Breakdown of Quasilocality in Long-Range Quantum Lattice Models

Salvatore R. Manmana
Institute for Theoretical Physics
Georg-August-University Göttingen

J. Eisert et al., PRL 111, 260401 (2013), Breakdown of quasilocality in long-range quantum lattice models

Funding:
Topic of this talk:

How does information or correlations spread after pushing a system out-of-equilibrium?
Optical Lattices & Lattice Models

Standing waves of laser light: periodic structures

Mechanism: Stark-Effect

Induced dipole moment in neutral atoms leads to a trapping force in the periodic potential: “Crystals of Light”
Typical lattice models:

Hubbard model (e.g., cuprates):

\[ H = -t \sum_{\langle i,j \rangle, \sigma} \left[ c_{i+1}^{\dagger, \sigma} c_{i,\sigma} + h.c. \right] + U \sum_i n_{i,\uparrow} n_{i,\downarrow} \]

Heisenberg exchange: 2\textsuperscript{nd} order perturbation theory for \( U \gg t \)

\[ J \vec{S}_1 \cdot \vec{S}_2 \]

\[ J = \frac{4t^2}{U} \]

Spinless Fermions' (e.g., fully polarized extended Hubbard model):

\[ \hat{H} = -t \sum_j \left( c_{j+1}^{\dagger} c_j + h.c. \right) + V \sum_j \hat{n}_j \hat{n}_{j+1} \]
Example: “time-of-flight measurements” (Bragg scattering)

“Quantum Quench”:
- sudden change of interaction strength

$U_0 \rightarrow U$

«Collapse and Revival of a Bose-Einstein-Condensate»

[M. Greiner et al., Nature (2002)]

e.g.,
S.R.M. et al., PRB (2009),
S.R.M. et al., PRL (2007),
Out-of-Equilibrium Dynamics: Quantum Quenches

Typical behavior after a quantum quench:

Relaxation (dephasing)

- Does the system reach a thermal state?
- On which time scale?

Lightcone effect

- Quasiparticles move ballistically through the system: lightcone

Thermal expectation value

Time average

[F. Essler et al., S.R.M., PRB (2014)]

[0.005]

[S.R.M. et al., PRB (2009)]

[0]

[P. Calabrese & J. Cardy, PRL (2006)]
Variable interaction range: Ions in a Trap

$^9\text{Be}^+$ ions in a Penning trap (NIST Boulder)
[J.W. Britton et al., Nature 484, 489 (2012)]

$^{171}\text{Yb}^+$ ions (JQI/NIST Maryland)
[K. Kim et al., Nature 465, 590 (2010);
R. Islam et al., Nature Comm. 2, 377 (2011);
NJP and more...]

Realization of Ising models with transverse field on variety of lattices:
Interactions $\sim 1/r^\alpha$
Two natural questions:

I. Time evolution of correlations?

II. Vary the exponent of the long-range interaction?

„Light-cone“ and emergence of a causal region vs. Instantaneous propagation of information
**Quasilocality:**

**Lieb-Robinson bound**

QM nonrelativistic:
local perturbations can have immediate effect everywhere

**But:** very small for short-range, finite-d systems:
light-cone, quasilocality & Lieb-Robinson-bound:

\[
\| [O_A(t), O_B(0)] \| \\
\leq C \| O_A \| \| O_B \| \min(|A|, |B|) e^{[v|t| - d(A, B)]/\xi}
\]

Long-range interactions ~ \( r^{-\alpha} \)?

\[
\| [O_A(t), O_B(0)] \| \leq C \| O_A \| \| O_B \| \frac{\min(|A|, |B|) (e^{v|t|} - 1)}{(d(A, B) + 1)^\alpha}
\]

(Koma&Hastings 2006)

Logarithmic behaviour

\[
v|t| > \ln \left[ 1 + \frac{e[1 + d(A, B)]^\alpha}{\min(|A|, |B|)} \right]
\]
Numerical Methods

How to treat correlated quantum systems on a computer?

My Main Tool: Matrix Product State Algorithms

Basic idea: data compression ("quantum version")

→ Graphics (acoustics, signal transmission, etc.)

Key aspect:
Ignore modes that cannot be resolved (by the ear, the screen, ...) – excellent quality with much smaller amount of data.

→ Control parameter here: entanglement.
MPS Algorithms: Key Aspects

Schmidt decomposition:

\[ |\psi\rangle = \sum_{j=1}^{\dim \mathcal{H}} w_j |\alpha\rangle_j |\beta\rangle_j \approx \sum_{j=1}^{m} w_j |\alpha\rangle_j |\beta\rangle_j \]

\(|\alpha\rangle_j, |\beta\rangle_j\) : eigenstates of the reduced density matrix of A or B

- very powerful in 1D
- nonequilibrium, finite-T linear-response dynamics

Approximation: \( m \ll \dim \mathcal{H} \)
(e.g., 1000 sites:
\( \dim \mathcal{H} = 2^{1000} > (1 \text{ googol})^3 \). Typical choice: \( m = 800 \))


Key: entanglement entropy \[ S = - \sum_j w_j^2 \log w_j^2 \]

- the larger the entanglement in the system, the larger \( m \)

Problem in 2D:
“area law of entanglement” - \( m \) grows exponentially with system size

\[ \Rightarrow \text{Frontier of today’s efforts.} \]
Horizons

Do we always see the lightcone?
Short-range systems: Nature of the light-cone


\[ \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle \]

- Dip moving ballistically through the system
- Onset of correlations moving “ballistically”
- Correlation length growing in time
Anticommutator: Correlation functions

\[ \langle \{ n_i(t) , n_j(0) \} \rangle_c \]

Commutator: Susceptibilities

\[ \langle [ n_i(t) , n_j(0) ] \rangle_c \]

- Lieb-Robinson theorem is – in general – only valid for commutators, i.e., susceptibilities.
- Correlation functions can also have a signal outside the lightcone – suppressed, if exponentially decaying in initial state (e.g., product state).
Light cone with Dipolar interactions

$\alpha=3$: looks quite linear!
Long-range Interactions: Causal Horizon vs. Immediate Spread

\[ \alpha = \frac{1}{4} \quad \alpha = \frac{3}{4} \quad \alpha = \frac{3}{2} \]

\[ \text{Ising, } L=1001 \]

\[ v |t| > \epsilon \delta^q \quad \text{power law shape} \]

\[ \text{XXZ, } L=40 \]

generic initial state: causal region appears for \( \alpha > D \)

product initial state: causal region appears for \( \alpha > D/2 \)
**Ion-Trap-Experiments**

Interactions $\sim 1/r^\alpha$

Not a linear ‘region of causality’, but curved!

[P. Richerme et al., Nature 511, 198 (2014)]
Algebraic bounds for causality?

Proposed behaviours:

- \( \alpha > 2D \): algebraic shape of the light-cone rather than logarithmic
- Becomes increasingly linear as \( \alpha \) grows
Conclusions & Outlook

Light cone effect:

Short range interactions

long range interactions

General form of the Lieb-Robinson bound for long-range interactions?

\[ \alpha = \frac{3}{4} \]