

Summary

Chapter 1 of Analogue TransPlanckian Simulation

- Trans-Planckian Problem of Inflationary Cosmology
  Experimentally Accessible in Dipolar BEC

- Violations of Scale Invariance Observable: First Such Analogue Gravity System!
  More Typically: No Violations
  \[ \text{cf Niemeyer and Parentani, PRD 64 (2001)} \]
  \[ \text{However: Even With "Roton Minimum"} \]

- Details of Transplanckian Dispersion Matter
  \[ \text{Ch¨ a & URF, PRL 118, 130404 (2017) [in Supplement]} \]

- Strong Quantum Fluctuations of Electric-Dipole BEC: Possible Influence on Early (Pre-Metric?) Stage of Cosmological Evolution
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- Creates Entanglement and Steering of Rotonic Quasiparticle Excitations
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▶ Dipolar Gas: Enhanced Potential for Nonlocal Quantum Correlations

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- Creates Entanglement and Steering of Rotonic Quasiparticle Excitations
- Dipolar Gas: Enhanced Potential for Nonlocal Quantum Correlations

- Ad: Analogue Gravity Postdoc Positions Available!

  Reversing Rutherford: “When a young man in my lab uses the word Universe, I tell him it is time for him to leave”
Queen’s Prophecy

Don’t stop me now (1978):
Queen’s Prophecy

Don’t stop me now (1978):

“I’m traveling at the speed of light
Queen’s Prophecy

Don’t stop me now (1978):

“I’m traveling at the speed of light
I wanna make a supersonic man out of you”
Queen’s Prophecy

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So: Don’t Stop Analogue Gravity Now!